Unit 1

First-Degree Equations and Inequalities

Focus

Use algebraic concepts and the relationships among them to better understand the structure of algebra.

CHAPTER 1

Equations and Inequalities

BIG Idea Manipulate symbols in order to solve problems and use algebraic skills to solve equations and inequalities in problem situations.

CHAPTER 2

Linear Relations and Functions

BIG Idea Use properties and attributes of functions and apply functions to problem situations.

BIG Idea Connect algebraic and geometric representations of functions.

CHAPTER 3

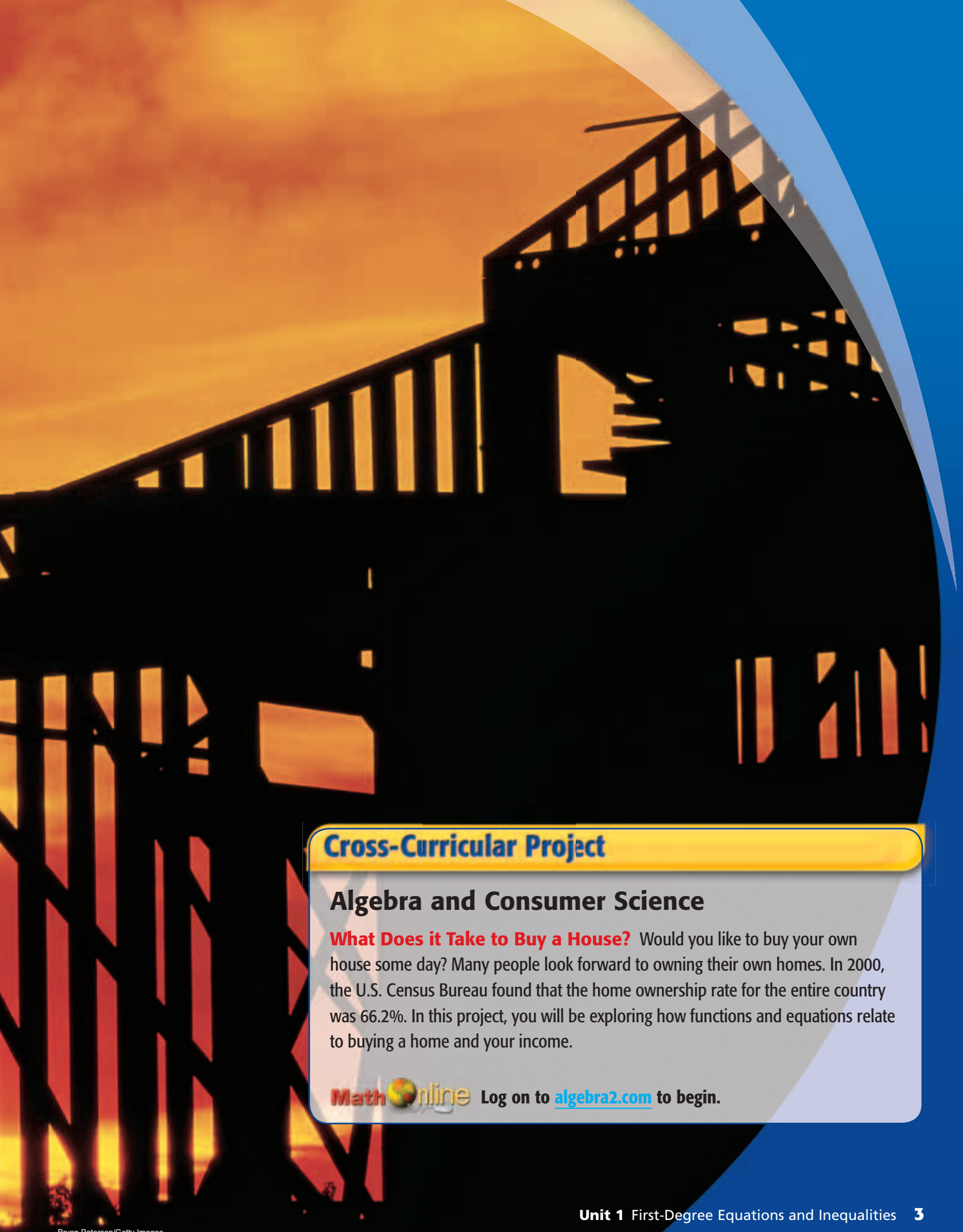
Systems of Equations and Inequalities

BIG Idea Formulate systems of equations and inequalities from problem situations, use a variety of methods to solve them, and analyze the solutions in terms of the situations.

CHAPTER 4

Matrices

BIG Idea Use matrices to organize data and solve systems of equations from problem situations.



Cross-Curricular Project

Algebra and Consumer Science

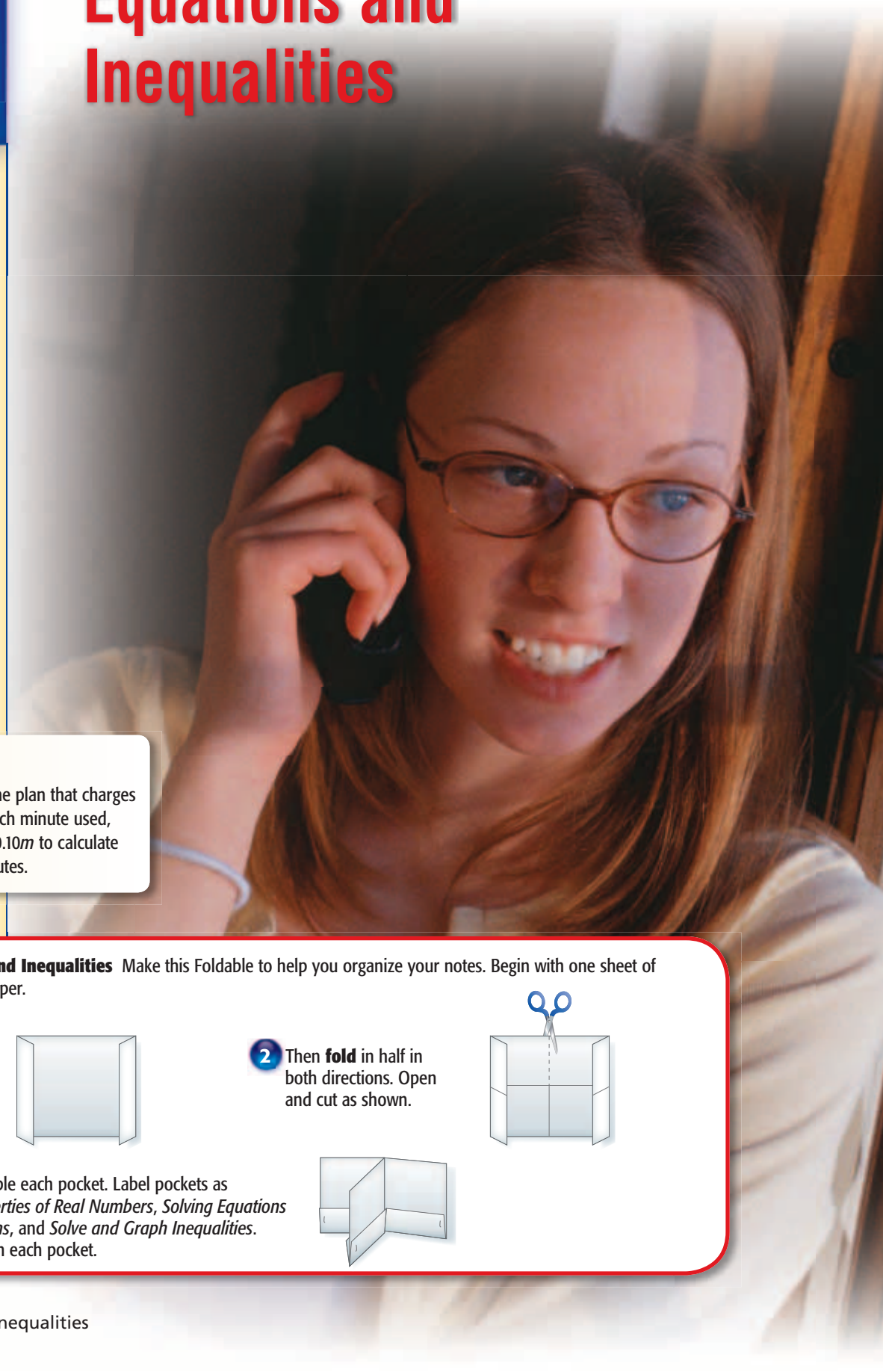
What Does it Take to Buy a House? Would you like to buy your own house some day? Many people look forward to owning their own homes. In 2000, the U.S. Census Bureau found that the home ownership rate for the entire country was 66.2%. In this project, you will be exploring how functions and equations relate to buying a home and your income.

MathOnline **Log on to algebra2.com to begin.**

Bryan Peterson/Getty Images

CHAPTER 1

Equations and Inequalities



BIG Ideas

- Simplify and evaluate algebraic expressions.
- Solve linear and absolute value equations.
- Solve and graph inequalities

Key Vocabulary

counterexample (p. 17)
equation (p. 18)
formula (p. 8)
solution (p. 19)

Real-World Link

Cell Phone Charges For a cell phone plan that charges a monthly fee of $10 plus $0.10 for each minute used, you can use the equation $C = 10 + 0.10m$ to calculate the monthly charges for using m minutes.

FOLDABLES Study Organizer

Equations and Inequalities Make this Foldable to help you organize your notes. Begin with one sheet of 11″ by 17″ paper.

1. **Fold** 2″ tabs on each of the short sides.

2. Then **fold** in half in both directions. Open and cut as shown.

3. **Refold** along the width. Staple each pocket. Label pockets as *Algebraic Expressions*, *Properties of Real Numbers*, *Solving Equations and Absolute Value Equations*, and *Solve and Graph Inequalities*. Place index cards for notes in each pocket.

GET READY for Chapter 1

Diagnose Readiness You have two options for checking Prerequisite Skills.

Option 2

MathOnline Take the Online Readiness Quiz at algebra2.com.

Option 1

Take the Quick Check below. Refer to the Quick Review for help.

QUICKCheck

Simplify. (Prerequisite Skill)

1. $20 - 0.16$
2. $12.2 + (-8.45)$
3. $\frac{1}{4} - \frac{2}{3}$
4. $\frac{3}{5} + (-6)$
5. $-7\frac{1}{2} + 5\frac{1}{3}$
6. $-11\frac{5}{8} - \left(-4\frac{3}{7}\right)$
7. $(0.15)(3.2)$
8. $2 \div (-0.4)$
9. $-4 \div \frac{3}{2}$
10. $\left(\frac{5}{4}\right)\left(-\frac{3}{10}\right)$
11. $\left(-2\frac{3}{4}\right)\left(-3\frac{1}{5}\right)$
12. $7\frac{1}{8} \div (-2)$
13. **LUNCH** Angela has \$11.56. She spends \$4.25 on lunch. How much money does Angela have left? (Prerequisite Skill)

Evaluate each power. (Prerequisite Skill)

14. 2^3
15. 5^3
16. $(-7)^2$
17. $(-1)^3$
18. $(-0.8)^2$
19. $-(1.2)^2$
20. $\left(\frac{2}{3}\right)^2$
21. $\left(\frac{5}{9}\right)^2$
22. $\left(-\frac{4}{11}\right)^2$
23. **GENEALOGY** In a family tree, you are generation "now." One generation ago, your 2 parents were born. Two generations ago your 4 grandparents were born. How many ancestors were born five generations ago? (Prerequisite Skill)

Identify each statement as *true* or *false*. (Prerequisite Skill)

24. $-5 < -7$
25. $6 > -8$
26. $-2 \geq -2$
27. $-3 \geq -3.01$
28. $-1 < -2$
29. $\frac{1}{5} < \frac{1}{8}$
30. $\frac{2}{5} \geq \frac{16}{40}$
31. $\frac{3}{4} > 0.8$

QUICKReview

EXAMPLE 1

Simplify $\left(-\frac{3}{5}\right)\left(\frac{13}{15}\right)$.

$\left(-\frac{3}{5}\right)\left(\frac{13}{15}\right) = -\frac{3(13)}{5(15)}$ Multiply the numerators and denominators.

$= -\frac{39}{75}$ Simplify.

$= -\frac{39 \div 3}{75 \div 3}$ Divide the numerator and denominator by their GCF, 3.

$= -\frac{13}{25}$ Simplify.

EXAMPLE 2

Evaluate $-(-10)^3$.

$-(-10)^3 = -[(-10)(-10)(-10)]$ $(-10)^3$ means -10 is a factor 3 times.

$= -[-1000]$ Evaluate inside the brackets.

$= 1000$ Simplify.

EXAMPLE 3

Identify $\frac{2}{7} < \frac{8}{28}$ as *true* or *false*.

$\frac{2}{7} \stackrel{?}{<} \frac{8 \div 4}{28 \div 4}$ Divide 8 and 28 by their GCF, 4.

$\frac{2}{7} \not< \frac{2}{7}$ Simplify.

False, $\frac{2}{7} \not< \frac{8}{28}$ because $\frac{2}{7} = \frac{8}{28}$.

1-1 Expressions and Formulas

Main Ideas

- Use the order of operations to evaluate expressions.
- Use formulas.

New Vocabulary

variable
algebraic expression
order of operations
monomial
constant
coefficient
degree
power
polynomial
term
like terms
trinomial
binomial
formula

GET READY for the Lesson

Nurses setting up intravenous or IV fluids must control the flow rate F, in drops per minute. They use the formula $F = \frac{V \times d}{t}$, where V is the volume of the solution in milliliters, d is the drop factor in drops per milliliter, and t is the time in minutes.

Suppose 1500 milliliters of saline are to be given over 12 hours. Using a drop factor of 15 drops per milliliter, the expression $\frac{1500 \times 15}{12 \times 60}$ gives the correct IV flow rate.

Order of Operations

Variables are symbols, usually letters, used to represent unknown quantities. Expressions that contain at least one variable are called **algebraic expressions**. You can evaluate an algebraic expression by replacing each variable with a number and then applying the **order of operations**.

KEY CONCEPT *Order of Operations*

Step 1 Evaluate expressions inside grouping symbols.

Step 2 Evaluate all powers.

Step 3 Multiply and/or divide from left to right.

Step 4 Add and/or subtract from left to right.

An algebraic expression that is a number, a variable, or the product of a number and one or more variables is called a **monomial**. Monomials cannot contain variables in denominators, variables with exponents that are negative, or variables under radicals.

Monomials	Not Monomials
$5b$	$\frac{1}{n^4}$
$-w$	$\sqrt[3]{x}$
23	$x + 8$
x^2	a^{-1}
$\frac{1}{3}x^3y^4$	

Constants are monomials that contain no variables, like 23 or -1. The numerical factor of a monomial is the **coefficient** of the variable(s). For example, the coefficient of m in $-6m$ is -6. The **degree** of a monomial is the sum of the exponents of its variables. For example, the degree of $12g^7h^4$ is $7 + 4$ or 11. The degree of a constant is 0. A **power** is an expression of the form x^n. The word *power* is also used to refer to the exponent itself.

A **polynomial** is a monomial or a sum of monomials. The monomials that make up a polynomial are called the **terms** of the polynomial. In a polynomial such as $x^2 + 2x + x + 1$, the two monomials $2x$ and x can be combined because they are **like terms**. The result is $x^2 + 3x + 1$. The polynomial $x^2 + 3x + 1$ is a **trinomial** because it has three unlike terms. A polynomial such as $xy + z^3$ is a **binomial** because it has two unlike terms.

EXAMPLE 1 Evaluate Algebraic Expressions

a. Evaluate $m + (n - 1)^2$ if $m = 3$ and $n = -4$.

$$\begin{aligned} m + (n - 1)^2 &= 3 + (-4 - 1)^2 && \text{Replace } m \text{ with 3 and } n \text{ with } -4. \\ &= 3 + (-5)^2 && \text{Add } -4 \text{ and } -1. \\ &= 3 + 25 && \text{Find } (-5)^2. \\ &= 28 && \text{Add 3 and 25.} \end{aligned}$$

b. Evaluate $x^2 - y(x + y)$ if $x = 8$ and $y = 1.5$.

$$\begin{aligned} x^2 - y(x + y) &= 8^2 - 1.5(8 + 1.5) && \text{Replace } x \text{ with 8 and } y \text{ with 1.5.} \\ &= 8^2 - 1.5(9.5) && \text{Add 8 and 1.5.} \\ &= 64 - 1.5(9.5) && \text{Find } 8^2. \\ &= 64 - 14.25 && \text{Multiply 1.5 and 9.5.} \\ &= 49.75 && \text{Subtract 14.25 from 64.} \end{aligned}$$

c. Evaluate $\dfrac{a^3 + 2bc}{c^2 - 5}$ if $a = 2$, $b = -4$, and $c = -3$.

$$\begin{aligned} \frac{a^3 + 2bc}{c^2 - 5} &= \frac{2^3 + 2(-4)(-3)}{(-3)^2 - 5} && a = 2, b = -4, \text{ and } c = -3 \\ &= \frac{8 + (-8)(-3)}{9 - 5} && \text{Evaluate the numerator and the denominator separately.} \\ &= \frac{8 + 24}{9 - 5} && \text{Multiply } -8 \text{ by } -3. \\ &= \frac{32}{4} \text{ or } 8 && \text{Simplify the numerator and the denominator. Then divide.} \end{aligned}$$

Study Tip

Fraction Bar

The fraction bar acts as both an operation symbol, indicating division, and as a grouping symbol. Evaluate the expressions in the numerator and denominator separately before dividing.

