

# Get Ready for Chapter 11

**Diagnose Readiness** You have two options for checking Prerequisite Skills.

## Text Option

Take the Quick Check below. Refer to the Quick Review for help.

### QuickCheck

Solve each proportion. (Lesson 2-6)

- $\frac{y}{3} = \frac{8}{9}$
- $\frac{5}{12} = \frac{x}{36}$
- $\frac{7}{2} = \frac{y}{3}$
- $\frac{5}{x} = \frac{10}{4}$

5. **DRAWING** Rosie is making a scale drawing. She is using the scale 1 inch = 3 feet. How many inches will represent 10 feet?

Find the GCF of each pair of monomials. (Lesson 8-1)

- $12ab, 18b$
  - $15cd^2, 25c^2d$
  - $60r^2, 45r^3$
  - $12xy, 16x^2y$
10. **GAMES** Fifty girls and 75 boys attend a sports club. For a game, boys and girls are going to split into groups. The number in each group has to be the same. How large can the groups be?

Factor each polynomial. (Lessons 8-2 and 8-4)

- $2x^2 - 4x$
  - $6x^2 - 5x - 4$
  - $6xy + 15x$
  - $2c^2d - 4c^2d^2$
15. **AREA** The area of a rectangle is  $x^2 + 5x + 6$ . What binomial expressions represent the side lengths of the rectangle?

$$A = x^2 + 5x + 6$$

### QuickReview

#### EXAMPLE 1

Solve  $\frac{3}{5} = \frac{x}{12}$ .

$$\frac{3}{5} = \frac{x}{12}$$

**Original equation**

$$3 \cdot 12 = 5 \cdot x$$

**Cross products**

$$36 = 5x$$

**Simplify.**

$$\frac{36}{5} = \frac{5x}{5}$$

**Divide each side by 5.**

$$\frac{36}{5} = x$$

**Simplify.**

#### EXAMPLE 2

Find the greatest common factor of 30 and 42.

$$2 \cdot 3 \cdot 5$$

**Prime factorization of 30**

$$2 \cdot 3 \cdot 7$$

**Prime factorization of 42**

$$2 \cdot 3 = 6$$

**Product of the common factors**

The greatest common factor of 30 and 42 is 6.

#### EXAMPLE 3

Factor  $x^2 + 4x - 45$ .

In this trinomial,  $b = 4$  and  $c = -45$ . Find factors of  $-45$  with a sum of 4. The correct factors are  $-5$  and 9.

$$x^2 + 4x - 45$$

**Original expression**

$$= (x + m)(x + p)$$

**Write the pattern.**

$$= (x - 5)(x + 9)$$

**$m = -5$  and  $p = 9$**

## Online Option

### Math Online

Take a self-check Chapter Readiness Quiz at [glencoe.com](http://glencoe.com).

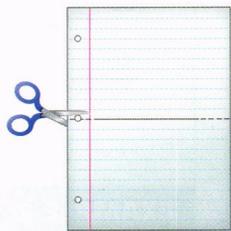
# Get Started on Chapter 11

You will learn several new concepts, skills, and vocabulary terms as you study Chapter 11. To get ready, identify important terms and organize your resources. You may wish to refer to **Chapter 0** to review prerequisite skills.

## FOLDABLES® Study Organizer

**Rational Functions and Equations** Make this Foldable to help you organize your Chapter 11 notes about rational functions and equations. Begin with 3 sheets of notebook paper.

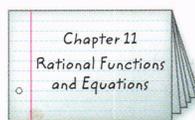
- 1 **Take** one sheet of paper and fold in half along the width. Cut 1 inch slits on each side of the paper.



- 2 **Stack** the two sheets of paper and fold in half along the width. Cut a slit through the center stopping 1 inch from each side.



- 3 **Insert** the first sheet through the second sheets and align the folds to form a booklet. Label the cover with the chapter title.



## Math Online [glencoe.com](http://glencoe.com)

- Study the chapter online
- Explore **Math in Motion**
- Get extra help from your own **Personal Tutor**
- Use **Extra Examples** for additional help
- Take a **Self-Check Quiz**
- Review Vocabulary** in fun ways

## New Vocabulary

English	Español
inverse variation • p. 670	variación inversa
product rule • p. 671	regla del producto
excluded value • p. 678	valores excluidos
rational function • p. 678	función racional
asymptote • p. 679	asíntota
rational expression • p. 684	expresión racional
least common multiple (LCM) • p. 707	mínimo común múltiplo (mcm)
least common denominator (LCD) • p. 708	mínimo común denominador (mcd)
complex fraction • p. 714	fracción compleja
mixed expression • p. 714	expresión mixta
rational equation • p. 720	ecuación racional
extraneous solutions • p. 721	soluciones extrañas
work problems • p. 722	problemas de trabajo
rate problems • p. 723	problemas de tasas

## Review Vocabulary

**direct variation** • p. 180 • **variación directa** an equation of the form  $y = kx$ , where  $k \neq 0$

**Quotient of Powers** • p. 408 • **cociente de potencia**

$$\frac{a^m}{a^n} = a^{m-n}$$

$$\frac{x^5}{x^3} = \frac{x \cdot x \cdot x \cdot x \cdot x}{x \cdot x \cdot x} = x \cdot x \text{ or } x^2$$

$$\frac{x^5}{x^3} = x^{5-3} \text{ or } x^2$$

**Zero Product Property** • p. 478 • **propiedad del producto de cero** if the product of two factors is 0, then at least one of the factors must be 0

**Multilingual eGlossary** [glencoe.com](http://glencoe.com)

You can use a data collection device to investigate the relationship between volume and pressure.

### Set Up The Lab

- Connect a syringe to the gas pressure sensor. Then connect the data collection device to both the sensor and the calculator as shown.
- Start the collection program and select the sensor.



### ACTIVITY

#### Collect Data

- Step 1** Open the valve between the atmosphere and the syringe. Set the inside ring of the syringe to 20 mL and close the valve. This ensures that the amount of air inside the syringe will be constant throughout the experiment.
- Step 2** Press the plunger of the syringe to the 5 mL mark. Wait for the pressure gauge to stop changing, then take the data reading. Enter 5 as the volume in the calculator. The pressure is measured in atmospheres (atm).
- Step 3** Repeat step 2, pressing the plunger to 7.5 mL, 10.0 mL, 12.5 mL, 15.0 mL, 17.5 mL, and 20.0 mL. Record the volume from each data reading.
- Step 4** After taking the last data reading, use **STAT PLOT** to create a line graph.

### Exercises

1. Does the pressure vary directly as the volume? Explain.
2. As the volume changes from 10 to 20 mL, what happens to the pressure?
3. Predict what the pressure of the gas in the syringe would be if the volume increased to 40 mL.
4. Add a column to the data table to find the product of the volume and the pressure for each data reading. What pattern do you observe?
5. **MAKE A CONJECTURE** The relationship between the pressure and volume of a gas is called Boyle's Law. Write an equation relating the volume  $v$  in milliliters and pressure  $p$  in atmospheres in your experiment. Compare your conjecture to those of two classmates. Formulate mathematical questions about their conjectures.

## Inverse Variation

### Why?

The time it takes a runner to finish a race is inversely proportional to the average pace of the runner.



**Identify and Use Inverse Variations** In the situation above, the runner's time decreases as the pace of the runner increases. So, these quantities are *inversely proportional*. An **inverse variation** can be represented by the equation  $y = \frac{k}{x}$  or  $xy = k$ .

### Then

You solved problems involving direct variation. (Lesson 3-4)

### Now

- Identify and use inverse variations.
- Graph inverse variations.

### New Vocabulary

inverse variation  
product rule

### Math Online

glencoe.com

- Extra Examples
- Personal Tutor
- Self-Check Quiz
- Homework Help

### Key Concept

### Inverse Variation

For Your

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$y$  varies inversely as  $x$  if there is some nonzero constant  $k$  such that

$$y = \frac{k}{x} \text{ or } xy = k, \text{ where } x \neq 0 \text{ and } y \neq 0.$$

In an inverse variation, the product of two values remains constant. Recall that a relationship of the form  $y = kx$  is a *direct variation*. For either a direct or indirect variations, the constant  $k$  is called the *constant of variation* or the *constant of proportionality*.

### EXAMPLE 1 Identify Inverse and Direct Variations

Determine whether each table or equation represents an *inverse* or a *direct variation*. Explain.

a.

$x$	$y$
1	16
2	8
4	4

In an inverse variation,  $xy$  equals a constant  $k$ . Find  $xy$  for each ordered pair in the table.

$$\begin{aligned} 1 \cdot 16 &= 16 \\ 2 \cdot 8 &= 16 \\ 4 \cdot 4 &= 16 \end{aligned}$$

The product is constant, so the table represents an inverse variation.

c.  $x = 2y$   
The equation can be written as  $y = \frac{1}{2}x$ . Therefore, it represents a direct variation.

b.

$x$	$y$
1	3
2	6
3	9

Notice that  $xy$  is not constant. So, the table does not represent an indirect variation.

$$\begin{aligned} 3 &= k(1) & 6 &= k(2) & 9 &= k(3) \\ 3 &= k & 3 &= k & 3 &= k \end{aligned}$$

The table of values represents the direct variation  $y = 3x$ .

d.  $2xy = 10$   
 $2xy = 10$   
 $xy = 5$

**Write the equation.**  
**Divide each side by 2.**

The equation represents an inverse variation.

### Check Your Progress

1A.

$x$	1	2	5
$y$	10	5	2

1B.  $-2x = y$

## ReadingMath

### Variation

**Equations** For direct variation equations, you say that  $y$  varies directly as  $x$ . For inverse variation equations, you say that  $y$  varies inversely as  $x$ .

You can use  $xy = k$  to write an inverse variation equation that relates  $x$  and  $y$ .

### EXAMPLE 2 Write an Inverse Variation

Assume that  $y$  varies inversely as  $x$ . If  $y = 18$  when  $x = 2$ , write an inverse variation equation that relates  $x$  and  $y$ .

$$\begin{array}{ll} xy = k & \text{Inverse variation equation} \\ 2(18) = k & x = 2 \text{ and } y = 18 \\ 36 = k & \text{Simplify.} \end{array}$$

The constant of variation is 36. So, an equation that relates  $x$  and  $y$  is  $xy = 36$  or  $y = \frac{36}{x}$ .

### Check Your Progress

2. Assume that  $y$  varies inversely as  $x$ . If  $y = 5$  when  $x = -4$ , write an inverse variation equation that relates  $x$  and  $y$ .

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If  $(x_1, y_1)$  and  $(x_2, y_2)$  are solutions of an inverse variation, then  $x_1y_1 = k$  and  $x_2y_2 = k$ .

$$\begin{array}{ll} x_1y_1 = k \text{ and } x_2y_2 = k & \\ x_1y_1 = x_2y_2 & \text{Substitute } x_2y_2 \text{ for } k. \end{array}$$

The equation  $x_1y_1 = x_2y_2$  is called the **product rule** for inverse variations.



### Key Concept

### Product Rule for Inverse Variations

For Your

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**Words** If  $(x_1, y_1)$  and  $(x_2, y_2)$  are solutions of an inverse variation, then the products  $x_1y_1$  and  $x_2y_2$  are equal.

**Symbols**  $x_1y_1 = x_2y_2$  or  $\frac{x_1}{x_2} = \frac{y_2}{y_1}$

### EXAMPLE 3 Solve for $x$ or $y$

Assume that  $y$  varies inversely as  $x$ . If  $y = 3$  when  $x = 12$ , find  $x$  when  $y = 4$ .

$$\begin{array}{ll} x_1y_1 = x_2y_2 & \text{Product rule for inverse variations} \\ 12 \cdot 3 = x_2 \cdot 4 & x_1 = 12, y_1 = 3, \text{ and } y_2 = 4 \\ 36 = x_2 \cdot 4 & \text{Simplify.} \\ \frac{36}{4} = x_2 & \text{Divide each side by 4.} \\ 9 = x_2 & \text{Simplify.} \end{array}$$

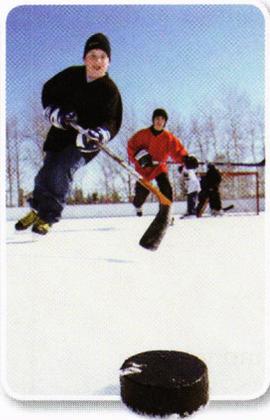
So, when  $y = 4$ ,  $x = 9$ .

### Check Your Progress

3. If  $y$  varies inversely as  $x$  and  $y = 4$  when  $x = -8$ , find  $y$  when  $x = -4$ .

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The product rule for inverse variations can be used to write an equation to solve real-world problems.



### Real-World EXAMPLE 4 Use Inverse Variations

**PHYSICS** The acceleration  $a$  of a hockey puck is inversely proportional to its mass  $m$ . Suppose a hockey puck with a mass of 164 grams is hit so that it accelerates  $122 \text{ m/s}^2$ . Find the acceleration of a 158-gram hockey puck if the same amount of force is applied.

Make a table to organize the information.

Let  $m_1 = 164$ ,  $a_1 = 122$ , and  $m_2 = 158$ . Solve for  $a_2$ .

$$m_1 a_1 = m_2 a_2 \quad \text{Use the product rule to write an equation.}$$

$$164 \cdot 122 = 158 a_2 \quad m_1 = 164, a_1 = 122, \text{ and } m_2 = 158$$

$$20,008 = 158 a_2 \quad \text{Simplify.}$$

$$126.6 \approx a_2 \quad \text{Divide each side by 158 and simplify.}$$

Puck	Mass	Acceleration
1	164 g	$122 \text{ m/s}^2$
2	158 g	$a_2$

The 158-gram puck has an acceleration of approximately  $126.6 \text{ m/s}^2$ .

#### Real-World Link

A standard hockey puck is 1 inch thick and 3 inches in diameter. Its mass is between approximately 156 and 170 grams.

Source: NHL Rulebook

#### Check Your Progress

4. **RACING** Manuel runs an average of 8 miles per hour and finishes a race in 0.39 hour. Dyani finished the race in 0.35 hour. What was her average pace?

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**Graph Inverse Variations** The graph of an inverse variation is not a straight line like the graph of a direct variation.

### EXAMPLE 5 Graph an Inverse Variation

Graph an inverse variation equation in which  $y = 8$  when  $x = 3$ .

- Step 1** Write an inverse variation equation.

$$xy = k \quad \text{Inverse variation equation}$$

$$3(8) = k \quad x = 3, y = 8$$

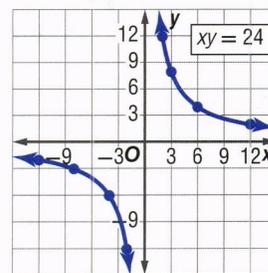
$$24 = k \quad \text{Simplify.}$$

The inverse variation equation is  $xy = 24$  or  $y = \frac{24}{x}$ .

- Step 2** Choose values for  $x$  and  $y$  that have a product of 24.

- Step 3** Plot each point and draw a smooth curve that connects the points.

$x$	$y$
-12	-2
-8	-3
-4	-6
-2	-12
0	undefined
2	12
3	8
6	4
12	2



Notice that since  $y$  is undefined when  $x = 0$ , there is no point on the graph when  $x = 0$ . This graph is called a hyperbola.

#### Check Your Progress

5. Graph an inverse variation equation in which  $y = 16$  when  $x = 4$ .

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#### Problem-Solving Tip

##### Solve a Simpler Problem

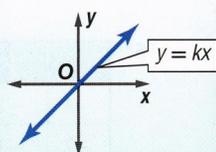
Sometimes it is necessary to break a problem into parts, solve each part, and then combine them to find the solution to the problem.

## Concept Summary

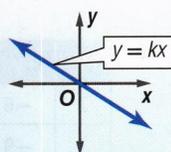
## Direct and Inverse Variations

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### Direct Variation



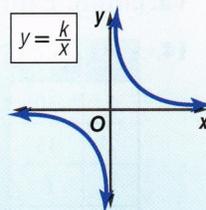
$$k > 0$$



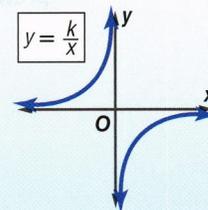
$$k < 0$$

- $y = kx$
- $y$  varies directly as  $x$ .
- The ratio  $\frac{y}{x}$  is a constant.

### Inverse Variation



$$k > 0$$



$$k < 0$$

- $y = \frac{k}{x}$
- $y$  varies inversely as  $x$ .
- The product  $xy$  is a constant.

## Check Your Understanding

### Example 1

p. 670

Determine whether each table or equation represents an *inverse* or a *direct* variation. Explain.

1. 

$x$	1	4	8	12
$y$	2	8	16	24

2. 

$x$	1	2	3	4
$y$	24	12	8	6

3.  $xy = 4$

4.  $y = \frac{x}{10}$

### Examples 2 and 5

pp. 671–672

Assume that  $y$  varies inversely as  $x$ . Write an inverse variation equation that relates  $x$  and  $y$ . Then graph the equation.

5.  $y = 8$  when  $x = 6$

6.  $y = 2$  when  $x = 5$

7.  $y = 3$  when  $x = -10$

8.  $y = -1$  when  $x = -12$

### Example 3

p. 671

Solve. Assume that  $y$  varies inversely as  $x$ .

9. If  $y = 8$  when  $x = 4$ , find  $x$  when  $y = 2$ .
10. If  $y = 7$  when  $x = 6$ , find  $y$  when  $x = -21$ .
11. If  $y = -5$  when  $x = 9$ , find  $y$  when  $x = 6$ .

### Example 4

p. 672

12. **RACING** The time it takes to complete a go-cart race course is inversely proportional to the average speed of the go-cart. One rider has an average speed of 73.3 feet per second and completes the course in 30 seconds. Another rider completes the course in 25 seconds. What was the average speed of the second rider?

13. **OPTOMETRY** When a person does not have clear vision, an optometrist can prescribe lenses to correct the condition. The power  $P$  of a lens, in a unit called diopters, is equal to 1 divided by the focal length  $f$ , in meters, of the lens.
- Graph the inverse variation  $P = \frac{1}{f}$ .
  - Find the powers of lenses with focal lengths  $+0.2$  to  $-0.4$  meters.

# Practice and Problem Solving

= **Step-by-Step Solutions** begin on page R12.  
**Extra Practice** begins on page 815.

**Example 1**  
p. 670

Determine whether each table or equation represents an *inverse* or a *direct* variation. Explain.

14. 

x	y
1	30
2	15
5	6
6	5

15. 

x	y
2	-6
3	-9
4	-12
5	-15

16. 

x	y
-4	-2
-2	-1
2	1
4	2

17. 

x	y
-5	8
-2	20
4	-10
8	-5

18.  $5x - y = 0$

19.  $xy = \frac{1}{4}$

20.  $x = 14y$

21.  $\frac{y}{x} = 9$

**Examples 2 and 5**  
pp. 671-672

Assume that  $y$  varies inversely as  $x$ . Write an inverse variation equation that relates  $x$  and  $y$ . Then graph the equation.

22.  $y = 2$  when  $x = 20$

23.  $y = 18$  when  $x = 4$

24.  $y = -6$  when  $x = -3$

25.  $y = -4$  when  $x = -3$

26.  $y = -4$  when  $x = 16$

27.  $y = 12$  when  $x = -9$

**Example 3**  
p. 671

Solve. Assume that  $y$  varies inversely as  $x$ .

28. If  $y = 12$  when  $x = 3$ , find  $x$  when  $y = 6$ .

29. If  $y = 5$  when  $x = 6$ , find  $x$  when  $y = 2$ .

30. If  $y = 4$  when  $x = 14$ , find  $x$  when  $y = -5$ .

31. If  $y = 9$  when  $x = 9$ , find  $y$  when  $x = -27$ .

32. If  $y = 15$  when  $x = -2$ , find  $y$  when  $x = 3$ .

33. If  $y = -8$  when  $x = -12$ , find  $y$  when  $x = 10$ .

**Example 4**  
p. 672

34. **EARTH SCIENCE** The water level in a river varies inversely with air temperature. When the air temperature was  $90^\circ$  Fahrenheit, the water level was 11 feet. If the air temperature was  $110^\circ$  Fahrenheit, what was the level of water in the river?

35. **MUSIC** When under equal tension, the frequency of a vibrating string in a piano varies inversely with the string length. If a string that is 420 millimeters in length vibrates at a frequency of 523 cycles a second, at what frequency will a 707-millimeter string vibrate?

Determine whether each situation is an example of an *inverse* or a *direct* variation. Justify your reasoning.

36. The drama club can afford to purchase 10 wigs at \$2 each or 5 wigs at \$4 each.

37. The Spring family buys several lemonades for \$1.50 each.

38. Nicole earns \$14 for babysitting 2 hours, and \$21 for babysitting 3 hours.

39. Thirty video game tokens are divided evenly among a group of friends.

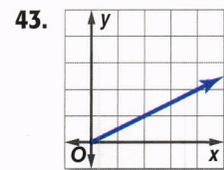
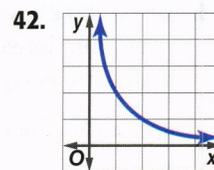
Determine whether each table or graph represents an *inverse* or a *direct* variation. Explain.

40. 

x	y
5	1
8	1.6
11	2.2

41. 

x	y
-3	-7
-2	-10.5
4	5.25



**Real-World Link**

A medium-sized piano has about 230 strings with a combined tension of 15 to 20 tons. A concert grand piano may have a combined string tension of up to 30 tons.

Source: Piano World

44. **PHYSICAL SCIENCE** When two people are balanced on a seesaw, their distances from the center of the seesaw are inversely proportional to their weights. If a 118-pound person sits 1.8 meters from the center of the seesaw, how far should a 125-pound person sit from the center to balance the seesaw?

**Solve.** Assume that  $y$  varies inversely as  $x$ .

45. If  $y = 9.2$  when  $x = 6$ , find  $x$  when  $y = 3$ .  
 46. If  $y = 3.8$  when  $x = 1.5$ , find  $x$  when  $y = 0.3$ .  
 47. If  $y = \frac{1}{5}$  when  $x = -20$ , find  $y$  when  $x = -\frac{8}{5}$ .  
 48. If  $y = -6.3$  when  $x = \frac{2}{3}$ , find  $y$  when  $x = 8$ .

49. **SWIMMING** Logan and Brianna each bought a pool membership. Their average cost per day is inversely proportional to the number of days that they go to the pool. Logan went to the pool 25 days for an average cost per day of \$5.60. Brianna went to the pool 35 days. What was her average cost per day?
50. **PHYSICAL SCIENCE** The amount of force required to do a certain amount of work in moving an object is inversely proportional to the distance that the object is moved. Suppose 90 N of force is required to move an object 10 feet. Find the force needed to move another object 15 feet if the same amount of work is done.
51. **DRIVING** Lina must practice driving 40 hours with a parent or guardian before she is allowed to take the test to get her driver's license. She plans to practice the same number of hours each week.

- a. Let  $h$  represent the number of hours per week that she practices driving. Make a table showing the number of weeks  $w$  that she will need to practice for the following values of  $h$ : 1, 2, 4, 5, 8, and 10.  
 b. Describe how the number of weeks changes as the number of hours per week increases.  
 c. Write and graph an equation that shows the relationship between  $h$  and  $w$ .



### Real-World Link

In 2007, Illinois began the Operation Teen Safe Driving program. The program challenges high schools to compete against each other to develop the most comprehensive and creative safe driving community.

Source: Ford Motor Company

### H.O.T. Problems

Use **H**igher-**O**rder **T**hinking Skills

52. **FIND THE ERROR** Christian and Trevor found an equation such that  $x$  and  $y$  vary inversely, and  $y = 10$  when  $x = 5$ . Is either of them correct? Explain.

*Christian*

$$k = \frac{y}{x}$$

$$= \frac{10}{2} \text{ or } 5$$

$$y = 5x$$

*Trevor*

$$k = xy$$

$$= (5)(10) \text{ or } 50$$

$$y = \frac{50}{x}$$

53. **CHALLENGE** Suppose  $f$  varies inversely with  $g$ , and  $g$  varies inversely with  $h$ . What is the relationship between  $f$  and  $h$ ?
54. **REASONING** Does  $xy = -k$  represent an inverse variation when  $k \neq 0$ ? Explain.
55. **OPEN ENDED** Give a real-world situation or phenomena that can be modeled by an inverse variation equation. Use the correct terminology to describe your example and explain why this situation is an inverse variation.
56. **WRITING IN MATH** Compare and contrast direct and inverse variation. Include a description of the relationship between slope and the graphs of a direct and inverse variation.

# 11-2

## Rational Functions

### Why?

Trina is reading a 300-page book. The average number of pages she reads each day  $y$  is given by  $y = \frac{300}{x}$ , where  $x$  is the number of days that she reads.

**Identify Excluded Values** The function  $y = \frac{300}{x}$  is an example of a **rational function**. This function is *nonlinear*.



### Then

You wrote inverse variation equations. (Lesson 11-1)

### Now

- Identify excluded values.
- Identify and use asymptotes to graph rational functions.

### New Vocabulary

rational function  
excluded value  
asymptote

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### Key Concept

### Rational Functions

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**Words** A rational function can be described by an equation of the form  $y = \frac{p}{q}$ , where  $p$  and  $q$  are polynomials and  $q \neq 0$ .

