Get Ready for Chapter 6

Diagnose Readiness You have two options for checking Prerequisite Skills.

QuickCheck

Name the ordered pair for each point on the coordinate plane. (Lesson 1-6)
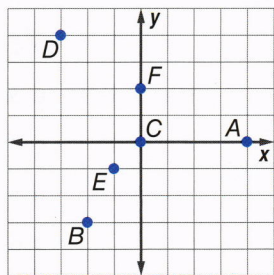
1. A
2. D
3. B
4. C
5. E
6. F

Solve each equation or formula for the variable specified. (Lesson 2-8)

7. $2x + 4y = 12$, for x

8. $x = 3y - 9$, for y

9. $m - 2n = 6$, for m

10. $y = mx + b$, for x

11. $P = 2\ell + 2w$, for ℓ

12. $5x - 10y = 40$, for y

13. **GEOMETRY** The formula for the area of a triangle is $A = \frac{1}{2}bh$, where A represents the area, b is the base, and h is the height of the triangle. Solve the equation for b.

QuickReview

EXAMPLE 1

Name the ordered pair for Q on the coordinate plane.
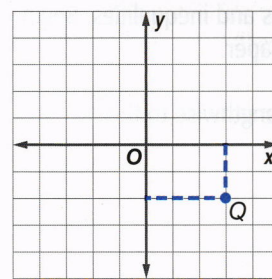
Follow a vertical line from the point to the x-axis. This gives the x-coordinate, 3.

Follow a horizontal line from the point to the y-axis. This gives the y-coordinate, -2.

The ordered pair is $(3, -2)$.

EXAMPLE 2

Solve $12x + 3y = 36$ for y.

$$12x + 3y = 36 \quad \text{Original equation}$$

$$12x + 3y - 12x = 36 - 12x \quad \text{Subtract 12x from each side.}$$

$$3y = 36 - 12x \quad \text{Simplify.}$$

$$\frac{3y}{3} = \frac{36 - 12x}{3} \quad \text{Divide each side by 3.}$$

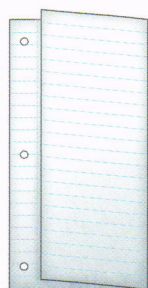$$y = 12 - 4x \quad \text{Simplify.}$$

Get Started on Chapter 6

You will learn several new concepts, skills, and vocabulary terms as you study Chapter 6. To get ready, identify important terms and organize your resources. You may wish to refer to **Chapter 0** to review prerequisite skills.
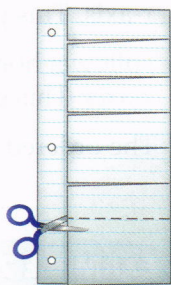
FOLDABLES® Study Organizer

Linear Functions Make this Foldable to help you organize your Chapter 6 notes about solving systems of equations and inequalities. Begin with a sheet of notebook paper.
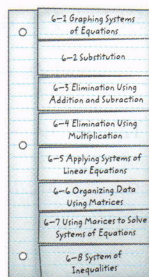
1 **Fold** lengthwise to the holes.

2 **Cut** 8 tabs.

3 **Label** the tabs using the lesson numbers and lesson titles.

6–1 Graphing Systems of Equations
6–2 Substitution
6–3 Elimination Using Addition and Subtraction
6–4 Elimination Using Multiplication
6–5 Applying Systems of Linear Equations
6–6 Organizing Data Using Matrices
6–7 Using Matrices to Solve Systems of Equations
6–8 System of Inequalities

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- Study the chapter online
- Explore **Math in Motion**
- Get extra help from your own **Personal Tutor**
- Use **Extra Examples** for additional help
- Take a **Self-Check Quiz**
- **Review Vocabulary** in fun ways

New Vocabulary

English		Español
system of equations	• p. 333 •	sistema de ecuaciones
consistent	• p. 333 •	consistente
independent	• p. 333 •	independiente
dependent	• p. 333 •	dependiente
inconsistent	• p. 333 •	inconsistente
substitution	• p. 342 •	sustitución
elimination	• p. 348 •	eliminación
matrix	• p. 369 •	matriz
element	• p. 369 •	elemento
dimension	• p. 369 •	dimensión
scalar	• p. 371 •	escalar
scalar multiplication	• p. 371 •	multiplicación por escalares
augmented matrix	• p. 376 •	matriz ampliada
row reduction	• p. 377 •	reducción de fila
identity matrix	• p. 377 •	matriz
system of inequalities	• p. 382 •	sistema de desigualdades