CHECK Your Progress

1A. Evaluate $m + (3 - n)^2$ if $m = 12$ and $n = -1$.

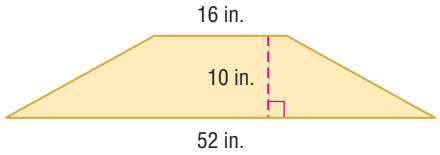
1B. Evaluate $x^2y + x(x - y)$ if $x = 4$ and $y = 0.5$.

1C. Evaluate $\dfrac{b^2 - 3a^2c}{b^3 + 2}$ if $a = -1$, $b = 2$, and $c = 8$.

Formulas A **formula** is a mathematical sentence that expresses the relationship between certain quantities. If you know the value of every variable in the formula except one, you can find the value of the remaining variable.

EXAMPLE 2 Use a Formula

GEOMETRY **The formula for the area A of a trapezoid is $A = \frac{1}{2}h(b_1 + b_2)$, where h represents the height, and b_1 and b_2 represent the measures of the bases. Find the area of the trapezoid shown below.**



The height is 10 inches. The bases are 16 inches and 52 inches. Substitute each value given into the formula. Then evaluate the expression using the order of operations.

$A = \frac{1}{2}h(b_1 + b_2)$ Area of a trapezoid

$= \frac{1}{2}(10)(16 + 52)$ Replace h with 10, b_1 with 16, and b_2 with 52.

$= \frac{1}{2}(10)(68)$ Add 16 and 52.

$= 5(68)$ Multiply $\frac{1}{2}$ and 10.

$= 340$ Multiply 5 by 68.

The area of the trapezoid is 340 square inches.

CHECK Your Progress

2. The formula for the volume V of a rectangular prism is $V = \ell wh$, where ℓ represents the length, w represents the width, and h represents the height. Find the volume of a rectangular prism with a length of 4 feet, a width of 2 feet, and a height of 3.5 feet.

Online **Personal Tutor at** algebra2.com

CHECK Your Understanding

Example 1 (p. 7)

Evaluate each expression if $x = 4$, $y = -2$, and $z = 3.5$.

1. $z - x + y$

2. $x + (y - 1)^3$

3. $x + [3(y + z) - y]$

4. $\frac{x^2 - y}{z + 2.5}$

5. $\frac{x + 2y^2}{x - z}$

6. $\frac{y^3 + 2xz}{x^2 - z}$

Example 2 (p. 8)

BANKING **For Exercises 7 and 8, use the following information.**
Simple interest is calculated using the formula $I = prt$, where p represents the principal in dollars, r represents the annual interest rate, and t represents the time in years. Find the simple interest I given each set of values.

7. $p = \$1800$, $r = 6\%$, $t = 4$ years

8. $p = \$31{,}000$, $r = 2\frac{1}{2}\%$, $t = 18$ months

Exercises

HOMEWORK HELP	
For Exercises	See Examples
9–22	1
23, 24	2

Evaluate each expression if $w = 6$, $x = 0.4$, $y = \frac{1}{2}$, and $z = -3$.

9. $w + x + z$ **10.** $w + 12 \div z$ **11.** $w(8 - y)$

12. $z(x + 1)$ **13.** $w - 3x + y$ **14.** $5x + 2z$

Evaluate each expression if $a = 3$, $b = 0.3$, $c = \frac{1}{3}$, and $d = -1$.

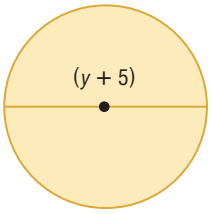
15. $\frac{a - d}{bc}$ **16.** $\frac{a + d}{c}$ **17.** $\frac{a^2c^2}{d}$

18. $\frac{a - 10b}{c^2d^2}$ **19.** $\frac{d + 4}{a^2 + 3}$ **20.** $\frac{1 - b}{3c - 3b}$

21. NURSING Determine the IV flow rate for the patient described at the beginning of the lesson by finding the value of $\frac{1500 \times 15}{12 \times 60}$.

22. BICYCLING Air pollution can be reduced by riding a bicycle rather than driving a car. To find the number of pounds of pollutants created by starting a typical car 10 times and driving it for 50 miles, find the value of the expression $\frac{(52.84 \times 10) + (5.955 \times 50)}{454}$.

23. GEOMETRY The formula for the area A of a circle with diameter d is $A = \pi\left(\frac{d}{2}\right)^2$. Write an expression to represent the area of the circle.



24. GEOMETRY The formula for the volume V of a right circular cone with radius r and height h is $V = \frac{1}{3}\pi r^2 h$. Write an expression for the volume of a cone with $r = 3x$ and $h = 2x$.

Evaluate each expression if $a = \frac{2}{5}$, $b = -3$, $c = 0.5$, and $d = 6$.

25. $b^4 - d$ **26.** $(5 - d)^2 + a$ **27.** $\frac{5ad}{b}$

28. $\frac{2b - 15a}{3c}$ **29.** $(a - c)^2 - 2bd$ **30.** $\frac{1}{c} + \frac{1}{d}$

31. Find the value of ab^n if $n = 3$, $a = 2000$, and $b = -\frac{1}{5}$.

Real-World Link

To estimate the width w in feet of a firework burst, use the formula $w = 20At$. In this formula, A is the estimated viewing angle of the fireworks display, and t is the time in seconds from the instant you see the light until you hear the sound.

Source: efg2.com

32. FIREWORKS Suppose you are about a mile from a fireworks display. You count 5 seconds between seeing the light and hearing the sound of the fireworks display. You estimate the viewing angle is about 4°. Using the information at the left, estimate the width of the firework display.

33. MONEY In 1960, the average price of a car was about $2500. This may sound inexpensive, but the average income in 1960 was much less than it is now. To compare dollar amounts over time, use the formula $V = \frac{A}{S}C$, where A is the old dollar amount, S is the starting year's Consumer Price Index (CPI), C is the converting year's CPI, and V is the current value of the old dollar amount. Buying a car for $2500 in 1960 was like buying a car for how much money in 2004?

Year	1960	1970	1980	1990	2000	2004
Average CPI	29.6	38.8	82.4	130.7	172.2	188.9

Source: U.S. Department of Labor

EXTRA PRACTICE
See pages 891, 926.
Math Online
Self-Check Quiz at algebra2.com

34. MEDICINE A patient must take blood pressure medication that is dispensed in 125-milligram tablets. The dosage is 15 milligrams per kilogram of body weight and is given every 8 hours. If the patient weighs 25 kilograms, how many tablets would be needed for a 30-day supply? Use the formula $n = [15b \div (125 \times 8)] \times 24d$, where n is the number of tablets, d is the number of days the supply should last, and b is body weight in kilograms.

35. QB RATING The formula for quarterback efficiency rating in the National Football League is $\left(\frac{\frac{C}{A} - 0.3}{0.2} + \frac{\frac{Y}{A} - 3}{4} + \frac{\frac{T}{A}}{0.05} + \frac{0.095 - \frac{I}{A}}{0.04}\right) \times \frac{100}{6}$, where C is the number of passes completed, A is the number of passes attempted, Y is passing yardage, T is the number of touchdown passes, and I is the number of interceptions. In 2005, Ben Roethlisberger of the Pittsburgh Steelers completed 168 of the 268 passes he attempted for 2385 yards. He threw 17 touchdowns and 9 interceptions. Find his efficiency rating for 2005.

H.O.T. Problems

36. OPEN ENDED Write an algebraic expression in which subtraction is performed before division, and the symbols ( ), [ ], or { } are not used.

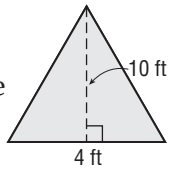
37. CHALLENGE Write expressions having values from one to ten using exactly four 4s. You may use any combination of the operation symbols +, −, ×, ÷, and/or grouping symbols, but no other digits are allowed. An example of such an expression with a value of zero is $(4 + 4) - (4 + 4)$.

38. REASONING Explain how to evaluate $a + b[(c + d) \div e]$, if you were given the values for a, b, c, d, and e.

39. *Writing in Math* Use the information about IV flow rates on page 6 to explain how formulas are used by nurses. Explain why a formula for the flow rate of an IV is more useful than a table of specific IV flow rates and describe the impact of using a formula, such as the one for IV flow rate, incorrectly.

STANDARDIZED TEST PRACTICE

40. ACT/SAT The following are the dimensions of four rectangles. Which rectangle has the same area as the triangle at the right?



A 1.6 ft by 25 ft
B 5 ft by 16 ft
C 3.5 ft by 4 ft
D 0.4 ft by 50 ft

41. REVIEW How many cubes that are 3 inches on each edge can be placed completely inside a box that is 9 inches long, 6 inches wide, and 27 inches tall?

F 12
G 36
H 54
J 72

GET READY for the Next Lesson

PREREQUISITE SKILL Evaluate each expression.

42. $\sqrt{9}$
43. $\sqrt{16}$
44. $\sqrt{100}$
45. $\sqrt{169}$
46. $-\sqrt{4}$
47. $-\sqrt{25}$
48. $\sqrt{\frac{4}{9}}$
49. $\sqrt{\frac{36}{49}}$

1-2 Properties of Real Numbers

Main Ideas

- Classify real numbers.
- Use the properties of real numbers to evaluate expressions.

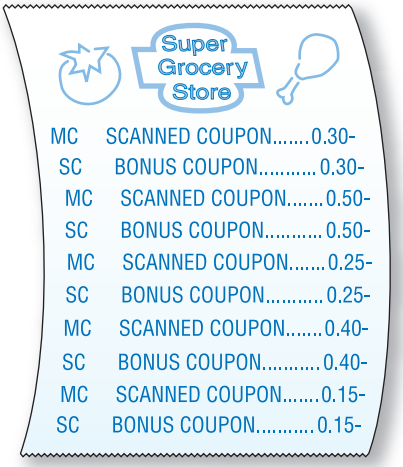
New Vocabulary

real numbers
rational numbers
irrational numbers

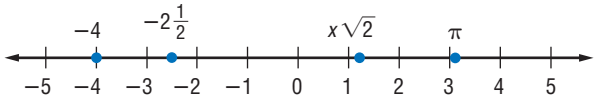
GET READY for the Lesson

Manufacturers often offer coupons to get consumers to try their products. Some grocery stores try to attract customers by doubling the value of manufacturers' coupons.

You can use the Distributive Property to calculate these savings.



Real Numbers The numbers that you use in everyday life are **real numbers**. Each real number corresponds to exactly one point on the number line, and every point on the number line represents exactly one real number.



Real numbers can be classified as either **rational** or **irrational**.

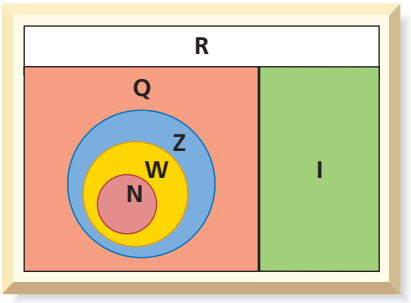
Review Vocabulary

Ratio the comparison of two numbers by division

KEY CONCEPT — *Real Numbers*

Words	A rational number can be expressed as a ratio $\frac{m}{n}$, where m and n are integers and n is not zero. The decimal form of a rational number is either a terminating or repeating decimal.
Examples	$\frac{1}{6}$, 1.9, 2.575757…, −3, $\sqrt{4}$, 0
Words	A real number that is not rational is irrational. The decimal form of an irrational number neither terminates nor repeats.
Examples	$\sqrt{5}$, π, 0.010010001…

The sets of natural numbers, {1, 2, 3, 4, 5, …}, whole numbers, {0, 1, 2, 3, 4, …}, and integers, {…, −3,−2,−1, 0, 1, 2, …} are all subsets of the rational numbers. The whole numbers are a subset of the rational numbers because every whole number n is equal to $\frac{n}{1}$.



The Venn diagram shows the relationships among these sets of numbers.

R = reals Q = rationals

I = irrationals Z = integers

W = wholes N = naturals

The square root of any whole number is either a whole number or it is irrational. For example, $\sqrt{36}$ is a whole number, but $\sqrt{35}$ is irrational and lies between 5 and 6.

Study Tip

Common Misconception

Do not assume that a number is irrational because it is expressed using the square root symbol. Find its value first.

EXAMPLE Classify Numbers

1 **Name the sets of numbers to which each number belongs.**

a. $\sqrt{16}$

$\sqrt{16} = 4$ naturals (N), wholes (W), integers (Z), rationals (Q), reals (R)

b. -18 integers (Z), rationals (Q), and reals (R)

c. $\sqrt{20}$ irrationals (I) and reals (R)

$\sqrt{20}$ lies between 4 and 5 so it is not a whole number.

d. $-\frac{7}{8}$ rationals (Q) and reals (R)

e. $0.\overline{45}$ rationals (Q) and reals (R)

The bar over the 45 indicates that those digits repeat forever.

CHECK Your Progress

1A. -185 **1B.** $-\sqrt{49}$ **1C.** $\sqrt{95}$

Properties of Real Numbers Some of the properties of real numbers are summarized below.

KEY CONCEPT *Real Number Properties*

For any real numbers *a*, *b*, and *c*:

Property	Addition	Multiplication
Commutative	$a + b = b + a$	$a \cdot b = b \cdot a$
Associative	$(a + b) + c = a + (b + c)$	$(a \cdot b) \cdot c = a \cdot (b \cdot c)$
Identity	$a + 0 = a = 0 + a$	$a \cdot 1 = 1 \cdot a$
Inverse	$a + (-a) = 0 = (-a) + a$	If $a \neq 0$, then $a \cdot \frac{1}{a} = 1 = \frac{1}{a} \cdot a$.
Distributive	$a(b + c) = ab + ac$ and $(b + c)a = ba + ca$	

Reading Math

Opposites

$-a$ is read *the opposite of a*.

EXAMPLE 2 Identify Properties of Real Numbers

Name the property illustrated by $(5 + 7) + 8 = 8 + (5 + 7)$.

Commutative Property of Addition

The Commutative Property says that the order in which you add does not change the sum.

CHECK Your Progress

2. Name the property illustrated by $2(x + 3) = 2x + 6$.

EXAMPLE 3 Additive and Multiplicative Inverses

Identify the additive inverse and multiplicative inverse for $-1\frac{3}{4}$.

Since $-1\frac{3}{4} + \left(1\frac{3}{4}\right) = 0$, the additive inverse of $-1\frac{3}{4}$ is $1\frac{3}{4}$.

Since $-1\frac{3}{4} = -\frac{7}{4}$ and $\left(-\frac{7}{4}\right)\left(-\frac{4}{7}\right) = 1$, the multiplicative inverse of $-1\frac{3}{4}$ is $-\frac{4}{7}$.

CHECK Your Progress

Identify the additive inverse and multiplicative inverse for each number.

3A. 1.25 **3B.** $2\frac{1}{2}$

Concepts in Motion
Animation algebra2.com

You can model the Distributive Property using algebra tiles.

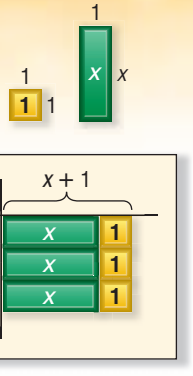
ALGEBRA LAB

Distributive Property

Step 1 A 1-tile is a square that is 1 unit wide and 1 unit long. Its area is 1 square unit. An x-tile is a rectangle that is 1 unit wide and x units long. Its area is x square units.

Step 2 To find the product $3(x + 1)$, model a rectangle with a width of 3 and a length of $x + 1$. Use your algebra tiles to mark off the dimensions on a product mat. Then make the rectangle with algebra tiles.

Step 3 The rectangle has 3 x-tiles and 3 1-tiles. The area of the rectangle is $x + x + x + 1 + 1 + 1$ or $3x + 3$. Thus, $3(x + 1) = 3x + 3$.



MODEL AND ANALYZE

Tell whether each statement is *true* or *false*. Justify your answer with algebra tiles and a drawing.

1. $4(x + 2) = 4x + 2$ **2.** $3(2x + 4) = 6x + 7$

3. $2(3x + 5) = 6x + 10$ **4.** $(4x + 1)5 = 4x + 5$

Real-World EXAMPLE

4 **FOOD SERVICE** **A restaurant adds a 20% tip to the bills of parties of 6 or more people. Suppose a server waits on five such tables. The bill without the tip for each party is listed in the table. How much did the server make in tips during this shift?**

Party 1	Party 2	Party 3	Party 4	Party 5
$185.45	$205.20	$195.05	$245.80	$262.00

There are two ways to find the total amount of tips received.

Method 1 Multiply each dollar amount by 20% or 0.2 and then add.

$$\begin{aligned} T &= 0.2(185.45) + 0.2(205.20) + 0.2(195.05) + 0.2(245.80) + 0.2(262) \\ &= 37.09 + 41.04 + 39.01 + 49.16 + 52.40 \\ &= 218.70 \end{aligned}$$

Method 2 Add all of the bills and then multiply the total by 0.2.

$$\begin{aligned} T &= 0.2(185.45 + 205.20 + 195.05 + 245.80 + 262) \\ &= 0.2(1093.50) \\ &= 218.70 \end{aligned}$$

The server made $218.70 during this shift.

Notice that both methods result in the same answer.