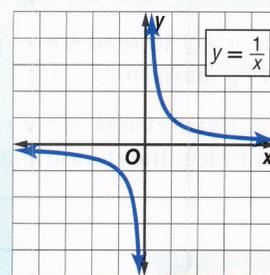
Parent function:  $f(x) = \frac{1}{x}$

Type of graph: hyperbola

Domain:  $\{x \mid x \neq 0\}$

Range:  $\{y \mid y \neq 0\}$

### Graph



Since division by zero is undefined, any value of a variable that results in a denominator of zero in a rational function is excluded from the domain of the function. These are called **excluded values** for the rational function.

### EXAMPLE 1 Find Excluded Values

State the excluded value for each function.

a.  $y = -\frac{2}{x}$

The denominator cannot equal 0. So, the excluded value is  $x = 0$ .

b.  $y = \frac{2}{x+1}$

$$x + 1 = 0$$

$$x = -1$$

Set the denominator equal to 0.

Subtract 1 from each side.

The excluded value is  $x = -1$ .

c.  $y = \frac{5}{4x-8}$

$$4x - 8 = 0$$

$$4x = 8$$

$$x = 2$$

Set the denominator equal to 0.

Add 8 to each side.

Divide each side by 4.

The excluded value is  $x = 2$ .



### Check Your Progress

1A.  $y = \frac{5}{2x}$

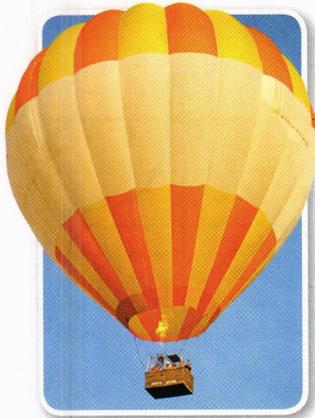
1B.  $y = \frac{x}{x-7}$

1C.  $y = \frac{4}{3x+9}$



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Depending on the real-world situation, in addition to excluding  $x$ -values that make a denominator zero from the domain of a rational function, additional values might have to be excluded from the domain as well.



### Real-World Link

As the temperature of the gas inside a hot air balloon increases, the density of the gas decreases. A hot air balloon rises because the density of the air inside it is less than the density of the air outside.

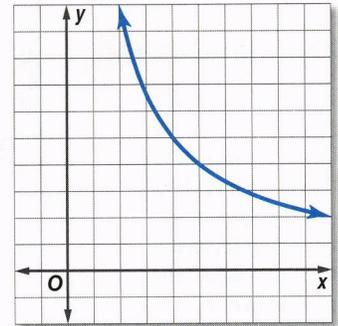
Source: Goddard Space Flight Center

### Real-World EXAMPLE 2 Graph Real-Life Rational Functions

**BALLOONS** If there are  $x$  people in the basket of a hot air balloon, the function  $y = \frac{20}{x}$  represents the average number of square feet  $y$  per person. Graph this function.

Since the number of people cannot be zero, it is reasonable to exclude negative values and only use positive values for  $x$ .

Number of People $x$	2	4	5	10
Square Feet per Person $y$	10	5	4	2



### Check Your Progress

2. **GEOMETRY** A rectangle has an area of 18 square inches. The function  $\ell = \frac{18}{w}$  shows the relationship between the length and width. Graph the function.

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**Identify and Use Asymptotes** In Example 2, an excluded value is  $x = 0$ . Notice that the graph approaches the vertical line  $x = 0$ , but never touches it.

The graph also approaches but never touches the horizontal line  $y = 0$ . The lines  $x = 0$  and  $y = 0$  are called *asymptotes*. An **asymptote** is a line that the graph of a function approaches.

### StudyTip

#### Use Asymptotes

Asymptotes are helpful for graphing rational functions. However, they are not part of the graph.



### Key Concept

### Asymptotes

For Your

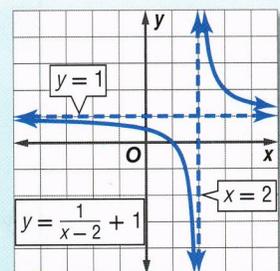
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#### Words

A rational function in the form  $y = \frac{a}{x - b} + c$ ,  $a \neq 0$ , has a vertical asymptote at the  $x$ -value that makes the denominator equal zero,  $x = b$ . It has a horizontal asymptote at  $y = c$ .

#### Model

#### Example



The domain of  $y = \frac{a}{x - b} + c$  is all real numbers except  $x = b$ . The range is all real numbers except  $y = c$ . Rational functions cannot be traced with a pencil that never leaves the paper, so choose  $x$ -values on both sides of the vertical asymptote to graph both portions of the function.



### Math History Link

**Evelyn Boyd Granville**  
(1924–)

Granville majored in mathematics and physics at Smith College in 1945, where she graduated summa cum laude. She earned an M.A. in mathematics and physics and a Ph.D. in mathematics from Yale University. Granville's doctoral work focused on functional analysis.

### EXAMPLE 3 Identify and Use Asymptotes to Graph Functions

Identify the asymptotes of each function. Then graph the function.

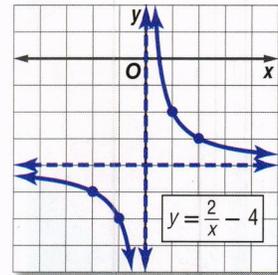
a.  $y = \frac{2}{x} - 4$

**Step 1** Identify and graph the asymptotes using dashed lines.

vertical asymptote:  $x = 0$   
horizontal asymptote:  $y = -4$

**Step 2** Make a table of values and plot the points. Then connect them.

$x$	-2	-1	1	2
$y$	-5	-6	-2	-3



b.  $y = \frac{1}{x+1}$

**Step 1** To find the vertical asymptote, find the excluded value.

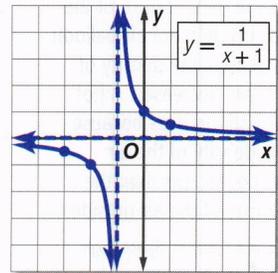
$$x + 1 = 0 \quad \text{Set the denominator equal to 0.}$$

$$x = -1 \quad \text{Subtract 1 from each side.}$$

vertical asymptote:  $x = -1$   
horizontal asymptote:  $y = 0$

**Step 2**

$x$	-3	-2	0	1
$y$	-0.5	-1	1	0.5



### Check Your Progress

3A.  $y = -\frac{6}{x}$

3B.  $y = \frac{1}{x-3}$

3C.  $y = \frac{2}{x+2} + 1$

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Four types of nonlinear functions are shown below.

Concept Summary		Families of Functions	
Quadratic	Exponential	Radical	Rational
Parent function: $y = x^2$	Parent function: varies	Parent function: $y = \sqrt{x}$	Parent function: $y = \frac{1}{x}$
General form: $y = ax^2 + bx + c$	General form: $y = ab^x$	General form: $y = \sqrt{x - b} + c$	General form: $y = \frac{a}{x - b} + c$

For Your  
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## Check Your Understanding

### Example 1 p. 678

State the excluded value for each function.

1.  $y = \frac{5}{x}$

2.  $y = \frac{1}{x+3}$

3.  $y = \frac{x+2}{x-1}$

4.  $y = \frac{x}{2x-8}$

### Example 2 p. 679

5. **PARTY PLANNING** The cost of decorations for a party is \$32. This is split among a group of friends. The amount each person pays  $y$  is given by  $y = \frac{32}{x}$ , where  $x$  is the number of people. Graph the function.

### Example 3 p. 680

Identify the asymptotes of each function. Then graph the function.

6.  $y = \frac{2}{x}$

7.  $y = \frac{3}{x} - 1$

8.  $y = \frac{1}{x-2}$

9.  $y = \frac{-4}{x+2}$

10.  $y = \frac{3}{x-1} + 2$

11.  $y = \frac{2}{x+1} - 5$

## Practice and Problem Solving

 = **Step-by-Step Solutions** begin on page R12.  
**Extra Practice** begins on page 815.

### Example 1 p. 678

State the excluded value for each function.

12.  $y = \frac{-1}{x}$

13.  $y = \frac{8}{x-8}$

14.  $y = \frac{x}{x+2}$

15.  $y = \frac{4}{x+6}$

16.  $y = \frac{x+1}{x-3}$

17.  $y = \frac{2x+5}{x+5}$

18.  $y = \frac{7}{5x-10}$

19.  $y = \frac{x}{2x+14}$

### Example 2 p. 679

20. **ANTELOPES** A pronghorn antelope can run 40 miles without stopping. The average speed is given by  $y = \frac{40}{x}$ , where  $x$  is the time it takes to run the distance.
- Graph  $y = \frac{40}{x}$ .
  - Describe the asymptotes.

21. **CYCLING** A cyclist rides 10 miles each morning. Her average speed  $y$  is given by  $y = \frac{10}{x}$ , where  $x$  is the time it takes her to ride 10 miles. Graph the function.

### Example 3 p. 680

Identify the asymptotes of each function. Then graph the function.

22.  $y = \frac{5}{x}$

23.  $y = \frac{-3}{x}$

24.  $y = \frac{2}{x} + 3$

25.  $y = \frac{1}{x} - 2$

26.  $y = \frac{1}{x+3}$

27.  $y = \frac{1}{x-2}$

28.  $y = \frac{-2}{x+1}$

29.  $y = \frac{4}{x-1}$

30.  $y = \frac{1}{x-2} + 1$

31.  $y = \frac{3}{x-1} - 2$

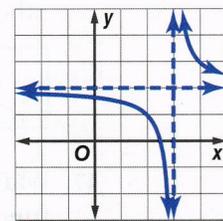
32.  $y = \frac{2}{x+1} - 4$

33.  $y = \frac{-1}{x+4} + 3$

34. **READING** Refer to the application at the beginning of the lesson.
- Graph the function.
  - Choose a point on the graph, and describe what it means in the context of the situation.

35. The graph shows a translation of the graph of  $y = \frac{1}{x}$ .

- Describe the asymptotes.
- Write a function for the graph.





### Real-World Link

The bird with the longest migration is the arctic tern. The arctic tern flies up to 20,000 miles per year, from the Arctic to the Antarctic and back.

Source: University of Wisconsin

36. **BIRDS** A long-tailed jaeger is a sea bird that can migrate 5000 miles or more each year. The average rate in miles per hour  $r$  can be given by the function  $r = \frac{5000}{t}$ , where  $t$  is the time in hours. Use the function to determine the average rate of the bird if it spends 250 hours flying.
37. **CLASS TRIP** The freshmen class is going to a science museum. As part of the trip, each person in the class is also contributing an equal amount of money to name a star.
- Write a verbal description for the cost per person.
  - Write an equation to represent the total cost  $y$  per person if  $p$  people go to the museum.
  - Use a graphing calculator to graph the equation.
  - Estimate the number of people needed for the total cost of the trip to be about \$15.



Graph each function. Identify the asymptotes.

38.  $y = \frac{4x + 3}{2x - 4}$

39.  $y = \frac{x^2}{x^2 - 1}$

40.  $y = \frac{x}{x^2 - 9}$

41. **GEOMETRY** The equation  $h = \frac{2(64)}{b_1 + 8}$  represents the height  $h$  of a quadrilateral with an area of 64 square units. The quadrilateral has two opposite sides that are parallel and  $h$  units apart; one is  $b_1$  units long and another is 8 units long.
- Describe a reasonable domain and range for the function.
  - Graph the function in the first quadrant.
  - Use the graph to estimate the value of  $h$  when  $b_1 = 10$ .

### H.O.T. Problems

Use Higher-Order Thinking Skills

42. **CHALLENGE** Graph  $y = \frac{1}{x^2 - 4}$ . State the domain and the range of the function.
43. **REASONING** Without graphing, describe the transformation that takes place between the graph of  $y = \frac{1}{x}$  and the graph of  $y = \frac{1}{x + 5} - 2$ .
44. **OPEN ENDED** Write a rational function if the asymptotes of the graph are at  $x = 3$  and  $y = 1$ . Explain how you found the function.
45. **REASONING** Is the following statement *true* or *false*? If false, give a counterexample.

*The graph of a rational function will have at least one intercept.*

46. **WHICH ONE DOESN'T BELONG** Identify the function that does not belong with the other three. Explain your reasoning.

$$y = \frac{4}{x}$$

$$y = \frac{6}{x + 1}$$

$$y = \frac{8}{x} + 1$$

$$y = \frac{10}{2x}$$

47. **WRITING IN MATH** Write a rule to find the vertical asymptotes of a rational function.

## Standardized Test Practice

48. Simplify  $\frac{2a^2d}{3bc} \cdot \frac{9b^2c}{16ad^2}$ .

A  $\frac{abd}{c}$

C  $\frac{6a}{4bd}$

B  $\frac{ab}{d}$

D  $\frac{3ab}{8d}$

49. **SHORT RESPONSE** One day Lola ran 100 meters in 15 seconds, 200 meters in 45 seconds, and 200 meters over low hurdles in one and a half minutes. How many more seconds did it take her to run 200 meters over low hurdles than the 200-meter dash?

50. Scott and Ian started a T-shirt printing business. The total start-up costs were \$450. It costs \$5.50 to print one T-shirt. Write a rational function  $A(x)$  for the average cost of producing  $x$  T-shirts.

F  $A(x) = \frac{450 + 5.5x}{x}$     H  $A(x) = 450x + 5.5$

G  $A(x) = \frac{450}{x} + 5.5$     J  $A(x) = 450 + 5.5x$

51. **GEOMETRY** Which of the following is a quadrilateral with exactly one pair of parallel sides?

A parallelogram

C square

B rectangle

D trapezoid

## Spiral Review

52. **TRAVEL** The Brooks family can drive to the beach, which is 220 miles away, in 4 hours if they drive 55 miles per hour. Kendra says that they would save at least a half an hour if they were to drive 65 miles per hour. Is Kendra correct? Explain. (Lesson 11-1)

Use a calculator to find the measure of each angle to the nearest degree. (Lesson 10-8)

53.  $\sin C = 0.9781$

54.  $\tan H = 0.6473$

55.  $\cos K = 0.7658$

56.  $\tan Y = 3.6541$

57.  $\cos U = 0.5000$

58.  $\sin N = 0.3832$

If  $c$  is the measure of the hypotenuse of a right triangle, find each missing measure. If necessary round to the nearest hundredth. (Lesson 10-5)

59.  $a = 15, b = 60, c = ?$

60.  $a = 17, c = 35, b = ?$

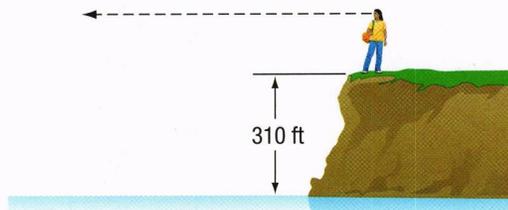
61.  $a = \sqrt{110}, b = 1, c = ?$

62.  $a = \sqrt{17}, b = \sqrt{12}, c = ?$

63.  $a = 6, c = 11, b = ?$

64.  $a = 9, b = 6, c = ?$

65. **SIGHT** The formula  $d = \sqrt{\frac{3h}{2}}$  represents the distance  $d$  in miles that a person  $h$  feet high can see. Irene is standing on a cliff that is 310 feet above sea level. How far can Irene see from the cliff? Write a simplified radical expression and a decimal approximation. (Lesson 10-3)



## Skills Review

Factor each trinomial. (Lessons 8-3 and 8-4)

66.  $x^2 + 11x + 24$

67.  $w^2 + 13w - 48$

68.  $p^2 - 2p - 35$

69.  $72 + 27a + a^2$

70.  $c^2 + 12c + 35$

71.  $d^2 - 7d + 10$

72.  $g^2 - 19g + 60$

73.  $n^2 + 3n - 54$

74.  $5x^2 + 27x + 10$

75.  $24b^2 - 14b - 3$

76.  $12a^2 - 13a - 35$

77.  $6x^2 - 14x - 12$

## Simplifying Rational Expressions

**Then**

You simplified expressions involving the quotient of monomials. (Lesson 7-2)

**Now**

- Identify values excluded from the domain of a rational expression.
- Simplify rational expressions.

**New Vocabulary**

rational expression

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**Why?**

Big-O is a “hubless” Ferris wheel in Tokyo, Japan. The *centripetal force*, or the force acting toward the center, is given by  $\frac{mv^2}{r}$ , where  $m$  is the mass of the Ferris wheel,  $v$  is the velocity, and  $r$  is the radius.

**Identify Excluded Values** The expression  $\frac{mv^2}{r}$  is an example of a rational expression.

A **rational expression** is an algebraic fraction whose numerator and denominator are polynomials. Since division by zero is undefined, the polynomial in the denominator cannot be 0.

**EXAMPLE 1 Find Excluded Values**

State the excluded values for each rational expression.

a.  $\frac{-8}{r^2 - 36}$

Exclude the values for which  $r^2 - 36 = 0$ .

$$r^2 - 36 = 0$$

**The denominator cannot be zero.**

$$(r - 6)(r + 6) = 0$$

**Factor.**

$$r - 6 = 0 \quad \text{or} \quad r + 6 = 0$$

**Zero Product Property**

$$r = 6$$

$$r = -6$$

Therefore,  $r$  cannot equal 6 or  $-6$ .

b.  $\frac{n^2}{n^2 + 4n - 5}$

Exclude the values for which  $n^2 + 4n - 5 = 0$ .

$$n^2 + 4n - 5 = 0$$

**The denominator cannot be zero.**

$$(n - 1)(n + 5) = 0$$

**Factor.**

$$n - 1 = 0 \quad \text{or} \quad n + 5 = 0$$

**Zero Product Property**

$$n = 1$$

$$n = -5$$

Therefore,  $n$  cannot equal 1 or  $-5$ .

**Check Your Progress**

1A.  $\frac{5x}{x^2 - 81}$

1B.  $\frac{3a - 2}{a^2 + 6a + 8}$

### StudyTip

**Cylinder** The volume of a cylinder is  $V = \pi r^2 h$ . The height of a cylinder with volume  $V$  and radius  $r$  is given by  $\frac{V}{\pi r^2}$ .

## Real-World EXAMPLE 2 Use Rational Expressions

**GEOMETRY** Find the height of a cylinder that has a volume of 821 cubic inches and a radius of 7 inches. Round to the nearest tenth.

**Understand** You have a rational expression with two variables,  $V$  and  $r$ .

**Plan** Substitute 821 for  $V$  and 7 for  $r$  and simplify.

**Solve** 
$$\frac{V}{\pi r^2} = \frac{821}{\pi(7)^2}$$
$$\approx 5.3$$

Replace  $V$  with 821 and  $r$  with 7.

The height of the cylinder is about 5.3 inches.

**Check** Use estimation to determine whether the answer is reasonable.

$$\frac{800}{3(50)} \approx 5 \checkmark$$

The solution is reasonable.

### Check Your Progress

2. Find the height of the cylinder that has a volume of 710 cubic inches and a diameter of 18 inches.

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**Simplify Expressions** A rational expression is in simplest form when the numerator and denominator have no common factors except 1. To simplify a rational expression, divide out any common factors of the numerator and denominator.



### Key Concept

### Simplifying Rational Expressions

For Your  
FOLDABLE

**Words** Let  $a$ ,  $b$ , and  $c$ , be polynomials with  $a \neq 0$  and  $c \neq 0$ .

**Symbols** 
$$\frac{ba}{ca} = \frac{b \cdot a}{c \cdot a} = \frac{b}{c}$$

**Example** 
$$\frac{3x - 9}{4x - 12} = \frac{3(x - 3)}{4(x - 3)} = \frac{3}{4}$$

Math in Motion, Interactive Lab [glencoe.com](http://glencoe.com)

### Test-TakingTip

#### Eliminate Possibilities

Since there is only one negative factor in the expression in Example 3, the simplified expression should have a negative sign. So, you can eliminate choices A and B.

## STANDARDIZED TEST EXAMPLE 3

Which expression is equivalent to  $\frac{(-3x^2)(4x^5)}{9x^6}$ ?

A  $\frac{4}{3}x$

B  $\frac{4}{3x}$

C  $-\frac{4}{3x}$

D  $-\frac{4}{3}x$

**Read the Test Item** The expression represents the product of two monomials and the division of that product by another monomial.

#### Solve the Test Item

**Step 1** Factor the numerator and denominator, using their GCF.

$$\frac{(3x^6)(-4x)}{(3x^6)(3)}$$

**Step 2** Simplify. The correct answer is D.

$$\frac{(3x^6)(-4x)}{(3x^6)(3)} \text{ or } -\frac{4}{3}x$$

### Check Your Progress

3. Which expression is equivalent to  $\frac{16c^2b^4}{8c^3b}$ ?

F  $\frac{2b^3}{c}$

G  $\frac{b^3}{2c}$

H  $\frac{1}{2b^3c}$

J  $2b^3c$

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You can use the same procedure to simplify a rational expression in which the numerator and denominator are polynomials.

### EXAMPLE 4 Simplify Rational Expressions

Simplify  $\frac{2r + 18}{r^2 + 8r - 9}$ . State the excluded values of  $r$ .

$$\frac{2r + 18}{r^2 + 8r - 9} = \frac{2(r + 9)}{(r + 9)(r - 1)}$$

Factor.

$$= \frac{\cancel{2}(\cancel{r + 9})}{(\cancel{r + 9})(r - 1)} \text{ or } \frac{2}{r - 1}$$

Divide the numerator and denominator by the GCF,  $r + 9$ .

Exclude the values for which  $r^2 + 8r - 9$  equals 0.

$$r^2 + 8r - 9 = 0$$

The denominator cannot equal zero.

$$(r + 9)(r - 1) = 0$$

Factor.

$$r = -9 \text{ or } r = 1$$

Zero Product Property

So,  $r \neq -9$  and  $r \neq 1$ .

#### Check Your Progress

Simplify each rational expression. State the excluded values of the variables.

4A.  $\frac{n + 3}{n^2 + 10n + 21}$

4B.  $\frac{y^2 + 9y - 10}{2y + 20}$

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#### StudyTip

##### Excluded Values

Determine the excluded values using the *original* expression rather than the simplified expression.

When simplifying rational expressions, look for binomials that are opposites. For example,  $5 - x$  and  $x - 5$  are opposites because  $5 - x = -1(x - 5)$ . So, you can write  $\frac{x - 5}{5 - x}$  as  $\frac{x - 5}{-1(x - 5)}$ .

### EXAMPLE 5 Recognize Opposites

Simplify  $\frac{36 - t^2}{5t - 30}$ . State the excluded values of  $t$ .

$$\frac{36 - t^2}{5t - 30} = \frac{(6 - t)(6 + t)}{5(t - 6)}$$

Factor.

$$= \frac{-1(t - 6)(6 + t)}{5(t - 6)}$$

Rewrite  $6 - t$  as  $-1(t - 6)$ .

$$= \frac{-1(\cancel{t - 6})(6 + t)}{5(\cancel{t - 6})} \text{ or } -\frac{6 + t}{5}$$

Divide out the common factor,  $t - 6$ .

Exclude the values for which  $5t - 30$  equals 0.

$$5t - 30 = 0$$

The denominator cannot equal zero.

$$5t = 30$$

Add 30 to each side.

$$t = 6$$

Zero Product Property

So,  $t \neq 6$ .

#### Check Your Progress

Simplify each expression. State the excluded values of  $x$ .

5A.  $\frac{12x + 36}{x^2 - x - 12}$

5B.  $\frac{x^2 - 2x - 35}{x^2 - 9x + 14}$

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**StudyTip**

**Zeros** Once simplified, a rational function has zeros at the values that make the numerator 0.

Recall that to find the zeros of a quadratic function, you need to find the values of  $x$  when  $f(x) = 0$ . The zeros of a rational function are found in the same way.

**EXAMPLE 6 Rational Functions**

Find the zeros of  $f(x) = \frac{x^2 + 3x - 18}{x - 3}$ .

$$f(x) = \frac{x^2 + 3x - 18}{x - 3} \quad \text{Original function}$$

$$0 = \frac{x^2 + 3x - 18}{x - 3} \quad f(x) = 0$$

$$0 = \frac{(x + 6)(x - 3)}{x - 3} \quad \text{Factor.}$$

$$0 = \frac{(x + 6)\cancel{(x - 3)}}{\cancel{x - 3}} \quad \text{Divide out common factors.}$$

$$0 = x + 6 \quad \text{Simplify.}$$

When  $x = -6$ , the numerator becomes 0, so  $f(x) = 0$ . Therefore, the zero of the function is  $-6$ .

**Check Your Progress**

Find the zeros of each function.

6A.  $f(x) = \frac{x^2 + 2x - 15}{x + 1}$

6B.  $f(x) = \frac{x^2 + 6x + 8}{x^2 + x - 2}$

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**Check Your Understanding****Example 1**

p. 684

State the excluded values for each rational expression.

1.  $\frac{8}{x^2 - 16}$

2.  $\frac{3m}{m^2 - 6m + 5}$

**Example 2**

p. 685

3. **PHYSICAL SCIENCE** A 0.16-kilogram ball attached to a string is being spun in a circle 7.26 meters per second. The expression  $\frac{mv^2}{r}$ , where  $m$  is the mass of the ball,  $v$  is the velocity, and  $r$  is the radius, can be used to find the force that keeps the ball spinning in a circle. If the circle has a radius of 0.5 meter, find the force that must be exerted to keep the ball spinning. Round to the nearest tenth.