Review Vocabulary

domain • p. 38 • **dominio** the set of the first numbers of the ordered pairs in a relation

intersection • p. 304 • **intersección** the graph of a compound inequality containing *and*; the solution is the set of elements common to both graphs

proportion • p. 111 • **proporción** an equation stating that two ratios are equal

Proportion

$$\div 6$$
$$\frac{24}{30} = \frac{4}{5}$$
$$\div 6$$

**Multilingual eGlossary** glencoe.com

Graphing Systems of Equations

Why?

The cost to begin production on a band's CD is $1500. Each CD costs $4 to produce and will sell for $10. The band wants to know how many CDs they will have to sell to earn a profit.

Graphing a system can show when a company makes a profit. The cost of producing the CD can be modeled by the equation $y = 4x + 1500$, where y represents the cost of production and x is the number of CDs produced.

The income from the CDs sold can be modeled by the equation $y = 10x$, where y represents the total income of selling the CDs, and x is the number of CDs sold.

If we graph these equations, we can see at which point the band begins making a profit. The point where the two graphs intersect is where the band breaks even. This happens when the band sells 250 CDs. If the band sells more than 250 CDs, they will make a profit.

Band's CD Sales
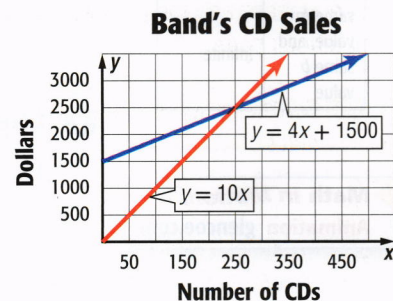
Possible Number of Solutions The two equations, $y = 4x + 1500$ and $y = 10x$, form a **system of equations**. The ordered pair that is a solution of both equations is the solution of the system. A system of two linear equations can have one solution, an infinite number of solutions, or no solution.

- If a system has at least one solution, it is said to be **consistent**. The graphs intersect at one point or are the same line.

- If a consistent system has exactly one solution, it is said to be **independent**. If it has an infinite number of solutions, it is **dependent**. This means that there are unlimited solutions that satisfy both equations.

- If a system has no solution, it is said to be **inconsistent**. The graphs are parallel.

Concept Summary   **Possible Solutions**   For Your FOLDABLE

| Number of Solutions	exactly one	infinite	no solution
Terminology	consistent and independent	consistent and dependent	inconsistent
Graph			

EXAMPLE 1 **Number of Solutions**

Use the graph at the right to determine whether each system is *consistent* or *inconsistent* and if it is *independent* or *dependent*.

a. $y = -2x + 3$
   $y = x - 5$

Since the graphs of these two lines intersect at one point, there is exactly one solution. Therefore, the system is consistent and independent.

b. $y = -2x - 5$
   $y = -2x + 3$

Since the graphs of these two lines are parallel, there is no solution of the system. Therefore, the system is inconsistent.
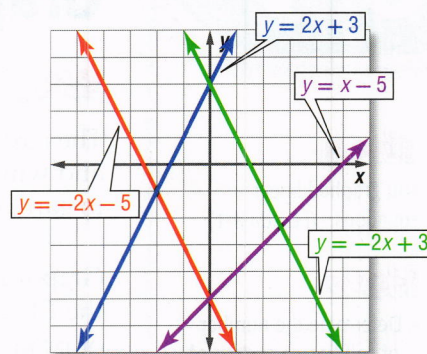
✓ **Check Your Progress**

1A. $y = 2x + 3$
    $y = -2x - 5$

1B. $y = x - 5$
    $y = -2x - 5$

▶ **Personal Tutor glencoe.com**

Solve by Graphing One method of solving a system of equations is to graph the equations carefully on the same coordinate grid and find their point of intersection. This point is the solution of the system.

EXAMPLE 2 **Solve by Graphing**

Graph each system and determine the number of solutions that it has. If it has one solution, name it.

a. $y = -3x + 10$
   $y = x - 2$

The graphs appear to intersect at the point (3, 1). You can check this by substituting 3 for x and 1 for y.