Real-World Link

Leaving a "tip" began in 18th century English coffee houses and is believed to have originally stood for "To Insure Promptness." Today, the American Automobile Association suggests leaving a 15% tip.

Source: Market Facts, Inc.

CHECK Your Progress

4. Kayla makes $8 per hour working at a grocery store. The number of hours Kayla worked each day in one week are 3, 2.5, 2, 1, and 4. How much money did Kayla earn this week?

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The properties of real numbers can be used to simplify algebraic expressions.

EXAMPLE Simplify an Expression

5 **Simplify $2(5m + n) + 3(2m - 4n)$.**

$$\begin{aligned} &2(5m + n) + 3(2m - 4n) \\ &= 2(5m) + 2(n) + 3(2m) - 3(4n) && \text{Distributive Property} \\ &= 10m + 2n + 6m - 12n && \text{Multiply.} \\ &= 10m + 6m + 2n - 12n && \text{Commutative Property (+)} \\ &= (10 + 6)m + (2 - 12)n && \text{Distributive Property} \\ &= 16m - 10n && \text{Simplify.} \end{aligned}$$

CHECK Your Progress

5. Simplify $3(4x - 2y) - 2(3x + y)$.

CHECK Your Understanding

Example 1 (p. 12) **Name the sets of numbers to which each number belongs.**

1. -4 **2.** 45 **3.** $6.\overline{23}$

Example 2 (p. 13) **Name the property illustrated by each question.**

4. $\frac{2}{3} \cdot \frac{3}{2} = 1$ **5.** $(a + 4) + 2 = a + (4 + 2)$ **6.** $4x + 0 = 4x$

Example 3 (p. 13) **Identify the additive inverse and multiplicative inverse for each number.**

7. -8 **8.** $\frac{1}{3}$ **9.** 1.5

Example 4 (p. 14) **FUND-RAISING For Exercises 10 and 11, use the table.**

Catalina is selling candy for $1.50 each to raise money for the band.

10. Write an expression to represent the total amount of money Catalina raised during this week.

11. Evaluate the expression from Exercise 10 by using the Distributive Property.

Catalina's Sales for One Week

Day	Bars Sold
Monday	10
Tuesday	15
Wednesday	12
Thursday	8
Friday	19
Saturday	22
Sunday	31

Example 5 (p. 14) **Simplify each expression.**

12. $3(5c + 4d) + 6(d - 2c)$

13. $\frac{1}{2}(16 - 4a) - \frac{3}{4}(12 + 20a)$

Exercises

HOMEWORK HELP	
For Exercises	See Examples
14–21	1
22–27	2
28–33	3
34, 35	4
36–43	5

Name the sets of numbers to which each number belongs.

14. $-\frac{2}{9}$ **15.** -4.55 **16.** $-\sqrt{10}$ **17.** $\sqrt{19}$

18. -31 **19.** $\frac{12}{2}$ **20.** $\sqrt{121}$ **21.** $-\sqrt{36}$

Name the property illustrated by each equation.

22. $5a + (-5a) = 0$ **23.** $-6xy + 0 = -6xy$

24. $[5 + (-2)] + (-4) = 5 + [-2 + (-4)]$ **25.** $(2 + 14) + 3 = 3 + (2 + 14)$

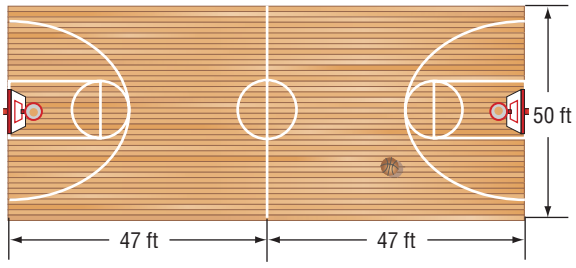
26. $\left(1\frac{2}{7}\right)\left(\frac{7}{9}\right) = 1$ **27.** $2\sqrt{3} + 5\sqrt{3} = (2 + 5)\sqrt{3}$

Identify the additive inverse and multiplicative inverse for each number.

28. -10 **29.** 2.5 **30.** -0.125

31. $-\frac{5}{8}$ **32.** $\frac{4}{3}$ **33.** $-4\frac{3}{5}$

34. BASKETBALL Illustrate the Distributive Property by writing two expressions for the area of the NCAA basketball court. Then find the area of the basketball court.



35. BAKING Mitena is making two types of cookies. The first recipe calls for $2\frac{1}{4}$ cups of flour, and the second calls for $1\frac{1}{8}$ cups of flour. If she wants to make 3 batches of the first recipe and 2 batches of the second recipe, how many cups of flour will she need? Use the properties of real numbers to show how Mitena could compute this amount mentally. Justify each step.

Simplify each expression.

36. $7a + 3b - 4a - 5b$

37. $3x + 5y + 7x - 3y$

38. $3(15x - 9y) + 5(4y - x)$

39. $2(10m - 7a) + 3(8a - 3m)$

40. $8(r + 7t) - 4(13t + 5r)$

41. $4(14c - 10d) - 6(d + 4c)$

42. $4(0.2m - 0.3n) - 6(0.7m - 0.5n)$

43. $7(0.2p + 0.3q) + 5(0.6p - q)$

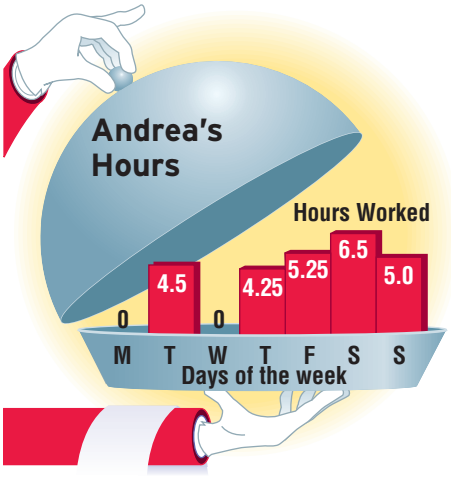
Real-World Link
Pythagoras (572–497 B.C.) was a Greek philosopher whose followers came to be known as the Pythagoreans. It was their knowledge of what is called the Pythagorean Theorem that led to the first discovery of irrational numbers.

Source: *A History of Mathematics*

WORK For Exercises 44 and 45, use the information below and in the graph.
Andrea works in a restaurant and is paid every two weeks.

44. If Andrea earns \$6.50 an hour, illustrate the Distributive Property by writing two expressions representing Andrea's pay last week.

45. Find the mean or average number of hours Andrea worked each day, to the nearest tenth of an hour. Then use this average to predict her pay for a two-week pay period.



NUMBER THEORY For Exercises 46–49, use the properties of real numbers to answer each question.

46. If $m + n = m$, what is the value of n?

47. If $m + n = 0$, what is the value of n? What is n's relationship to m?

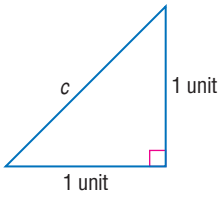
48. If $mn = 1$, what is the value of n? What is n's relationship to m?

49. If $mn = m$ and $m \neq 0$, what is the value of n?

MATH HISTORY For Exercises 50–52, use the following information.
The Greek mathematician Pythagoras believed that all things could be described by numbers. By *number* he meant a positive integer.

50. To what set of numbers was Pythagoras referring when he spoke of *numbers*?

51. Use the formula $c = \sqrt{2s^2}$ to calculate the length of the hypotenuse c, or longest side, of this right triangle using s, the length of one leg.



52. Explain why Pythagoras could not find a "number" for the value of c.

Name the sets of numbers to which each number belongs.

53. 0

54. $\frac{3\pi}{2}$

55. $-2\sqrt{7}$

56. Name the sets of numbers to which all of the following numbers belong. Then arrange the numbers in order from least to greatest.

$$2.\overline{49},\ 2.4\overline{9},\ 2.4,\ 2.49,\ 2.\overline{9}$$

EXTRA PRACTICE
See pages 891, 926.

Math Online
Self-Check Quiz at algebra2.com

H.O.T. Problems

OPEN ENDED **Give an example of a number that satisfies each condition.**

57. integer, but not a natural number

58. integer with a multiplicative inverse that is an integer

CHALLENGE Determine whether each statement is *true* or *false*. If *false*, give a counterexample. A **counterexample** is a specific case that shows that a statement is false.

59. Every whole number is an integer.

60. Every integer is a whole number.

61. Every real number is irrational.

62. Every integer is a rational number.

63. REASONING Is the Distributive Property also true for division? In other words, does $\frac{b+c}{a} = \frac{b}{a} + \frac{c}{a}$, $a \neq 0$? If so, give an example and explain why it is true. If not true, give a counterexample.

64. *Writing in Math* Use the information about coupons on page 11 to explain how the Distributive Property is useful in calculating store savings. Include an explanation of how the Distributive Property could be used to calculate the coupon savings listed on a grocery receipt.

STANDARDIZED TEST PRACTICE

65. ACT/SAT If a and b are natural numbers, then which of the following must also be a natural number?

I. $a - b$ II. ab III. $\frac{a}{b}$

A I only **C** III only

B II only **D** I and II only

66. REVIEW Which equation is equivalent to $4(9 - 3x) = 7 - 2(6 - 5x)$?

F $8x = 41$ **H** $22x = 41$

G $8x = 24$ **J** $22x = 24$

Spiral Review

Evaluate each expression. (Lesson 1-1)

67. $9(4 - 3)^5$

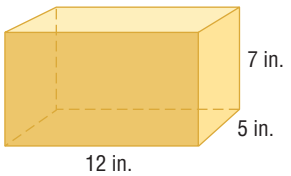
68. $5 + 9 \div 3(3) - 8$

Evaluate each expression if $a = -5$, $b = 0.25$, $c = \frac{1}{2}$, and $d = 4$. (Lesson 1-1)

69. $a + 2b - c$

70. $b + 3(a + d)^3$

71. GEOMETRY The formula for the surface area SA of a rectangular prism is $SA = 2\ell w + 2\ell h + 2wh$, where ℓ represents the length, w represents the width, and h represents the height. Find the surface area of the rectangular prism. (Lesson 1-1)



GET READY for the Next Lesson

PREREQUISITE SKILL **Evaluate each expression if $a = 2$, $b = -\frac{3}{4}$, and $c = 1.8$.** (Lesson 1-1)

72. $8b - 5$

73. $\frac{2}{5}b + 1$

74. $1.5c - 7$

75. $-9(a - 6)$

1-3 Solving Equations

Main Ideas

- Translate verbal expressions into algebraic expressions and equations, and vice versa.
- Solve equations using the properties of equality.

New Vocabulary

open sentence
equation
solution

GET READY for the Lesson

An important statistic for pitchers is the earned run average (ERA). To find the ERA, divide the number of earned runs allowed R by the number of innings pitched I. Then multiply the quotient by 9.

$$\text{ERA} = \frac{R \text{ runs}}{I \text{ innings}} \times \frac{9 \text{ innings}}{1 \text{ game}}$$
$$= \frac{9R}{I} \text{ runs per game}$$

Verbal Expressions to Algebraic Expressions Verbal expressions can be translated into algebraic or mathematical expressions. Any letter can be used as a variable to represent a number that is not known.

EXAMPLE 1 Verbal to Algebraic Expression

Write an algebraic expression to represent each verbal expression.

a. three times the square of a number $3x^2$

b. twice the sum of a number and 5 $2(y + 5)$

CHECK Your Progress

1A. the cube of a number increased by 4 times the same number

1B. three times the difference of a number and 8

A mathematical sentence containing one or more variables is called an **open sentence**. A mathematical sentence stating that two mathematical expressions are equal is called an **equation**.

EXAMPLE 2 Algebraic to Verbal Sentence

Write a verbal sentence to represent each equation.

a. $n + (-8) = -9$ The sum of a number and -8 is -9.

b. $\frac{n}{6} = n^2$ A number divided by 6 is equal to that number squared.

CHECK Your Progress

2A. $g - 5 = -2$

2B. $2c = c^2 - 4$

Open sentences are neither true nor false until the variables have been replaced by numbers. Each replacement that results in a true sentence is called a **solution** of the open sentence.

Properties of Equality To solve equations, we can use properties of equality. Some of these properties are listed below.

Vocabulary Link
Symmetric
Everyday Use having two identical sides
Math Use The two sides of an equation are equal, so the sides can be switched.

KEY CONCEPT *Properties of Equality*

Property	Symbols	Examples
Reflexive	For any real number a, $a = a$.	$-7 + n = -7 + n$
Symmetric	For all real numbers a and b, if $a = b$, then $b = a$.	If $3 = 5x - 6$, then $5x - 6 = 3$.
Transitive	For all real numbers a, b, and c, if $a = b$ and $b = c$, then $a = c$.	If $2x + 1 = 7$ and $7 = 5x - 8$, then $2x + 1 = 5x - 8$.
Substitution	If $a = b$, then a may be replaced by b and b may be replaced by a.	If $(4 + 5)m = 18$, then $9m = 18$.

EXAMPLE **Identify Properties of Equality**

3 **Name the property illustrated by each statement.**

a. **If $3m = 5n$ and $5n = 10p$, then $3m = 10p$.**
Transitive Property of Equality

b. **If $12m = 24$, then $(2 \cdot 6)m = 24$.**
Substitution

CHECK Your Progress

3. If $-11a + 2 = -3a$, then $-3a = -11a + 2$.

Sometimes an equation can be solved by adding the same number to each side, or by subtracting the same number from each side, or by multiplying or dividing each side by the same number.

KEY CONCEPT *Properties of Equality*

Addition and Subtraction

Symbols For any real numbers a, b, and c, if $a = b$, then $a + c = b + c$ and $a - c = b - c$.

Examples If $x - 4 = 5$, then $x - 4 + 4 = 5 + 4$.
If $n + 3 = -11$, then $n + 3 - 3 = -11 - 3$.

Multiplication and Division

Symbols For any real numbers a, b, and c, if $a = b$, then $a \cdot c = b \cdot c$, and if $c \neq 0$, $\frac{a}{c} = \frac{b}{c}$.

Examples If $\frac{m}{4} = 6$, then $4 \cdot \frac{m}{4} = 4 \cdot 6$. If $-3y = 6$, then $\frac{-3y}{-3} = \frac{6}{-3}$.

EXAMPLE Solve One-Step Equations

4 **Solve each equation. Check your solution.**

a. $a + 4.39 = 76$

$$a + 4.39 = 76$$ Original equation

$$a + 4.39 - 4.39 = 76 - 4.39$$ Subtract 4.39 from each side.

$$a = 71.61$$ Simplify.

The solution is 71.61.

CHECK

$$a + 4.39 = 76$$ Original equation

$$71.61 + 4.39 \stackrel{?}{=} 76$$ Substitute 71.61 for a.

$$76 = 76 \checkmark$$ Simplify.

b. $-\frac{3}{5}d = 18$

$$-\frac{3}{5}d = 18$$ Original equation

$$-\frac{5}{3}\left(-\frac{3}{5}\right)d = -\frac{5}{3}(18)$$ Multiply each side by $-\frac{5}{3}$, the multiplicative inverse of $-\frac{3}{5}$.

$$d = -30$$ Simplify.

The solution is -30.

CHECK

$$-\frac{3}{5}d = 18$$ Original equation

$$-\frac{3}{5}(-30) \stackrel{?}{=} 18$$ Substitute -30 for d.

$$18 = 18 \checkmark$$ Simplify.

Study Tip

Multiplication and Division Properties of Equality

Example 4b could also have been solved using the Division Property of Equality. Note that dividing each side of the equation by $-\frac{3}{5}$ is the same as multiplying each side by $-\frac{5}{3}$.

CHECK Your Progress

4A. $x - 14.29 = 25$ **4B.** $\frac{2}{3}y = -18$

EXAMPLE Solve a Multi-Step Equation

5 **Solve $2(2x + 3) - 3(4x - 5) = 22$.**

$$2(2x + 3) - 3(4x - 5) = 22$$ Original equation

$$4x + 6 - 12x + 15 = 22$$ Apply the Distributive Property.

$$-8x + 21 = 22$$ Simplify the left side.

$$-8x = 1$$ Subtract 21 from each side to isolate the variable.

$$x = -\frac{1}{8}$$ Divide each side by -8.

The solution is $-\frac{1}{8}$.

CHECK Your Progress

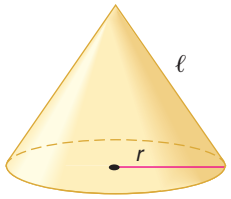
Solve each equation.

5A. $-10x + 3(4x - 2) = 6$ **5B.** $2(2x - 1) - 4(3x + 1) = 2$

You can use properties to solve an equation or formula for a variable.

EXAMPLE Solve for a Variable

6 **GEOMETRY** **The formula for the surface area S of a cone is $S = \pi r\ell + \pi r^2$, where ℓ is the slant height of the cone and r is the radius of the base. Solve the formula for ℓ.**



$S = \pi r\ell + \pi r^2$ Surface area formula

$S - \pi r^2 = \pi r\ell + \pi r^2 - \pi r^2$ Subtract πr^2 from each side.

$S - \pi r^2 = \pi r\ell$ Simplify.

$\frac{S - \pi r^2}{\pi r} = \frac{\pi r\ell}{\pi r}$ Divide each side by πr.

$\frac{S - \pi r^2}{\pi r} = \ell$ Simplify.