**Examples 3–5**

pp. 685–686

Simplify each expression. State the excluded values of the variables.

4.  $\frac{28ab^3}{16a^2b}$

5.  $\frac{(-3r)(10r^4)}{6r^5}$

6.  $\frac{5d + 15}{d^2 - d - 12}$

7.  $\frac{x^2 + 11x + 28}{x + 4}$

8.  $\frac{2r - 12}{r^2 - 36}$

9.  $\frac{3y - 27}{81 - y^2}$

**Example 6**

p. 687

Find the zeros of each function.

10.  $f(x) = \frac{x^2 - x - 12}{x - 2}$

11.  $f(x) = \frac{x^2 - x - 6}{x^2 + 8x + 12}$

# Practice and Problem Solving

= **Step-by-Step Solutions** begin on page R12.  
**Extra Practice** begins on page 815.

**Example 1**  
p. 684

State the excluded values for each rational expression.

12.  $\frac{-n}{n^2 - 49}$

13.  $\frac{5x + 1}{x^2 - 1}$

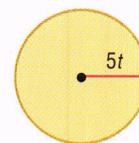
14.  $\frac{12a}{a^2 - 3a - 10}$

15.  $\frac{k^2 - 4}{k^2 + 5k - 24}$

**Example 2**  
p. 685

16. **GEOMETRY** The volume of a rectangular prism is  $3x^3 + 34x^2 + 72x - 64$ . If the height is  $x + 4$ , what is the area of the base of the prism?

17. **GEOMETRY** Use the circle at the right to write the ratio  $\frac{\text{circumference}}{\text{area}}$ . Then simplify. State the excluded value of the variable.



**Examples 3–5**  
pp. 685–686

Simplify each expression. State the excluded values of the variables.

18.  $\frac{15x^4y^2}{40x^3y^3}$

19.  $\frac{32n^2p}{2n^4p}$

20.  $\frac{(4t^3)(2t)}{20t^2}$

21.  $\frac{(7c^2)(-6c^3)}{21c^4}$

22.  $\frac{4x - 24}{x^2 - 12x + 36}$

23.  $\frac{a^2 + 3a}{a^2 - 3a - 18}$

24.  $\frac{n^2 + 7n - 18}{n - 2}$

25.  $\frac{x^2 + 4x - 32}{x + 8}$

26.  $\frac{x^2 - 25}{x^2 + 5x}$

27.  $\frac{2p^2 - 14p}{p^2 - 49}$

28.  $\frac{2x - 10}{25 - x^2}$

29.  $\frac{64 - c^2}{c^2 - 7c - 8}$

**Example 6**  
p. 687

Find the zeros of each function.

30.  $f(x) = \frac{x^2 - x - 12}{x^2 + 2x - 35}$

31.  $f(x) = \frac{x^2 + 3x - 4}{x^2 + 9x + 20}$

32.  $f(x) = \frac{2x^2 + 11x - 40}{2x + 5}$

33.  $f(x) = \frac{3x^2 - 18x + 24}{x - 6}$

34.  $f(x) = \frac{x^3 + x^2 - 6x}{x - 1}$

35.  $f(x) = \frac{x^3 - 4x^2 - 12x}{x + 2}$

36. **PYRAMIDS** The perimeter of the base of the Pyramid of the Sun is  $4\pi$  times the height. The perimeter of the base of the Great Pyramid of Giza is  $2\pi$  times the height. Write and simplify each ratio comparing the base perimeters.

- a. Pyramid of the Sun to the Great Pyramid
- b. Great Pyramid to the Pyramid of the Sun

Pyramid	Height (ft)
Pyramid of the Sun (Mexico)	233.5
Great Pyramid (Egypt)	481.4

Source: Nexus Network Journal

37. **FERRIS WHEELS** Refer to the Real-World Link.

- a. To find the speed traveled by a car located on the wheel, you can find the circumference of a circle and divide by the time it takes for one rotation. Write a rational expression for the speed of a car rotating in time  $t$ .
- b. Suppose the first Ferris wheel rotated once every 5 minutes. What was the speed of a car on the circumference in feet per minute?



**Real-World Link**

George Ferris built the first Ferris wheel for the World's Columbian Exposition in 1893. It had a diameter of 250 feet.

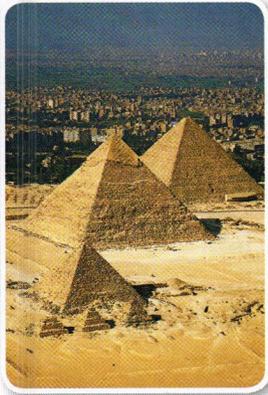
Source: *The New York Times*

Simplify each expression. State the excluded values of the variables.

38.  $\frac{3a^2b^4 + 9a^3b - 6a^5b}{3a^2b}$

39.  $\frac{8x^5 - 10xy^2}{2xy^3}$

40.  $\frac{x + 5}{3x^2 + 14x - 5}$

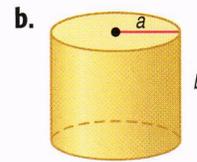
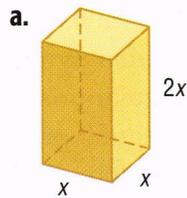


### Real-World Link

The ancient Egyptians probably used levers to help them maneuver the giant blocks they used to build the pyramids. An estimated 20,000 to 30,000 workers built the Pyramids at Giza over 80 years.

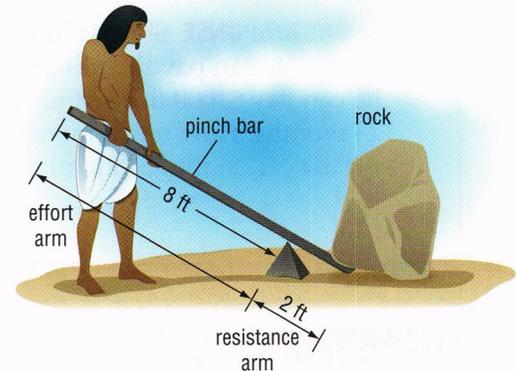
Source: National Geographic

- 41. PACKAGING** To minimize packaging expenses, a company uses packages that have the least surface area to volume ratio. For each figure, write a ratio comparing the surface area to the volume. Then simplify. State the excluded values of the variables.



- 42. HISTORY** The diagram shows how a lever may have been used to move blocks.

- a. The mechanical advantage of a lever is  $\frac{L_A}{L_R}$ , where  $L_A$  is the length of the effort arm and  $L_R$  is the length of the resistance arm. Find the mechanical advantage of the lever shown.



- b. The force placed on the rock is the product of the mechanical advantage and the force applied to the end of the lever. If the Egyptian worker can apply a force of 180 pounds, what is the greatest weight he can lift with the lever?
- c. To lift a 535-pound rock using a 7-foot lever with the fulcrum 2 feet from the rock, how much force will have to be used?

### H.O.T. Problems

Use **H**igher-**O**rders **T**hinking Skills

- 43. FIND THE ERROR** Colleen and Sanson examined  $\frac{12x + 36}{x^2 - x - 12}$  and found the excluded value(s). Is either of them correct? Explain.

*Colleen*

$$\frac{12x + 36}{x^2 - x - 12} = \frac{12(x + 3)}{(x - 4)(x + 3)}$$

The excluded values are  
4 and -3.

*Sanson*

$$\begin{aligned} \frac{12x + 36}{x^2 - x - 12} &= \frac{12(x + 3)}{(x - 4)(x + 3)} \\ &= \frac{12(x + 3)}{(x - 4)(x + 3)} \\ &= \frac{12}{x - 4} \end{aligned}$$

The excluded value is 4.

- 44. CHALLENGE** Compare and contrast the graphs of  $y = x - 2$  and  $y = \frac{x^2 + 5x - 14}{x + 7}$ .
- 45. REASONING** Explain why every polynomial is also a rational expression.
- 46. OPEN ENDED** Write a rational expression with excluded values  $-2$  and  $2$ . Explain how you found the expression.
- 47. REASONING** Is  $\frac{2x^2 - 4x}{x - 2}$  in simplest form? Justify your answer.
- 48. WRITING IN MATH** List the steps you would use to simplify  $\frac{x^2 + x - 20}{x + 5}$ . State the excluded value.

## Standardized Test Practice

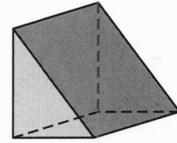
49. Simplify  $\frac{2x+4}{2}$ .

- A  $x + 1$
- B  $x$
- C  $x + 2$
- D  $\frac{x}{2}$

50. **SHORT RESPONSE** Shiro is buying a car for \$5800. He can pay the full amount in cash, or he can pay \$1000 down and \$230 a month for 24 months. How much more would he pay for the car on the second plan?

51. **GEOMETRY** What is the name of the figure?

- F triangular pyramid
- G triangular prism
- H rectangular prism
- J triangularon



52. A rectangle has a length of 10 inches and a width of 5 inches. Another rectangle has the same area as the first rectangle but its width is 2 inches. Find the length of the second rectangle.

- A 30 in.
- B 60 in.
- C 20 in.
- D 25 in.

## Spiral Review

State the excluded value for each function. (Lesson 11-2)

53.  $y = \frac{6}{x}$

54.  $y = \frac{2}{x-5}$

55.  $y = \frac{x-4}{x-3}$

56.  $y = \frac{3x}{2x+6}$

Solve. Assume that  $y$  varies inversely as  $x$ . (Lesson 11-1)

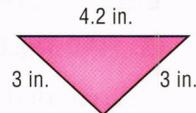
57. If  $y = 10$  when  $x = 4$ , find  $x$  when  $y = 2$ .

58. If  $y = 12$  when  $x = 3$ , find  $x$  when  $y = 6$ .

59. If  $y = -5$  when  $x = 3$ , find  $x$  when  $y = -3$ .

60. If  $y = 21$  when  $x = -6$ , find  $x$  when  $y = 7$ .

61. **CRAFTS** Melinda is working on a quilt using the pattern shown. She has several triangular pieces of material with two sides that measure 6 inches. If these pieces are similar to the pattern shown, what is the length of the third side? (Lesson 10-7)



Find the distance between each pair of points whose coordinates are given.

(Lesson 10-6)

62.  $(12, 3), (-8, 3)$

63.  $(0, 0), (5, 12)$

64.  $(6, 8), (3, 4)$

65.  $(-8, -4), (-3, -8)$

Simplify. (Lesson 10-2)

66.  $\sqrt{20}$

67.  $\sqrt{18}$

68.  $\sqrt{2} \cdot \sqrt{8}$

69.  $2\sqrt{32}$

70.  $\sqrt{5} \cdot \sqrt{6}$

71.  $\sqrt{40a^2}$

72.  $\sqrt{\frac{t}{8}}$

73.  $\sqrt{\frac{2}{7}} \cdot \sqrt{\frac{7}{3}}$

74. **FINANCIAL LITERACY** Determine the amount of an investment if \$250 is invested at an interest rate of 7.3% compounded quarterly for 40 years. (Lesson 9-7)

## Skills Review

Find the greatest common factor for each set of monomials. (Lesson 8-1)

75.  $2x, 8x^2$

76.  $3y^2, 7y^3$

77.  $7g, 10h$

78.  $21c^2d^3, 14cd^2$

79.  $9qt^2, 18q^2t^2, 27qt$

80.  $10ab, 25a^2b^2, 30a^2b$

# Graphing Technology Lab

## Simplifying Rational Expressions

- Other Calculator Keystrokes
- Graphing Technology Personal Tutor

When simplifying rational expressions, you can use a graphing calculator to support your answer. If the graphs of the original expression and the simplified expression overlap, they are equivalent. You can also use the graphs to see excluded values.

### ACTIVITY Simplify a Rational Expression

Simplify  $\frac{x^2 - 16}{x^2 + 8x + 16}$ .

**Step 1** Factor the numerator and denominator.

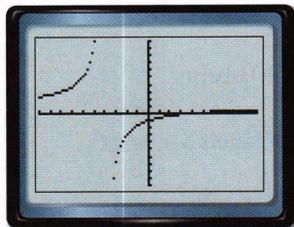
$$\begin{aligned} \bullet \frac{x^2 - 16}{x^2 + 8x + 16} &= \frac{(x - 4)(x + 4)}{(x + 4)(x + 4)} \\ &= \frac{(x - 4)}{(x + 4)} \end{aligned}$$

When  $x = -4$ ,  $x + 4 = 0$ . Therefore,  $x$  cannot equal  $-4$  because you cannot divide by zero.

**Step 2** Graph the original expression.

- Set the calculator to Dot mode.
- Enter  $\frac{x^2 - 16}{x^2 + 8x + 16}$  as Y1 and graph.

KEYSTROKES: **MODE**  $\blacktriangledown$   $\blacktriangledown$   $\blacktriangledown$   $\blacktriangledown$   $\blacktriangleright$  **ENTER**  
 $\text{Y=}$  ( **X,T,θ,n**  $x^2$   $-$  16 )  
 $\div$  ( **X,T,θ,n**  $x^2$   $+$  8  
**X,T,θ,n**  $+$  16 ) **ZOOM** 6

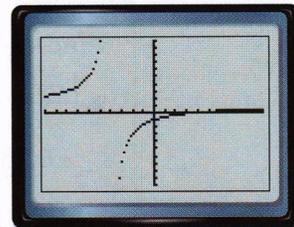


$[-10, 10]$  scl: 1 by  $[-10, 10]$  scl: 1

**Step 3** Graph the simplified expression.

- Enter  $\frac{(x - 4)}{(x + 4)}$  as Y2 and graph.

KEYSTROKES:  $\text{Y=}$   $\blacktriangledown$  ( **X,T,θ,n**  $-$  4 )  
 $\div$  ( **X,T,θ,n**  $+$  4 )  
**GRAPH**



$[-10, 10]$  scl: 1 by  $[-10, 10]$  scl: 1

Since the graphs overlap, the two expressions are equivalent.

### Exercises

Simplify each expression. Then verify your answer graphically. Name the excluded values.

1.  $\frac{5x + 15}{x^2 + 10x + 21}$

2.  $\frac{x^2 - 8x + 12}{x^2 + 7x - 18}$

3.  $\frac{2x^2 + 6x + 4}{3x^2 + 9x + 6}$

4. a. Simplify  $\frac{3x - 8}{6x^2 - 16x}$ .

b. How can you use the **TABLE** function to verify that the original expression and the simplified expression are equivalent?

c. How does the **TABLE** function show you that an  $x$ -value is an excluded value?

# 11-4

## Multiplying and Dividing Rational Expressions

### Then

You multiplied and divided polynomials. (Lesson 7-7 and 7-2)

### Now

- Multiply rational expressions.
- Divide rational expressions.

### Math Online

[glencoe.com](http://glencoe.com)

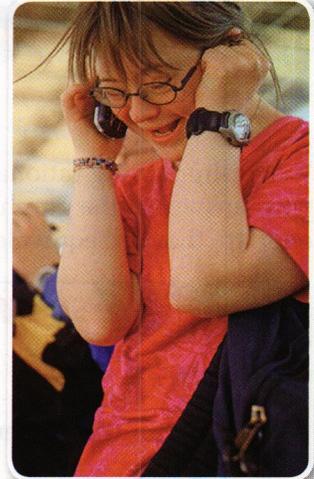
- Extra Examples
- Personal Tutor
- Self-Check Quiz
- Homework Help

### Why?

A recent survey showed 10- to 17-year olds talk on their cell phones an average of 3.75 hours per day during the summer. The expression below can be used to find the average number of minutes youth talk on their phones during summer, approximately 90 days.

$$90 \text{ days} \cdot \frac{3.75 \text{ hours}}{\text{day}} \cdot \frac{60 \text{ minutes}}{1 \text{ hour}} = 20,250 \text{ minutes}$$

**Multiply Rational Expressions** To multiply fractions, you multiply numerators and multiply denominators. Use this same method to multiply rational expressions.



### Key Concept

### Multiplying Rational Expressions

For Your

**FOLDABLE**

**Words** Let  $a$ ,  $b$ ,  $c$ , and  $d$  be polynomials with  $b \neq 0$  and  $d \neq 0$ .

$$\text{Then, } \frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$$

**Example** 
$$\frac{x}{2x-3} \cdot \frac{4x^2}{5} = \frac{4x^3}{5(2x-3)}$$

### EXAMPLE 1 Multiply Expressions Involving Monomials

Find each product.

a. 
$$\frac{r^2x}{9t^3} \cdot \frac{3t^4}{r}$$

Divide by the common factors before multiplying.

$$\begin{aligned} \frac{r^2x}{9t^3} \cdot \frac{3t^4}{r} &= \frac{\overset{r}{\cancel{r^2}}x}{\underset{3}{\cancel{9}}\underset{1}{\cancel{t^3}}} \cdot \frac{\underset{3}{\cancel{3}}\underset{1}{\cancel{t^4}}}{\underset{r}{\cancel{r}}} \\ &= \frac{rxt}{3} \end{aligned}$$

Divide by the common factors 3,  $r$ , and  $t^3$ .

Simplify.

b. 
$$\frac{a+4}{a^2} \cdot \frac{a}{a^2+2a-8}$$

$$\frac{a+4}{a^2} \cdot \frac{a}{a^2+2a-8} = \frac{a+4}{a^2} \cdot \frac{a}{(a+4)(a-2)}$$

Factor the denominator.

$$= \frac{\overset{1}{\cancel{a+4}}}{\underset{a^2}{\cancel{a^2}}} \cdot \frac{\underset{1}{\cancel{a}}}{\underset{1}{\cancel{(a+4)}}(a-2)}$$

The GCF is  $a(a+4)$ .

$$= \frac{1}{a(a-2)} \text{ or } \frac{1}{a^2-2a}$$

Simplify.

### Check Your Progress

1A. 
$$\frac{3x}{16x^2} \cdot \frac{8x^2}{3}$$

1B. 
$$\frac{x+3}{x} \cdot \frac{5}{x^2+7x+12}$$

1C. 
$$\frac{y^2-3y-4}{y+5} \cdot \frac{y+5}{y^2-4y}$$

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### StudyTip

#### Rational Expressions

From this point on, assume that no denominator of a rational expression has a value of zero.

When you multiply fractions that involve units of measure, you can divide by the units in the same way that you divide by variables. Recall that this process is called *dimensional analysis*. You can use dimensional analysis to convert units of measure within a system and between systems.

### Real-World EXAMPLE 2 Dimensional Analysis

**SKI RACING** Ann Proctor won the 2007 World Waterski Racing Championship race in her category when she finished the 88-kilometer course in 51.23 minutes. What was her average speed in miles per hour? (Hint: 1 km  $\approx$  0.62 mi)

$$\begin{aligned}\frac{88 \text{ km}}{51.23 \text{ min}} \cdot \frac{0.62 \text{ mi}}{1 \text{ km}} \cdot \frac{60 \text{ min}}{1 \text{ h}} &= \frac{88 \cancel{\text{ km}}}{51.23 \cancel{\text{ min}}} \cdot \frac{0.62 \text{ mi}}{1 \cancel{\text{ km}}} \cdot \frac{60 \cancel{\text{ min}}}{1 \text{ h}} \\ &= \frac{88 \cdot 0.62 \text{ mi} \cdot 60}{51.23 \cdot 1 \cdot 1 \text{ h}} && \text{Simplify.} \\ &= \frac{3273.6 \text{ mi}}{51.23 \text{ h}} && \text{Multiply.} \\ &\approx \frac{63.9 \text{ mi}}{\text{h}} && \text{Divide the numerator and the denominator by 51.23.}\end{aligned}$$

Her average speed was 63.9 miles per hour.

#### Check Your Progress

2. **SKI RACING** What was Ann Proctor's speed in feet per second?

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**Divide Rational Expressions** To divide by a fraction, you multiply by the reciprocal. You can use this same method to divide by a rational expression.

### Key Concept

#### Dividing Rational Expressions

For Your  
**FOLDABLE**

**Symbols** Let  $a$ ,  $b$ ,  $c$ , and  $d$  be polynomials with  $b \neq 0$ ,  $c \neq 0$ , and  $d \neq 0$ . Then,  $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}$ .

**Example**  $\frac{x-3}{x} \div \frac{2x^2}{5} = \frac{x-3}{x} \cdot \frac{5}{2x^2} = \frac{5(x-3)}{2x^3}$

### EXAMPLE 3 Divide by a Rational Expression

Find  $\frac{4}{15n^3} \div \frac{12}{25n}$ .

$$\begin{aligned}\frac{4}{15n^3} \div \frac{12}{25n} &= \frac{4}{15n^3} \cdot \frac{25n}{12} && \text{Multiply by } \frac{25n}{12}, \text{ the reciprocal of } \frac{12}{25n} \\ &= \frac{\overset{1}{\cancel{4}}}{\underset{3n^2}{\cancel{15n^3}}} \cdot \frac{\overset{5}{\cancel{25n}}}{\underset{3}{\cancel{12}}} && \text{Divide by common factors 4, 5, and } n. \\ &= \frac{5}{9n^2} && \text{Simplify.}\end{aligned}$$

#### Check Your Progress

Find each quotient.

3A.  $\frac{15y^2}{4x} \div \frac{5y}{8x^3}$

3B.  $\frac{12a^2}{5b} \div \frac{25a}{6b^2}$

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## ReadingMath

### Rational Expressions

In a rational expression, the fraction bar acts as a grouping symbol.

In Example 4a,  $\frac{2x+6}{x^2}$  is read *the quantity two x plus six divided by x squared*.

## EXAMPLE 4 Divide by Rational Expressions and Polynomials

Find each quotient.

a.  $\frac{2x+6}{x^2} \div (x+3)$

$$\begin{aligned} \frac{2x+6}{x^2} \div (x+3) &= \frac{2x+6}{x^2} \div \frac{x+3}{1} \\ &= \frac{2x+6}{x^2} \cdot \frac{1}{x+3} \\ &= \frac{2(x+3)}{x^2} \cdot \frac{1}{x+3} \\ &= \frac{\cancel{2(x+3)}}{x^2} \cdot \frac{1}{\cancel{x+3}} \text{ or } \frac{2}{x^2} \end{aligned}$$

Write the binomial as a fraction.

Multiply by the reciprocal of  $x+3$ .

Factor  $4x+6$ .

Divide out the common factor and simplify.

b.  $\frac{a-2}{4a+4} \div \frac{a+5}{a+1}$

$$\begin{aligned} \frac{a-2}{4a+4} \div \frac{a+5}{a+1} &= \frac{a-2}{4a+4} \cdot \frac{a+1}{a+5} \\ &= \frac{a-2}{4(\cancel{a+1})} \cdot \frac{\cancel{a+1}}{a+5} \\ &= \frac{a-2}{4(a+5)} \end{aligned}$$

Multiply by the reciprocal.

Factor  $4a+4$ .

The GCF is  $a+1$  and simplify.

### Check Your Progress

4A.  $\frac{4d-8}{2d-6} \div \frac{2d-4}{d-4}$

4B.  $\frac{b+4}{3b+2} \div \frac{3b+12}{b+1}$

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Sometimes you must factor a quadratic expression before you can simplify the quotient of rational expressions.

## StudyTip

**Canceling** Remember that only factors can be canceled, not individual terms.

## EXAMPLE 5 Expression Involving Polynomials

Find  $\frac{y-3}{y^2-10y+16} \div \frac{y^2-9}{y-8}$ .

$$\begin{aligned} \frac{y-3}{y^2-10y+16} \div \frac{y^2-9}{y-8} &= \frac{y-3}{y^2-10y+16} \cdot \frac{y-8}{y^2-9} \\ &= \frac{y-3}{(y-2)(y-8)} \cdot \frac{y-8}{(y-3)(y+3)} \\ &= \frac{\cancel{y-3}}{(y-2)\cancel{(y-8)}} \cdot \frac{\cancel{y-8}}{\cancel{(y-3)}(y+3)} \\ &= \frac{1}{(y-2)(y+3)} \end{aligned}$$

Multiply by the reciprocal,  $\frac{y-8}{y^2-9}$ .

Factor  $y^2-10y+16$  and  $y^2-9$ .

The GCF is  $(y-3)(y-8)$ .

Simplify.

### Check Your Progress

Find each quotient.

5A.  $\frac{p^2-4}{5p} \div \frac{p-2}{p+q}$

5B.  $\frac{q^2+3q+2}{12} \div \frac{q+1}{q^2+4}$

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## Check Your Understanding

### Example 1

p. 692

Find each product.

1.  $\frac{2x^3}{7x} \cdot \frac{14}{x}$

3.  $\frac{t^2}{(t-5)(t+5)} \cdot \frac{t+5}{6t}$

2.  $\frac{3ab}{4c^4} \cdot \frac{16c^2}{9b}$

4.  $\frac{8}{r+1} \cdot \frac{r^2-1}{2}$

### Example 2

p. 693

5. **SLOTHS** The slowest land mammal is the three-toed sloth. It travels 0.07 mile per hour on the ground. What is this speed in feet per minute?
6. **EXERCISE** One hour of moderate inline skating burns approximately 330 Calories. If Nelia plans to do inline skating for 3 hours a week, how many Calories will she burn in a year from the skating?