CHECK $y = -3x + 10$ **Original equation**
$1 \stackrel{?}{=} -3(3) + 10$ **Substitution**
$1 \stackrel{?}{=} -9 + 10$ **Multiply.**
$1 = 1$ ✔

$y = x - 2$ **Original equation**
$1 \stackrel{?}{=} 3 - 2$ **Substitution**
$1 = 1$ ✔   **Multiply.**

The solution is (3, 1).

b. $2x - y = -1$
   $4x - 2y = 6$

The lines have the same slope but different y-intercepts, so the lines are parallel. Since they do not intersect, there is no solution of this system. The system is inconsistent.

Graph each system and determine the number of solutions that it has. If it has one solution, name it.

2A. $x - y = 2$
$3y + 2x = 9$

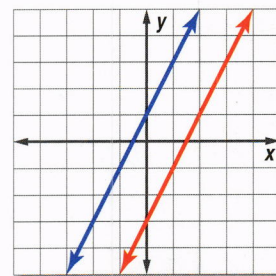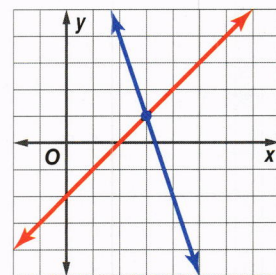**2B.** $y = -2x - 3$
$6x + 3y = -9$

We can use what we know about systems of equations to solve many real-world problems that involve two or more different functions.

🌐 Real-World EXAMPLE 3　Write and Solve a System of Equations

SPORTS The number of girls participating in high school soccer and track and field has steadily increased over the past few years. Use the information in the table to predict the approximate year when the number of girls participating in these two sports will be the same.

High School Sport	Number of Girls Participating in 2004 (thousands)	Average rate of increase (thousands per year)
soccer	309	8
track and field	418	3

Source: National Federation of State High School Associations

Words	Number of girls participating	equals	rate of increase	times	number of years after 2004	plus	number participating in 2004.

Variables　Let y = number of girls competing. Let x = number of years after 2004.

Equations

Soccer: $y = 8 \cdot x + 309$

Track and field: $y = 3 \cdot x + 418$

Real-World Link

In 2004, 2.9 million girls participated in high school sports. This was an all-time high for female participation.

Source: National Federation of State High School Associations

Graph $y = 8x + 309$ and $y = 3x + 418$. The graphs appear to intersect at approximately (22, 485).

CHECK　Use substitution to check this answer.

$y = 8x + 309$ 　　　 $y = 3x + 418$

$485 \stackrel{?}{=} 8(22) + 309$ 　 $485 \stackrel{?}{=} 3(22) + 418$

$485 = 485$ ✔ 　　　 $485 \approx 484$ ✔

The solution means that approximately 22 years after 2004, or in 2026, the number of girls participating in high school soccer and track and field will be the same, about 485,000.
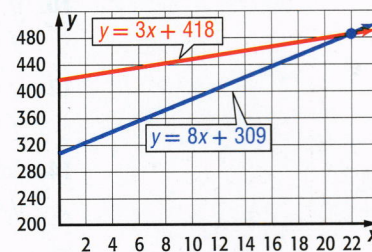
3. VIDEO GAMES Joe and Josh each want to buy a video game. Joe has $14 and saves $10 a week. Josh has $26 and saves $7 a week. In how many weeks will they have the same amount?

Example 1
p. 334

Use the graph at the right to determine whether each system is *consistent* or *inconsistent* and if it is *independent* or *dependent*.

1. $y = -3x + 1$
$y = 3x + 1$

2. $y = 3x + 1$
$y = x - 3$

3. $y = x - 3$
$y = x + 3$

4. $y = x + 3$
$x - y = -3$

5. $x - y = -3$
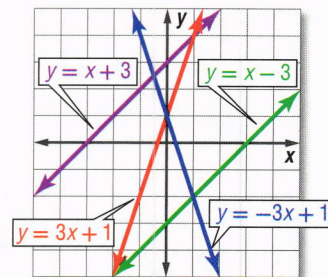$y = -3x + 1$

6. $y = -3x + 1$
$y = x - 3$

Example 2
p. 334

Graph each system and determine the number of solutions that it has. If it has one solution, name it.