CHECK Your Progress

6. The formula for the surface area S of a cylinder is $S = 2\pi r^2 + 2\pi rh$, where r is the radius of the base, and h is the height of the cylinder. Solve the formula for h.

STANDARDIZED TEST EXAMPLE Apply Properties of Equality

7 **If $3n - 8 = \frac{9}{5}$, what is the value of $3n - 3$?**

A $\frac{34}{5}$ **B** $\frac{49}{15}$ **C** $-\frac{16}{5}$ **D** $-\frac{27}{5}$

Test-Taking Tip

Using Properties
If a problem seems to require lengthy calculations, look for a shortcut. There may be a quicker way to solve it. Try using properties of equality.

Read the Test Item

You are asked to find the value of $3n - 3$. Your first thought might be to find the value of n and then evaluate the expression using this value. Notice that you are *not* required to find the value of n. Instead, you can use the Addition Property of Equality.

Solve the Test Item

$3n - 8 = \frac{9}{5}$ Original equation

$3n - 8 + 5 = \frac{9}{5} + 5$ Add 5 to each side.

$3n - 3 = \frac{34}{5}$ $\frac{9}{5} + 5 = \frac{9}{5} + \frac{25}{5}$ or $\frac{34}{5}$

The answer is A.

CHECK Your Progress

7. If $5y + 2 = \frac{8}{3}$, what is the value of $5y - 6$?

F $\frac{-20}{3}$ **G** $\frac{-16}{3}$ **H** $\frac{16}{3}$ **J** $\frac{32}{3}$

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To solve a word problem, it is often necessary to define a variable and write an equation. Then solve by applying the properties of equality.

Real-World Link

Previously occupied homes account for approximately 85% of all U.S. home sales. Most homeowners remodel within 18 months of purchase. The top two remodeling projects are kitchens and baths.

Source: National Association of Remodeling Industry

Real-World EXAMPLE Write an Equation

8 **HOME IMPROVEMENT** **Josh spent $425 of his $1685 budget for home improvements. He would like to replace six interior doors next. What can he afford to spend on each door?**

Explore Let c represent the cost to replace each door.

Plan Write and solve an equation to find the value of c.

The number of doors	times	the cost to replace each door	plus	previous expenses	equals	the total cost.
6	$\cdot$	c	$+$	425	$=$	1685

Solve

$6c + 425 = 1685$ Original equation

$6c + 425 - 425 = 1685 - 425$ Subtract 425 from each side.

$6c = 1260$ Simplify.

$\frac{6c}{6} = \frac{1260}{6}$ Divide each side by 6.

$c = 210$ Simplify.

Josh can afford to spend $210 on each door.

Check The total cost to replace six doors at $210 each is 6(210) or $1260. Add the other expenses of $425 to that, and the total home improvement bill is 1260 + 425 or $1685. Thus, the answer is correct.

CHECK Your Progress

8. A radio station had 300 concert tickets to give to its listeners as prizes. After 1 week, the station had given away 108 tickets. If the radio station wants to give away the same number of tickets each day for the next 8 days, how many tickets must be given away each day?

MathOnline **Problem Solving Handbook at** algebra2.com

CHECK Your Understanding

Example 1 (p. 18) **Write an algebraic expression to represent each verbal expression.**

1. five increased by four times a number

2. twice a number decreased by the cube of the same number

Example 2 (p. 18) **Write a verbal expression to represent each equation.**

3. $9n - 3 = 6$

4. $5 + 3x^2 = 2x$

Example 3 (p. 19) **Name the property illustrated by each statement.**

5. $(3x + 2) - 5 = (3x + 2) - 5$

6. If $4c = 15$, then $4c + 2 = 15 + 2$.

Examples 4–5 (p. 20)

Solve each equation. Check your solution.

7. $y + 14 = -7$ **8.** $3x = 42$ **9.** $16 = -4b$

10. $4(q - 1) - 3(q + 2) = 25$ **11.** $1.8a - 5 = -2.3$ **12.** $-\frac{3}{4}n + 1 = -11$

Example 6 (p. 21)

Solve each equation or formula for the specified variable.

13. $4y - 2n = 9$, for y **14.** $I = prt$, for p

Example 7 (p. 21)

15. STANDARDIZED TEST PRACTICE If $4x + 7 = 18$, what is the value of $12x + 21$?

A 2.75 **B** 32 **C** 33 **D** 54

Example 8 (p. 22)

16. BASEBALL During the 2005 season, Jacque Jones and Matthew LeCroy of the Minnesota Twins hit a combined total of 40 home runs. Jones hit 6 more home runs than LeCroy. How many home runs did each player hit? Define a variable, write an equation, and solve the problem.

Exercises

HOMEWORK HELP	
For Exercises	See Examples
17–22	1
23–26	2
27–30	3
31, 32	4
33–36	5
37–40	6
41	7
42, 43	8

Write an algebraic expression to represent each verbal expression.

17. the sum of 5 and three times a number

18. seven more than the product of a number and 10

19. four less than the square of a number

20. the product of the cube of a number and -6

21. five times the sum of 9 and a number

22. twice the sum of a number and 8

Write a verbal expression to represent each equation.

23. $x - 5 = 12$ **24.** $2n + 3 = -1$

25. $y^2 = 4y$ **26.** $3a^3 = a + 4$

Name the property illustrated by each statement.

27. If $[3(-2)]z = 24$, then $-6z = 24$. **28.** If $5 + b = 13$, then $b = 8$.

29. If $2x = 3d$ and $3d = -4$, then $2x = -4$. **30.** If $y - 2 = -8$, then $3(y - 2) = 3(8)$.

Solve each equation. Check your solution.

31. $2p = 14$ **32.** $-14 + n = -6$

33. $7a - 3a + 2a - a = 16$ **34.** $x + 9x - 6x + 4x = 20$

35. $27 = -9(y + 5) + 6(y + 8)$ **36.** $-7(p + 7) + 3(p - 4) = -17$

Solve each equation or formula for the specified variable.

37. $d = rt$, for r **38.** $x = \frac{-b}{2a}$, for a

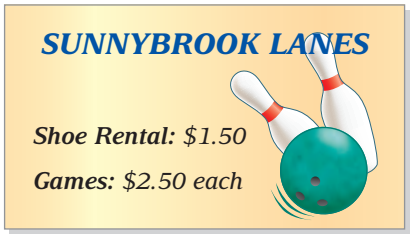
39. $V = \frac{1}{3}\pi r^2 h$, for h **40.** $A = \frac{1}{2}h(a + b)$, for b

41. If $3a + 1 = \frac{13}{3}$, what is the value of $3a - 3$?

For Exercises 42 and 43, define a variable, write an equation, and solve the problem.

42. BOWLING Omar and Morgan arrive at Sunnybrook Lanes with $16.75. What is the total number of games they can afford if they each rent shoes?

43. GEOMETRY The perimeter of a regular octagon is 124 inches. Find the length of each side.



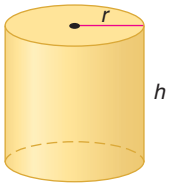
Write an algebraic expression to represent each verbal expression.

44. the square of the quotient of a number and 4

45. the cube of the difference of a number and 7

GEOMETRY For Exercises 46 and 47, use the following information.

The formula for the surface area of a cylinder with radius r and height h is π times twice the product of the radius and height plus twice the product of π and the square of the radius.



46. Write this as an algebraic expression.

47. Write an equivalent expression using the Distributive Property.

Write a verbal expression to represent each equation.

48. $\frac{b}{4} = 2(b + 1)$

49. $7 - \frac{1}{2}x = \frac{3}{x^2}$

Solve each equation or formula for the specified variable.

50. $\frac{a(b-2)}{c-3} = x$, for b

51. $x = \frac{y}{y+4}$, for y

Solve each equation. Check your solution.

52. $\frac{1}{9} - \frac{2}{3}b = \frac{1}{18}$

53. $3f - 2 = 4f + 5$

54. $4(k + 3) + 2 = 4.5(k + 1)$

55. $4.3n + 1 = 7 - 1.7n$

56. $\frac{3}{11}a - 1 = \frac{7}{11}a + 9$

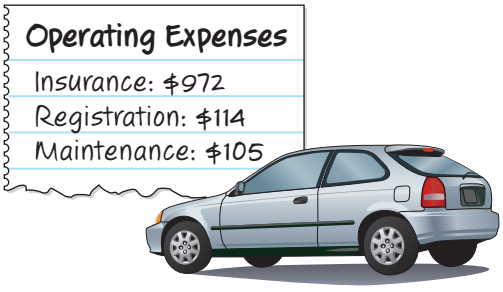
57. $\frac{2}{5}x + \frac{3}{7} = 1 - \frac{4}{7}x$

Cross-Curricular Project

You can write and solve equations to determine the monthly payment for a home. Visit algebra2.com to continue work on your project.

For Exercises 58–63, define a variable, write an equation, and solve the problem.

58. CAR EXPENSES Benito spent $1837 to operate his car last year. Some of these expenses are listed at the right. Benito's only other expense was for gasoline. If he drove 7600 miles, what was the average cost of the gasoline per mile?



59. SCHOOL A school conference room can seat a maximum of 83 people. The principal and two counselors need to meet with the school's student athletes to discuss eligibility requirements. If each student must bring a parent with them, how many students can attend each meeting?

Real-World Career

Industrial Design

Industrial designers use research on product use, marketing, materials, and production methods to create functional and appealing packaging designs.

Math Online

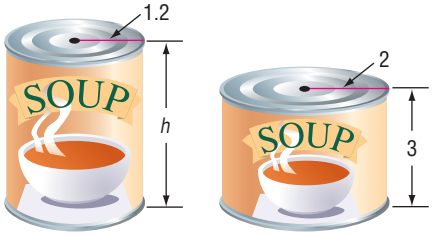
For more information, go to algebra2.com.

60. AGES Chun-Wei's mother is 8 more than twice his age. His father is three years older than his mother is. If the three family members have lived a total of 94 years, how old is each family member?

61. SCHOOL TRIP A Parent Teacher Organization has raised $1800 to help pay for a trip to an amusement park. They ask that there be one adult for every five students attending. Adult tickets are $45 and student tickets are $30. If the group wants to take 50 students, how much will each student need to pay so that adults agreeing to chaperone pay nothing?

62. BUSINESS A trucking company is hired to deliver 125 lamps for $12 each. The company agrees to pay $45 for each lamp that is broken during transport. If the trucking company needs to receive a minimum payment of $1364 for the shipment to cover their expenses, find the maximum number of lamps they can afford to break during the trip.

63. PACKAGING Two designs for a soup can are shown at the right. If each can holds the same amount of soup, what is the height of can A?



Can A Can B

RAILROADS **For Exercises 64–66, use the following information.**

The First Transcontinental Railroad was built by two companies. The Central Pacific began building eastward from Sacramento, California, while the Union Pacific built westward from Omaha, Nebraska. The two lines met at Promontory, Utah, in 1869, approximately 6 years after construction began.

64. The Central Pacific Company laid an average of 9.6 miles of track per month. Together the two companies laid a total of 1775 miles of track. Determine the average number of miles of track laid per month by the Union Pacific Company.

65. About how many miles of track did each company lay?

66. Why do you think the Union Pacific was able to lay track so much more quickly than the Central Pacific?

EXTRA PRACTICE
See pages 891, 926.
Math Online
Self-Check Quiz at algebra2.com

67. MONEY Allison is saving money to buy a video game system. In the first week, her savings were $8 less than $\frac{2}{5}$ the price of the system. In the second week, she saved 50 cents more than $\frac{1}{2}$ the price of the system. She was still $37 short. Find the price of the system.

H.O.T. Problems

68. FIND THE ERROR Crystal and Jamal are solving $C = \frac{5}{9}(F - 32)$ for F. Who is correct? Explain your reasoning.

Crystal

$C = \frac{5}{9}(F - 32)$

$C + 32 = \frac{5}{9}F$

$\frac{9}{5}(C + 32) = F$

Jamal

$C = \frac{5}{9}(F - 32)$

$\frac{9}{5}C = F - 32$

$\frac{9}{5}C + 32 = F$

69. OPEN ENDED Write a two-step equation with a solution of -7.

70. REASONING Determine whether the following statement is *sometimes*, *always*, or *never* true. Explain your reasoning.

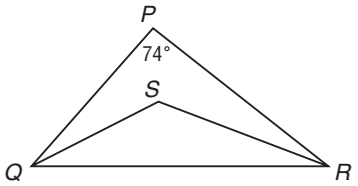
Dividing each side of an equation by the same expression produces an equivalent equation.

71. CHALLENGE Compare and contrast the Symmetric Property of Equality and the Commutative Property of Addition.

72. *Writing in Math* Use the information about ERA on page 18 to find the number of earned runs allowed for a pitcher who has an ERA of 2.00 and who has pitched 180 innings. Explain when it would be desirable to solve a formula like the one given for a specified variable.

STANDARDIZED TEST PRACTICE

73. ACT/SAT In triangle PQR, $\overline{QS}$ and $\overline{SR}$ are angle bisectors and angle $P = 74°$. How many degrees are there in angle QSR?



A 106 **C** 125

B 121 **D** 127

74. REVIEW Which of the following best describes the graph of the equations below?

$8y = 2x + 13$

$24y = 6x + 13$

F The lines have the same y-intercept.

G The lines have the same x-intercept.

H The lines are perpendicular.

J The lines are parallel.

Spiral Review

Simplify each expression. (Lesson 1-2)

75. $2x + 9y + 4z - y - 8x$

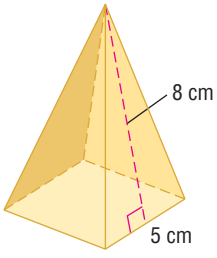
76. $4(2a + 5b) - 3(4b - a)$

Evaluate each expression if $a = 3$, $b = -2$, and $c = 1.2$. (Lesson 1-1)

77. $a - [b(a - c)]$

78. $c^2 - ab$

79. GEOMETRY The formula for the surface area S of a regular pyramid is $S = \frac{1}{2}P\ell + B$, where P is the perimeter of the base, ℓ is the slant height, and B is the area of the base. Find the surface area of the square pyramid at the right. (Lesson 1-1)



GET READY for the Next Lesson

PREREQUISITE SKILL Identify the additive inverse for each number or expression. (Lesson 1-2)

80. 2.5 **81.** $\frac{1}{4}$ **82.** $-3x$ **83.** $5 - 6y$

1-4 Solving Absolute Value Equations

Main Ideas

- Evaluate expressions involving absolute values.
- Solve absolute value equations.

New Vocabulary

absolute value

empty set

GET READY for the Lesson

Seismologists use the Richter scale to express the magnitudes of earthquakes. This scale ranges from 1 to 10, with 10 being the highest. The uncertainty in the estimate of a magnitude E is about plus or minus 0.3 unit. This means that an earthquake with a magnitude estimated at 6.1 on the Richter scale might actually have a magnitude as low as 5.8 or as high as 6.4. These extremes can be described by the absolute value equation $|E - 6.1| = 0.3$.

Robert Yager/Getty Images

Absolute Value Expressions The **absolute value** of a number is its distance from 0 on the number line. Since distance is nonnegative, the absolute value of a number is always nonnegative. The symbol $|x|$ is used to represent the absolute value of a number x.

KEY CONCEPT *Absolute Value*

Words For any real number a, if a is positive or zero, the absolute value of a is a. If a is negative, the absolute value of a is the opposite of a.

Symbols For any real number a, $|a| = a$ if $a \geq 0$, and $|a| = -a$ if $a < 0$.

When evaluating expressions, absolute value bars act as a grouping symbol. Perform any operations inside the absolute value bars first.

EXAMPLE **Evaluate an Expression with Absolute Value**

1 **Evaluate $1.4 + |5y - 7|$ if $y = -3$.**

$1.4 + |5y - 7| = 1.4 + |5(-3) - 7|$ Replace y with -3.

$= 1.4 + |-15 - 7|$ Simplify $5(-3)$ first.

$= 1.4 + |-22|$ Subtract 7 from -15.

$= 1.4 + 22$ $|-22| = 22$

$= 23.4$ Add.

CHECK Your Progress

1A. Evaluate $\left|4x + 3\right| - 3\frac{1}{2}$ if $x = -2$.

1B. Evaluate $1\frac{1}{3} - \left|2y + 1\right|$ if $y = -\frac{2}{3}$.

Absolute Value Equations Some equations contain absolute value expressions. The definition of absolute value is used in solving these equations. For any real numbers a and b, where $b \geq 0$, if $|a| = b$, then $a = b$ or $-a = b$. This second case is often written as $a = -b$.

EXAMPLE Solve an Absolute Value Equation

2 **Solve $|x - 18| = 5$. Check your solutions.**

Case 1 $a = b$ or **Case 2** $a = -b$

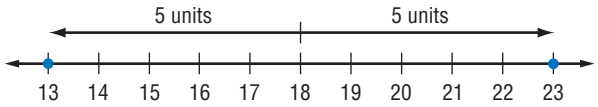
$$x - 18 = 5 \qquad x - 18 = -5$$
$$x - 18 + 18 = 5 + 18 \qquad x - 18 + 18 = -5 + 18$$
$$x = 23 \qquad x = 13$$

CHECK

$$|x - 18| = 5 \qquad |x - 18| = 5$$
$$|23 - 18| \stackrel{?}{=} 5 \qquad |13 - 18| \stackrel{?}{=} 5$$
$$|5| \stackrel{?}{=} 5 \qquad |-5| \stackrel{?}{=} 5$$
$$5 = 5 \checkmark \qquad 5 = 5 \checkmark$$

The solutions are 23 and 13. Thus, the solution set is {13, 23}.