### Examples 3–5

pp. 693–694

Find each quotient.

7.  $\frac{8}{3x^2} \div \frac{4}{x}$

9.  $\frac{b^2+6b+5}{6b+6} \div (b+5)$

8.  $\frac{c^5}{2} \div \frac{c^3}{6d^2}$

10.  $\frac{2x+8}{x+3} \div \frac{x+4}{x^2+6x+9}$

## Practice and Problem Solving

= **Step-by-Step Solutions** begin on page R12.  
**Extra Practice** begins on page 815.

### Example 1

p. 692

Find each product.

11.  $\frac{10n^2}{4} \cdot \frac{2}{n}$

13.  $\frac{x^5y}{2z^3} \cdot \frac{18z^4}{xy}$

15.  $\frac{9}{t-2} \cdot \frac{(t+2)(t-2)}{3}$

17.  $\frac{(k+6)(k-1)}{k+2} \cdot \frac{(k+1)(k+2)}{(k+1)(k-1)}$

19.  $\frac{n^2+n-2}{n+2} \cdot \frac{4n}{n-1}$

12.  $\frac{12c^3}{21b} \cdot \frac{14b^2}{6c}$

14.  $\frac{5c^3d}{c^4d} \cdot \frac{f^2d^3c}{10cf^4}$

16.  $\frac{(a+4)(a-5)}{a^2} \cdot \frac{6a}{a+4}$

18.  $\frac{(r-8)(r+3)}{r} \cdot \frac{2r}{(r+8)(r+3)}$

20.  $\frac{y^2-1}{y^2-49} \cdot \frac{y-7}{y+1}$

### Example 2

p. 693

21. **FINANCIAL LITERACY** A scarf bought in Italy cost 18 Euros. The exchange rate at the time was 1 U.S. dollar = 0.73 Euro.
- How much did the scarf cost in U.S. dollars?
  - If the exchange rate at the time was 1 Canadian dollar = 0.69 Euro, how much did the scarf cost in Canadian dollars?
22. **ROLLER COASTERS** A roller coaster has 6 trains. Each train has 3 cars, and each car seats 4 people. Write and simplify an expression including units to find the total number of people that can ride the roller coaster at one time.

### Examples 3–5

pp. 693–694

Find each quotient.

23.  $\frac{x^5}{y} \div \frac{x}{y^2}$

26.  $\frac{f^4g^2h}{x^2y} \div f^3g$

29.  $\frac{5x^2}{x^2-5x+4} \div \frac{10x}{x-1}$

31.  $\frac{r+2}{r+1} \div \frac{4}{r^2+3r+2}$

24.  $\frac{3r^4}{k^2} \div \frac{18r^3}{k}$

27.  $\frac{6b-12}{b+5} \div (12b+18)$

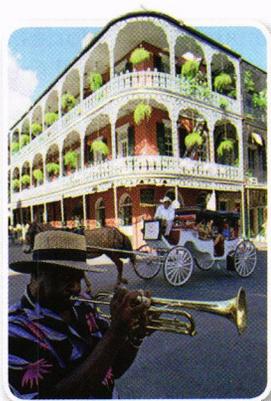
30.  $\frac{n^2+7n+12}{16n^2} \div \frac{n+3}{2n}$

32.  $\frac{3a}{a^2+2a+1} \div \frac{a-1}{a+1}$

25.  $\frac{21b^3}{4c^2} \div \frac{7}{6c^2}$

28.  $\frac{k+3}{k+2} \div \frac{k}{5k+10}$

33. **BEARS** A grizzly bear runs 110 feet in 5 seconds. What is the average speed of the bear in miles per hour?
34. **SEWING** The fabric that Megan wants to buy for a costume she is making costs \$7.50 per yard. How many yards can she buy with \$24?
35. **TRAVEL** An airplane is making a 1250-mile trip. Its average speed is 540 miles per hour.
- Write a division expression you can use to find the number of hours that the trip will take. Include the units.
  - Find the quotient. Round to the nearest tenth.
36. **VOLUNTEERING** Tyree is passing out orange drink from a 3.5-gallon cooler. If each cup of orange drink is 4.25 ounces, about how many cups can he hand out? (*Hint:* There are 128 ounces in a gallon.)



### Real-World Link

Much of New Orleans sits 11 feet below sea level. Parts of the French Quarter have sunk as much as 2 feet in the past six decades.

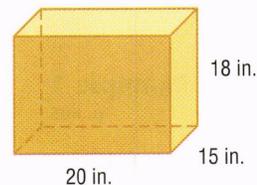
Source: Imaginova

37. **LAND** Louisiana loses about 30 square miles of land each year to coastal erosion, hurricanes, and other natural and human causes. Approximately how many square yards of land are lost per month? (*Hint:* Use 1 square mile = 3,097,600 square yards.)
38. **GEOMETRY** Write an expression to represent the length of the rectangle.

$$A = x^2 + 2x - 24 \quad \frac{x-4}{x-3}$$

Convert each rate. Round to the nearest tenth.

39. 46 feet per second to miles per hour
40. 29.5 meters per second to kilometers per hour
41. 28 milliliters per second to cups per minute. (*Hint:* 1 liter  $\approx$  0.908 quarts)
42. 32.4 meters per second to miles per hour. (*Hint:* 1 mile  $\approx$  1.609 kilometers)
43. **LIFE SCIENCE** A human heart pumps about a cup of blood each time it beats. On average, a person's heart beats about 70 times a minute. Write and simplify an expression to find how many gallons of blood are pumped per hour.
44. **GEOMETRY** Refer to the prism at the right.
- Find the volume in cubic inches.
  - Use the ratio  $\frac{1 \text{ foot}^3}{1728 \text{ inches}^3}$  to write a multiplication expression to convert the volume to cubic feet. Then convert the volume.



Find each product. Describe what the final answer represents.

45.  $\frac{\$9.80}{1 \text{ hour}} \cdot \frac{15 \text{ hours}}{1 \text{ week}} \cdot \frac{52 \text{ weeks}}{1 \text{ year}}$
46.  $\frac{\$2.85}{1 \text{ gallon of gasoline}} \cdot \frac{15 \text{ gallons of gasoline}}{1 \text{ fill-up}} \cdot \frac{3 \text{ fill-ups}}{1 \text{ month}} \cdot \frac{1 \text{ month}}{30 \text{ days}}$
47.  $\frac{32 \text{ meters}}{1 \text{ second}} \cdot \frac{60 \text{ seconds}}{1 \text{ minute}} \cdot \frac{60 \text{ minutes}}{1 \text{ hour}} \cdot \frac{1 \text{ kilometer}}{1000 \text{ meters}} \cdot \frac{1 \text{ mile}}{1.609 \text{ kilometers}}$
48.  $\frac{\$32,000}{1 \text{ year}} \cdot \frac{1 \text{ year}}{52 \text{ weeks}} \cdot \frac{1 \text{ week}}{40 \text{ hours}}$
49. **SPACE** The highest speed at which any spacecraft has ever escaped from Earth is 35,800 miles per hour by the *New Horizons* probe, which was launched in 2006. Convert this speed to feet per second. Round to the nearest tenth.



### Real-World Link

The largest cylindrical aquarium ever built is located in a hotel lobby in Berlin. It is 25 meters high and contains 900,000 liters of seawater and 2500 tropical fish.

Source: South Travels

50. **ELECTRICITY** Simplify the expression below to find the cost of running a 3500-watt air conditioner for one week.

$$3500 \text{ watts} \cdot \frac{1 \text{ kilowatt}}{1000 \text{ watts}} \cdot \frac{168 \text{ hours}}{1 \text{ week}} \cdot \frac{10 \text{ cents}}{1 \text{ kilowatt} \cdot \text{hours}} \cdot \frac{1 \text{ dollar}}{100 \text{ cents}}$$

51. **AMUSEMENT PARKS** In an amusement park ride, riders stand along the wall of a circular room with a radius of 3.1 meters. The room completes 27 rotations per minute.

- Write an expression for the number of meters the room moves per second. (*Hint: The circumference of a circle is  $2\pi r$ .*)
- Simplify the expression you wrote in Part a and describe what it means.

52. **AQUARIUMS** An aquarium is a rectangular prism 30 inches long, 15 inches wide, and 18 inches high.

- Sketch and label a diagram of the aquarium. Then find the volume of the tank in cubic inches.
- Describe how to use the ratio  $\frac{1 \text{ ft}^3}{1728 \text{ in}^3}$  to find the volume of the tank in cubic feet. Then find the volume. Round to the nearest tenth.
- Water weighs 62 pounds per cubic foot. How much would the water in the tank weigh if the tank were filled?

### H.O.T. Problems

Use Higher-Order Thinking Skills

53. **FIND THE ERROR** Mei and Tamika are finding  $\frac{2x+6}{x+5} \div \frac{2}{x+5}$ . Is either of them correct? Explain.

*Mei*

$$\begin{aligned} \frac{2x+6}{x+5} \div \frac{2}{x+5} \\ = \frac{2x+6}{x+5} \cdot \frac{x+5}{2} \\ = x+6 \end{aligned}$$

*Tamika*

$$\begin{aligned} \frac{2x+6}{x+5} \div \frac{2}{x+5} \\ = \frac{2(x+3)}{x+5} \cdot \frac{2}{x+5} \\ = \frac{4(x+3)}{(x+5)^2} \end{aligned}$$

54. **REASONING** Find the missing term. Justify your answer.

$$\frac{?}{21} \div \frac{10x^3}{21} = \frac{3}{2x}$$

55. **CHALLENGE** Find  $\frac{x^2-3x-10}{x^2+2x-35} \cdot \frac{x^2+4x-21}{x^2+9x+14}$ . Write in simplest form.

56. **WRITING IN MATH** Give an example and describe how you could use dimensional analysis to solve a real-world problem involving rational expressions.

57. **OPEN ENDED** Give an example of a real-world situation that could be modeled by the quotient of two rational expressions. Provide an example of this quotient.

58. **WRITE A QUESTION** A classmate found that the product of two rational expressions is  $\frac{9x-3}{(x+3)(3x+1)}$ . She wants to find the excluded values. Write a question to help her solve the problem.

59. **WRITING IN MATH** Describe how to use dimensional analysis to find the number of hours in one year.

## Standardized Test Practice

**60. GEOMETRY** The perimeter of a rectangle is 30 inches. Its area is 54 square inches. Find the length of the longest side.

- A 6 inches
- B 9 inches
- C 12 inches
- D 30 inches

**61.** Find  $\frac{c^2 - c - 6}{2c - 10} \div \frac{2c + 4}{3c - 15}$ .

F  $\frac{3(c-3)}{4}$

H  $\frac{4(c-3)}{3}$

G  $\frac{c+5}{c-3}$

J  $\frac{c-3}{c-5}$

**62. EXTENDED RESPONSE** The weekly salaries of six employees at a fast food restaurant are \$140, \$220, \$90, \$180, \$140, \$200.

- a. What is the mean of the six salaries?
- b. What is the median of the six salaries?
- c. What is the mode of the six salaries?

**63.** Tito has three times as many CDs as Dasan. Dasan has two thirds as many CDs as Brant. Brant has 27 CDs. How many CDs does Tito have?

A 54

C 18

B 27

D 32

## Spiral Review

Simplify each expression. State the excluded values of the variables. (Lesson 11-3)

**64.**  $\frac{20x^2y}{25xy}$

**65.**  $\frac{14g^3h^2}{42gh^3}$

**66.**  $\frac{64qt}{16q^2t^3}$

**67.**  $\frac{y^2 + 10y + 16}{y + 2}$

**68.**  $\frac{p^2 - 9}{p^2 - 5p + 6}$

**69.**  $\frac{z^2 + z - 2}{z^2 - 3z + 2}$

Identify the asymptotes of each function. (Lesson 11-2)

**70.**  $y = \frac{2}{x}$

**71.**  $y = \frac{3}{x} + 5$

**72.**  $y = \frac{1}{x-5} - 4$

**73.**  $y = \frac{1}{x+3}$

**74.**  $y = \frac{-1}{x+6} + 7$

**75.**  $y = \frac{2}{x-8} - 3$

**76. FORESTRY** The number of board feet  $B$  that a log will yield can be estimated by using the formula  $B = \frac{L}{16}(D^2 - 8D + 16)$ , where  $D$  is the diameter in inches and  $L$  is the log length in feet. For logs that are 16 feet long, what diameter will yield approximately 256 board feet? (Lesson 8-6)

Find the degree of each polynomial. (Lesson 7-4)

**77.** 2

**78.**  $-3a$

**79.**  $5x^2 + 3x$

**80.**  $d^4 - 6c^2$

**81.**  $2x^3 - 4z + 8xz$

**82.**  $3d^4 + 5d^3 - 4c^2 + 1$

**83. DRIVING** Tires should be kept within 2 pounds per square inch (psi) of the manufacturer's recommended tire pressure. If the recommendation for a tire is 30 psi, what is the range of acceptable pressures? (Lesson 5-5)

## Skills Review

Factor each polynomial. (Lessons 8-3 and 8-4)

**84.**  $x^2 - 18x - 40$

**85.**  $x^2 - 5x + 6$

**86.**  $x^2 - 2x - 24$

**87.**  $3x^2 + 7x - 20$

**88.**  $2x^2 + x - 15$

**89.**  $8x^2 - 4x - 40$

1. Determine whether the table represents an inverse variation. Explain. (Lesson 11-1)

$x$	$y$
2	8
4	4
8	2
16	1

Assume that  $y$  varies inversely as  $x$ . Write an inverse variation equation that relates  $x$  and  $y$ . (Lesson 11-1)

2.  $y = 5$  when  $x = 10$   
 3.  $y = -2$  when  $x = 12$

Solve. Assume that  $y$  varies inversely as  $x$ . (Lesson 11-1)

4. If  $y = 6$  when  $x = 3$ , find  $x$  when  $y = 5$ .  
 5. If  $y = 3$  when  $x = 2$ , find  $y$  when  $x = 4$ .

State the excluded value for each function. (Lesson 11-2)

6.  $y = \frac{2}{x}$   
 7.  $y = \frac{1}{x-6}$

Identify the asymptotes of each function. (Lesson 11-2)

8.  $y = \frac{3}{2x+4}$   
 9.  $y = \frac{2}{x-4}$   
 10. **MULTIPLE CHOICE** Jorge has  $x^2 + 8x + 15$  square yards of carpet. He wants to carpet rooms that have areas of  $x^2 + 5x + 6$  square yards. Write and simplify an expression to show how many rooms he can carpet. (Lesson 11-3)
- A  $\frac{x+5}{x+3}$   
 B  $\frac{x+5}{x+2}$   
 C  $\frac{x+3}{x+2}$   
 D  $\frac{x+5}{x+6}$

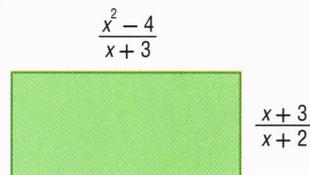
Simplify each expression. State the excluded values of the variables. (Lesson 11-3)

11.  $\frac{16x^2y^3}{8xy}$   
 12.  $\frac{z-5}{z^2-7z+10}$   
 13.  $\frac{3x-15}{x^2-25}$

Find each product. (Lesson 11-4)

14.  $\frac{(x+5)(x-3)}{x^3} \cdot \frac{5x}{x-3}$   
 15.  $\frac{a^2+2a+1}{a+1} \cdot \frac{a-1}{a^2-1}$   
 16.  $\frac{m}{m^2+3m+2} \cdot \frac{m+2}{m^2}$

17. **MULTIPLE CHOICE** Find the area of the rectangle. (Lesson 11-4)



- F  $\frac{x+2}{x-2}$   
 G  $\frac{x+3}{x-2}$   
 H 1  
 J  $x-2$

Find each quotient. (Lesson 11-4)

18.  $\frac{x^4}{y^2} \div \frac{x}{y}$   
 19.  $\frac{x+3}{2x+6} \div \frac{3x-6}{4x-8}$   
 20.  $\frac{x^2+7x+12}{x^2-25} \div \frac{x^2-9}{2x+10}$

21. **MOTOR VEHICLES** In 2005, the U.S. produced 4,411,300 motor vehicles. This was 10% of the total motor vehicle production for the whole world. How many motor vehicles were produced worldwide in 2005?

## Dividing Polynomials

### Then

You divided rational expressions.

(Lesson 11-5)

### Now

- Divide a polynomial by a monomial.
- Divide a polynomial by a binomial.

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### Why?

The equation below describes the distance  $d$  a horse travels when its initial velocity is 4 m/s, its final velocity is  $v$  m/s, and its acceleration is  $a$  m/s<sup>2</sup>.

$$d = \frac{v^2 - 4^2}{2a}$$

There are different ways to simplify the expression.

**Keep as one fraction.**

$$\frac{v^2 - 4^2}{2a} = \frac{v^2 - 16}{2a}$$

**Divide each term by  $2a$ .**

$$\begin{aligned} \frac{v^2 - 4^2}{2a} &= \frac{v^2}{2a} - \frac{4^2}{2a} \\ &= \frac{v^2}{2a} - \frac{8}{a} \end{aligned}$$



**Divide Polynomials by Monomials** To divide a polynomial by a monomial, divide each term of the polynomial by the monomial.

### EXAMPLE 1 Divide Polynomials by Monomials

Find each quotient.

a.  $(2x^2 + 16x) \div 2x$

$$(2x^2 + 16x) \div 2x = \frac{2x^2 + 16x}{2x}$$

**Write as a fraction.**

$$= \frac{2x^2}{2x} + \frac{16x}{2x}$$

**Divide each term by  $2x$ .**

$$= \frac{\overset{x}{\cancel{2x^2}}}{\underset{1}{\cancel{2x}}} + \frac{\overset{8}{\cancel{16x}}}{\underset{1}{\cancel{2x}}}$$

**Divide out common factors.**

$$= x + 8$$

**Simplify.**

b.  $(b^2 + 12b - 14) \div 3b$

$$(b^2 + 12b - 14) \div 3b = \frac{b^2 + 12b - 14}{3b}$$

**Write as a fraction.**

$$= \frac{b^2}{3b} + \frac{12b}{3b} - \frac{14}{3b}$$

**Divide each term by  $3b$ .**

$$= \frac{\overset{b}{\cancel{b^2}}}{\underset{3}{\cancel{3b}}} + \frac{\overset{4}{\cancel{12b}}}{\underset{1}{\cancel{3b}}} - \frac{14}{3b}$$

**Divide out common factors.**

$$= \frac{b}{3} + 4 - \frac{14}{3b}$$

**Simplify.**

### Check Your Progress

1A.  $(3q^3 - 6q) \div 3q$

1B.  $(4t^5 - 5t^2 - 12) \div 2t^2$

1C.  $(4r^6 + 3r^4 - 2r^2) \div 2r$

1D.  $(6w^3 - 3w) \div 4w^2$

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**Divide Polynomials by Binomials** You can also divide polynomials by binomials. When a polynomial can be factored and common factors can be divided out, write the division as a rational expression and simplify.

**EXAMPLE 2 Divide a Polynomial by a Binomial**

Find  $(h^2 + 9h + 18) \div (h + 6)$ .

$$\begin{aligned} (h^2 + 9h + 18) \div (h + 6) &= \frac{h^2 + 9h + 18}{h + 6} \\ &= \frac{(h + 3)(h + 6)}{h + 6} \\ &= \frac{(h + 3)\cancel{(h + 6)}}{\cancel{h + 6}} \\ &= h + 3 \end{aligned}$$

Write as a rational expression.

Factor the numerator.

Divide out common factors.

Simplify.

**Check Your Progress**

Find each quotient.

2A.  $(b^2 - 2b - 15) \div (b + 3)$

2B.  $(x^2 + 11x + 24) \div (x + 8)$

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If the polynomial cannot be factored or if there are no common factors by which to divide, you must use long division.

**EXAMPLE 3 Use Long Division**

Find  $(y^2 + 4y + 12) \div (y + 3)$  by using long division.

**Step 1** Divide the first term of the dividend,  $y^2$ , by the first term of the divisor,  $y$ .

$$\begin{array}{r} y \\ y + 3 \overline{) y^2 + 4y + 12} \\ \underline{(-) y^2 + 3y} \phantom{+ 12} \\ 1y + 12 \end{array} \quad \begin{array}{l} y^2 \div y = y \\ \text{Multiply } y \text{ and } y + 3 \\ \text{Subtract. Bring down the 12.} \end{array}$$

**Step 2** Divide the first term of the partial dividend,  $1y$ , by the first term of the divisor,  $y$ .

$$\begin{array}{r} y + 1 \\ y + 3 \overline{) y^2 + 4y + 12} \\ \underline{(-) y^2 + 3y} \phantom{+ 12} \\ 1y + 12 \\ \underline{(-) y + 3} \phantom{+ 12} \\ 9 \end{array} \quad \begin{array}{l} \text{Subtract. Bring down the 12.} \\ \text{Multiply 1 and } y + 3. \\ \text{Subtract.} \end{array}$$

So,  $(y^2 + 4y + 12) \div (y + 3)$  is  $y + 1$  with a remainder of 9. This answer can be written as  $y + 1 + \frac{9}{y + 3}$ .

**Check Your Progress**

3A.  $(3x^2 + 9x - 15) \div (x + 5)$

3B.  $(n^2 + 6n + 2) \div (n - 2)$

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**Watch Out!**

**Polynomials** When using long division, be sure the dividend is written in standard form. That is, the terms are written so that the exponents decrease from left to right.

$y^2 + 4y + 12$     yes  
 $4y + y^2 + 12$     no

### Real-World EXAMPLE 4 Divide Polynomials to Solve a Problem

**PARTIES** The expression  $5x + 250$  represents the cost of renting a picnic shelter and food for  $x$  people. The total cost is divided evenly among all the people except for the two who bought decorations. Find  $(5x + 250) \div (x - 2)$  to determine how much each person pays.

$$\begin{array}{r} 5 \\ x - 2 \overline{) 5x + 250} \\ \underline{(-) 5x - 10} \phantom{0} \\ 260 \phantom{0} \end{array}$$

So,  $5 + \frac{260}{x-2}$  represents the amount each person pays.

#### Check Your Progress

4. **GEOMETRY** The area of a rectangle is  $(2x^2 + 10x - 1)$  square units, and the width is  $(x + 1)$  units. What is the length?

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When a dividend is written in standard form and a power is missing, add a term of that power with a coefficient of zero.

### EXAMPLE 5 Insert Missing Terms

Find  $(c^3 + 5c - 6) \div (c - 1)$ .

$$\begin{array}{r} c^2 + c + 6 \\ c - 1 \overline{) c^3 + 0c^2 + 5c - 6} \\ \underline{(-) c^3 - c^2} \phantom{0} \\ c^2 + 5c \phantom{0} \\ \underline{(-) c^2 - c} \phantom{0} \\ 6c - 6 \phantom{0} \\ \underline{(-) 6c - 6} \\ 0 \end{array}$$

Insert a  $c^2$ -term that has a coefficient of 0.

Multiply  $c^2$  and  $c - 1$ .

Subtract. Bring down the  $5c$ .

Multiply  $c$  and  $c - 1$ .

Subtract. Bring down the  $-6$ .

Multiply  $6$  and  $c - 1$ .

Subtract.

So,  $(c^3 + 5c - 6) \div (c - 1) = c^2 + c + 6$ .

#### Check Your Progress Find each quotient.

5A.  $(2r^3 + 2r^2 - 4) \div (r - 1)$

5B.  $(x^4 + 2x^3 + 6x - 10) \div (x + 2)$

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## Check Your Understanding

### Examples 1 and 2 pp. 700-701

Find each quotient.

1.  $(8a^2 + 20a) \div 4a$

2.  $(4z^3 + 1) \div 2z$

3.  $(12n^3 - 6n^2 + 15) \div 6n$

4.  $(t^2 + 5t + 4) \div (t + 4)$

5.  $(x^2 + 3x - 28) \div (x + 7)$

6.  $(x^2 + x - 20) \div (x - 4)$

### Example 4 p. 702

7. **CHEMISTRY** The formula  $y = \frac{400 + 3x}{50 + x}$  describes a mixture when  $x$  liters of a 25% solution are added to a 90% solution. Find  $(400 + 3x) \div (50 + x)$ .

### Examples 3 and 5 pp. 701-702

Find each quotient. Use long division.

8.  $(n^2 + 3n + 10) \div (n - 1)$

9.  $(4y^2 + 8y + 3) \div (y + 2)$

10.  $(4h^3 + 6h^2 - 3) \div (2h + 3)$

11.  $(9n^3 - 13n + 8) \div (3n - 1)$

# Practice and Problem Solving

**Step-by-Step Solutions** begin on page R12.  
**Extra Practice** begins on page 815.

## Examples 1 and 2

pp. 700–701

Find each quotient.