7. $y = x + 4$
$y = -x - 4$

8. $y = x + 3$
$y = 2x + 4$

Example 3
p. 335

9. READING Alberto and Ashanti are reading a graphic novel.

a. Write an equation to represent the pages each boy has read.

b. Graph each equation.

c. How long will it be before Alberto has read more pages than Ashanti? Check and interpret your solution.

Alberto
35 pages read;
20 pages each day

Ashanti
85 pages read;
10 pages each day

Practice and Problem Solving

● = **Step-by-Step Solutions** begin on page R12.
Extra Practice begins on page 815.

Example 1
p. 334

Use the graph at the right to determine whether each system is *consistent* or *inconsistent* and if it is *independent* or *dependent*.

10. $y = 6$
$y = 3x + 4$

11. $y = 3x + 4$
$y = -3x + 4$

12. $y = -3x + 4$
$y = -3x - 4$

13 $y = -3x - 4$
$y = 3x - 4$

14. $3x - y = -4$
$y = -3x + 4$

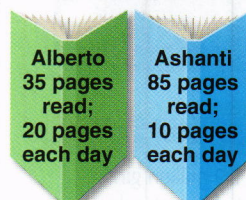**15.** $3x - y = 4$
$3x + y = 4$
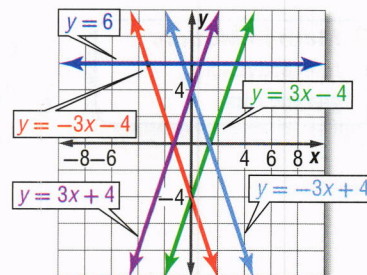
Example 2
p. 334

Graph each system and determine the number of solutions that it has. If it has one solution, name it.

16. $y = -3$
$y = x - 3$

17. $y = 4x + 2$
$y = -2x - 3$

18. $y = x - 6$
$y = x + 2$

19. $x + y = 4$
$3x + 3y = 12$

20. $x - y = -2$
$-x + y = 2$

21. $x + 2y = 3$
$x = 5$

22. $2x + 3y = 12$
$2x - y = 4$

23. $2x + y = -4$
$y + 2x = 3$

24. $2x + 2y = 6$
$5y + 5x = 15$

25. SCHOOL DANCE Akira and Jen are competing to see who can sell the most tickets for the Winter Dance. On Monday, Akira sold 22 and then sold 30 per day after that. Jen sold 53 one Monday and then sold 20 per day after that.

a. Write equations for the number of tickets each person has sold.

b. Graph each equation.

c. Solve the system of equations. Check and interpret your solution.

26. TRAVEL If x is the number of years since 2000 and y is the percent of people using travel services, the following equations represent the percent of people using travel agents and the percent of the people using the Internet to plan travel.

Travel agents: $y = -2x + 30$ Internet: $y = 6x + 41$

a. Graph the system of equations.

b. Estimate the year travel agents and the Internet were used equally.

Graph each system and determine the number of solutions that it has. If it has one solution, name it.

27. $y = \frac{1}{2}x$
$y = x + 2$

28. $y = 6x + 6$
$y = 3x + 6$

29. $y = 2x - 17$
$y = x - 10$

30. $8x - 4y = 16$
$-5x - 5y = 5$

31. $3x + 5y = 30$
$3x + y = 18$

32. $-3x + 4y = 24$
$4x - y = 7$

33. $2x - 8y = 6$
$x - 4y = 3$

34. $4x - 6y = 12$
$-2x + 3y = -6$

35. $2x + 3y = 10$
$4x + 6y = 12$

36. $3x + 2y = 10$
$2x + 3y = 10$

37. $3y - x = -2$
$y - \frac{1}{3}x = 2$

38. $\frac{8}{5}y = \frac{2}{5}x + 1$
$\frac{2}{5}y = \frac{1}{10}x + \frac{1}{4}$

39. $\frac{1}{3}x + \frac{1}{3}y = 1$
$x + y = 1$

40. $\frac{3}{4}x + \frac{1}{2}y = \frac{1}{4}$
$\frac{2}{3}x + \frac{1}{6}y = \frac{1}{2}$

41. $\frac{5}{6}x + \frac{2}{3}y = \frac{1}{2}$
$\frac{2}{5}x + \frac{1}{5}y = \frac{3}{5}$

42. PHOTOGRAPHY Suppose x represents the number of cameras sold and y represents the number of years since 2000. Then the number of digital cameras sold each year since 2000, in millions, can be modeled by the equation $y = 12.5x + 10.9$. The number of film cameras sold each year since 2000, in millions, can be modeled by the equation $y = -9.1x + 78.8$.

a. Graph each equation.

b. In which year did digital camera sales surpass film camera sales?

c. In what year will film cameras stop selling altogether?

d. What are the domain and range of each of the functions in this situation?