On the number line, we can see that each answer is 5 units away from 18.



CHECK Your Progress

Solve each equation. Check your solutions.

2A. $9 = |x + 12|$ **2B.** $8 = |y + 5|$

Study Tip

Symbols

The empty set is symbolized by { } or ∅.

Because the absolute value of a number is always positive or zero, an equation like $|x| = -5$ is never true. Thus, it has no solution. The solution set for this type of equation is the **empty set**.

EXAMPLE No Solution

3 **Solve $|5x - 6| + 9 = 0$.**

$|5x - 6| + 9 = 0$ Original equation

$|5x - 6| = -9$ Subtract 9 from each side.

This sentence is *never* true. So the solution set is ∅.

CHECK Your Progress

3A. Solve $-2|3a - 2| = 6$. **3B.** Solve $|4b + 1| + 8 = 0$.

It is important to check your answers when solving absolute value equations. Even if the correct procedure for solving the equation is used, the answers may not be actual solutions of the original equation.

EXAMPLE One Solution

4 **Solve $|x + 6| = 3x - 2$. Check your solutions.**

Case 1 $a = b$

$x + 6 = 3x - 2$

$6 = 2x - 2$

$8 = 2x$

$4 = x$

or

Case 2 $a = -b$

$x + 6 = -(3x - 2)$

$x + 6 = -3x + 2$

$4x + 6 = 2$

$4x = -4$

$x = -1$

There appear to be two solutions, 4 and -1.

CHECK Substitute each value in the original equation.

$|x + 6| = 3x - 2$

$|4 + 6| \stackrel{?}{=} 3(4) - 2$

$|10| \stackrel{?}{=} 12 - 2$

$10 = 10$ ✓

$|x + 6| = 3x - 2$

$|-1 + 6| \stackrel{?}{=} 3(-1) - 2$

$|5| \stackrel{?}{=} -3 - 2$

$5 \neq -5$

Since $5 \neq -5$, the only solution is 4. Thus, the solution set is {4}.

CHECK Your Progress

Solve each equation. Check your solutions.

4A. $2|x + 1| - x = 3x - 4$ **4B.** $3|2x + 2| - 2x = x + 3$

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CHECK Your Understanding

Example 1 (p. 27)

Evaluate each expression if $a = -4$ and $b = 1.5$.

1. $|a + 12|$ **2.** $|-6b|$ **3.** $-|a + 21| + 6.2$

Example 2 (p. 28)

FOOD For Exercises 4–6, use the following information.

Most meat thermometers are accurate to within plus or minus 2°F.

4. If a meat thermometer reads 160°F, write an equation to determine the least and greatest possible temperatures of the meat.

5. Solve the equation you wrote in Exercise 4.

6. Ham needs to reach an internal temperature of 160°F to be fully cooked. To what temperature reading should you cook a ham to ensure that the minimum temperature is reached? Explain.

Examples 2–4 (pp. 28–29)

Solve each equation. Check your solutions.

7. $|x + 4| = 17$ **8.** $|b + 15| = 3$

9. $20 = |a - 9|$ **10.** $34 = |y - 2|$

11. $|2w + 3| + 6 = 2$ **12.** $|3n + 2| + 4 = 0$

13. $|c - 2| = 2c - 10$ **14.** $|h - 5| = 3h - 7$

Exercises

HOMEWORK HELP

For Exercises	See Examples
15–22	1
23–32	2–4
33–34	2

Evaluate each expression if $a = -5$, $b = 6$, and $c = 2.8$.

15. $|-3a|$ **16.** $|-4b|$ **17.** $|a + 5|$ **18.** $|2 - b|$

19. $|2b - 15|$ **20.** $|4a + 7|$ **21.** $-|18 - 5c|$ **22.** $-|2c - a|$

Solve each equation. Check your solutions.

23. $|x - 25| = 17$ **24.** $|y + 9| = 21$

25. $33 = |a + 12|$ **26.** $11 = |3x + 5|$

27. $8|w - 7| = 72$ **28.** $2|b + 4| = 48$

29. $0 = |2z - 3|$ **30.** $|6c - 1| = 0$

31. $-12|9x + 1| = 144$ **32.** $1 = |5x + 9| + 6$

33. COFFEE Some say that to brew an excellent cup of coffee, you must have a brewing temperature of 200°F, plus or minus 5 degrees. Write and solve an equation describing the maximum and minimum brewing temperatures for an excellent cup of coffee.

34. SURVEYS Before an election, a company conducts a telephone survey of likely voters. Based on their survey data, the polling company states that an amendment to the state constitution is supported by 59% of the state's residents and that 41% of the state's residents do not approve of the amendment. According to the company, the results of their survey have a margin of error of 3%. Write and solve an equation describing the maximum and minimum percent of the state's residents that support the amendment.

Solve each equation. Check your solutions.

35. $35 = 7|4x - 13|$ **36.** $-9 = -3|2n + 5|$

37. $-6 = |a - 3| - 14$ **38.** $3|p - 5| = 2p$

39. $3|2a + 7| = 3a + 12$ **40.** $|3x - 7| - 5 = -3$

41. $16t = 4|3t + 8|$ **42.** $-2m + 3 = |15 + m|$

Evaluate each expression if $x = 6$, $y = 2.8$, and $z = -5$.

43. $9 - |-2x + 8|$ **44.** $3|z - 10| + |2z|$ **45.** $|z - x| - |10y - z|$

46. MANUFACTURING A machine fills bags with about 16 ounces of sugar each. After the bags are filled, another machine weighs them. If the bag weighs 0.3 ounce more or less than the desired weight, the bag is rejected. Write an equation to find the heaviest and lightest bags the machine will approve.

EXTRA PRACTICE
See pages 892, 926.

Math Online
Self-Check Quiz at algebra2.com

47. METEOROLOGY The *troposphere* is the layer of atmosphere closest to Earth. The average upper boundary of the layer is about 13 kilometers above Earth's surface. This height varies with latitude and with the seasons by as much as 5 kilometers. Write and solve an equation describing the maximum and minimum heights of the upper bound of the troposphere.

H.O.T. Problems

48. OPEN ENDED Write an absolute value equation and graph the solution set.

CHALLENGE For Exercises 49–51, determine whether each statement is *sometimes*, *always*, or *never* true. Explain your reasoning.

49. If a and b are real numbers, then $|a + b| = |a| + |b|$.

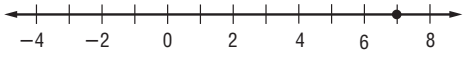
50. If a, b, and c are real numbers, then $c|a + b| = |ca + cb|$.

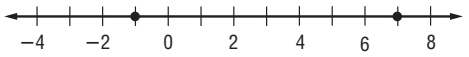
51. For all real numbers a and b, $a \neq 0$, the equation $|ax + b| = 0$ will have exactly one solution.

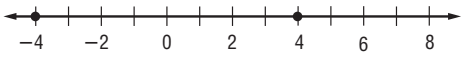
52. ***Writing in Math*** Use the information on page 27 to explain how an absolute value equation can describe the magnitude of an earthquake. Include a verbal and graphical explanation of how $|E - 6.1| = 0.3$ describes the possible magnitudes.

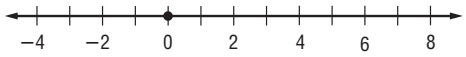
STANDARDIZED TEST PRACTICE

53. ACT/SAT Which graph represents the solution set for $|x - 3| - 4 = 0$?

A 

B 

C 

D 

54. REVIEW For a party, Lenora bought several pounds of cashews and several pounds of almonds. The cashews cost \$8 per pound, and the almonds cost \$6 per pound. Lenora bought a total of 7 pounds and paid a total of \$48. How many pounds of cashews did she buy?

F 2 pounds
G 3 pounds
H 4 pounds
J 5 pounds

Spiral Review

Solve each equation. Check your solution. (Lesson 1-3)

55. $3x + 6 = 22$

56. $7p - 4 = 3(4 + 5p)$

57. $\frac{5}{7}y - 3 = \frac{3}{7}y + 1$

Name the property illustrated by each equation. (Lesson 1-2)

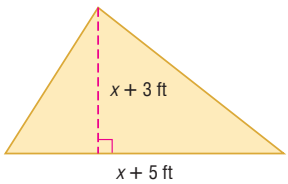
58. $(5 + 9) + 13 = 13 + (5 + 9)$

59. $m(4 - 3) = m \cdot 4 - m \cdot 3$

GEOMETRY For Exercises 60 and 61, use the following information.
The formula for the area A of a triangle is $A = \frac{1}{2}bh$, where b is the measure of the base and h is the measure of the height. (Lesson 1-1)

60. Write an expression to represent the area of the triangle.

61. Evaluate the expression you wrote in Exercise 60 for $x = 23$.



GET READY for the Next Lesson

PREREQUISITE SKILL Solve each equation. (Lesson 1-3)

62. $14y - 3 = 25$

63. $4.2x + 6.4 = 40$

64. $7w + 2 = 3w - 6$

65. $2(a - 1) = 8a - 6$

CHAPTER 1

Mid-Chapter Quiz

Lessons 1-1 through 1-4

Evaluate each expression if $a = -2$, $b = \frac{1}{3}$, and $c = -12$. (Lesson 1-1)

1. $a^3 + b(9 - c)$
2. $b(a^2 - c)$
3. $\frac{3ab}{c}$
4. $\frac{a - c}{a + c}$
5. $\frac{a^3 - c}{b^2}$
6. $\frac{c + 3}{ab}$

7. **ELECTRICITY** Find the amount of current I (in amperes) produced if the electromotive force E is 2.5 volts, the circuit resistance R is 1.05 ohms, and the resistance r within a battery is 0.2 ohm. Use the formula $I = \frac{E}{R + r}$. (Lesson 1-1)

Name the sets of numbers to which each number belongs. (Lesson 1-2)

8. 3.5
9. $\sqrt{100}$

Name the property illustrated by each equation. (Lesson 1-2)

10. $bc + (-bc) = 0$
11. $\left(\frac{4}{7}\right)\left(1\frac{3}{4}\right) = 1$
12. $3 + (x - 1) = (3 + x) + (-1)$

Name the additive inverse and multiplicative inverse for each number. (Lesson 1-2)

13. $\frac{6}{7}$
14. $-\frac{4}{3}$

15. Simplify $4(14x - 10y) - 6(x + 4y)$. (Lesson 1-2)

Write an algebraic expression to represent each verbal expression. (Lesson 1-3)

16. twice the difference of a number and 11
17. the product of the square of a number and 5

Solve each equation. Check your solution. (Lesson 1-3)

18. $-2(a + 4) = 2$
19. $2d + 5 = 8d + 2$
20. $4y - \frac{1}{10} = 3y + \frac{4}{5}$

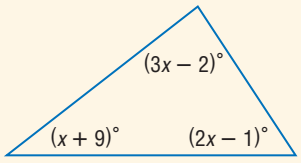
21. Solve $s = \frac{1}{2}gt^2$ for g. (Lesson 1-3)

22. **MULTIPLE CHOICE** Karissa has \$10 per month to spend text messaging on her cell phone. The phone company charges \$4.95 for the first 100 messages and \$0.10 for each additional message. How many text messages can Karissa afford to send each month? (Lesson 1-3)

A 50 **C** 150

B 100 **D** 151

23. **GEOMETRY** Use the information in the figure to find the value of x. Then state the degree measures of the three angles of the triangle. (Lesson 1-3)



Solve each equation. Check your solutions. (Lesson 1-4)

24. $|a + 4| = 3$
25. $|3x + 2| = 1$
26. $|3m - 2| = -4$
27. $|2x + 5| - 7 = 4$
28. $|h + 6| + 9 = 8$
29. $|5x - 2| - 6 = -3$

30. **CARNIVAL GAMES** Julian will win a prize if the carnival worker cannot guess his weight to within 3 pounds. Julian weighs 128 pounds. Write an equation to find the highest and lowest weights that the carnival guesser can guess to keep Julian from winning a prize. (Lesson 1-4)

1-5 Solving Inequalities

Main Ideas

- Solve inequalities with one operation.
- Solve multi-step inequalities.

New Vocabulary

set-builder notation

GET READY for the Lesson

Kuni is trying to decide between two rate plans offered by a wireless phone company.

	Plan 1	Plan 2
Monthly Access Fee	$35.00	$55.00
Minutes Included	400	650
Additional Minutes	40¢	35¢

To compare these two rate plans, we can use inequalities. The monthly access fee for Plan 1 is less than the fee for Plan 2, $35 < $55. However, the additional minutes fee for Plan 1 is greater than that of Plan 2, $0.40 > $0.35.

Solve Inequalities with One Operation For any two real numbers, a and b, exactly one of the following statements is true.

$$a < b \qquad a = b \qquad a > b$$

This is known as the **Trichotomy Property.**

Adding the same number to, or subtracting the same number from, each side of an inequality does not change the truth of the inequality.

KEY CONCEPT *Properties of Inequality*

Addition Property of Inequality

Words For any real numbers, a, b, and c:
If $a > b$, then $a + c > b + c$.
If $a < b$, then $a + c < b + c$.

Example
$3 < 5$
$3 + (-4) < 5 + (-4)$
$-1 < 1$

Subtraction Property of Inequality

Words For any real numbers, a, b, and c:
If $a > b$, then $a - c > b - c$.
If $a < b$, then $a - c < b - c$.

Example
$2 > -7$
$2 - 8 > -7 - 8$
$-6 > -15$

These properties are also true for $\leq$, $\geq$, and $\neq$.

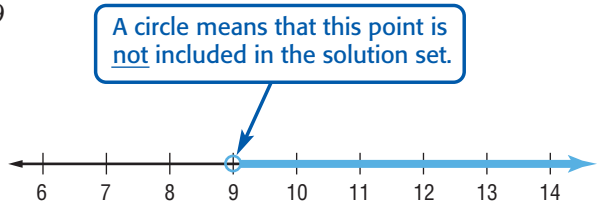
These properties can be used to solve inequalities. The solution sets of inequalities in one variable can then be graphed on number lines. Graph using a circle with an arrow to the left for $<$ and an arrow to the right for $>$. Graph using a dot with an arrow to the left for $\leq$ and an arrow to the right for $\geq$.

EXAMPLE 1 Solve an Inequality Using Addition or Subtraction

Solve $7x - 5 > 6x + 4$. Graph the solution set on a number line.

$7x - 5 > 6x + 4$	Original inequality
$7x - 5 + (-6x) > 6x + 4 + (-6x)$	Add $-6x$ to each side.
$x - 5 > 4$	Simplify.
$x - 5 + 5 > 4 + 5$	Add 5 to each side.
$x > 9$	Simplify.

Any real number greater than 9 is a solution of this inequality. The graph of the solution set is shown at the right.



CHECK Substitute a number greater than 9 for x in $7x - 5 > 6x + 4$. The inequality should be true.

CHECK Your Progress

1. Solve $4x + 7 \leq 3x + 9$. Graph the solution set on a number line.

Multiplying or dividing each side of an inequality by a positive number does not change the truth of the inequality. However, multiplying or dividing each side of an inequality by a *negative* number requires that the order of the inequality be *reversed*. For example, to reverse $\leq$, replace it with $\geq$.

KEY CONCEPT — Properties of Inequality

Multiplication Property of Inequality

Words		Examples
For any real numbers, a, b, and c, where		
c is positive:	if $a > b$, then $ac > bc$.	$-2 < 3$
	if $a < b$, then $ac < bc$.	$4(-2) < 4(3)$
		$-8 < 12$
c is negative:	if $a > b$, then $ac < bc$.	$5 > -1$
	if $a < b$, then $ac > bc$.	$(-3)(5) < (-3)(21)$
		$-15 < 3$

Division Property of Inequality

Words		Examples
For any real numbers, a, b, and c, where		
c is positive:	if $a > b$, then $\frac{a}{c} > \frac{b}{c}$.	$-18 < -9$
	if $a < b$, then $\frac{a}{c} < \frac{b}{c}$.	$\frac{-18}{3} < \frac{-9}{3}$
		$-6 < -3$
c is negative:	if $a > b$, then $\frac{a}{c} < \frac{b}{c}$.	$12 > 8$
	if $a < b$, then $\frac{a}{c} > \frac{b}{c}$.	$\frac{12}{-2} < \frac{8}{-2}$
		$-6 < -4$

These properties are also true for $\leq$, $\geq$, and $\neq$.

Reading Math

Set-Builder Notation
$\{x \mid x > 9\}$ is read *the set of all numbers x such that x is greater than 9.*

The solution set of an inequality can be expressed by using **set-builder notation.** For example, the solution set in Example 1 can be expressed as $\{x \mid x > 9\}$.

EXAMPLE 2 Solve an Inequality Using Multiplication or Division

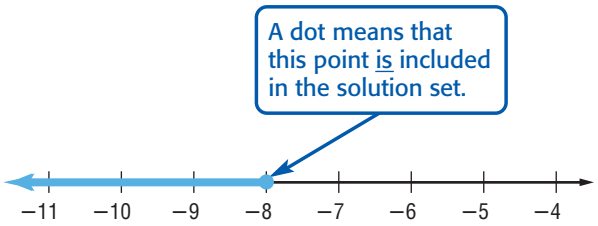
Solve $-0.25y \geq 2$. Graph the solution set on a number line.