12.  $(14x^2 + 7x) \div 7x$

14.  $(5q^3 + q) \div q$

16.  $(8k^2 - 6) \div 2k$

18.  $(a^2 + a - 12) \div (a - 3)$

20.  $(r^2 - 12r + 11) \div (r - 1)$

22.  $(y^2 - 36) \div (y^2 + 6y)$

24.  $(c^3 - 27) \div (c - 3)$

26.  $(6x^3 + 16x^2 - 60x + 39) \div (2x + 10)$

13.  $(a^3 + 4a^2 - 18a) \div a$

15.  $(6n^2 - 12n + 3) \div 3n$

17.  $(9m^2 + 5m) \div 6m$

19.  $(x^2 - 6x - 16) \div (x + 2)$

21.  $(k^2 - 5k - 24) \div (k - 8)$

23.  $(a^3 - 4a^2) \div (a - 4)$

25.  $(4t^2 - 1) \div (2t + 1)$

27.  $(2h^3 + 8h^2 - 3h - 12) \div (h + 4)$

## Example 4

p. 702

28. **GEOMETRY** The area of a rectangle is  $(x^3 - 4x^2)$  square units, and the width is  $(x - 4)$  units. What is the length?

29. **MANUFACTURING** The expression  $-n^2 + 18n + 850$  represents the number of baseball caps produced by  $n$  workers. Find  $(-n^2 + 18n + 850) \div n$  to write an expression for average number of caps produced per person.

## Examples 3 and 5

pp. 701–702

Find each quotient. Use long division.

30.  $(b^2 + 3b - 9) \div (b + 5)$

32.  $(2y^2 - 3y + 1) \div (y - 2)$

34.  $(p^3 - 4p^2 + 9) \div (p - 1)$

36.  $(6x^3 + 5x^2 + 9) \div (2x + 3)$

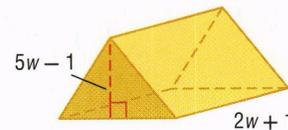
31.  $(a^2 + 4a + 3) \div (a - 1)$

33.  $(4n^2 - 3n + 6) \div (n - 2)$

35.  $(t^3 - 2t - 4) \div (t + 4)$

37.  $(8c^3 + 6c - 5) \div (4c - 2)$

38. **GEOMETRY** The volume of a prism with a triangular base is  $10w^3 + 23w^2 + 5w - 2$ . The height of the prism is  $2w + 1$ , and the height of the triangle is  $5w - 1$ . What is the measure of the base of the triangle? (*Hint:  $V = Bh$* )



Use long division to find the expression that represents the missing length.

39. 
$$\begin{array}{|c|} \hline A = x^2 - 3x - 18 \\ \hline \end{array} \quad ?$$
  
 $x - 6$

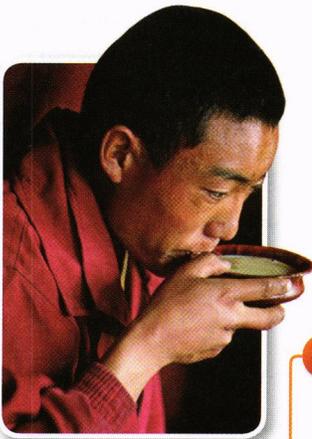
40. 
$$\begin{array}{|c|} \hline A = \\ 4x^2 + 16x + 16 \\ \hline \end{array} \quad 2x + 4$$
  
 $?$

41. Determine the quotient when  $x^3 + 11x + 14$  is divided by  $x + 2$ .

42. What is  $14y^5 + 21y^4 - 6y^3 - 9y^2 + 32y + 48$  divided by  $2y + 3$ ?

43. **FUNCTIONS** Consider  $f(x) = \frac{3x + 4}{x - 1}$ .

- Rewrite the function as a quotient plus a remainder. Then graph the quotient, ignoring the remainder.
- Graph the original function using a graphing calculator.
- How are the graphs of the function and quotient related?
- What happens to the graph near the excluded value of  $x$ ?

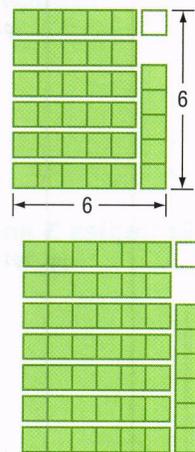


### Real-World Link

The higher the altitude, the lower the temperature at which water boils. Tibet is at a high altitude. The temperature of boiling water in Tibet is lower than that of boiling water in the United States. Tibetans drink their tea while it is still boiling.

Source: *Did You Know?*

- 44. ROAD TRIP** The first Ski Club van has been on the road for 20 minutes, and the second van has been on the road for 35 minutes.
- Write an expression for the amount of time that each van has spent on the road after an additional  $t$  minutes.
  - Write a ratio for the first van's time on the road to the second van's time on the road and use long division to rewrite this ratio as an expression. Then find the ratio of the first van's time on the road to the second van's time on the road after 60 minutes, 200 minutes.
- 45. BOILING POINT** The temperature at which water boils decreases by about  $0.9^\circ\text{F}$  for every 500 feet above sea level. The boiling point at sea level is  $212^\circ\text{F}$ .
- Write an equation for the temperature  $T$  at which water boils  $x$  feet above sea level.
  - Mount Whitney, the tallest point in California, is 14,494 feet above sea level. At approximately what temperature does water boil on Mount Whitney?
- 46. MULTIPLE REPRESENTATIONS** In this problem, you will use picture models to help divide expressions.
- ANALYTICAL** The first figure models  $6^2 \div 7$ . Notice that the square is divided into seven equal parts. What are the quotient and the remainder? What division problem does the second figure model?
  - CONCRETE** Draw figures for  $3^2 \div 4$  and  $2^2 \div 3$ .
  - VERBAL** Do you observe a pattern in the previous exercises? Express this pattern algebraically.
  - ANALYTICAL** Use long division to find  $x^2 \div (x + 1)$ . Does this result match your expression from part c?



### H.O.T. Problems

Use **H**igher-**O**rders **T**hinking Skills

- 47. FIND THE ERROR** Alvin and Andrea are dividing  $c^3 + 6c - 4$  by  $c + 2$ . Is either of them correct? Explain your reasoning.

*Alvin*

$$\begin{array}{r}
 c^2 + 4c - 12 \\
 c + 2 \overline{) c^3 + 6c - 4} \\
 \underline{c^3 + 2c^2} \phantom{- 4} \\
 4c^2 - 4 \\
 \underline{4c^2 + 8c} \phantom{- 4} \\
 -12c - 4 \\
 \underline{-12c - 24} \\
 24
 \end{array}$$

*Andrea*

$$\begin{array}{r}
 c^2 - 2c + 10 \\
 c + 2 \overline{) c^3 + 0c^2 + 6c - 4} \\
 \underline{c^3 + 2c^2} \phantom{- 4} \\
 -2c^2 + 6c - 4 \\
 \underline{-2c^2 - 4c} \phantom{- 4} \\
 10c - 4 \\
 \underline{10c + 20} \\
 -24
 \end{array}$$

- 48. CHALLENGE** The quotient of two polynomials is  $4x^2 - x - 7 + \frac{11x + 15}{x^2 + x + 2}$ . What are the polynomials?
- 49. OPEN ENDED** Write a division problem involving polynomials that you would solve by using long division. Explain your answer.
- 50. WRITING IN MATH** Describe the steps to find  $(w^2 - 2w - 30) \div (w + 7)$ .

## Standardized Test Practice

51. Simplify  $\frac{21x^3 - 35x^2}{7x}$ .

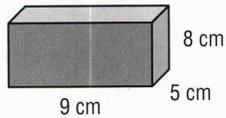
A  $3x^2 - 5x$

C  $3x - 5$

B  $4x^2 - 6x$

D  $5x - 3$

52. **EXTENDED RESPONSE** The box shown is designed to hold rice.



- a. How much rice would fit in the box?  
b. What is the area of the label on the box, if the label covers all surfaces?

53. Simplify  $\frac{x^2 + 7x + 12}{x^2 + 5x + 6}$ .

F  $x + 4$

H  $x + 2$

G  $\frac{x+4}{x+2}$

J  $\frac{x+2}{x+4}$

54. Susana bought cards at 6 for \$10. She decorated them and sold them at 4 for \$10. She made \$60 in profit. How many cards did she sell?

A 53

C 60

B 25

D 72

## Spiral Review

Find each product. (Lesson 11-4)

55.  $\frac{3x^3}{8x} \cdot \frac{16}{x}$

56.  $\frac{3ad}{4c^4} \cdot \frac{8c^2}{6d}$

57.  $\frac{t^2}{(t-4)(t+4)} \cdot \frac{t-4}{6t}$

58.  $\frac{10}{r-2} \cdot \frac{r^2-4}{2}$

Find the zeros of each function. (Lesson 11-3)

59.  $f(x) = \frac{x+2}{x^2-6x+8}$

60.  $f(x) = \frac{x^2-3x-4}{x^2-x-12}$

61.  $f(x) = \frac{x^2+6x+9}{x^2-9}$

62. **SHADOWS** A 25-foot flagpole casts a shadow that is 10 feet long and a nearby building casts a shadow that is 26 feet long. How tall is the building? (Lesson 10-7)

Solve each equation. Check your solution. (Lesson 10-4)

63.  $\sqrt{h} = 9$

64.  $\sqrt{x+3} = -5$

65.  $3 + 5\sqrt{n} = 18$

66.  $\sqrt{x-5} = 2\sqrt{6}$

Solve each equation by using the Quadratic Formula. Round to the nearest tenth if necessary. (Lesson 9-5)

67.  $v^2 + 12v + 20 = 0$

68.  $3t^2 - 7t - 20 = 0$

69.  $5y^2 - y - 4 = 0$

70.  $2x^2 + 98 = 28x$

71.  $2n^2 - 7n - 3 = 0$

72.  $2w^2 = -(7w + 3)$

73. **THEATER** The drama club is building a backdrop using arches with a shape that can be represented by the function  $f(x) = -x^2 + 2x + 8$ , where  $x$  is the length of the arch in feet. The region under each arch is to be covered with fabric. (Lesson 9-2)

- a. Graph the quadratic function and determine its  $x$ -intercepts.  
b. What is the height of the arch?

## Skills Review

Find each sum. (Lesson 7-5)

74.  $(3a^2 + 2a - 12) + (8a + 7 - 2a^2)$

75.  $(2c^3 + 3cd - d^2) + (-5cd - 2c^3 + 2d^2)$

Find the least common multiple for each set of numbers. (Lesson 8-1)

76. 2, 4, 6

77. 3, 6, 8

78. 5, 12, 15

79. 14, 18, 24

## Adding and Subtracting Rational Expressions

### Then

You added and subtracted polynomials. (Lesson 7-5)

### Now

- Add and subtract rational expressions with like denominators.
- Add and subtract rational expressions with unlike denominators.

### New Vocabulary

least common multiple (LCM)  
least common denominator (LCD)

### Math Online

glencoe.com

- Extra Examples
- Personal Tutor
- Self-Check Quiz
- Homework Help

### Why?

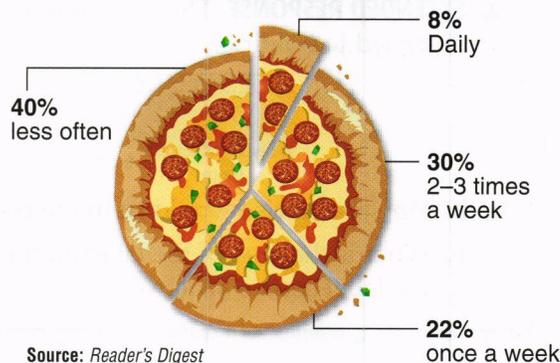
A survey asked families how often they eat takeout. To determine the fraction of those surveyed who eat takeout more than once a week, you can add. Remember that percents can be written as fractions with denominators of 100.

$$\begin{array}{l} \text{2-3 times} \\ \text{a week} \end{array} \text{ plus } \begin{array}{l} \text{daily} \\ \text{8\%} \end{array} \text{ equals } \begin{array}{l} \text{more than} \\ \text{once a week.} \\ \text{38\%} \end{array}$$

$$\frac{30}{100} + \frac{8}{100} = \frac{38}{100}$$

Thus,  $\frac{38}{100}$  or 38% eat takeout more than once a week.

### How Many Times a Week Families Eat Takeout



**Add and Subtract Rational Expressions with Like Denominators** To add or subtract rational expressions that have the same denominator, add or subtract the numerators and write the sum or difference over the common denominator.

### Key Concept

For Your FOLDABLE

#### Add or Subtract Rational Expressions with Like Denominators

Let  $a$ ,  $b$ , and  $c$  be polynomials with  $c \neq 0$ .

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c} \qquad \frac{a}{c} - \frac{b}{c} = \frac{a-b}{c}$$

### EXAMPLE 1 Add Rational Expressions with Like Denominators

Find  $\frac{5n}{n+3} + \frac{15}{n+3}$ .

$$\begin{aligned} \frac{5n}{n+3} + \frac{15}{n+3} &= \frac{5n+15}{n+3} \\ &= \frac{5(n+3)}{n+3} \\ &= \frac{5(\cancel{n+3})}{\cancel{n+3}} \\ &= \frac{5}{1} \text{ or } 5 \end{aligned}$$

The common denominator is  $n+3$ .

Factor the numerator.

Divide by the common factor,  $n+3$ .

Simplify.

### Check Your Progress

Find each sum.

1A.  $\frac{8c}{6} + \frac{5c}{6}$

1B.  $\frac{4t}{5xy} + \frac{7}{5xy}$

1C.  $\frac{3y}{3+y} + \frac{y^2}{3+y}$

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**Watch Out!****Common Terms**

Remember that every term of the numerator and the denominator must be multiplied or divided by the same number for the fraction to be equivalent to the original.

**EXAMPLE 2 Subtract Rational Expressions with Like Denominators**

Find  $\frac{3m - 5}{m + 4} - \frac{4m + 2}{m + 4}$ .

$$\begin{aligned} \frac{3m - 5}{m + 4} - \frac{4m + 2}{m + 4} &= \frac{(3m - 5) - (4m + 2)}{m + 4} \\ &= \frac{(3m - 5) + [-(4m + 2)]}{m + 4} \\ &= \frac{3m - 5 - 4m - 2}{m + 4} \\ &= \frac{-m - 7}{m + 4} \end{aligned}$$

The common denominator is  $m + 4$ .

The additive inverse of  $(4m + 2)$  is  $-(4m + 2)$ .

Distributive Property

Simplify.

**Check Your Progress**

Find each difference.

2A.  $\frac{2h + 4}{h + 1} - \frac{5 + h}{h + 1}$

2B.  $\frac{17h + 4}{15h - 5} - \frac{2h - 6}{15h - 5}$

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You can sometimes use additive inverses to form like denominators.

**EXAMPLE 3 Inverse Denominators**

Find  $\frac{3n}{n - 4} + \frac{6n}{4 - n}$ .

$$\begin{aligned} \frac{3n}{n - 4} + \frac{6n}{4 - n} &= \frac{3n}{n - 4} + \frac{6n}{-(n - 4)} \\ &= \frac{3n}{n - 4} - \frac{6n}{n - 4} \\ &= \frac{3n - 6n}{n - 4} \text{ or } -\frac{3n}{n - 4} \end{aligned}$$

Rewrite  $4 - n$  as  $-(n - 4)$ .

Rewrite so the denominators are the same.

Subtract the numerators and simplify.

**Check Your Progress**

Find each sum or difference.

3A.  $\frac{t^2}{t - 3} + \frac{3}{3 - t}$

3B.  $\frac{2p}{p - 1} - \frac{2p}{1 - p}$

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**Add and Subtract with Unlike Denominators** The **least common multiple (LCM)** is the least number that is a multiple of two or more numbers or polynomials.

**EXAMPLE 4 LCMs of Polynomials**

Find the LCM of each pair of polynomials.

a.  $6x$  and  $4x^3$

**Step 1** Find the prime factors of each expression.

$$6x = 2 \cdot 3 \cdot x$$

$$4x^3 = 2 \cdot 2 \cdot x \cdot x \cdot x$$

**Step 2** Use each prime factor, 2, 3, and  $x$ , the greatest number of times it appears in either of the factorizations.

$$6x = 2 \cdot 3 \cdot x$$

$$4x^3 = 2 \cdot 2 \cdot x \cdot x \cdot x$$

$$\text{LCM} = 2 \cdot 2 \cdot 3 \cdot x \cdot x \cdot x \text{ or } 12x^3$$

## Review Vocabulary

### Factored Form

A monomial is in factored form when it is expressed as the product of prime numbers and variables, and no variable has an exponent greater than 1. (Lesson 8-1)

b.  $n^2 + 5n + 4$  and  $(n + 1)^2$

$$n^2 + 5n + 4 = (n + 1)(n + 4) \quad \text{Factor each expression.}$$

$$(n + 1)^2 = (n + 1)(n + 1)$$

$(n + 1)$  is a factor twice in the second expression.  $(n + 4)$  is a factor once.

$$\text{LCM} = (n + 1)(n + 1)(n + 4) \text{ or } (n + 1)^2(n + 4)$$

### Check Your Progress

4A.  $8m^2t$  and  $12m^2t^3$

4B.  $x^2 - 2x - 8$  and  $x^2 - 5x - 14$

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To add or subtract fractions with unlike denominators, you need to rename the fractions using the least common multiple of the denominators, called the **least common denominator (LCD)**.

## Key Concept

For Your  
**FOLDABLE**

### Add or Subtract Rational Expressions with Unlike Denominators

**Step 1** Find the LCD.

**Step 2** Write each rational expression as an equivalent expression with the LCD as the denominator.

**Step 3** Add or subtract the numerators and write the result over the common denominator.

**Step 4** Simplify if possible.

## StudyTip

### Checking Answers

You can check whether you have simplified a rational expression correctly by substituting values, but this does not guarantee that the expressions are always equal. If the results are different, check for an error.

### EXAMPLE 5 Add Rational Expressions with Unlike Denominators

Find  $\frac{3t + 2}{t^2 - 2t - 3} + \frac{t + 1}{t - 3}$ .

Find the LCD. Since  $t^2 - 2t - 3 = (t - 3)(t + 1)$ , the LCD is  $(t - 3)(t + 1)$ .

$$\frac{3t + 2}{t^2 - 2t - 3} + \frac{t + 1}{t - 3} = \frac{3t + 2}{(t - 3)(t + 1)} + \frac{t + 1}{t - 3}$$

Factor  $t^2 - 2t - 3$ .

$$= \frac{3t + 2}{(t - 3)(t + 1)} + \frac{t + 1}{t - 3} \left( \frac{t + 1}{t + 1} \right)$$

Write  $\frac{t + 1}{t - 3}$  using the LCD.

$$= \frac{3t + 2}{(t - 3)(t + 1)} + \frac{t^2 + 2t + 1}{(t - 3)(t + 1)}$$

Simplify.

$$= \frac{3t + 2 + t^2 + 2t + 1}{(t - 3)(t + 1)}$$

Add the numerators.

$$= \frac{t^2 + 5t + 3}{(t - 3)(t + 1)}$$

Simplify.

### Check Your Progress

 Find each sum.

5A.  $\frac{4d^2}{d} + \frac{d + 2}{d^2}$

5B.  $\frac{b + 3}{b} + \frac{b - 5}{b + 1}$

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The formula  $\text{time} = \frac{\text{distance}}{\text{rate}}$  is helpful in solving real-world applications.

### Real-World EXAMPLE 6 Add Rational Expressions

**HANG GLIDING** For the first 5000 meters, a hang glider travels at a rate of  $x$  meters per minute. Then, due to a stronger wind, it travels 6000 meters at a speed that is 3 times as fast.

- a. Write an expression to represent how much time the hang glider is flying.

**Understand** For the first 5000 meters, the hang glider's speed is  $x$ . For the last 6000 meters, the hang glider's speed is  $3x$ .

**Plan** Use the formula  $d = r \times t$  or  $t = \frac{d}{r}$  to represent the time  $t$  of each section of the hang glider's trip, with rate  $r$  and distance  $d$ .

**Solve** Time to fly 5000 meters:  $\frac{d}{r} = \frac{5000}{x}$   $d = 5000, r = x$   
 Time to fly 6000 meters:  $\frac{d}{r} = \frac{6000}{3x}$   $d = 6000, r = 3x$

Total flying time:  $\frac{5000}{x} + \frac{6000}{3x}$

$$\begin{aligned} \frac{5000}{x} + \frac{6000}{3x} &= \frac{5000}{x} \left(\frac{3}{3}\right) + \frac{6000}{3x} \\ &= \frac{15,000}{3x} + \frac{6000}{3x} \\ &= \frac{21,000}{3x} \text{ or } \frac{7000}{x} \end{aligned}$$

**Check**  $\frac{5000}{x} + \frac{6000}{3x} = \frac{5000}{1} + \frac{6000}{3(1)}$   
 $= 5000 + 2000$  or  $7000$   
 $\frac{7000}{x} = \frac{7000}{1}$  or  $7000$

The LCD is  $3x$ .

Multiply.

Simplify.

Let  $x = 1$  in the original expression.

Simplify.

Let  $x = 1$  in the answer expression. Simplify.

Since the expressions have the same value for  $x = 1$ , the answer is reasonable. ✓

- b. If the hang glider is flying at a rate of 600 meters per minute for the first 5000 meters, find the total amount of time that the hang glider is flying.

$\frac{7000}{x} = \frac{7000}{600}$  Substitute 600 for  $x$  in the expression.  
 $\approx 11.7$  Simplify.

So, the hang glider is flying for approximately 11.7 minutes.

- c. If the hang glider flew for approximately 15 minutes, find the rate the hang glider flew for the first 5000 meters.

$\frac{7000}{x} = 15$  Set the expression equal to 15.  
 $7000 = 15x$  Multiply each side by  $x$ .  
 $466.7 \approx x$  Divide each side by 15 and simplify.

The hang glider was flying at a rate of 466.7 meters per minute.

### Check Your Progress

6. **TRAINS** A train travels 5 miles from Lynbrook to Long Beach and then back. The train travels about 1.2 times as fast returning from Long Beach. If  $r$  is the train's speed from Lynbrook to Long Beach, write and simplify an expression for the total time of the round trip.

### Real-World Link

The distance a hang glider can travel is determined by its *glide ratio*, or the ratio of the forward distance traveled to the vertical distance dropped.

Source: HowStuffWorks

To subtract rational expressions with unlike denominators, rename the expressions using the LCD. Then subtract the numerators.

### EXAMPLE 7 Subtract Rational Expressions with Unlike Denominators

Find  $\frac{5}{x} - \frac{2x+1}{4x}$ .

$$\frac{5}{x} - \frac{2x+1}{4x} = \frac{5\left(\frac{4}{4}\right) - \frac{2x+1}{4x}}$$

$$= \frac{20}{4x} - \frac{2x+1}{4x}$$

$$= \frac{20 - (2x+1)}{4x}$$

$$= \frac{20 - 2x - 1}{4x} \text{ or } \frac{19 - 2x}{4x}$$

Write  $\frac{5}{x}$  using the LCD,  $4x$ .

Simplify.

Subtract the numerators.

Simplify.

#### StudyTip

##### Simplifying Answers

When simplifying a rational expression, you can leave the denominator in factored form, or multiply the terms.

#### Check Your Progress

Find each difference.

7A.  $\frac{6}{t+3} - \frac{7}{t}$

7B.  $\frac{y}{y-3} - \frac{2}{y^2+y-12}$

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### Check Your Understanding

#### Examples 1–3

pp. 706–707

Find each sum or difference.

1.  $\frac{3}{7n} + \frac{2}{7n}$

2.  $\frac{x+8}{2} + \frac{x}{2}$

3.  $\frac{14r}{9-r} - \frac{2r}{r-9}$

4.  $\frac{7}{5t} - \frac{3+t}{5t}$

#### Example 4

pp. 707–708

Find the LCM of each pair of polynomials.

5.  $3t, 8t^2$

6.  $5m + 15, 2m + 6$

7.  $(x^2 - 8x + 7), (x^2 + x - 2)$

#### Examples 5 and 7

pp. 708 and 710

Find each sum or difference.

8.  $\frac{6}{n^4} + \frac{2}{n^2}$

9.  $\frac{3}{4x} + \frac{2}{5y}$

10.  $\frac{4}{5n} - \frac{1}{10n^3}$

11.  $\frac{8}{3c} - \frac{-5}{6d}$

12.  $\frac{a}{a+4} + \frac{6}{a+2}$

13.  $\frac{x}{x-3} - \frac{3}{x+2}$

#### Example 6

p. 709

14. **EXERCISE** Joseph walks 10 times around the track at a rate of  $x$  laps per hour. He runs 8 times around the track at a rate of  $3x$  laps per hour. Write and simplify an expression for the total time it takes him to go around the track 18 times.

### Practice and Problem Solving

= **Step-by-Step Solutions** begin on page R12.  
**Extra Practice** begins on page 815.

#### Examples 1–3

pp. 706–707

Find each sum or difference.

15.  $\frac{a}{4} + \frac{3a}{4}$

16.  $\frac{1}{6m} + \frac{5m}{6m}$

17.  $\frac{5y}{6} - \frac{y}{6}$

18.  $\frac{11}{4r} - \frac{-1}{4r}$

19.  $\frac{8b}{ab} + \frac{3a}{ab}$

20.  $\frac{t+2}{3} + \frac{t+5}{3}$

21.  $\frac{3c-7}{2c-1} + \frac{2c+1}{1-2c}$

22.  $\frac{15x}{33x-9} + \frac{3}{9-33x}$

23.  $\frac{n+6}{10} - \frac{n+1}{10}$

24.  $\frac{5x+2}{2x+5} - \frac{x-8}{2x+5}$

25.  $\frac{w+2}{8w} - \frac{2w-3}{8w}$

26.  $\frac{3a+1}{a-1} - \frac{a+4}{a-1}$

**Example 4**  
pp. 707–708

Find the LCM of each pair of polynomials.