Graph each system and determine the number of solutions that it has. If it has one solution, name it.

43. $2y = 1.2x - 10$

$4y = 2.4x$

44. $x = 6 - \frac{3}{8}y$

$4 = \frac{2}{3}x + \frac{1}{4}y$
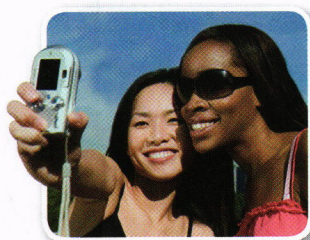
45 **WEB SITES** Personal publishing site *Lookatme* had 2.5 million visitors in 2005. Each year after that, the number of visitors rose by 13.1 million. Online auction site *Buyourstuff* had 59 million visitors in 2005, but each year after that the number of visitors fell by 2 million.
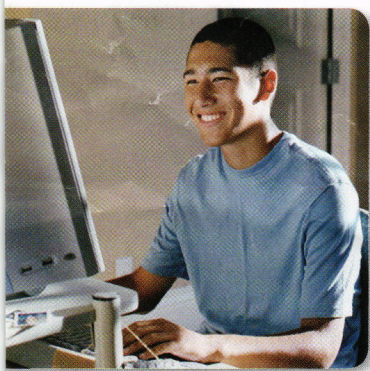
   a. Write an equation for each of the companies.

   b. Make a table of values for 5 years for each of the companies.

   c. Graph each equation.

   d. When will *Lookatme* and *Buyourstuff's* sites have the same number of visitors?

   e. Name the domain and range of these functions in this situation.

46. **MULTIPLE REPRESENTATIONS** In this problem, you will explore different methods for finding the intersection of the graphs of two linear equations.

   a. **ALGEBRAIC** Use algebra to solve the equation $\frac{1}{2}x + 3 = -x + 12$.

   b. **GRAPHICAL** Use a graph to solve $y = \frac{1}{2}x + 3$ and $y = -x + 12$.

   c. **ANALYTICAL** How is the equation in part **a** related to the system in part **b**?

   d. **VERBAL** Explain how to use the graph in part b to solve the equation in part a.

H.O.T. Problems   Use **H**igher-**O**rder **T**hinking Skills

47. **FIND THE ERROR** Store A is offering a 10% discount on the purchase of all electronics in their store. Store B is offering $10 off all the electronics in their store. Francisca and Alan are deciding which offer will save them more money. Is either of them correct? Explain your reasoning.

Francisca	Alan
You can't determine which store has the better offer unless you know the price of the items you want to buy.	Store A has the better offer because 10% of the sale price is a greater discount than $10.

48. **CHALLENGE** Use graphing to find the solution of the system of equations $2x + 3y = 5$, $3x + 4y = 6$, and $4x + 5y = 7$.

49. **REASONING** Determine whether a system of two linear equations with (0, 0) and (2, 2) as solutions *sometimes*, *always*, or *never* has other solutions. Explain.

50. **WHICH ONE DOESN'T BELONG?** Which one of the following systems of equations doesn't belong with the other three? Explain your reasoning.

$4x - y = 5$	$-x + 4y = 8$	$4x + 2y = 14$	$3x - 2y = 1$
$-2x + y = -1$	$3x - 6y = 6$	$12x + 6y = 18$	$2x + 3y = 18$

51. **OPEN ENDED** Write three equations such that they form three systems of equations with $y = 5x - 3$. The three systems should be inconsistent, consistent and independent, and consistent and dependent, respectively.

52. **WRITING IN MATH** Describe the advantages and disadvantages to solving systems of equations by graphing.

53. SHORT RESPONSE Certain bacteria can reproduce every 20 minutes, doubling the population. If there are 450,000 bacteria in a population at 9:00 A.M., how many bacteria will be in the population at 2:00 P.M.?