$-0.25y \geq 2$ Original inequality

$\frac{-0.25y}{-0.25} \leq \frac{2}{-0.25}$ Divide each side by -0.25, reversing the inequality symbol.

$y \leq -8$ Simplify.

The solution set is $\{y \mid y \leq -8\}$. The graph of the solution set is shown below.



CHECK Your Progress

2. Solve $-\frac{1}{3}x < 1$. Graph the solution set on a number line.

Study Tip

Solutions to Inequalities

When solving an inequality,

- if you arrive at a false statement, such as $3 > 5$, then the solution set for that inequality is the empty set, $\emptyset$.
- if you arrive at a true statement such as $3 > -1$, then the solution set for that inequality is the set of all real numbers.

Solve Multi-Step Inequalities Solving multi-step inequalities is similar to solving multi-step equations.

EXAMPLE 3 Solve a Multi-Step Inequality

Solve $-m \leq \frac{m+4}{9}$. Graph the solution set on a number line.

$-m \leq \frac{m+4}{9}$ Original inequality

$-9m \leq m + 4$ Multiply each side by 9.

$-10m \leq 4$ Add $-m$ to each side.

$m \geq -\frac{4}{10}$ Divide each side by -10, reversing the inequality symbol.

$m \geq -\frac{2}{5}$ Simplify.

The solution set is $\left\{m \mid m \geq -\frac{2}{5}\right\}$ and is graphed below.

CHECK Your Progress

3. Solve $3(2q - 4) > 6$. Graph the solution set on a number line.

Real-World EXAMPLE Write an Inequality

4 **DELIVERIES** **Craig is delivering boxes of paper. Each box weighs 64 pounds, and Craig weighs 160 pounds. If the maximum capacity of the elevator is 2000 pounds, how many boxes can Craig safely take on each trip?**

Explore Let b = the number of boxes Craig can safely take on each trip. A maximum capacity of 2000 pounds means that the total weight must be less than or equal to 2000.

Plan The total weight of the boxes is $64b$. Craig's weight plus the total weight of the boxes must be less than or equal to 2000. Write an inequality.

Craig's weight		plus		the weight of the boxes		is less than or equal to		2000.
160		+		$64b$		$\leq$		2000

Solve

$160 + 64b \leq 2000$ Original inequality

$64b \leq 1840$ Subtract 160 from each side.

$b \leq 28.75$ Divide each side by 64.

Check Since Craig cannot take a fraction of a box, he can take no more than 28 boxes per trip and still meet the safety requirements.

Study Tip

Math Symbols

- $<$ is less than; is fewer than
- $>$ is greater than; is more than
- $\leq$ is at most; is no more than; is less than or equal to
- $\geq$ is at least; is no less than; is greater than or equal to

CHECK Your Progress

4. Sophia's goal is to score at least 200 points this basketball season. If she has already scored 122 points, how many points does Sophia have to score on average for the last 6 games to reach her goal?

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You can use a graphing calculator to solve inequalities.

GRAPHING CALCULATOR LAB

Solving Inequalities

The inequality symbols in the TEST menu on the TI-83/84 Plus are called *relational operators.* They compare values and return 1 if the test is true or 0 if the test is false.

You can use these relational operators to solve an inequality in one variable.

THINK AND DISCUSS

1. Clear the Y= list. Enter $11x + 3 \geq 2x - 6$ as Y1. Put your calculator in DOT mode. Then, graph in the standard viewing window. Describe the graph.
2. Using the TRACE function, investigate the graph. What values of x are on the graph? What values of y are on the graph?
3. Based on your investigation, what inequality is graphed?
4. Solve $11x + 3 \geq 2x - 6$ algebraically. How does your solution compare to the inequality you wrote in Exercise 3?

CHECK Your Understanding

Examples 1–3 (pp. 34–35)

Solve each inequality. Then graph the solution set on a number line.

1. $a + 2 < 3.5$
2. $11 - c \leq 8$
3. $5 \geq 3x$
4. $-0.6p < -9$
5. $2w + 19 < 5$
6. $4y + 7 > 31$
7. $n \leq \frac{n - 4}{5}$
8. $\frac{3z + 6}{11} < z$

Example 4 (p. 36)

9. **SCHOOL** The final grade for a class is calculated by taking 75% of the average test score and adding 25% of the score on the final exam. If all scores are out of 100 and a student has a 76 test average, what score does the student need on the final exam to have a final grade of at least 80?

Exercises

HOMEWORK HELP	
For Exercises	See Examples
10, 11	1
12–15	2
16–26	3
27–32	4

Solve each inequality. Then graph the solution set on a number line.

10. $n + 4 \geq -7$
11. $b - 3 \leq 15$
12. $5x < 35$
13. $\frac{d}{2} > -4$
14. $\frac{g}{-3} \geq -9$
15. $-8p \geq 24$
16. $13 - 4k \leq 27$
17. $14 > 7y - 21$
18. $-27 < 8m + 5$
19. $6b + 11 \geq 15$
20. $2(4t + 9) \leq 18$
21. $90 \geq 5(2r + 6)$
22. $\frac{3t + 6}{2} < 3t + 6$
23. $\frac{k + 7}{3} - 1 < 0$
24. $\frac{2n - 6}{5} + 1 > 0$

25. **PART-TIME JOB** David earns \$6.40 an hour working at Box Office Videos. Each week 25% of his total pay is deducted for taxes. If David wants his take-home pay to be at least \$120 a week, solve $6.4x - 0.25(6.4x) \geq 120$ to determine how many hours he must work.

26. **STATE FAIR** Admission to a state fair is \$12 per person. Bus parking costs \$20. Solve $12n + 20 \leq 600$ to determine how many people can go to the fair if a group has \$600 and uses only one bus.

Define a variable and write an inequality for each problem. Then solve.

27. The product of 12 and a number is greater than 36.
28. Three less than twice a number is at most 5.
29. The sum of a number and 8 is more than 2.
30. The product of -4 and a number is at least 35.
31. The difference of one half of a number and 7 is greater than or equal to 5.
32. One more than the product of -3 and a number is less than 16.

Solve each inequality. Then graph the solution set on a number line.

33. $14 - 8n \leq 0$
34. $-4(5w - 8) < 33$
35. $0.02x + 5.58 < 0$
36. $1.5 - 0.25c < 6$
37. $6d + 3 \geq 5d - 2$
38. $9z + 2 > 4z + 15$
39. $2(g + 4) < 3g - 2(g - 5)$
40. $3(a + 4) - 2(3a + 4) \leq 4a - 1$
41. $y < \frac{-y + 2}{9}$
42. $\frac{1 - 4p}{5} < 0.2$
43. $\frac{4x + 2}{6} < \frac{2x + 1}{3}$
44. $12\left(\frac{1}{4} - \frac{n}{3}\right) \leq -6n$

CAR SALES **For Exercises 45 and 46, use the following information.**
Mrs. Lucas earns a salary of $34,000 per year plus 1.5% commission on her sales. If the average price of a car she sells is $30,500, about how many cars must she sell to make an annual income of at least $50,000?

45. Write an inequality to describe this situation.

46. Solve the inequality and interpret the solution.

Define a variable and write an inequality for each problem. Then solve.

47. Twice the sum of a number and 5 is no more than 3 times that same number increased by 11.

48. 9 less than a number is at most that same number divided by 2.

49. CHILD CARE By Ohio law, when children are napping, the number of children per childcare staff member may be as many as twice the maximum listed at the right. Write and solve an inequality to determine how many staff members are required to be present in a room where 17 children are napping and the youngest child is 18 months old.

Maximum Number of Children Per Child Care Staff Member
At least one child care staff member caring for:
Every 5 infants less than 12 months old (or 2 for every 12)
Every 6 infants who are at least 12 months old, but less than 18 months old
Every 7 toddlers who are at least 18 months old, but less than 30 months old
Every 8 toddlers who are at least 30 months old, but less than 3 years old

Source: Ohio Department of Job and Family Services

EXTRA PRACTICE
See pages 892, 926.
MathOnline
Self-Check Quiz at algebra2.com

TEST GRADES **For Exercises 50 and 51, use the following information.**
Flavio's scores on the first four of five 100-point history tests were 85, 91, 89, and 94.

50. If a grade of at least 90 is an A, write an inequality to find the score Flavio must receive on the fifth test to have an A test average.

51. Solve the inequality and interpret the solution.

Graphing Calculator

Use a graphing calculator to solve each inequality.

52. $-5x - 8 < 7$ **53.** $-4(6x - 3) \leq 60$ **54.** $3(x + 3) \geq 2(x + 4)$

H.O.T. Problems

55. OPEN ENDED Write an inequality for which the solution set is the empty set.

56. REASONING Explain why it is not necessary to state a division property for inequalities.

57. CHALLENGE Which of the following properties hold for inequalities? Explain your reasoning or give a counterexample.

a. Reflexive **b.** Symmetric **c.** Transitive

58. CHALLENGE Write a multi-step inequality requiring multiplication or division, the solution set is graphed below.

59. ***Writing in Math*** Use the information about phone rate plans on page 33 to explain how inequalities can be used to compare phone plans. Include an explanation of how Kuni might determine when Plan 2 might be cheaper than Plan 1 if she typically uses more than 400 but less than 650 minutes.

STANDARDIZED TEST PRACTICE

60. ACT/SAT If $a < b$ and $c < 0$, which of the following are true?

I. $ac > bc$

II. $a + c < b + c$

III. $a - c > b - c$

A I only

B II only

C III only

D I and II only

61. REVIEW What is the complete solution to the equation $|8 - 4x| = 40$?

F $x = 8; x = 12$

G $x = 8; x = -12$

H $x = -8; x = -12$

J $x = -8; x = 12$

Spiral Review

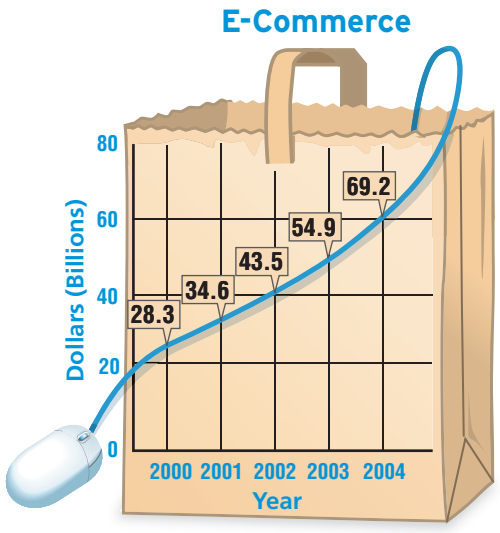
Solve each equation. Check your solutions. (Lesson 1-4)

62. $|x - 3| = 17$

63. $8|4x - 3| = 64$

64. $|x + 1| = x$

65. E-COMMERCE On average, by how much did the amount spent on online purchases increase each year from 2000 to 2004? Define a variable, write an equation, and solve the problem. (Lesson 1-3)



Name the sets of numbers to which each number belongs. (Lesson 1-2)

66. 31

67. $-4.\overline{2}$

68. $\sqrt{7}$

69. BABY-SITTING Jenny baby-sat for $5\frac{1}{2}$ hours on Friday night and 8 hours on Saturday. She charges $4.25 per hour. Use the Distributive Property to write two equivalent expressions that represent how much money Jenny earned. (Lesson 1-2)

GET READY for the Next Lesson

PREREQUISITE SKILL **Solve each equation. Check your solutions.** (Lesson 1-4)

70. $|x| = 7$

71. $|x + 5| = 18$

72. $|5y - 8| = 12$

73. $14 = |2x - 36|$

74. $10 = 2|w + 6|$

75. $|x + 4| + 3 = 17$

READING MATH

Interval Notation

The solution set of an inequality can be described by using **interval notation.** The infinity symbols below are used to indicate that a set is unbounded in the positive or negative direction, respectively.

Read as *positive infinity.* → $+\infty$ $-\infty$ ← Read as *negative infinity.*

To indicate that an endpoint is *not* included in the set, a parenthesis, (or), is used.

$x < 2$

interval notation
$(-\infty, 2)$

A bracket is used to indicate that the endpoint, −2, *is* included in the solution set below. Parentheses are always used with the symbols $+\infty$ and $-\infty$, because they do not include endpoints.

$x \geq -2$

interval notation
$[-2, +\infty)$

In interval notation, the symbol for the union of the two sets is ∪. The solution set of the compound inequality $y \leq -7$ or $y > -1$ is written as $(-\infty, -7] \cup (-1, +\infty)$.

Reading to Learn

Describe each set using interval notation.

1. $\{a|a \leq -3\}$

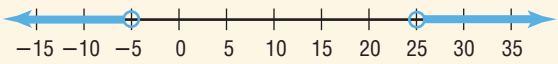
2. $\{n|n > -8\}$

3. $\{y|y < 2 \text{ or } y \geq 14\}$

4. $\{b|b \leq -9 \text{ or } b > 1\}$

5.

6.



Graph each solution set on a number line.

7. $(-1, +\infty)$

8. $(-\infty, 4]$

9. $(-\infty, 5] \cup (7, +\infty)$

10. Write in words the meaning of $(-\infty, 3) \cup [10, +\infty)$. Then write the compound inequality that has this solution set.

1-6 Solving Compound and Absolute Value Inequalities

Main Ideas

- Solve compound inequalities.
- Solve absolute value inequalities.

New Vocabulary

compound inequality
intersection
union

GET READY for the Lesson

One test used to determine whether a patient is diabetic is a glucose tolerance test. Patients start the test in a *fasting state*, meaning they have had no food or drink except water for at least 10, but no more than 16, hours. The acceptable number of hours h for fasting can be described by the following compound inequality.

$$h \geq 10 \text{ and } h \leq 16$$

Compound Inequalities A **compound inequality** consists of two inequalities joined by the word *and* or the word *or*. To solve a compound inequality, you must solve each part of the inequality. The graph of a compound inequality containing *and* is the **intersection** of the solution sets of the two inequalities. Compound inequalities involving the word *and* are called *conjunctions*. Compound inequalities involving the word *or* are called *disjunctions*.

Vocabulary Link
Intersection
Everyday Use the place where two streets meet
Math Use the set of elements common to two sets

KEY CONCEPT "And" Compound Inequalities

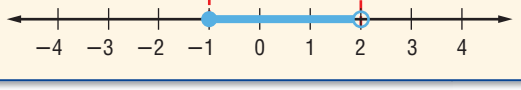
Words A compound inequality containing the word *and* is true if and only if *both* inequalities are true.

Example

$x \geq -1$

$x < 2$

$x \geq -1$ and $x < 2$



Another way of writing $x \geq -1$ and $x < 2$ is $-1 \leq x < 2$.
Both forms are read *x is greater than or equal to −1 and less than 2.*

EXAMPLE 1 Solve an "and" Compound Inequality

Solve $13 < 2x + 7 \leq 17$. Graph the solution set on a number line.

Method 1

Write the compound inequality using the word *and*. Then solve each inequality.

$13 < 2x + 7$ and $2x + 7 \leq 17$
$6 < 2x$ $2x \leq 10$
$3 < x$ $x \leq 5$
$3 < x \leq 5$

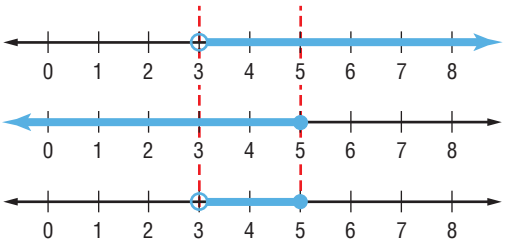
Method 2

Solve both parts at the same time by subtracting 7 from each part. Then divide each part by 2.

$13 < 2x + 7 \leq 17$
$6 < 2x \leq 10$
$3 < x \leq 5$

(continued on the next page)

Graph the solution set for each inequality and find their intersection.



The solution set is $\{x|3 < x \leq 5\}$.

Concepts in Motion
Animation algebra2.com

CHECK Your Progress

1. Solve $8 \leq 3x - 4 < 11$. Graph the solution set on a number line.

The graph of a compound inequality containing *or* is the **union** of the solution sets of the two inequalities.

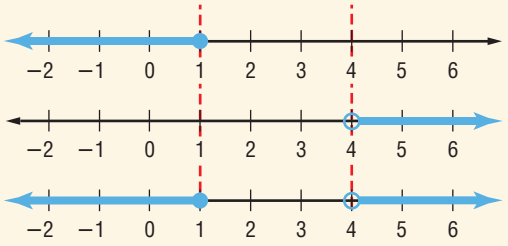
KEY CONCEPT "Or" Compound Inequalities

Words A compound inequality containing the word *or* is true if one or more of the inequalities is true.

Examples $x \leq 1$

$x > 4$

$x \leq 1$ or $x > 4$



Vocabulary Link
Union
Everyday Use something formed by combining parts or members
Math Use the set of elements belonging to one or more of a group of sets

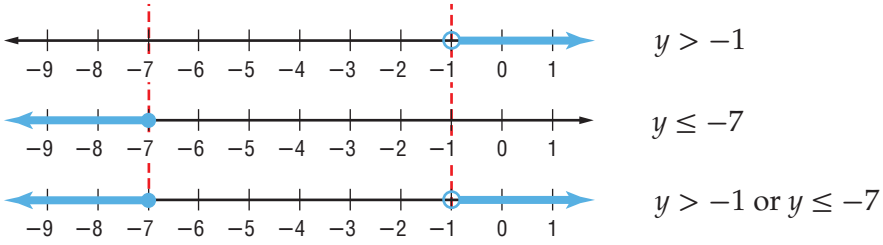
EXAMPLE 2 Solve an "or" Compound Inequality

Solve $y - 2 > -3$ or $y + 4 \leq -3$. Graph the solution set on a number line.