27.  $x^3y, x^2y^2$

28.  $5ab, 10b$

29.  $(3r - 1), (r + 2)$

30.  $2n - 10, 4n - 20$

31.  $(x^2 + 9x + 18), x + 3$

32.  $(k^2 - 2k - 8), (k + 2)^2$

**Examples 5 and 7**  
pp. 708 and 710

Find each sum or difference.

33.  $\frac{5}{4x} + \frac{1}{10x}$

34.  $\frac{6}{r} + \frac{2}{r^2}$

35.  $\frac{3}{2a} + \frac{1}{5b}$

36.  $\frac{6g}{g+5} - \frac{g-2}{2g}$

37.  $\frac{7}{4k+8} - \frac{k}{k+2}$

38.  $\frac{5}{2d+2} - \frac{d}{d+5}$

39.  $\frac{-2}{7r} + \frac{4}{t}$

40.  $\frac{n}{n-2} + \frac{n}{n+1}$

41.  $\frac{d}{d+5} + \frac{7}{d-1}$

42.  $\frac{4}{a} - \frac{1}{3a}$

43.  $\frac{6}{5t^2} - \frac{2}{3t}$

44.  $\frac{7}{4r} - \frac{3}{t}$

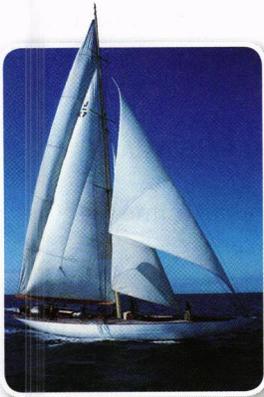
45.  $\frac{w-3}{w^2-w-20} + \frac{w}{w+4}$

46.  $\frac{n}{2n+10} + \frac{1}{n^2-25}$

47.  $\frac{2x}{x^2+8x+15} - \frac{x+3}{x+5}$

48.  $\frac{r-3}{r^2+6r+9} - \frac{r-9}{r^2-9}$

**Example 6**  
p. 709



**Real-World Link**

In a recent year, there were nearly 12.8 million boats registered in the U.S.

Source: USCG Boating

49. **TRAVEL** Grace walks to her friend's house 2 miles away and then jogs back home. Her jogging speed is 2.5 times her walking speed  $w$ .

- Write and simplify an expression to represent the amount of time Grace spends going to and coming from her friend's house.
- If Grace walks about 3.5 miles per hour, how many minutes did she spend going to and from her friend's house?

50. **BOATS** A boat travels 3 miles downstream at a rate 2 miles per hour faster than the current, or  $x + 2$  miles per hour. It then travels 6 miles upstream at a rate 2 miles per hour slower than the current, or  $x - 2$  miles per hour.

- Write and simplify an expression to represent the total time it takes the boat to travel 3 miles downstream and 6 miles upstream.
- If the rate of the current  $x$  is 4 miles per hour, how long did it take the boat to travel the 9 miles?

51. **SCHOOL** Mr. Kim had 18 more geometry tests to grade than algebra tests. He graded 12 tests on Saturday and 20 tests on Sunday. Write an expression for the fraction of tests he graded if  $a$  represents the number of algebra tests.

52. **PLAYS** A total of 1248 people attended the school play. The same number  $x$  attended each of the two Sunday performances. There were twice as many people at the Saturday performance than at both Sunday performances. Write an expression to represent the fraction of people who attended on Saturday.

Find each sum or difference.

53.  $\frac{x+5}{x^2-4} - \frac{3}{x^2-4}$

55.  $\frac{k^2-26}{k-5} - \frac{1}{5-k}$

57.  $\frac{2}{x-1} + \frac{3}{x+1} - \frac{4x-2}{x^2-1}$

59.  $\frac{a^2-5a}{3a-18} - \frac{7a-36}{3a-18}$

61.  $\frac{x^2-16}{x^3} + \frac{x^3+1}{x^4}$

63.  $\frac{5x}{3x^2+19x-14} - \frac{1}{9x^2-12x+4}$

54.  $\frac{18y}{9y+2} - \frac{-4}{-2-9y}$

56.  $\frac{8}{c-1} + \frac{c}{1-c}$

58.  $\frac{x^2-x-12}{x^2-11x+30} - \frac{x-4}{18-x}$

60.  $\frac{8n-3}{n^2+8n+12} - \frac{5n-9}{n^2+8n+12}$

62.  $\frac{x}{7x-3} + \frac{x+2}{15x+30}$

64.  $\frac{2x+7}{x^2-y^2} + \frac{-5}{x^2-2xy+y^2}$



### Real-World Link

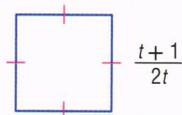
A regular triathlon includes a 1.5-kilometer swim, a 40-kilometer bike ride, and a 10-kilometer run. The first Olympic triathlons were held in 2000 in Sydney, Australia.

Source: USA Triathlon

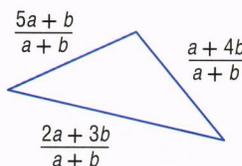
65. **TRIATHLONS** In a sprint triathlon, athletes swim 400 meters, bike 20 kilometers, and run 5 kilometers. An athlete bikes 12 times as fast as she swims and runs 5 times as fast as she swims.
- Simplify  $\frac{400}{x} + \frac{20,000}{12x} + \frac{5000}{5x}$ , an expression that represents the time it takes the athlete to complete the sprint triathlon.
  - If the athlete swims 40 meters per minute, find the total time it takes her to complete the triathlon.

**GEOMETRY** Write an expression for the perimeter of each figure.

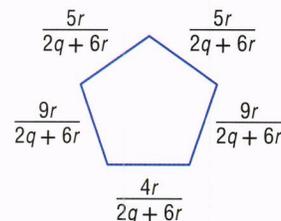
66.



67.



68.



69. **BIKES** Marina rides her bike at an average rate of 10 miles per hour. On one day, she rides 9 miles and then rides around a large loop  $x$  miles long. On the second day, she rides 5 miles and then rides around the loop three times.
- Write an expression to represent the total time she spent riding her bike on those two days. (*Hint: Use  $t = \frac{d}{r}$ , where  $t$  is time,  $d$  is distance, and  $r$  is rate.*) Then simplify the expression.
  - If the loop is 2 miles long, how long did Marina ride on those two days?
70. **TRAVEL** The Showalter family drives 80 miles to a college football game. On the trip home, their average speed is about 3 miles per hour slower.
- Let  $x$  represent the average speed of the car on the way to the game. Write and simplify an expression to represent the total time it took driving to the game and then back home.
  - If their average speed on the way to the game was 68 miles per hour, how long did it take the Showalter family to drive to the game and back? Round to the nearest tenth.

### H.O.T. Problems

Use **H**igher-**O**rders **T**hinking Skills

71. **CHALLENGE** Find  $\left(\frac{4}{7y-2} + \frac{7y}{2-7y}\right)\left(\frac{y+5}{6} - \frac{y+3}{6}\right)$ .
72. **WRITING IN MATH** Describe in words the steps you use to find the LCM in an addition or subtraction of rational expressions with unlike denominators.
73. **CHALLENGE** Is the following statement *sometimes*, *always*, or *never* true? Explain.  

$$\frac{a}{x} + \frac{b}{y} = \frac{ay + bx}{xy}; x \neq 0, y \neq 0$$
74. **OPEN ENDED** Describe a real-life situation that could be expressed by adding two rational expressions that are fractions. Explain what the denominator and numerator represent in both expressions.
75. **WRITING IN MATH** Describe how to add rational expressions with denominators that are additive inverses.

## Standardized Test Practice

**76. SHORT RESPONSE** An object is launched upwards at 19.6 meters per second from a 58.8-meter-tall platform. The equation for the object's height  $h$ , in meters, at time  $t$  seconds after launch is  $h(t) = -4.9t^2 + 19.6t + 58.8$ . How long after the launch does the object strike the ground?

77. Simplify  $\frac{2}{5} + \frac{3}{25} + \frac{1}{10}$ .

A  $\frac{2}{5}$

B  $\frac{3}{5}$

C  $\frac{31}{50}$

D  $\frac{5}{31}$

**78. STATISTICS** Courtney has grades of 84, 65, and 76 on three math tests. What grade must she earn on the next test to have an average of exactly 80 for the four tests?

F 84

G 80

H 98

J 95

79. Simplify  $\frac{2}{x} + \frac{3}{x^2} + \frac{1}{2x}$ .

A  $\frac{3x+2}{x^2}$

B  $\frac{6}{2x^2}$

C  $\frac{5x+6}{2x^2}$

D  $\frac{6+x}{x^2}$

## Spiral Review

Find each quotient. (Lesson 11-5)

80.  $(6x^2 + 10x) \div 2x$

81.  $(15y^3 + 14y) \div 3y$

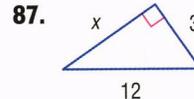
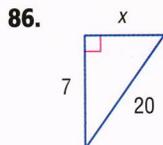
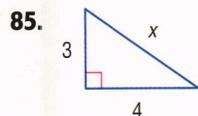
82.  $(10a^3 - 20a^2 + 5a) \div 5a$

Convert each rate. Round to the nearest tenth if necessary. (Lesson 11-4)

83. 23 feet per second to miles per hour

84. 118 milliliters per second to quarts per hour (Hint: 1 liter  $\approx$  1.06 quarts)

Find the length of the missing side. If necessary, round to the nearest hundredth. (Lesson 10-5)



**88. AMUSEMENT RIDE** The height  $h$  in feet of a car above the exit ramp of a free-fall ride can be modeled by  $h(t) = -16t^2 + s$ .  $t$  is the time in seconds after the car drops, and  $s$  is the starting height of the car in feet. If the designer wants the ride to last 3 seconds, what should be the starting height in feet? (Lesson 8-6)

Express each number in scientific notation. (Lesson 7-3)

89. 12,300

90. 0.0000375

91. 1,255,000

**92. FINANCIAL LITERACY** Ruben has \$13 to order pizza. The pizza costs \$7.50 plus \$1.25 per topping. He plans to tip 15% of the total cost. Write and solve an inequality to find out how many toppings he can order. (Lesson 5-3)

## Skills Review

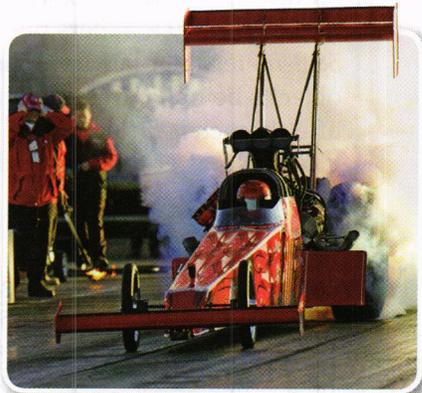
Find each quotient. (Lesson 11-4)

93.  $\frac{12}{3x^2} \div \frac{6}{x}$

94.  $\frac{g^4}{2} \div \frac{g^3}{8d^2}$

95.  $\frac{4y-8}{y+1} \div (y-2)$

# Mixed Expressions and Complex Fractions



## Then

You simplified rational expressions. (Lesson 11-3)

## Now

- Simplify mixed expressions.
- Simplify complex fractions.

## New Vocabulary

mixed expression  
complex fraction

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- Self-Check Quiz
- Homework Help

## Why?

A Top Fuel dragster can cover  $\frac{1}{4}$  mile in  $4\frac{2}{5}$  seconds. The average speed in miles per second can be described by the expression below. It is called a *complex fraction*.

$$\frac{\frac{1}{4} \text{ mile}}{4\frac{2}{5} \text{ seconds}}$$

**Simplify Mixed Expressions** An expression like  $2 + \frac{4}{x+1}$  is called a **mixed expression** because it contains the sum of a monomial, 2, and a rational expression,  $\frac{4}{x+1}$ . You can use the LCD to change a mixed expression to a rational expression.

### EXAMPLE 1 Change Mixed Expression to Rational Expressions

Write  $2 + \frac{4}{x-1}$  as a rational expression.

$$\begin{aligned} 2 + \frac{4}{x-1} &= \frac{2(x-1)}{x-1} + \frac{4}{x-1} && \text{The LCD is } x-1. \\ &= \frac{2(x-1) + 4}{x-1} && \text{Add the numerators.} \\ &= \frac{2x - 2 + 4}{x-1} && \text{Distributive Property} \\ &= \frac{2x + 2}{x-1} && \text{Simplify.} \end{aligned}$$

### Check Your Progress

Write each mixed expression as a rational expression.

1A.  $2 + \frac{5}{x}$

1B.  $\frac{6y}{4y+8} + 5y$

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**Simplify Complex Fractions** A **complex fraction** has one or more fractions in the numerator or denominator. You can simplify by using division.

numerical complex fraction

$$\begin{aligned} \frac{\frac{2}{3}}{\frac{5}{8}} &= \frac{2}{3} \div \frac{5}{8} \\ &= \frac{2}{3} \times \frac{8}{5} \\ &= \frac{16}{15} \end{aligned}$$

algebraic complex fraction

$$\begin{aligned} \frac{\frac{a}{b}}{\frac{c}{d}} &= \frac{a}{b} \div \frac{c}{d} \\ &= \frac{a}{b} \times \frac{d}{c} \\ &= \frac{ad}{bc} \end{aligned}$$

To simplify a complex fraction, write it as a division expression. Then find the reciprocal of the second expression and multiply.

## Real-World EXAMPLE 2 Use Complex Fractions to Solve Problems

**RACING** Refer to the application at the beginning of the lesson. Find the average speed of the Top Fuel dragster in miles per minute.

$$\frac{\frac{1}{4} \text{ mile}}{4\frac{2}{5} \text{ seconds}} = \frac{\frac{1}{4} \text{ mile}}{4\frac{2}{5} \text{ seconds}} \times \frac{60 \text{ seconds}}{1 \text{ minute}}$$

Convert seconds to minutes. Divide by common units.

$$= \frac{1}{4} \times \frac{60}{4\frac{2}{5}}$$

Simplify.

$$= \frac{60}{4\frac{2}{5}}$$

Express each term as an improper fraction.

$$= \frac{15}{4} \times \frac{5}{22}$$

Use the rule  $\frac{a}{b} \div \frac{c}{d} = \frac{ad}{bc}$ .

$$= \frac{75}{22} \text{ or } 3\frac{9}{22}$$

Simplify.

So, the average speed of the Top Fuel dragster is  $3\frac{9}{22}$  miles per minute.

### Check Your Progress

2. **RACING** Refer to the information about the Jr. Dragster at the left. What is the average speed of the car in feet per second?

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To simplify complex fractions, you can either use the rule as in Example 2, or you can rewrite the fraction as a division expression, as shown below.

### EXAMPLE 3 Complex Fractions Involving Monomials

Simplify  $\frac{\frac{8t^2}{v}}{\frac{4t}{v^3}}$ .

$$\frac{\frac{8t^2}{v}}{\frac{4t}{v^3}} = \frac{8t^2}{v} \div \frac{4t}{v^3}$$

Write as a division expression.

$$= \frac{8t^2}{v} \times \frac{v^3}{4t}$$

To divide, multiply by the reciprocal.

$$= \frac{8t^2}{\cancel{v}} \times \frac{\cancel{v^3}}{4\cancel{t}} \text{ or } 2tv^2$$

Divide by the common factors  $4t$  and  $v$  and simplify.

### Check Your Progress

Simplify each expression.

3A.  $\frac{\frac{g^3h}{b}}{\frac{gh^3}{b^2}}$

3B.  $\frac{\frac{-24m^3t^5}{p^2h}}{\frac{16pm^2}{t^4h}}$

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### Real-World Link

A Jr. Dragster is a half-scale version of a Top Fuel dragster. This car, which can go  $\frac{1}{8}$  mile in  $7\frac{9}{10}$  seconds, is designed to be driven by kids ages 8–17 in the NHRA Jr. Drag Racing League.

Source: NHRA

Complex fractions may also involve polynomials.

### EXAMPLE 4 Complex Fractions Involving Polynomials

Simplify each expression.

a. 
$$\frac{\frac{2}{y+3}}{\frac{5}{y^2-9}}$$

$$\frac{\frac{2}{y+3}}{\frac{5}{y^2-9}} = \frac{2}{y+3} \div \frac{5}{y^2-9}$$

Write as a division expression.

$$= \frac{2}{y+3} \times \frac{y^2-9}{5}$$

To divide, multiply by the reciprocal.

$$= \frac{2}{y+3} \times \frac{(y-3)(y+3)}{5}$$

Factor  $y^2 - 9$ .

$$= \frac{2}{\cancel{y+3}} \times \frac{(y-3)\cancel{(y+3)}}{5}$$

Divide by the GCF,  $y + 3$ .

$$= \frac{2(y-3)}{5}$$

Simplify.

#### StudyTip

**Factoring** When simplifying fractions involving polynomials, factor the numerator and the denominator of each expression if possible.

b. 
$$\frac{\frac{n^2+7n-18}{n^2-2n+1}}{\frac{n^2-81}{n-1}}$$

$$\frac{\frac{n^2+7n-18}{n^2-2n+1}}{\frac{n^2-81}{n-1}} = \frac{n^2+7n-18}{n^2-2n+1} \div \frac{n^2-81}{n-1}$$

Write as a division expression.

$$= \frac{n^2+7n-18}{n^2-2n+1} \times \frac{n-1}{n^2-81}$$

Multiply by the reciprocal.

$$= \frac{(n-2)(n+9)}{(n-1)(n-1)} \times \frac{n-1}{(n-9)(n+9)}$$

Factor the polynomials.

$$= \frac{(n-2)\cancel{(n+9)}}{(n-1)\cancel{(n-1)}} \times \frac{\cancel{n-1}}{(n-9)\cancel{(n+9)}}$$

Divide out the common factors.

$$= \frac{n-2}{(n-1)(n-9)}$$

Simplify.

#### Check Your Progress

4A. 
$$\frac{\frac{a+7}{4}}{\frac{a^2-49}{10}}$$

4B. 
$$\frac{\frac{x+4}{x-1}}{\frac{x^2+6x+8}{2x-2}}$$

4C. 
$$\frac{\frac{c-d}{j+p}}{\frac{c^2-d^2}{j^2-p^2}}$$

4D. 
$$\frac{\frac{n^2+4n-21}{n^2-9n+18}}{\frac{n^2+3n-28}{n^2-10n+24}}$$

## Check Your Understanding

### Example 1 p. 714

Write each mixed expression as a rational expression.

1.  $\frac{2}{n} + 4$

2.  $r + \frac{1}{3r}$

3.  $6 + \frac{5}{t+1}$

4.  $\frac{x+7}{2x} - 5x$

### Example 2 p. 715

5. **ROWING** Rico rowed a canoe  $2\frac{1}{2}$  miles in  $\frac{1}{3}$  hour.

- Write an expression to represent his speed in miles per hour.
- Simplify the expression to find his average speed.

### Examples 3 and 4 pp. 715–716

Simplify each expression.

6.  $\frac{2\frac{1}{3}}{1\frac{2}{5}}$

7.  $\frac{\frac{4}{5}}{6\frac{2}{3}}$

8.  $\frac{\frac{a^2}{b^3}}{\frac{b^5}{a}}$

9.  $\frac{\frac{y^4}{x^2}}{\frac{xy^2}{2x^2}}$

10.  $\frac{\frac{6}{x-2}}{\frac{3}{x^2-x-2}}$

11.  $\frac{\frac{r+s}{x^2-y^2}}{\frac{(r+s)^2}{x-y}}$

12.  $\frac{\frac{2+q}{q^2-4}}{\frac{q+4}{q^2-6q+8}}$

13.  $\frac{\frac{p+3}{p^2+p-6}}{\frac{p^2+4p+3}{p^2+6p+9}}$

## Practice and Problem Solving

 = **Step-by-Step Solutions** begin on page R12.  
**Extra Practice** begins on page 815.

### Example 1 p. 714

Write each mixed expression as a rational expression.

14.  $10 + \frac{6}{f}$

15.  $p - \frac{7}{2p}$

16.  $5a - \frac{2a}{b}$

17.  $3h + \frac{1+h}{h}$

18.  $t + \frac{v+w}{v-w}$

19.  $n^2 + \frac{n-1}{n+4}$

20.  $(k+2) + \frac{k-1}{k-2}$

21.  $(d-6) + \frac{d+1}{d-7}$

22.  $\frac{h-3}{h+5} - (h+2)$

### Example 2 p. 715

23. **READING** Ebony reads  $6\frac{3}{4}$  pages of a book in 9 minutes. What is her average reading rate in pages per minute?

24. **HORSES** A thoroughbred can run  $\frac{1}{2}$  mile in about  $\frac{3}{4}$  minute. What is the horse's speed in miles per hour?

### Examples 3 and 4 pp. 715–716

Simplify each expression.

25.  $\frac{2\frac{2}{9}}{3\frac{1}{3}}$

26.  $\frac{5\frac{3}{5}}{2\frac{1}{7}}$

27.  $\frac{\frac{g^2}{h}}{\frac{g^5}{h^2}}$

28.  $\frac{\frac{5n^4}{p^3}}{\frac{6n}{5p}}$

29.  $\frac{\frac{2}{a}}{\frac{1}{a+6}}$

30.  $\frac{\frac{t+5}{9}}{\frac{t^2-t-30}{12}}$

31.  $\frac{\frac{j^2-16}{j^2+10j+16}}{\frac{15}{j+8}}$

32.  $\frac{\frac{x-3}{x^2+3x+2}}{\frac{x^2-9}{x+1}}$

33. **COOKING** The Centralville High School Cooking Club has  $12\frac{1}{2}$  pounds of flour with which to make tortillas. There are  $3\frac{3}{4}$  cups of flour in a pound, and it takes about  $\frac{1}{3}$  cup of flour per tortilla. How many tortillas can they make?



### Real-World Career

#### Lab Technician

Lab technicians work with scientists, running experiments, conducting research projects, and running routine diagnostic samples. Lab technicians in any field need at least a two-year associate degree.

- 34. SCOOTER** The speed  $v$  of an object spinning in a circle equals the circumference of the circle divided by the time  $T$  it takes the object to complete one revolution.
- Use the variables  $v$ ,  $r$  (the radius of the circle), and  $T$  to write a formula describing the speed of a spinning object.
  - A scooter has tires with a radius of  $3\frac{1}{2}$  inches. The tires make one revolution every  $\frac{1}{10}$  second. Find the speed in miles per hour. Round to the nearest tenth.

- 35. SCIENCE** The *density* of an object equals  $\frac{m}{V}$ , where  $m$  is the mass of the object and  $V$  is the volume. The densities of four metals are shown in the table. Identify the metal of each ball described below. (*Hint:* The volume of a sphere is  $V = \frac{4}{3}\pi r^3$ .)

Metal	Density (kg/m <sup>3</sup> )
copper	8900
gold	19,300
iron	7800
lead	11,300

- A metal ball has a mass of 15.6 kilograms and a radius of 0.0748 meter.
  - A metal ball has a mass of 285.3 kilograms and a radius of 0.1819 meter.
- 36. SIRENS** As an ambulance approaches, the siren sounds different than if it were sitting still. If the ambulance is moving toward you at  $v$  miles per hour and blowing the siren at a frequency of  $f$ , then you hear the siren as if it were blowing at a frequency  $h$ . This can be described by the equation  $h = \frac{f}{1 - \frac{v}{s}}$ , where  $s$  is the speed of sound, approximately 760 miles per hour.
- Simplify the complex fraction in the formula.
  - Suppose a siren blows at 45 cycles per minute and is moving toward you at 65 miles per hour. Find the frequency of the siren as you hear it.

Simplify each expression.

37.  $15 - \frac{17x + 5}{5x + 10}$

38.  $\frac{\frac{b}{b+3} + 2}{b^2 - 2b - 8}$

39.  $1 + \frac{2c^2 - 6c - 10}{c + 7} \cdot \frac{1}{2c + 1}$

40.  $\frac{\frac{y - 12}{y - 4}}{y - \frac{18}{y - 3}}$

41.  $\frac{\frac{x^2 - 4x - 32}{x + 1}}{\frac{x^2 + 6x + 8}{x^2 - 1}}$

42.  $\frac{\frac{r^2 - 9r}{r^2 + 7r + 10}}{\frac{r^2 + 5r}{r^2 + r - 2}}$

### H.O.T. Problems

Use Higher-Order Thinking Skills

- 43. REASONING** Describe the first step to simplify the expression below.

$$\frac{\left(\frac{y}{x} - \frac{x}{y}\right)}{\frac{x+y}{xy}}$$

- 44. REASONING** Is  $\frac{n}{1 - \frac{5}{p}} + \frac{n}{\frac{5}{p} - 1}$  sometimes, always, or never equal to 0? Explain.

- 45. CHALLENGE** Simplify the rational expression below.

$$\frac{\frac{1}{t-1} + \frac{1}{t+1}}{\frac{1}{t} - \frac{1}{t^2}}$$

- 46. OPEN ENDED** Write a complex fraction that, when simplified, results in  $\frac{1}{x}$ .