54. GEOMETRY An 84-centimeter piece of wire is cut into equal segments and then attached at the ends to form the edges of a cube. What is the volume of the cube?
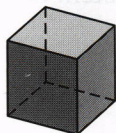
A 294 cm^3 C 1158 cm^3

B 343 cm^3 D 2744 cm^3

55. What is the solution of the inequality $-9 < 2x + 3 < 15$?

F $-x \geq 0$ H $-6 < x < 6$

G $x \leq 0$ J $-5 < x < 5$

56. What is the solution of the system of equations?

$$x + 2y = -1$$
$$2x + 4y = -2$$

A $(-1, -1)$ C no solution

B $(2, 1)$ D infinitely many solutions

Spiral Review

Graph each inequality. (Lesson 5-6)

57. $3x + 6y > 0$

58. $4x - 2y < 0$

59. $3y - x \leq 9$

60. $4y - 3x \geq 12$

61. $y < -4x - 8$

62. $3x - 1 > y$

63. LIBRARY To get a grant from the city's historical society, the number of history books must be within 25 of 1500. What is the range of the number of historical books that must be in the library? (Lesson 5-5)

64. SCHOOL Camilla's scores on three math tests are shown in the table. The fourth and final test of the grading period is tomorrow. She needs an average of at least 92 to receive an A for the grading period. (Lesson 5-3)

Test	Score
1	91
2	95
3	88

a. If m represents her score on the fourth math test, write an inequality to represent this situation.

b. If Camilla wants an A in math, what must she score on the test?

c. Is your solution reasonable? Explain.

Write the slope-intercept form of an equation for the line that passes through the given point and is perpendicular to the graph of the equation. (Lesson 4-4)

65. $(-3, 1), y = \frac{1}{3}x + 2$

66. $(6, -2), y = \frac{3}{5}x - 4$

67. $(2, -2), 2x + y = 5$

68. $(-3, -3), -3x + y = 6$

Skills Review

Find the solution of each equation using the given replacement set. (Lesson 1-5)

69. $f - 14 = 8; \{12, 15, 19, 22\}$

70. $15(n + 6) = 165; \{3, 4, 5, 6, 7\}$

71. $23 = \frac{d}{4}; \{91, 92, 93, 94, 95\}$

72. $36 = \frac{t - 9}{2}; \{78, 79, 80, 81\}$

Evaluate each expression if $a = 2$, $b = -3$, and $c = 11$. (Lesson 1-2)

73. $a + 6b$ **74.** $7 - ab$ **75.** $(2c + 3a) \div 4$ **76.** $b^2 + (a^3 - 8)5$

EXTEND

6-1

Graphing Technology Lab
Systems of Equations

Math Online > glencoe.com
- Other Calculator Keystrokes
- Graphing Technology Personal Tutor

You can use a graphing calculator to graph and solve a system of equations.

ACTIVITY 1   Solve a System of Equations

Solve the system of equations. State the decimal solution to the nearest hundredth.

$$5.23x + y = 7.48$$
$$6.42x - y = 2.11$$

Step 1   Solve each equation for y to enter them into the calculator.

$5.23x + y = 7.48$	**First equation**
$5.23x + y - 5.23x = 7.48 - 5.23x$	**Subtract 5.23x from each side.**
$y = 7.48 - 5.23x$	**Simplify.**
$6.42x - y = 2.11$	**Second equation**
$6.42x - y - 6.42x = 2.11 - 6.42x$	**Subtract 6.42x from each side.**
$-y = 2.11 - 6.42x$	**Simplify.**
$(-1)(-y) = (-1)(2.11 - 6.42x)$	**Multiply each side by −1.**
$y = -2.11 + 6.42x$	**Simplify.**

Step 2   Enter these equations in the **Y=** list and graph.

KEYSTROKES: *Review on pages 167–168.*
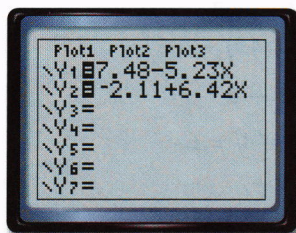
Step 3   Use the **CALC** menu to find the point of intersection.