Solve each inequality separately.

$y - 2 > -3$ or $y + 4 \leq -3$

$y > -1$ $\qquad$ $y \leq -7$



The solution set is $\{y|y > -1 \text{ or } y \leq -7\}$.

CHECK Your Progress

2. Solve $y + 5 \leq 7$ or $y - 6 > 2$. Graph the solution set on a number line.

Reading Math

When solving problems involving inequalities,

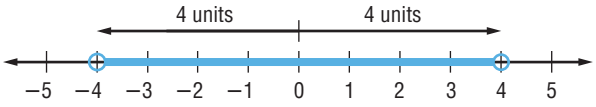
- *within* is meant to be inclusive. Use $\leq$ or $\geq$.
- *between* is meant to be exclusive. Use $<$ or $>$.

Absolute Value Inequalities In Lesson 1-4, you learned that the absolute value of a number is its distance from 0 on the number line. You can use this definition to solve inequalities involving absolute value.

EXAMPLE Solve an Absolute Value Inequality (<)

3 **Solve $|a| < 4$. Graph the solution set on a number line.**

$|a| < 4$ means that the distance between a and 0 on a number line is less than 4 units. To make $|a| < 4$ true, substitute numbers for a that are fewer than 4 units from 0.



Notice that the graph of $|a| < 4$ is the same as the graph of $a > -4$ and $a < 4$.

All of the numbers between -4 and 4 are less than 4 units from 0. The solution set is $\{a \mid -4 < a < 4\}$.

CHECK Your Progress

3. Solve $|x| \leq 3$. Graph the solution set on a number line.

Study Tip

Absolute Value Inequalities

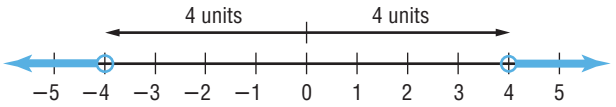
Because the absolute value of a number is never negative,

- the solution of an inequality like $|a| < -4$ is the empty set.
- the solution of an inequality like $|a| > -4$ is the set of all real numbers.

EXAMPLE Solve an Absolute Value Inequality (>)

4 **Solve $|a| > 4$. Graph the solution set on a number line.**

$|a| > 4$ means that the distance between a and 0 on a number line is greater than 4 units.



Notice that the graph of $|a| > 4$ is the same as the graph of $\{a > 4$ *or* $a < -4\}$.

The solution set is $\{a \mid a > 4 \text{ or } a < -4\}$.

CHECK Your Progress

4. Solve $|x| \geq 3$. Graph the solution set on a number line.

An absolute value inequality can be solved by rewriting it as a compound inequality.

KEY CONCEPT *Absolute Value Inequalities*

Symbols For all real numbers a and b, $b > 0$, the following statements are true.
1. If $|a| < b$, then $-b < a < b$.
2. If $|a| > b$, then $a > b$ or $a < -b$

Examples If $|2x + 1| < 5$, then $-5 < 2x + 1 < 5$
If $|2x + 1| > 5$, then $2x + 1 > 5$ or $2x + 1 < -5$.

These statements are also true for $\leq$ and $\geq$, respectively.

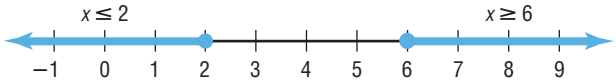
EXAMPLE Solve a Multi-Step Absolute Value Inequality

5 **Solve $|3x - 12| \geq 6$. Graph the solution set on a number line.**

$|3x - 12| \geq 6$ is equivalent to $3x - 12 \geq 6$ or $3x - 12 \leq -6$.
Solve the inequality.

$3x - 12 \geq 6$	or	$3x - 12 \leq -6$	Rewrite the inequality.
$3x \geq 18$		$3x \leq 6$	Add 12.
$x \geq 6$		$x \leq 2$	Divide by 3.

The solution set is $\{x \mid x \geq 6 \text{ or } x \leq 2\}$.



CHECK Your Progress

5. Solve $|3x + 4| < 10$. Graph the solution set on a number line.

Real-World Link

When executives in a recent survey were asked to name one quality that impressed them the most about a candidate during a job interview, 32 percent said honesty and integrity.

Source: careerexplorer.net

Real-World EXAMPLE Write an Absolute Value Inequality

6 **JOB HUNTING** **To prepare for a job interview, Megan researches the position's requirements and pay. She discovers that the average starting salary for the position is $38,500, but her actual starting salary could differ from the average by as much as $2450.**

a. Write an absolute value inequality to describe this situation.
Let x equal Megan's starting salary.

Her starting salary could differ from the average	by as much as	$2450.
$\lvert 38{,}500 - x \rvert$	$\leq$	2450

b. Solve the inequality to find the range of Megan's starting salary.
Rewrite the absolute value inequality as a compound inequality. Then solve for x.

$$\begin{aligned} -2450 \leq\ & 38{,}500 - x \leq 2450 \\ -2450 - 38{,}500 \leq\ & 38{,}500 - x - 38{,}500 \leq 2450 - 38{,}500 \\ -40{,}950 \leq\ & -x \leq -36{,}050 \\ 40{,}950 \geq\ & x \geq 36{,}050 \end{aligned}$$

The solution set is $\{x \mid 36{,}050 \leq x \leq 40{,}950\}$. Thus, Megan's starting salary will fall within $36,050 and $40,950.

CHECK Your Progress

6. The ideal pH value for water in a swimming pool is 7.5. However, the pH may differ from the ideal by as much as 0.3 before the water will cause discomfort to swimmers or damage to the pool. Write an absolute value inequality to describe this situation. Then solve the inequality to find the range of acceptable pH values for the water.

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CHECK Your Understanding

Examples 1–5 (pp. 41–44)

Solve each inequality. Graph the solution set on a number line.

1. $3 < d + 5 < 8$
2. $-4 \leq 3x - 1 < 14$
3. $y - 3 > 1$ or $y + 2 < 1$
4. $p + 6 < 8$ or $p - 3 > 1$
5. $|a| \geq 5$
6. $|w| \geq -2$
7. $|h| < 3$
8. $|b| < -2$
9. $|4k - 8| < 20$
10. $|g + 4| \leq 9$

Example 6 (p. 44)

11. **FLOORING** Deion is considering several types of flooring for his kitchen. He estimates that he will need between 55 and 60 12-inch by 12-inch tiles to retile the floor. The table below shows the price per tile for each type of tile Deion is considering.

Tile Type	Price per Tile
Vinyl	\$0.99
Slate	\$2.34
Porcelain	\$3.88
Marble	\$5.98

Write a compound inequality to determine how much he could be spending.

Exercises

HOMEWORK HELP	
For Exercises	See Examples
12, 13	1
14, 15	2
16, 17	3
18, 19	4
20, 21	5
22, 23	6

Solve each inequality. Graph the solution set on a number line.

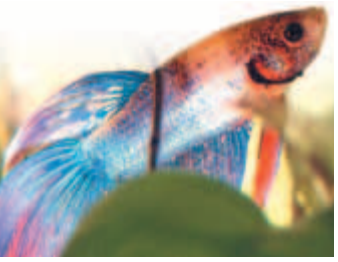
12. $9 < 3t + 6 < 15$
13. $-11 < -4x + 5 < 13$
14. $3p + 1 \leq 7$ or $2p - 9 \geq 7$
15. $2c - 1 < -5$ or $3c + 2 \geq 5$
16. $|g| \leq 9$
17. $|3k| < 0$
18. $|2m| \geq 8$
19. $|b - 4| > 6$
20. $|3w + 2| \leq 5$
21. $|6r - 3| < 21$

SPEED LIMITS **For Exercises 22 and 23, use the following information.**
On some interstate highways, the maximum speed a car may drive is 65 miles per hour. A tractor-trailer may not drive more than 55 miles per hour. The minimum speed for all vehicles is 45 miles per hour.

22. Write an inequality to represent the allowable speed for a car on an interstate highway.
23. Write an inequality to represent the speed at which a tractor-trailer may travel on an interstate highway.

Solve each inequality. Graph the solution set on a number line.

24. $-4 < 4f + 24 < 4$
25. $a + 2 > -2$ or $a - 8 < 1$
26. $|-5y| < 35$
27. $|7x| + 4 < 0$
28. $|n| \geq n$
29. $|n| \leq n$
30. $\frac{|2n - 7|}{3} \leq 0$
31. $\frac{|n - 3|}{2} < n$



Real-World Link

Adult Male Size: 3 inches

Water pH: 6.8–7.4

Temperature: 75–86°F

Diet: omnivore, prefers live foods

Tank Level: top dweller

Difficulty of Care: easy to intermediate

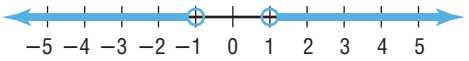
Life Span: 2–3 years

Source: www.about.com

32. FISH A Siamese Fighting Fish, better known as a Betta fish, is one of the most recognized and colorful fish kept as a pet. Using the information at the left, write a compound inequality to describe the acceptable range of water pH levels for a male Betta.

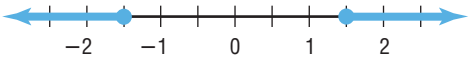
Write an absolute value inequality for each graph.

33.



34.

35.



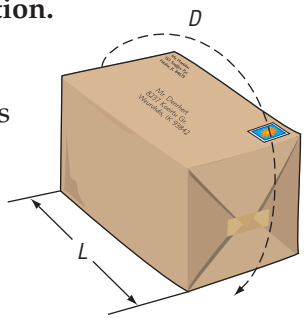
36.

37.

38.

39. HEALTH *Hypothermia* and *hyperthermia* are similar words but have opposite meanings. Hypothermia is defined as a lowered body temperature. Hyperthermia means an extremely high body temperature. Both conditions are potentially dangerous and occur when a person's body temperature fluctuates by more than 8° from the normal body temperature of 98.6°F. Write and solve an absolute value inequality to describe body temperatures that are considered potentially dangerous.

MAIL For Exercises 40 and 41, use the following information.
The U.S. Postal Service defines an oversized package as one for which the length L of its longest side plus the distance D around its thickest part is more than 108 inches and less than or equal to 130 inches.



40. Write a compound inequality to describe this situation.

41. If the distance around the thickest part of a package you want to mail is 24 inches, describe the range of lengths that would classify your package as oversized.

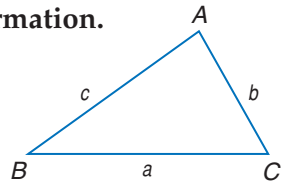
AUTO RACING For Exercises 42 and 43, use the following information.
The shape of a car used in NASCAR races is determined by NASCAR rules. The rules stipulate that a car must conform to a set of 32 templates, each shaped to fit a different contour of the car. The biggest template fits over the center of the car from front to back. When a template is placed on a car, the gap between it and the car cannot exceed the specified tolerance. Each template is marked on its edge with a colored line that indicates the tolerance for the template.

42. Suppose a certain template is 24.42 inches long. Use the information in the table at the right to write an absolute value inequality for templates with each line color.

43. Find the acceptable lengths for that part of a car if the template has each line color.

Line Color	Tolerance (in.)
Red	0.07
Blue	0.25
Green	0.5

GEOMETRY **For Exercises 44 and 45, use the following information.**
The *Triangle Inequality Theorem* states that the sum of the measures of any two sides of a triangle is greater than the measure of the third side.



44. Write three inequalities to express the relationships among the sides of $\triangle ABC$.

45. Write a compound inequality to describe the range of possible measures for side c in terms of a and b. Assume that $a > b > c$. (*Hint:* Solve each inequality you wrote in Exercise 44 for c.)

Graphing Calculator

LOGIC MENU **For Exercises 46–49, use the following information.**
You can use the operators in the LOGIC menu on the TI-83/84 Plus to graph compound and absolute value inequalities. To display the LOGIC menu, press 2nd [TEST] ▶.

46. Clear the Y= list. Enter $(5x + 2 > 12)$ and $(3x - 8 < 1)$ as Y1. With your calculator in DOT mode and using the standard viewing window, press GRAPH. Make a sketch of the graph displayed.

47. Using the TRACE function, investigate the graph. Based on your investigation, what inequality is graphed?

48. Write the expression you would enter for Y1 to find the solution set of the compound inequality $5x + 2 \geq 3$ or $5x + 2 \leq -3$. Then use the graphing calculator to find the solution set.

49. A graphing calculator can also be used to solve absolute value inequalities. Write the expression you would enter for Y1 to find the solution set of the inequality $|2x - 6| > 10$. Then use the graphing calculator to find the solution set. (*Hint:* The absolute value operator is item 1 on the MATH NUM menu.)

EXTRA PRACTICE
See pages 892, 926.
MathOnline
Self-Check Quiz at algebra2.com

H.O.T. Problems

50. **OPEN ENDED** Write a compound inequality for which the graph is the empty set.

51. **FIND THE ERROR** Sabrina and Isaac are solving $|3x + 7| > 2$. Who is correct? Explain your reasoning.

Sabrina

$|3y + 7| > 2$

$3y + 7 > 2$ or $3y + 7 < -2$

$3y > -5$ $\quad$ $3y < -9$

$y > -\frac{5}{3}$ $\quad$ $y < -3$

Isaac

$|3x + 7| > 2$

$-2 < 3x + 7 < 2$

$-9 < 3x < -5$

$-3 < x < -\frac{5}{3}$

52. **CHALLENGE** **Graph each set on a number line.**

a. $-2 < x < 4$

b. $x < -1$ or $x > 3$

c. $(-2 < x < 4)$ and $(x < -1$ or $x > 3)$ (*Hint:* This is the intersection of the graphs in part **a** and part **b**.)

d. Solve $3 < |x + 2| \leq 8$. Explain your reasoning and graph the solution set.

53. ***Writing in Math*** Use the information about fasting on page 41 to explain how compound inequalities are used in medicine. Include an explanation of an acceptable number of hours for this fasting state and a graph to support your answer.

STANDARDIZED TEST PRACTICE

54. ACT/SAT If $5 < a < 7 < b < 14$, then which of the following best describes $\frac{a}{b}$?

A $\frac{5}{7} < \frac{a}{b} < \frac{1}{2}$

B $\frac{5}{14} < \frac{a}{b} < \frac{1}{2}$

C $\frac{5}{7} < \frac{a}{b} < 1$

D $\frac{5}{14} < \frac{a}{b} < 1$

55. REVIEW What is the solution set of the inequality $-20 < 4x - 8 < 12$?

F $-7 < x < 1$

G $-3 < x < 5$

H $-7 < x < 5$

J $-3 < x < 1$

Spiral Review

Solve each inequality. Then graph the solution set on a number line. (Lesson 1-5)

56. $2d + 15 \geq 3$ **57.** $7x + 11 > 9x + 3$ **58.** $3n + 4\,(n + 3) < 5(n + 2)$

59. CONTESTS To get a chance to win a car, you must guess the number of keys in a jar to within 5 of the actual number. Those who are within this range are given a key to try in the ignition of the car. Suppose there are 587 keys in the jar. Write and solve an equation to determine the highest and lowest guesses that will give contestants a chance to win the car. (Lesson 1-4)

Solve each equation. Check your solutions. (Lesson 1-4)

60. $5\,|x - 3| = 65$ **61.** $|2x + 7| = 15$ **62.** $|8c + 7| = -4$

Name the property illustrated by each statement. (Lesson 1-3)

63. If $3x = 10$, then $3x + 7 = 10 + 7$.

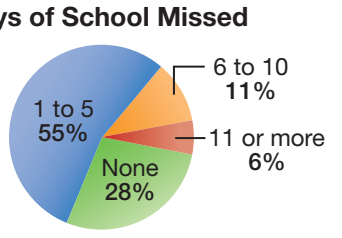
64. If $-5 = 4y - 8$, then $4y - 8 = -5$.

65. If $-2x - 5 = 9$ and $9 = 6x + 1$, then $-2x - 5 = 6x + 1$.

SCHOOL For Exercises 66 and 67, use the graph at the right.

66. Illustrate the Distributive Property by writing two expressions to represent the number of students at a high school who missed 5 or fewer days of school if the school enrollment is 743.

67. Evaluate the expressions from Exercise 66.



Source: Centers for Disease Control and Prevention

Simplify each expression. (Lesson 1-2)

68. $6a - 2b - 3a + 9b$ **69.** $-2(m - 4n) - 3(5n + 6)$

Find the value of each expression. (Lesson 1-1)

70. $6(5 - 8) \div 9 + 4$ **71.** $(3 + 7)^2 - 16 \div 2$ **72.** $\frac{7(1 - 4)}{8 - 5}$

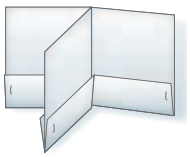
CHAPTER 1 Study Guide and Review

Download Vocabulary Review from algebra2.com

FOLDABLES Study Organizer

GET READY to Study

Be sure the following Key Concepts are noted in your Foldable.



Key Concepts

Expressions and Formulas (Lesson 1-1)

- Use the order of operations and the properties of equality to solve equations.