- 47. WRITING IN MATH** Explain how complex fractions can be used to solve a problem involving distance, rate, and time. Give an example.

## Standardized Test Practice

48. A number is between 44 squared and 45 squared. 5 squared is one of its factors, and it is a multiple of 13. Find the number.

- A 1950
- B 2000
- C 2025
- D 1975

49. **SHORT RESPONSE** Bernard is reading a 445-page book. He has already read 157 pages. If he reads 24 pages a day, how long will it take him to finish the book?

50. **GEOMETRY** Angela wanted a round rug to fit her room that is 16 feet wide. The rug should just meet the edges. What is the area of the rug rounded to the nearest tenth?

- F 100.5 ft
- G 804.2 ft<sup>2</sup>
- H 50.3 ft
- J 201.1 ft<sup>2</sup>

51. Simplify  $7x + \frac{10}{2xy}$ .

- A  $\frac{7x + 10}{2xy}$
- B  $\frac{7x^2y + 5}{xy}$
- C  $\frac{17x}{2xy}$
- D  $\frac{7xy + 5}{x^2y}$

## Spiral Review

Find each sum or difference. (Lesson 11-6)

52.  $\frac{6}{7x} - \frac{5+x}{7x}$

53.  $\frac{4}{d-1} + \frac{d}{1-d}$

54.  $\frac{3q+2}{2q+1} + \frac{q-5}{2q+1}$

55.  $\frac{2}{5m} - \frac{1}{15m^3}$

56.  $\frac{10}{3g} - \frac{-3}{4h}$

57.  $\frac{b}{b+3} + \frac{6}{b-2}$

Find each quotient. Use long division. (Lesson 11-5)

58.  $(x^2 - 2x - 30) \div (x + 7)$

59.  $(a^2 + 4a - 22) \div (a - 3)$

60.  $(3q^2 + 20q + 11) \div (q + 6)$

61.  $(3y^3 + 8y^2 + y - 7) \div (y + 2)$

62.  $(6t^3 - 9t^2 + 6) \div (2t - 3)$

63.  $(9h^3 + 5h - 8) \div (3h - 2)$

64. **GEOMETRY** Triangle  $ABC$  has vertices  $A(7, -4)$ ,  $B(-1, 2)$ , and  $C(5, -6)$ . Determine whether the triangle has three, two, or no sides that are equal in length. (Lesson 10-6)

Graph each function. Determine the domain and range. (Lesson 10-1)

65.  $y = 2\sqrt{x}$

66.  $y = -3\sqrt{x}$

67.  $y = \frac{1}{4}\sqrt{x}$

Factor each polynomial. If the polynomial cannot be factored, write *prime*. (Lesson 8-5)

68.  $x^2 - 81$

69.  $a^2 - 121$

70.  $n^2 + 100$

71.  $-25 + 4y^2$

72.  $p^4 - 16$

73.  $4t^4 - 4$

74. **PARKS** A youth group traveling in two vans visited Mammoth Cave in Kentucky. The number of people in each van and the total cost of the cave are shown. Find the adult price and the student price of the tour. (Lesson 6-3)

Van	Number of Adults	Number of Students	Total Cost
A	2	5	\$77
B	2	7	\$95

## Skills Review

Solve each equation. (Lesson 2-2 and 2-3)

75.  $6x = 24$

76.  $5y - 1 = 19$

77.  $2t + 7 = 21$

78.  $\frac{p}{3} = -4.2$

79.  $\frac{2m+1}{4} = -5.5$

80.  $\frac{3}{4}g = \frac{1}{2}$

## Rational Equations

### Then

You solved proportions.  
(Lesson 2-6)

### Now

- Solve rational equations.
- Use rational equations to solve problems.

### New Vocabulary

rational equation  
extraneous solution  
work problem  
rate problem

### Math Online

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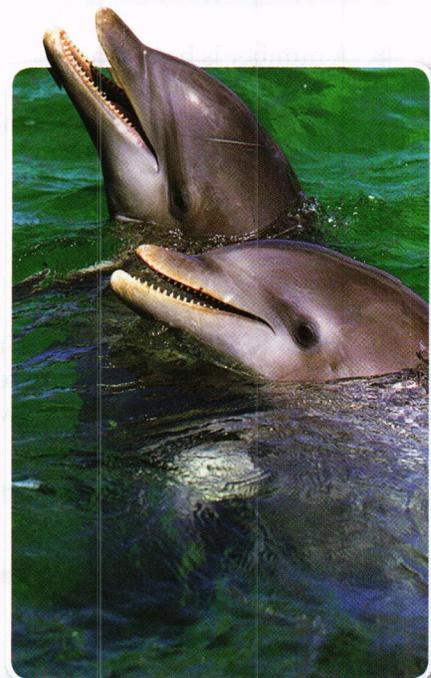
### Why?

Oceanic species of dolphins can swim 5 miles per hour faster than coastal species of dolphins. An oceanic dolphin can swim 3 miles in the same time that it takes a coastal dolphin to swim 2 miles.

Dolphins			
Species	Distance	Rate	Time
coastal	2 miles	$x$ mph	$t$ hours
oceanic	3 miles	$x + 5$ mph	$t$ hours

Since time =  $\frac{\text{distance}}{\text{rate}}$ , the equation below represents this situation.

$$\begin{array}{ccc} \text{Time an oceanic} & \text{equals} & \text{time a coastal} \\ \text{dolphin swims} & & \text{dolphin swims} \\ \text{3 miles} & & \text{2 miles.} \\ \hline \text{distance} \rightarrow \frac{3}{\text{rate} \rightarrow x + 5} & = & \frac{2}{x} \leftarrow \text{distance} \\ & & \leftarrow \text{rate} \end{array}$$



**Solve Rational Equations** A rational equation contains one or more rational expressions. When a rational equation is a proportion, you can use cross products to solve it.

### Real-World EXAMPLE 1 Use Cross Products to Solve Equations

**DOLPHINS** Refer to the information above. Solve  $\frac{3}{x+5} = \frac{2}{x}$  to find the speed of a coastal dolphin. Check the solution.

$$\frac{3}{x+5} = \frac{2}{x}$$

Original equation

$$3x = 2(x+5)$$

Find the cross products.

$$3x = 2x + 10$$

Distributive Property

$$x = 10$$

Subtract  $2x$  from each side.

So, a coastal dolphin can swim 10 miles per hour.

**CHECK**  $\frac{3}{x+5} = \frac{2}{x}$

Original equation

$$\frac{3}{10+5} \stackrel{?}{=} \frac{2}{10}$$

Replace  $x$  with 10.

$$\frac{3}{15} \stackrel{?}{=} \frac{1}{5}$$

Simplify.

$$\frac{1}{5} = \frac{1}{5} \checkmark$$

Simplify.

### Check Your Progress

Solve each equation. Check the solution.

1A.  $\frac{7}{y-3} = \frac{3}{y+1}$

1B.  $\frac{13}{10} = \frac{2f+0.2}{7}$

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Another method that can be used to solve any rational equation is to find the LCD of all the fractions in the equation. Then multiply each side of the equation by the LCD to eliminate the fractions.

### EXAMPLE 2 Use the LCD to Solve Rational Equations

Solve  $\frac{4}{y} + \frac{5y}{y+1} = 5$ . Check the solution.

**Step 1** Find the LCD.

The LCD of  $\frac{4}{y}$  and  $\frac{5y}{y+1}$  is  $y(y+1)$ .

**Step 2** Multiply each side of the equation by the LCD.

$$\frac{4}{y} + \frac{5y}{y+1} = 5$$

$$y(y+1) \left( \frac{4}{y} + \frac{5y}{y+1} \right) = y(y+1)(5)$$

$$\left( \frac{\cancel{1}y(y+1)}{1} \cdot \frac{4}{\cancel{y}} \right) + \left( \frac{y(\cancel{y+1})}{1} \cdot \frac{5y}{\cancel{y+1}} \right) = y(y+1)(5)$$

$$(y+1)4 + y(5y) = y(y+1)(5)$$

$$4y + 4 + 5y^2 = 5y^2 + 5y$$

$$4y + 4 + 5y^2 - 5y^2 = 5y^2 - 5y^2 + 5y$$

$$4y + 4 = 5y$$

$$4y - 4y + 4 = 5y - 4y$$

$$4 = y$$

Original equation

Multiply each side by the LCD,  $y(y+1)$ .

Distributive Property

Simplify.

Multiply.

Subtract  $5y^2$  from each side.

Simplify.

Subtract  $4y$  from each side.

Simplify.

**CHECK**  $\frac{4}{y} + \frac{5y}{y+1} = 5$

$$\frac{4}{4} + \frac{5(4)}{4+1} \stackrel{?}{=} 5$$

$$1 + 4 \stackrel{?}{=} 5$$

$$5 = 5 \checkmark$$

Original equation

Replace  $y$  with 4.

Simplify.

Simplify.

### Check Your Progress

Solve each equation. Check your solutions.

2A.  $\frac{2b-5}{b-2} - 2 = \frac{3}{b+2}$

2B.  $1 + \frac{1}{c+2} = \frac{28}{c^2+2c}$

2C.  $\frac{y+2}{y-2} - \frac{2}{y+2} = -\frac{7}{3}$

2D.  $\frac{n}{3n+6} - \frac{n}{5n+10} = \frac{2}{5}$

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Recall that any value of a variable that makes the denominator of a rational expression zero must be excluded from the domain.

In the same way, when a solution of a rational equation results in a zero in the denominator, that solution must be excluded. Such solutions are called **extraneous solutions**.

$$\frac{4+x}{x-5} + \frac{1}{x} = \frac{2}{x+1}$$

5, 0, and  $-1$  cannot be solutions.

### StudyTip

**Solutions** It is important to check the solutions of rational equations to be sure that they satisfy the original equation.

### Vocabulary Link

#### extraneous

**Everyday Use**  
irrelevant or unimportant

#### extraneous solution

**Math Use** a result that is not a solution of the original equation

**EXAMPLE 3 Extraneous Solutions**Solve  $\frac{2n}{n-5} + \frac{4n-30}{n-5} = 5$ . State any extraneous solutions.

$$\frac{2n}{n-5} + \frac{4n-30}{n-5} = 5$$

**Original equation**

$$(n-5)\left(\frac{2n}{n-5} + \frac{4n-30}{n-5}\right) = (n-5)5$$

**Multiply each side by the LCD,  $n-5$ .**

$$\left(\frac{\cancel{n-5}^1}{1} \cdot \frac{2n}{\cancel{n-5}_1}\right) + \left(\frac{\cancel{n-5}^1}{1} \cdot \frac{4n-30}{\cancel{n-5}_1}\right) = (n-5)5$$

**Distributive Property**

$$2n + 4n - 30 = 5n - 25$$

**Simplify.**

$$6n - 30 = 5n - 25$$

**Add like terms.**

$$6n - 5n - 30 = 5n - 5n - 25$$

**Subtract  $5n$  from each side.**

$$n - 30 = -25$$

**Simplify.**

$$n - 30 + 30 = -25 + 30$$

**Add 30 to each side.**

$$n = 5$$

**Simplify.**

Since  $n = 5$  results in a zero in the denominator of the original equation, it is an extraneous solution. So, the equation has no solution.

**Check Your Progress**3. Solve  $\frac{n^2 - 3n}{n^2 - 4} - \frac{10}{n^2 - 4} = 2$ . State any extraneous solutions. **Personal Tutor** [glencoe.com](http://glencoe.com)

**Use Rational Equations to Solve Problems** You can use rational equations to solve **work problems**, or problems involving work rates.

**Real-World EXAMPLE 4 Work Problem**

**JOBS** At his part-time job at the zoo, Ping can clean the bird area in 2 hours. Natalie can clean the same area in 1 hour and 15 minutes. How long would it take them if they worked together?

**Understand** It takes Ping 2 hours to complete the job and Natalie  $1\frac{1}{4}$  hours.

You need to find the rate that each person works and the total time  $t$  that it will take them if they work together.

**Plan** Find the fraction of the job that each person can do in an hour.

$$\text{Ping's rate} \rightarrow \frac{1 \text{ job}}{2 \text{ hours}} = \frac{1}{2} \text{ job per hour}$$

$$\text{Natalie's rate} \rightarrow \frac{1 \text{ job}}{1\frac{1}{4} \text{ hours}} \text{ or } \frac{1 \text{ job}}{\frac{5}{4} \text{ hours}} = \frac{4}{5} \text{ job per hour}$$

Since  $\text{rate} \cdot \text{time} = \text{fraction of job done}$ , multiply each rate by the time  $t$  to represent the amount of the job done by each person.

**StudyTip**

**Solutions** It is possible to get both a valid solution and an extraneous solution when solving a rational equation.

### StudyTip

#### Work Problems

When solving work problems, remember that each term should represent the portion of a job completed in one unit of time.

**Solve** Fraction of job Ping completes plus fraction of job Natalie completes equals 1 job.

$$\frac{1}{2}t + \frac{4}{5}t = 1$$

**Multiply each side by the LCD, 10.**

$$10\left(\frac{1}{2}t + \frac{4}{5}t\right) = 10(1)$$

**Distributive Property**

$$10\left(\frac{1}{2}t\right) + 10\left(\frac{4}{5}t\right) = 10$$

**Simplify.**

$$5t + 8t = 10$$

**Add like terms and divide each side by 13.**

$$t = \frac{10}{13}$$

So, it would take them  $\frac{10}{13}$  hour or about 46 minutes to complete the job if they work together.

**Check** In  $\frac{10}{13}$  hour, Ping would complete  $\frac{1}{2} \cdot \frac{10}{13}$  or  $\frac{5}{13}$  of the job and Natalie would complete  $\frac{4}{5} \cdot \frac{10}{13}$  or  $\frac{8}{13}$  of the job. Together, they complete  $\frac{5}{13} + \frac{8}{13}$  or 1 whole job. So, the answer is reasonable. ✓

### Check Your Progress

4. **RAKING** Jenna can rake the leaves in 2 hours. It takes her brother Benjamin 3 hours. How long would it take them if they worked together?

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Rational equations can also be used to solve **rate problems**.

### Real-World EXAMPLE 5 Rate Problem

**AIRPLANES** An airplane takes off and flies an average of 480 miles per hour. Another plane leaves 15 minutes later and flies to the same city traveling 560 miles per hour. How long will it take the second plane to pass the first plane?

Record the information that you know in a table.

Plane	Distance	Rate	Time
1	$d$ miles	480 mi/h	$t$ hours
2	$d$ miles	560 mi/h	$t - \frac{1}{4}$ hours

← Plane 2 took off 15 minutes, or  $\frac{1}{4}$  hour, after Plane 1

Since both planes will have traveled the same distance when Plane 2 passes Plane 1, you can write the following equation.

Distance for Plane 1 = Distance for Plane 2

$$480 \cdot t = 560 \cdot \left(t - \frac{1}{4}\right) \quad \text{distance} = \text{rate} \cdot \text{time}$$

$$480t = (560 \cdot t) - \left(560 \cdot \frac{1}{4}\right) \quad \text{Distributive Property}$$

$$480t = 560t - 140 \quad \text{Simplify.}$$

$$-80t = -140 \quad \text{Subtract } 560t \text{ from each side.}$$

$$t = 1.75 \quad \text{Divide each side by } -80.$$

So, the second plane passes the first plane after 1.75 hours.

### Check Your Progress

5. Lenora leaves the house walking at 3 miles per hour. After 10 minutes, her mother leaves the house riding a bicycle at 10 miles per hour. In how many minutes will Lenora's mother catch her?

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### Real-World Link

The longest nonstop commercial flight was 13,422 miles from Hong Kong Airport in China to London Heathrow in the United Kingdom. It took 22 hours and 42 minutes.

Source: Guinness Book of World Records

Math in Motion,  
Animation [glencoe.com](http://glencoe.com)

## Check Your Understanding

### Examples 1–3 pp. 720–722

Solve each equation. State any extraneous solutions.

- $\frac{2}{x+1} = \frac{4}{x}$
- $\frac{t+3}{5} = \frac{2t+3}{9}$
- $\frac{a+3}{a} - \frac{6}{5a} = \frac{1}{a}$
- $4 - \frac{p}{p-1} = \frac{2}{p-1}$
- $\frac{2t}{t+1} + \frac{4}{t-1} = 2$
- $\frac{x+3}{x^2-1} - \frac{2x}{x-1} = 1$

### Example 4 pp. 722–723

- WEEDING** Maurice can weed the garden in 45 minutes. Olinda can weed the garden in 50 minutes. How long would it take them to weed the garden if they work together?

### Example 5 p. 723

- LANDSCAPING** Hunter is filling a 3.5-gallon bucket to water plants at a faucet that flows at a rate of 1.75 gallons a minute. If he were to add a hose that flows at a rate of 1.45 gallons per minute, how many minutes would it take him to fill the bucket? Round to the nearest tenth.

## Practice and Problem Solving

 = **Step-by-Step Solutions** begin on page R12.  
**Extra Practice** begins on page 815

### Examples 1–3 pp. 720–722

Solve each equation. State any extraneous solutions.

- $\frac{8}{n} = \frac{3}{n-5}$
- $\frac{5h}{4} + \frac{1}{2} = \frac{3h}{8}$
- $\frac{x-1}{x+1} - \frac{2x}{x-1} = -1$
- $\frac{a}{a+3} + \frac{a^2}{a+3} = 2$
- $\frac{3n}{n-1} + \frac{6n-9}{n-1} = 6$
- $\frac{6}{t+2} = \frac{4}{t}$
- $\frac{2}{3w} = \frac{2}{15} + \frac{12}{5w}$
- $\frac{y+4}{y-2} + \frac{6}{y-2} = \frac{1}{y+3}$
- $\frac{12}{a+3} + \frac{6}{a^2-9} = \frac{8}{a+3}$
- $\frac{n^2-n-6}{n^2-n} - \frac{n-5}{n-1} = \frac{n-3}{n^2-n}$
- $\frac{3g+2}{12} = \frac{g}{2}$
- $\frac{c-4}{c+1} = \frac{c}{c-1}$

### Example 4 pp. 722–723

- PAINTING** It takes Noah 3 hours to paint one side of a fence. It takes Gilberto 5 hours. How long would it take them if they worked together?
- DISHWASHING** Ron works as a dishwasher and can wash 500 plates in two hours and 15 minutes. Chris can finish the 500 plates in 3 hours. About how long would it take them to finish all of the plates if they work together?

### Example 5 p. 723

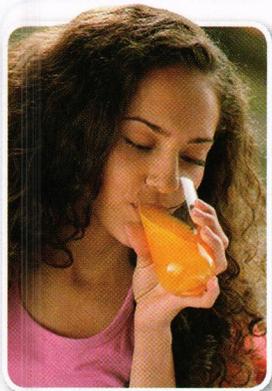
- ICE** A hotel has two ice machines in its kitchen. How many hours would it take both machines to make 60 pounds of ice? Round to the nearest tenth.
- CYCLING** Two cyclists travel in opposite directions around a 5.6-mile circular trail. They start at the same time. The first cyclist completes the trail in 22 minutes and the second in 28 minutes. At what time do they pass each other?



**GRAPHING CALCULATOR** For each function, a) describe the shape of the graph, b) use factoring to simplify the function, and c) find the zeros of the function.

- $f(x) = \frac{x^2 - x - 30}{x - 6}$
- $f(x) = \frac{x^3 + x^2 - 2x}{x + 2}$
- $f(x) = \frac{x^3 + 6x^2 + 12x}{x}$

- PAINTING** Morgan can paint a standard-sized house in about 5 days. For his latest job, Morgan hires two assistants. At what rate must these assistants work for Morgan to meet a deadline of two days?



### Real-World Link

Orange juice contains more than 10 times as much vitamin C as apple juice. Drinking an 8-ounce glass counts as one of the five recommended fruit and vegetable servings for the day.

Source: Ask Dr. Sears

- 29. AIRPLANES** Headwinds push against a plane and reduce its total speed, while tailwinds push on a plane and increase its total speed. Let  $w$  equal the speed of the wind,  $r$  equal the speed set by the pilot, and  $s$  equal the total speed.
- Write an equation for the total speed with a headwind and an equation for the total speed with a tailwind.
  - Use the rate formula to write an equation for the distance traveled by a plane with a headwind and another equation for the distance traveled by a plane with a tailwind. Then solve each equation for time instead of distance.
- 30. MIXTURES** A pitcher of fruit juice has 3 pints of pineapple juice and 2 pints of orange juice. Erin wants to add more orange juice so that the fruit juice mixture is 60% orange juice. Let  $x$  equal the pints of orange juice that she needs to add.

- Copy and complete the table below.

Juice	Pints of Orange Juice	Total Pints of Juice	Percent of Orange Juice
original mixture		5	
final mixture	$2 + x$		0.6

- Write and solve an equation to find the pints of orange juice to add.

- 31. DORMITORIES** The number of hours  $h$  it takes to clean a dormitory varies inversely with the number of people cleaning it  $c$  and directly with the number of people living there  $p$ .
- Write an equation showing how  $h$ ,  $c$ , and  $p$  are related. (*Hint:* Include the constant  $k$ .)
  - It takes 8 hours for 5 people to clean the dormitory when there are 100 people there. How long will it take to clean the dormitory if there are 10 people cleaning and the number of people living in the dorm stays the same?

Solve each equation. State any extraneous solutions.

**32.**  $\frac{4b+2}{b^2-3b} + \frac{b+2}{b} = \frac{b-1}{b}$

**33.**  $\frac{x^2-x-6}{x+2} + \frac{x^3+x^2}{x} = 3$

**34.**  $\frac{y^2+5y-6}{y^3-2y^2} = \frac{5}{y} - \frac{6}{y^3-2y^2}$

**35.**  $\frac{x-\frac{6}{5}}{x} - \frac{x-10\frac{1}{2}}{x-5} = \frac{x+21}{x^2-5x}$

### H.O.T. Problems

Use **H**igher-**O**rders **T**hinking Skills

**36. CHALLENGE** Solve  $\frac{2x}{x-2} + \frac{x^2+3x}{(x+1)(x-2)} = \frac{2}{(x+1)(x-2)}$ .

- 37. REASONING** How is an excluded value of a rational expression related to an extraneous solution of a corresponding rational equation? Explain.

- 38. OPEN ENDED** Write a problem about a real-world situation where work is being done. Write an equation that models the situation.

- 39. REASONING** Find a counterexample for the following statement.

*The solution of a rational equation can never be zero.*

- 40. WRITING IN MATH** Describe the steps for solving a rational equation that is not a proportion.

## Standardized Test Practice

41. It takes Cheng 4 hours to build a fence. If he hires Odell to help him, they can do the job in 3 hours. If Odell built the same fence alone, how long would it take him?
- A  $1\frac{5}{7}$  hours      C 8 hours  
 B  $3\frac{2}{3}$  hours      D 12 hours
42. In the 1000-meter race, Zoe finished 35 meters ahead of Taryn and 53 meters ahead of Evan. How far was Taryn ahead of Evan?
- F 18 m    G 35 m    H 53 m    J 88 m
43. Twenty gallons of lemonade were poured into two containers of different sizes. Express the amount of lemonade poured into the smaller container in terms of  $g$ , the amount poured into the larger container.
- A  $g + 20$       C  $g - 20$   
 B  $20 + g$       D  $20 - g$
44. **GRIDDED RESPONSE** The gym has 2-kilogram and 5-kilogram disks for weight lifting. They have fourteen disks in all. The total weight of the 2-kilogram disks is the same as the total weight of the 5-kilogram disks. How many 2-kilogram disks are there?

## Spiral Review

Simplify each expression. (Lesson 11-7)

45.  $\frac{\frac{c^2}{d}}{\frac{c^3}{d^2}}$

46.  $\frac{\frac{5g^3}{h^2}}{\frac{6g}{5h}}$

47.  $\frac{\frac{2}{b}}{\frac{4}{b-3}}$

48.  $\frac{\frac{q-2}{9}}{\frac{q^2-6q+8}{12}}$

Find the LCM of each pair of polynomials. (Lesson 11-6)

49.  $2h, 4h^2$

50.  $5c^2, 12c^3$

51.  $x - 4, x + 2$

52.  $p - 7, 2(p - 14)$

Look for a pattern in each table of values to determine which kind of model best describes the data. (Lesson 9-9)

53.

$x$	0	1	2	3	4
$y$	4	5	6	7	8

54.

$x$	1	2	3	4	5
$y$	2	4	8	16	32

55.

$x$	-3	-2	-1	0	1
$y$	14	9	6	5	6

56.

$x$	3	4	5	6	7
$y$	3	5	7	9	11

57. **GENETICS** Brown genes  $B$  are dominant over blue genes  $b$ . A person with genes  $BB$  or  $Bb$  has brown eyes. Someone with genes  $bb$  has blue eyes. Mrs. Dunn has brown eyes with genes  $Bb$ , and Mr. Dunn has blue eyes. Write an expression for the possible eye coloring of their children. Then find the probability that a child would have blue eyes. (Lesson 7-8)

Solve each inequality. Check your solution. (Lesson 5-2)

58.  $\frac{b}{10} \leq 5$

59.  $-7 > -\frac{r}{7}$

60.  $\frac{5}{8}y \geq -15$

## Skills Review

Determine the probability of each event if you randomly select a marble from a bag containing 9 red marbles, 6 blue marbles, and 5 yellow marbles. (Lesson 0-11)

61.  $P(\text{blue})$

62.  $P(\text{red})$

63.  $P(\text{not yellow})$

## Chapter Summary

### Key Concepts

#### Inverse Variation (Lesson 11-1)

- You can use  $\frac{x_1}{x_2} = \frac{y_2}{y_1}$  to solve problems involving inverse variation.