Properties of Real Numbers (Lesson 1-2)

- Real numbers can be classified as rational (Q) or irrational (I). Rational numbers can be classified as natural numbers (N), whole numbers (W), integers (Z), and/or quotients of these.

Solving Equations (Lesson 1-3 and 1-4)

- Verbal expressions can be translated into algebraic expressions.
- The absolute value of a number is the number of units it is from 0 on a number line.
- For any real numbers a and b, where $b \geq 0$, if $|a| = b$, then $a = b$ or $-a = b$.

Solving Inequalities (Lessons 1-5 and 1-6)

- Adding or subtracting the same number from each side of an inequality does not change the truth of the inequality.
- When you multiply or divide each side of an inequality by a negative number, the direction of the inequality symbol must be *reversed*.
- The graph of an *and* compound inequality is the intersection of the solution sets of the two inequalities. The graph of an *or* compound inequality is the union of the solution sets of the two inequalities.
- An *and* compound inequality can be expressed in two different ways. For example, $-2 \leq x \leq 3$ is equivalent to $x \geq -2$ and $x \leq 3$.
- For all real numbers a and b, where $b > 0$, the following statements are true.
  1. If $|a| < b$ then $-b < a < b$.
  2. If $|a| > b$ then $a > b$ or $a < -b$.

Key Vocabulary

absolute value (p. 27)
algebraic expression (p. 6)
coefficient (p. 7)
counterexample (p. 17)
empty set (p. 28)
equation (p. 18)
formula (p. 8)
intersection (p. 41)
irrational numbers (p. 11)
like terms (p. 7)
monomial (p. 6)
polynomial (p. 7)
rational numbers (p. 11)
real numbers (p. 11)
solution (p. 19)
trinomial (p. 7)
union (p. 42)

Vocabulary Check

Choose the term from the list above that best completes each statement.

1. The _____ contains no elements.
2. A polynomial with exactly three terms is called a _____.
3. The set of _____ includes terminating and repeating decimals but does not include π.
4. _____ can be combined by adding or subtracting their coefficients.
5. The _____ of a number is never negative.
6. The set of _____ contains the rational and the irrational numbers.
7. The _____ of the term $-6xy$ is -6.
8. A(n) _____ to an equation is a value that makes the equation true.
9. A(n) _____ is a statement that two expressions have the same value.
10. $\sqrt{2}$ belongs to the set of _____ but $\frac{1}{2}$ does not.

Lesson-by-Lesson Review

1–1 Expressions and Formulas (pp. 6–10)

Evaluate each expression.

11. $10 + 16 \div 4 + 8$

12. $[21 - (9 - 2)] \div 2$

13. $\frac{1}{2}(5^2 + 3)$

14. $\frac{14(8 - 15)}{2}$

Evaluate each expression if $a = 12$, $b = 0.5$, $c = -3$, and $d = \frac{1}{3}$.

15. $6b - 5c$

16. $c^3 + ad$

17. $\frac{9c + ab}{c}$

18. $a[b^2(b + a)]$

19. DISTANCE The formula to evaluate distance is $d = r \times t$, where d is distance, r is rate, and t is time. How far can Tosha drive in 4 hours if she is driving at 65 miles per hour?

Example 1 Evaluate $(10 - 2) \div 2^2$.

$(10 - 2) \div 2^2 = 8 \div 2^2$ First subtract 2 from 10.

$= 8 \div 4$ Then square 2.

$= 2$ Finally, divide 8 by 4.

Example 2 Evaluate $\frac{y^3}{3ab + 2}$ if $y = 4$, $a = -2$, and $b = -5$.

$\frac{y^3}{3ab + 2} = \frac{4^3}{3(-2)(-5) + 2}$ $y = 4$, $a = -2$, and $b = -5$

$= \frac{64}{3(10) + 2}$ Evaluate the numerator and denominator separately.

$= \frac{64}{32}$ or 2 Simplify.

1–2 Properties of Real Numbers (pp. 11–17)

Name the sets of numbers to which each value belongs.

20. $-\sqrt{9}$

21. $1.\overline{6}$

22. $\sqrt{18}$

Simplify each expression.

23. $2m + 7n - 6m - 5n$

24. $-5(a - 4b) + 4b$

25. $2(5x + 4y) - 3(x + 8y)$

CLOTHING **For Exercises 26 and 27, use the following information.**
A department store sells shirts for $12.50 each. Dalila buys 2, Latisha buys 3, and Pilar buys 1.

26. Illustrate the Distributive Property by writing two expressions to represent the cost of these shirts.

27. Use the Distributive Property to find how much money the store received from selling these shirts.

Example 3 Name the sets of numbers to which $\sqrt{25}$ belongs.

$\sqrt{25} = 5$ naturals (N), wholes (W), integers (Z), rationals (Q), and reals (R)

Example 4 Simplify $3(x + 2) + 4x - 3y$.

$3(x + 2) + 4x - 3y$

$= 3(x) + 3(2) + 4x - 3y$ Distributive Property

$= 3x + 6 + 4x - 3y$ Multiply.

$= 7x - 3y + 6$ Simplify.

Mixed Problem Solving
For mixed problem-solving practice, see page 926.

1–3 Solving Equations (pp. 18–26)

Solve each equation. Check your solution.

28. $x - 6 = -20$

29. $-\frac{2}{3}a = 14$

30. $7 + 5n = -58$

31. $3w + 14 = 7w + 2$

32. $\frac{n}{4} + \frac{n}{3} = \frac{1}{2}$

33. $5y + 4 = 2(y - 4)$

34. MONEY If Tabitha has 98 cents and you know she has 2 quarters, 1 dime, and 3 pennies, how many nickels does she have?

Solve each equation or formula for the specified variable.

35. $Ax + By = C$ for x

36. $\frac{a - 4b^2}{2c} = d$ for a

37. $A = p + prt$ for p

38. $d = b^2 - 4ac$ for c

39. GEOMETRY Alex wants to find the radius of the circular base of a cone. He knows the height of the cone is 8 inches and the volume of the cone is 18.84 cubic inches. Use the formula for volume of a cone, $V = \frac{1}{3}\pi r^2 h$, to find the radius.

Example 5 Solve $4(a + 5) - 2(a + 6) = 3$.

$4(a + 5) - 2(a + 6) = 3$	Original equation
$4a + 20 - 2a - 12 = 3$	Distributive Property
$4a - 2a + 20 - 12 = 3$	Commutative Property
$2a + 8 = 3$	Distributive and Substitution Properties
$2a = -5$	Subtraction Property
$a = -2.5$	Division Property

Example 6 Solve $A = \frac{h(a + b)}{2}$ for b.

$2A = h(a + b)$	Multiply each side by 2.
$\frac{2A}{h} = a + b$	Divide each side by h.
$\frac{2A}{h} - a = b$	Subtract a from each side.

1–4 Solving Absolute Value Equations (pp. 27–31)

Solve each equation. Check your solution.

40. $|x + 11| = 42$

41. $3|x + 6| = 36$

42. $|4x - 5| = -25$

43. $|x + 7| = 3x - 5$

44. $|y - 5| - 2 = 10$

45. $4|3x + 4| = 4x + 8$

46. BIKING Paloma's training goal is to ride four miles on her bicycle in 15 minutes. If her actual time is always within plus or minus 3 minutes of her preferred time, how long are her shortest and longest rides?

Example 7 Solve $|2x + 9| = 11$.

Case 1: $a = b$	**Case 2:** $a = -b$
$2x + 9 = 11$	$2x + 9 = -11$
$2x = 2$	$2x = -20$
$x = 1$	$x = -10$

The solutions are 1 and -10.

1–5 Solving Inequalities (pp. 33–39)

Solve each inequality. Describe the solution set using set builder notation. Then graph the solution set on a number line.

47. $-7w > 28$

48. $3x + 4 \geq 19$

49. $\frac{n}{12} + 5 \leq 7$

50. $3(6 - 5a) < 12a - 36$

51. $2 - 3z \geq 7(8 - 2z) + 12$

52. $8(2x - 1) > 11x - 17$

53. PIZZA A group has \$75 to order 6 large pizzas each with the same amount of toppings. Each pizza costs \$9 plus \$1.25 per topping. Write and solve an inequality to determine how many toppings the group can order on each pizza.

Example 8 **Solve $5 - 4a > 8$. Graph the solution set on a number line.**

$5 - 4a > 8$ Original inequality

$-4a > 3$ Subtract 5 from each side.

$a < -\frac{3}{4}$ Divide each side by -4, reversing the inequality symbol.

The solution set is $\left\{a \mid a < -\frac{3}{4}\right\}$.

The graph of the solution set is shown below.

1–6 Solving Compound and Absolute Value Inequalities (pp. 41–48)

Solve each inequality. Graph the solution set on a number line.

54. $4x + 3 < 11$ or $2x - 1 > 9$

55. $-1 < 3a + 2 < 14$

56. $-1 < 3(d - 2) \leq 9$

57. $5y - 4 > 16$ or $3y + 2 < 1$

58. $|x| + 1 > 12$

59. $|2y - 9| \leq 27$

60. $|5n - 8| > -4$

61. $|3b + 11| > 1$

62. FENCING Don is building a fence around a rectangular plot and wants the perimeter to be between 17 and 20 yards. The width of the plot is 5 yards. Write and solve a compound inequality to describe the range of possible measures for the length of the fence.

Example 9 **Solve each inequality. Graph the solution set on a number line.**

a. $-19 < 4d - 7 \leq 13$

$-19 < 4d - 7 \leq 13$ Original inequality

$-12 < 4d \leq 20$ Add 7 to each part.

$-3 < d \leq 5$ Divide each part by 4.

The solution set is $\{d \mid -3 < d \leq 5\}$.

b. $|2x + 4| \geq 12$

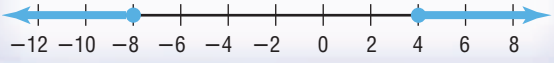
$|2x + 4| \geq 12$ is equivalent to $2x + 4 \geq 12$ or $2x + 4 \leq -12$.

$2x + 4 \geq 12$ or $2x + 4 \leq -12$

$2x \geq 8$ $\quad$ $2x \leq -16$ Subtract.

$x \geq 4$ $\quad$ $x \leq -8$ Divide.

The solution set is $\{x \mid x \geq 4 \text{ or } x \leq -8\}$.



CHAPTER 1

Practice Test

Find the value of each expression.

1. $[(3 + 6)^2 \div 3] \times 4$
2. $\dfrac{20 + 4 \times 3}{11 - 3}$
3. $0.5(2.3 + 25) \div 1.5$

Evaluate each expression if $a = -9$, $b = \frac{2}{3}$, $c = 8$, and $d = -6$.

4. $\dfrac{db + 4c}{a}$
5. $\dfrac{a}{b^2} + c$

Name the sets of numbers to which each number belongs.

6. $\sqrt{17}$
7. 0.86
8. $\sqrt{64}$

Name the property illustrated by each equation or statement.

9. $(7 \cdot s) \cdot t = 7 \cdot (s \cdot t)$
10. If $(r + s)t = rt + st$, then $rt + st = (r + s)t$.
11. $\left(3 \cdot \frac{1}{3}\right) \cdot 7 = \left(3 \cdot \frac{1}{3}\right) \cdot 7$
12. $(6 - 2)a - 3b = 4a - 3b$
13. $(4 + x) + y = y + (4 + x)$
14. If $5(3) + 7 = 15 + 7$ and $15 + 7 = 22$, then $5(3) + 7 = 22$.

Solve each equation. Check your solution(s).

15. $5t - 3 = -2t + 10$
16. $2x - 7 - (x - 5) = 0$
17. $5m - (5 + 4m) = (3 + m) - 8$
18. $|8w + 2| + 2 = 0$
19. $12\left|\frac{1}{2}y + 3\right| = 6$
20. $2|2y - 6| + 4 = 8$

Solve each inequality. Then graph the solution set on a number line.

21. $4 > b + 1$
22. $3q + 7 \geq 13$
23. $|5 + k| \leq 8$
24. $-12 < 7d - 5 \leq 9$

Solve each inequality. Then graph the solution set on a number line.

25. $|3y - 1| > 5$
26. $5(3x - 5) + x < 2(4x - 1) + 1$

For Exercises 27 and 28, define a variable, write an equation or inequality, and solve the problem.

27. **CAR RENTAL** Ms. Denney is renting a car that gets 35 miles per gallon. The rental charge is \$19.50 a day plus 18¢ per mile. Her company will reimburse her for \$33 of this portion of her travel expenses. Suppose Ms. Denney rents the car for 1 day. Find the maximum number of miles that will be paid for by her company.

28. **SCHOOL** To receive a B in his English class, Nick must have an average score of at least 80 on five tests. What must he score on the last test to receive a B in the class?

Test	Score
1	87
2	89
3	76
4	77

29. **MULTIPLE CHOICE** If $\frac{a}{b} = 8$ and $ac - 5 = 11$, then $bc =$

A 93

B 2

C $\frac{5}{8}$

D cannot be determined

30. **MULTIPLE CHOICE** At a veterinarian's office, 2 cats and 4 dogs are seen in a random order. What is the probability that the 2 cats are seen in a row?

F $\frac{1}{3}$

G $\frac{2}{3}$

H $\frac{1}{2}$

J $\frac{3}{5}$

CHAPTER 1

Standardized Test Practice

Chapter 1

Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.

1. Lucas determined that the total cost C to rent a car for the weekend could be represented by the equation $C = 0.35m + 125$, where m is the number of miles that he drives. If the total cost to rent the car was \$363, how many miles did he drive?

A 125

B 238

C 520

D 680

TEST-TAKING TIP

Question 1 On multiple choice questions, try to compute the answer first. Then compare your answer to the given answer choices. If you don't find your answer among the choices, check your calculations.

2. Leo sells T-shirts at a local swim meet. It costs him \$250 to set up the stand and rent the machine. It costs him an additional \$5 to make each T-shirt. If he sells each T-shirt for \$15, how many T-shirts does he have to sell before he can make a profit?

F 10

G 15

H 25

J 50

3. GRIDDABLE Malea sells engraved necklaces over the Internet. She purchases 50 necklaces for \$400, and it costs her an additional \$3 for each personalized engraving. If she charges \$20 each, how many necklaces will she need to sell in order to make a profit of at least \$225?

4. If the surface area of a cube is increased by a factor of 9, what is the change in the length of the sides of the cube?

A The length is 2 times the original length.

B The length is 3 times the original length.

C The length is 6 times the original length.

D The length is 9 times the original length.

5. The profit p that Selena's Shirt store makes in a day can be represented by the inequality $10t + 200 < p < 15t + 250$, where t represents the number of shirts sold. If the store sold 45 shirts on Friday, which of the following is a reasonable amount that the store made?

F \$200.00

G \$625.00

H \$850.00

J \$950.00

6. Solve the equation $4x - 5 = 2x + 5 - 3x$ for x.

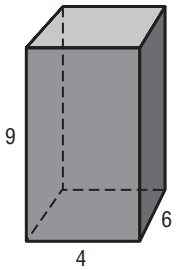
A -2

B -1

C 1

D 2

7. Which set of dimensions corresponds to a rectangular prism that is similar to the one shown below?



F 12 units by 18 units by 27 units

G 12 units by 18 units by 18 units

H 8 units by 12 units by 9 units

J 8 units by 10 units by 18 units

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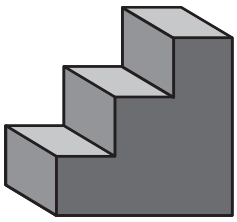
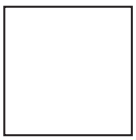
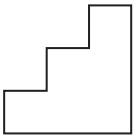
8. Which of the following best represents the side view of the solid shown below?



A

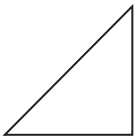


C



B

D



9. Given: Two angles are complementary. The measure of one angle is 10 less than the measure of the other angle.
Conclusion: The measures of the angles are 85 degrees and 95 degrees.
This conclusion:

F is contradicted by the first statement given.

G is verified by the first statement given.

H invalidates itself because there is no angle complementary to an 85 degree angle

J verifies itself because one angle is 10 degrees less than the other

10. A rectangle has a width of 8 inches and a perimeter of 30 inches. What is the perimeter, in inches, of a similar rectangle with a width of 12 inches?

A 40

B 45

C 48

D 360

11. Marvin and his younger brother like to bike together. Marvin rides his bike at a speed of 21 miles per hour and can ride his training loop 10 times in the time that it takes his younger brother to complete the training loop 8 times. Which is a reasonable estimate for Marvin's younger brother's speed?

F between 14 mph and 15 mph

G between 15 mph and 16 mph

H between 16 mph and 17 mph

J between 17 mph and 18 mph

Pre-AP

Record your answers on a sheet of paper Show your work.

12. Amanda's hours at her summer job for one week are listed in the table below. She earns $6 per hour.

Amanda's Work Hours	
Sunday	0
Monday	6
Tuesday	4
Wednesday	0
Thursday	2
Friday	6
Saturday	8

a. Write an expression for Amanda's total weekly earnings.

b. Evaluate the expression from Part **a** by using the Distributive Property.

c. Michael works with Amanda and also earns $6 per hour. If Michael's earnings were $192 this week, write and solve an equation to find how many more hours Michael worked than Amanda.

NEED EXTRA HELP?												
If You Missed Question...	1	2	3	4	5	6	7	8	9	10	11	12
Go to Lesson...	1-3	1-3	1-5	1-2	1-5	1-3	1-1	1-1	1-3	1-1	1-3	1-3