#### Rational Functions (Lesson 11-2)

- Excluded values are values of a variable that result in a denominator of zero.
- If vertical asymptotes occur, it will be at excluded values.

#### Rational Expressions (Lessons 11-3 and 11-4)

- Multiplying rational expressions is similar to multiplying rational numbers.
- Divide rational expressions by multiplying by the reciprocal of the divisor.

#### Dividing Polynomials (Lesson 11-5)

- To divide a polynomial by a monomial, divide each term of the polynomial by the monomial.

#### Adding and Subtracting Rational Expressions (Lesson 11-6)

- Rewrite rational expressions with unlike denominators using the least common denominator (LCD). Then add or subtract.

#### Complex Fractions (Lesson 11-7)

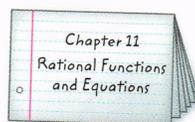
- Simplify complex fractions by writing them as division problems.

#### Solving Rational Equations (Lesson 11-8)

- Use cross products to solve rational equations with a single fraction on each side of the equals sign.

## FOLDABLES® Study Organizer

Be sure the Key Concepts are noted in your Foldable.



## Key Vocabulary

**asymptote** (p. 679)

**complex fraction** (p. 714)

**excluded value** (p. 678)

**extraneous solution** (p. 721)

**inverse variation** (p. 670)

**least common denominator (LCD)** (p. 708)

**least common multiple (LCM)** (p. 707)

**mixed expression** (p. 714)

**product rule** (p. 671)

**rate problems** (p. 723)

**rational equation** (p. 720)

**rational expression** (p. 684)

**rational function** (p. 678)

**work problems** (p. 722)

## Vocabulary Check

State whether each sentence is *true* or *false*. If *false*, replace the underlined word, phrase, expression, or number to make a true sentence.

1. The least common multiple for  $x^2 - 25$  and  $x - 5$  is  $x - 5$ .
2. If the product of two variables is a nonzero constant, the relationship is an inverse variation.
3. If the line  $x = a$  is a vertical asymptote of a rational function, then  $a$  is an excluded value.
4. A rational expression is a fraction in which the numerator and denominator are fractions.
5. The excluded values for  $\frac{x}{x^2 + 5x + 6}$  are  $-2$  and  $-3$ .
6. The equation  $\frac{3x}{x-2} = \frac{6}{x-2}$  has an extraneous solution, 2.
7. A rational expression has one or more fractions in the numerator and denominator.
8. The expression  $\frac{1}{\frac{2}{3} \cdot \frac{4}{4}}$  can be simplified to  $\frac{2}{3}$ .
9. A direct variation can be represented by an equation of the form  $k = xy$ , where  $k$  is a nonzero constant.
10. The rational function  $y = \frac{2}{x-1} + 3$  has a horizontal asymptote at  $y = 3$ .

## Lesson-by-Lesson Review

## 11-1 Inverse Variation (pp. 670–676)

Solve. Assume that  $y$  varies inversely as  $x$ .

- If  $y = 4$  when  $x = 1$ , find  $x$  when  $y = 12$
- If  $y = -1$  when  $x = -3$ , find  $y$  when  $x = -9$
- If  $y = 1.5$  when  $x = 6$ , find  $y$  when  $x = -16$
- PHYSICS** A 135-pound person sits 5 feet from the center of a seesaw. How far from the center should a 108-pound person sit to balance the seesaw?

## EXAMPLE 1

If  $y$  varies inversely as  $x$  and  $y = 28$  when  $x = 42$ , find  $y$  when  $x = 56$ .

Let  $x_1 = 42$ ,  $x_2 = 56$ , and  $y_1 = 28$ . Solve for  $y_2$ .

$$\frac{x_1}{x_2} = \frac{y_2}{y_1} \quad \text{Proportion for inverse variation}$$

$$\frac{42}{56} = \frac{y_2}{28} \quad \text{Substitution}$$

$$1176 = 56y_2 \quad \text{Cross multiply.}$$

$$21 = y_2$$

Thus,  $y = 21$  when  $x = 56$ .

## 11-2 Rational Functions (pp. 678–683)

State the excluded value for each function.

$$15. y = \frac{1}{x-3} \quad 16. y = \frac{2}{2x-5}$$

$$17. y = \frac{3}{3x-6} \quad 18. y = \frac{-1}{2x+8}$$

- PIZZA PARTY** Katelyn ordered pizza and soda for her study group for \$38. The cost per person  $y$  is given by  $y = \frac{38}{x}$ , where  $x$  is the number of people in the study group. Graph the function and describe the asymptotes.

## EXAMPLE 2

State the excluded value for the function  $y = \frac{1}{4x+16}$ .

Set the denominator equal to zero.

$$4x + 16 = 0$$

$$4x + 16 - 16 = 0 - 16 \quad \text{Subtract 16 from each side.}$$

$$4x = -16 \quad \text{Simplify.}$$

$$x = -4 \quad \text{Divide each side by 4.}$$

## 11-3 Simplifying Rational Expressions (pp. 684–691)

Simplify each expression.

$$20. \frac{2xy^2}{16xyz} \quad 21. \frac{x+4}{x^2+12x+32}$$

$$22. \frac{x^2+10x+21}{x^3+x^2-42x} \quad 23. \frac{y^2-25}{y^2+3y-10}$$

$$24. \frac{3x^3}{3x^3+6x^2} \quad 25. \frac{4y^2}{8y^4+16y^3}$$

State the excluded values for each function.

$$26. y = \frac{x}{x^2+9x+18} \quad 27. y = \frac{10}{6x^2+7x-3}$$

## EXAMPLE 3

Simplify  $\frac{a^2-7a+12}{a^2-13a+36}$ .

Factor and simplify.

$$\frac{a^2-7a+12}{a^2-13a+36} = \frac{(a-3)(a-4)}{(a-9)(a-4)} \quad \text{Factor.}$$

$$= \frac{a-3}{a-9} \quad \text{Simplify.}$$

**11-4** Multiplying and Dividing Rational Expressions (pp. 692–698)

Find each product or quotient.

28.  $\frac{6x^2y^4}{12} \cdot \frac{3x^3y^2}{xy}$

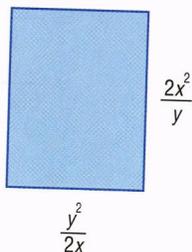
29.  $\frac{3x-6}{x^2-9} \cdot \frac{x+3}{x^2-2x}$

30.  $\frac{x^2}{x+4} \div \frac{3x}{x^2-16}$

31.  $\frac{3b-12}{b+4} \div (b^2-6b+8)$

32.  $\frac{2a^2+7a-15}{a+5} \div \frac{9a^2-4}{3a+2}$

33. **GEOMETRY** Find the area of the rectangle shown. Write the answer in simplest form.



**EXAMPLE 4**

Find  $\frac{7b^2}{9} \cdot \frac{6a^2}{b}$ .

$$\frac{7b^2}{9} \cdot \frac{6a^2}{b} = \frac{42a^2b^2}{9b}$$

**Multiply.**

$$= \frac{14a^2b}{3}$$

**Simplify.**

**EXAMPLE 5**

Find  $\frac{x^2-25}{x^2-9} \div \frac{x+5}{x-3}$ .

$$\frac{x^2-25}{x^2-9} \div \frac{x+5}{x-3} = \frac{(x+5)(x-5)}{(x+3)(x-3)} \div \frac{x+5}{x-3}$$

**Factor.**

$$= \frac{\overset{1}{\cancel{x+5}}(x-5)}{(x+3)\overset{1}{\cancel{x-3}}} \cdot \frac{\overset{1}{\cancel{x-3}}}{\overset{1}{\cancel{x+5}}}$$

**Multiply by the reciprocal.**  
**Simplify.**

$$= \frac{x-5}{x+3}$$

**11-5** Dividing Polynomials (pp. 700–705)

Find each quotient.

34.  $(x^3 - 2x^2 - 22x + 21) \div (x - 3)$

35.  $(x^3 + 7x^2 + 10x - 6) \div (x + 3)$

36.  $(5x^2y^2 - 10x^2y + 5xy) \div 5xy$

37.  $(48y^2 + 8y + 7) \div (12y - 1)$

38. **GEOMETRY** The area of a rectangle is  $x^2 + 7x + 13$ . If the length is  $(x + 4)$ , what is the width of the rectangle?

**EXAMPLE 6**

Find  $(4x^2 + 17x - 1) \div (4x + 1)$ .

$$\begin{array}{r} x + 4 \\ 4x + 1 \overline{) 4x^2 + 17x - 1} \\ \underline{4x^2 + x} \phantom{- 1} \\ 16x - 1 \\ \underline{16x + 4} \\ -5 \end{array}$$

**Multiply x and 4x + 1.**

**Subtract, bring down -1.**

**Multiply 4 and 4x + 1.**

**Subtract.**

The quotient is  $x + 4 - \frac{5}{4x + 1}$ .

**11-6** Adding and Subtracting Rational Expressions (pp. 706–713)

Find each sum or difference.

39.  $\frac{5a}{b} - \frac{2a}{b}$

40.  $\frac{-3}{2n-3} + \frac{2n}{2n-3}$

41.  $\frac{3}{y+1} - \frac{y}{y-3}$

42.  $\frac{1}{x+1} + \frac{3}{x-2}$

43. **DESIGN** Miguel is decorating a model of a room that is  $\frac{2x}{x+4}$  feet long and  $\frac{8}{x+4}$  feet wide. What is the perimeter of the room?

**EXAMPLE 7**

Find  $\frac{x^2}{x+1} + \frac{2x+1}{x+1}$ .

$$\frac{x^2}{x+1} + \frac{2x+1}{x+1} = \frac{x^2+2x+1}{x+1}$$

**Add the numerators.**

$$= \frac{(x+1)(x+1)}{x+1}$$

**Factor.**

$$= x + 1$$

**Simplify.**

## 11-7 Mixed Expressions and Complex Fractions (pp. 714-719)

Simplify each expression.

44.  $\frac{\frac{a^2b^4}{c}}{\frac{a^3b}{c^2}}$

45.  $\frac{x - \frac{35}{x+2}}{x + \frac{42}{x+13}}$

46.  $\frac{\frac{x^2 - 25}{x+2}}{\frac{x-5}{x^2-4}}$

47.  $\frac{y+9 - \frac{6}{y+4}}{y+4 + \frac{2}{y+1}}$

48. **FABRICS** Donna makes tablecloths to sell at craft fairs. A small one takes one-half yard of fabric, a medium one takes five-eighths yard, and a large one takes one and one-quarter yard.

- How many yards of fabric does she need to make a tablecloth of each size?
- One bolt of fabric contains 30 yards of fabric. Can she use the entire bolt of fabric by making an equal number of each type of tablecloth? Explain.

## EXAMPLE 8

Simplify  $\frac{\frac{x+3}{6}}{\frac{x^2-2x-15}{x}}$ .

Write as a division expression.

$$\begin{aligned} \frac{\frac{x+3}{6}}{\frac{x^2-2x-15}{x}} &= \frac{x+3}{6} \div \frac{x^2-2x-15}{x} \\ &= \frac{x+3}{6} \cdot \frac{x}{x^2-2x-15} \\ &= \frac{x+3}{6} \cdot \frac{x}{(x+3)(x-5)} \\ &= \frac{x}{6(x-5)} \end{aligned}$$

## 11-8 Rational Equations (pp. 720-726)

Solve each equation. State any extraneous solutions.

49.  $\frac{5n}{6} + \frac{1}{n-2} = \frac{n+1}{3(n-2)}$

50.  $\frac{4x}{3} + \frac{7}{2} = \frac{7x}{12} - 14$

51.  $\frac{11}{2x} + \frac{2}{4x} = \frac{1}{4}$

52.  $\frac{1}{x+4} - \frac{1}{x-1} = \frac{2}{x^2+3x-4}$

53.  $\frac{1}{n-2} = \frac{n}{8}$

54. **PAINTING** Anne can paint a room in 6 hours. Oljay can paint a room in 4 hours. How long will it take them to paint the room working together?

## EXAMPLE 9

Solve  $\frac{3}{x^2+3x} + \frac{x+2}{x+3} = \frac{1}{x}$ .

$$\begin{aligned} \frac{3}{x^2+3x} + \frac{x+2}{x+3} &= \frac{1}{x} \\ x(x+3)\left(\frac{3}{x(x+3)}\right) + x(x+3)\left(\frac{x+2}{x+3}\right) &= x(x+3)\left(\frac{1}{x}\right) \\ 3 + x(x+2) &= 1(x+3) \\ 3 + x^2 + 2x &= x + 3 \\ x^2 + x &= 0 \\ x(x+1) &= 0 \\ x = 0 \text{ or } x = -1 \end{aligned}$$

The solution is  $-1$ , and there is an extraneous solution of  $0$ .

Determine whether each table represents an inverse variation. Explain.

1.

x	y
2	10
4	12
8	14

2.

x	y
2	2
4	1
8	$\frac{1}{2}$

Find each product or quotient.

3.  $\frac{(x+6)(x-2)}{x^3} \cdot \frac{7x^2}{x-3}$

4.  $\frac{(x+3)}{y^2} \div \frac{x^2-9}{y}$

Solve. Assume that  $y$  varies inversely as  $x$ .

5. If  $y = 3$  when  $x = 9$ , find  $x$  when  $y = 1$ .

6. If  $y = 2$  when  $x = 0.5$ , find  $y$  when  $x = 3$ .

Simplify each expression. State the excluded values of the variables.

7.  $\frac{z-6}{z^2-3z-18}$

8.  $\frac{4x-28}{x^2-49}$

9. **MULTIPLE CHOICE** The area of a rectangle is  $x^2 + 5x + 6$  square feet. If the width is  $x + 2$ , what is the length of the rectangle?



A  $x + 2$

B  $x + 3$

C 1

D 3

Simplify each expression.

10.  $\frac{2\frac{1}{3}}{3\frac{1}{2}}$

11.  $\frac{\frac{x^2-25}{x-2}}{\frac{x-5}{x-2}}$

12.  $\frac{\frac{a-4}{a^2+6a+8}}{\frac{a^2-3a-4}{a^2-a-6}}$

13.  $\frac{\frac{y^2+10y+24}{y^2-9}}{\frac{3y^2+17y-6}{2y^2-11y+15}}$

Find each quotient.

14.  $(2x^2 + 10x) \div 2x$

15.  $(4x^2 - 8x + 5) \div (2x + 1)$

16.  $(3x^2 - 14x - 3) \div (x - 5)$

Assume that  $y$  varies inversely as  $x$ . Write an inverse variation equation that relates  $x$  and  $y$ .

17.  $y = 2$  when  $x = 8$

18.  $y = -3$  when  $x = 1$

Find each sum or difference.

19.  $\frac{3}{x} + \frac{6}{x}$

20.  $\frac{t-5}{t-6} + \frac{t+8}{t-6}$

21.  $\frac{1}{x-6} + \frac{3}{x-2}$

22.  $\frac{5}{x^2-2x-24} + \frac{x}{x-6}$

State the excluded value or values for each function.

23.  $y = \frac{6}{x-1}$

24.  $y = \frac{5}{x^2-5x-24}$

Identify the asymptotes of each function.

25.  $y = \frac{2}{(x-4)(x+2)}$

26.  $y = \frac{4}{x^2+3x-28} + 2$

27. **MULTIPLE CHOICE** Lee can shovel the driveway in 3 hours, and Susan can shovel the driveway in 2 hours. How long will it take them working together?

F 6 hours

G 5 hours

H  $\frac{3}{2}$  hour

J  $\frac{6}{5}$  hours

28. **PAINTING** Sydney can paint a 60-square foot wall in 40 minutes. Working with her friend Cleveland, the two of them can paint the wall in 25 minutes. How long would it take Cleveland to do the job himself?

## Model with an Equation

In order to successfully solve some standardized test questions, you will need to be able to write equations to model different situations. Use this lesson to practice solving these types of problems.

### Strategies for Modeling with Equations

#### Step 1

Read the problem statement carefully.

#### Ask yourself:

- What am I being asked to solve?
- What information is given in the problem?
- What is the unknown quantity that I need to find?

#### Step 2

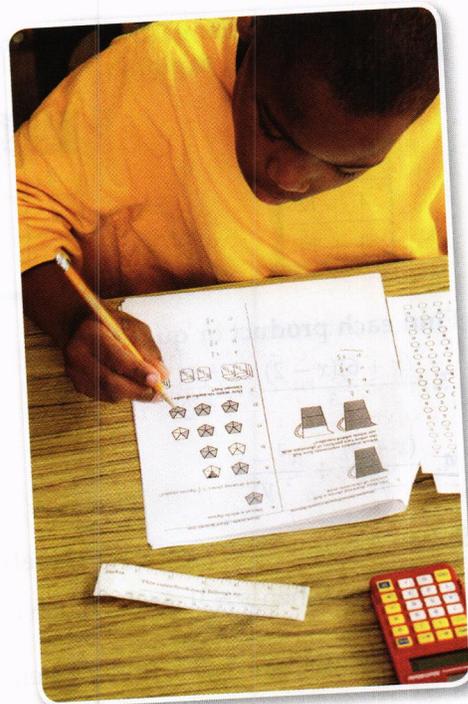
Translate the problem statement into an equation.

- Assign a variable to the unknown quantity.
- Write the word sentence as a mathematical number sentence.
- Look for keywords such as *is*, *is the same as*, *is equal to*, or *is identical to* that indicate where to place the equals sign.

#### Step 3

Solve the equation.

- Solve for the unknown in the equation.
- Check your answer to be sure it is reasonable and that it answers the question in the problem statement.



### EXAMPLE

Read the problem. Identify what you need to know. Then use the information in the problem to solve.

It takes Craig 75 minutes to paint a small room. If Delsin can paint the same room in 60 minutes, how long would it take them to paint the room if they work together? Round to the nearest tenth.

- A about 33.3 minutes                      C about 45.1 minutes  
B about 38.4 minutes                      D about 50.3 minutes

Read the problem carefully. You know how long it takes Craig and Delsin to paint a room individually. Model the situation with an equation to find how long it would take them to paint the room if they work together.

Find the rate that each person works when painting individually.

$$\text{Craig's rate: } \frac{1 \text{ job}}{75 \text{ minutes}} = \frac{1}{75} \text{ job per minute}$$

$$\text{Delsin's rate: } \frac{1 \text{ job}}{60 \text{ minutes}} = \frac{1}{60} \text{ job per minute}$$

Let  $t$  represent the number of minutes it would take them to complete the job working together. Multiply each rate by the time  $t$  to represent the portion of the job done by each painter. Add these expressions and set them equal to 1 job. Then solve for  $t$ .

<b>Portion that Craig completes</b>	plus	<b>portion that Delsin completes</b>	equals	<b>1 job.</b>
$\frac{1}{75}t$	+	$\frac{1}{60}t$	=	1

Solve for  $t$ :

$$\frac{1}{75}t + \frac{1}{60}t = 1 \quad \text{Original equation}$$

$$300\left(\frac{1}{75}t + \frac{1}{60}t\right) = 1(300) \quad \text{Multiply each side by the LCD, 300.}$$

$$4t + 5t = 300 \quad \text{Simplify.}$$

$$9t = 300 \quad \text{Combine like terms.}$$

$$t \approx 33.3 \quad \text{Divide each side by 9.}$$

So, it would take Craig and Delsin about 33.3 minutes to paint the room working together. The correct answer is A.

## Exercises

Read each problem. Identify what you need to know. Then use the information in the problem to solve.

1. Hana can finish a puzzle in 6 hours, while Eric can finish one in 5 hours. How long would it take them to finish a puzzle together? Round to the nearest tenth.
  - A about 1.8 hours
  - B about 2.4 hours
  - C about 2.5 hours
  - D about 2.7 hours
2. Roberto wants to print 500 flyers for his landscaping business. His printer can complete the job in 35 minutes, and his brother's printer can print them in 45 minutes. How long would it take to print the flyers using both printers? Round to the nearest whole minute.
  - F about 15 minutes
  - G about 18 minutes
  - H about 20 minutes
  - J about 23 minutes

## Multiple Choice

Read each problem. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.

1. What is the inverse variation equation for the numbers shown in the table?

x	y
-8	16
-4	32
2	-64
8	-16
16	-8

- A  $y = -2x$   
 B  $y = 8x$   
 C  $xy = 24$   
 D  $xy = -128$
2. Suppose a square has a side length given by the expression  $\frac{x+5}{8x}$ . What is the perimeter of the square?
- F  $\frac{4x+20}{5x}$   
 G  $\frac{2x+10}{x}$   
 H  $\frac{x+5}{4x}$   
 J  $\frac{x+5}{2x}$
3. Find the distance between  $(3, -6)$  and  $(1, 4)$  on a coordinate grid. Round to the nearest tenth.
- A 8.1  
 B 8.5  
 C 9.6  
 D 10.2

## Test-Taking Tip

**Question 1** Sometimes you can eliminate answer choices as unreasonable because they are not in the proper form. Choices A and B show direct variation equations, so they can be eliminated.

4. In 1985, the population of a country was about 3.66 million people. By 2005, this number had grown to about 4.04 million people. What was the annual rate of change in population from 1985 to 2005?

- F about 15,000 people per year  
 G about 19,000 people per year  
 H about 24,000 people per year  
 J about 38,000 people per year

5. Ricky's Rentals rented 12 more bicycles than scooters last weekend for a total revenue of \$2,125. How many scooters were rented?

Item	Rental Fee
Bicycle	\$20
Scooter	\$45

- A 26  
 B 29  
 C 37  
 D 41
6. The table shows the relationship between calories and fat grams contained in orders of french fries from various restaurants.

Calories	Fat Grams
240	14
280	15
310	16
260	12
340	16
350	18
300	13

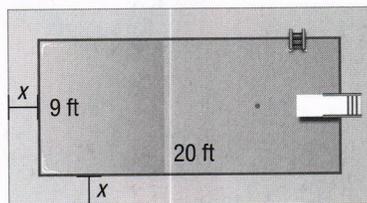
Assuming the data can best be described by a linear model, how many fat grams would be expected to be contained in a 315-calorie order of french fries?

- F 15 fat grams  
 G 16 fat grams  
 H 17 fat grams  
 J 18 fat grams

### Short Response/Gridded Response

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

7. Suppose the first term of a geometric sequence is 3 and the fourth term is 192.
- What is the common ratio of the sequence?
  - Write an equation that can be used to find the  $n$ th term of the sequence.
  - What is the sixth term of the sequence?
8. **GRIDDED RESPONSE** Peggy is having a cement walkway installed around the perimeter of her swimming pool with the dimensions shown below. If  $x = 3$  find the area, in square feet, of the pool and walkway.



9. Use the equation  $y = 2(4 + x)$  to answer each question.
- Complete the following table for the different values of  $x$ .
  - Plot the points from the table on a coordinate grid. What do you notice about the points?

$x$	$y$
1	
2	
3	
4	
5	
6	

10. Jason received a \$50 gift certificate for his birthday. He wants to buy a DVD and a poster from a media store. (Assume that sales tax is included in the prices.) Write and solve a linear inequality to show how much he would have left to spend after making these purchases.

**Weekend Blowout Sale**

- \* All DVDs only **\$14.95**
- \* All CDs only **\$11.25**
- \* All posters only **\$10.99**

11. Simplify the complex fraction. Show your work.

$$\frac{\frac{5}{x-3}}{\frac{x-6}{x^2-x-6}}$$

### Extended Response

Record your answers on a sheet of paper. Show your work.

12. Carl's father is building a tool chest that is shaped like a rectangular prism. He wants the tool chest to have a surface area of 62 square feet. The height of the chest will be 1 foot shorter than the width. The length will be 3 feet longer than the height.
- Sketch a model to represent the problem.
  - Write a polynomial that represents the surface area of the tool chest.
  - What are the dimensions of the tool chest?

#### Need Extra Help?

If you missed Question...

Go to Lesson or Page...

1	2	3	4	5	6	7	8	9	10	11	12
11-1	11-6	10-6	3-3	6-2	4-5	9-8	7-7	1-4	5-1	11-7	8-6

# CHAPTER 12

# Statistics and Probability

## Then

In chapter 0, you calculated simple probability.

## Now

In Chapter 12, you will:

- Design surveys and evaluate results.
- Use permutations and combinations.
- Find probabilities of compound events.
- Design and use simulations.

## Why?

 **RESTAURANTS** A restaurant may ask their customers to complete a survey about their visit. The survey data can be analyzed using statistical methods. The restaurant staff can learn more about their customers and how to improve their experiences in the restaurant.



Statistics and Probability

Activity

Cathy's Cafe asks their customers to fill out the following survey.



**Cathy's Cafe**

We welcome your comments. Please fill out this questionnaire. Thank You!

1. Please rate the quality of the service you received from the server.
2. Please rate the quality of your main dish.
3. Please rate the quality of your Desserts.
4. Please rate the cleanliness of your booth.
5. Please rate your overall Dinner experience.

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