KEYSTROKES: [2nd] [CALC] 5 [ENTER] [ENTER] [ENTER]
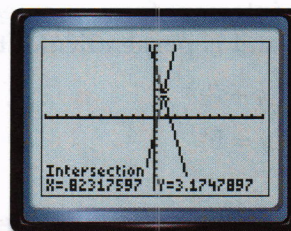
[−10, 10] scl: 1 by [−10, 10] scl: 1

The solution is approximately (0.82, 3.17).

One method you can use to solve an equation with one variable is by graphing and solving a system of equations based on the equation. To do this, write a system using both sides of the equation. Then use a graphing calculator to solve the system.

ACTIVITY 2   **Use a System to Solve a Linear Equation**

Use a system of equations to solve $5x + 6 = -4$.

Step 1 Write a system of equations.
Set each side of the equation equal to y.

$y = 5x + 6$ **First equation**

$y = -4$ **Second equation**

Step 2 Enter these equations in the Y= list and graph.

Step 3 Use the **CALC** menu to find the point of intersection.
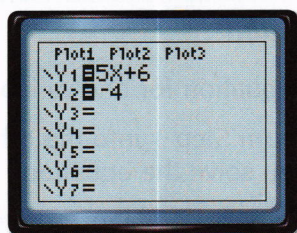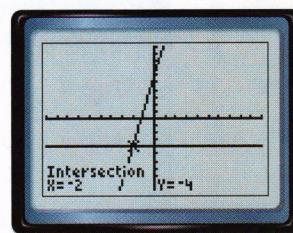
[−10, 10] scl: 1 by [−10, 10] scl: 1

The solution is −2.

Exercises

Use a graphing calculator to solve each system of equations. Write decimal solutions to the nearest hundredth.

1. $y = 2x - 3$
$y = -0.4x + 5$

2. $y = 6x + 1$
$y = -3.2x - 4$

3. $x + y = 9.35$
$5x - y = 8.75$

4. $2.32x - y = 6.12$
$4.5x + y = -6.05$

5. $5.2x - y = 4.1$
$1.5x + y = 6.7$

6. $1.8 = 5.4x - y$
$y = -3.8 - 6.2x$

7. $7x - 2y = 16$
$11x + 6y = 32.3$

8. $3x + 2y = 16$
$5x + y = 9$

9. $0.62x + 0.35y = 1.60$
$-1.38x + y = 8.24$

10. $75x - 100y = 400$
$33x - 10y = 70$

Use a graphing calculator to solve each equation. Write decimal solutions to the nearest hundredth.

11. $4x - 2 = -6$

12. $3 = 1 + \frac{x}{2}$

13. $\frac{x + 4}{-2} = -1$

14. $\frac{x}{7} - 3 = -2$

15. $-9 = 7 + 3x$

16. $-2 + 10x = 8x - 1$

17. WRITING IN MATH Explain why you can solve an equation like $r = ax + b$ by solving the system of equations $y = r$ and $y = ax + b$.

Substitution

Why?

Two movies were released at the same time. Movie A earned $31 million in its opening week, but fell to $15 million the following week. Movie B opened earning $21 million and fell to $11 million the following week. If the earnings for each movie continue to decrease at the same rate, when will they earn the same amount?
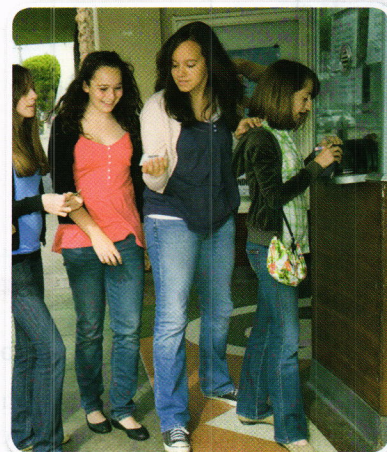
Solve by Substitution You can use a system of equations to find when the movie earnings are the same. One method of finding an exact solution of a system of equations is called **substitution**.

Key Concept Solving by Substitution	For Your **FOLDABLE**

Step 1 When necessary, solve at least one equation for one variable.

Step 2 Substitute the resulting expression from Step 1 into the other equation to replace the variable. Then solve the equation.

Step 3 Substitute the value from Step 2 into either equation, and solve for the other variable. Write the solution as an ordered pair.

EXAMPLE 1   **Solve a System by Substitution**

Use substitution to solve the system of equations.

$y = 2x + 1$ ←   **Step 1**   One equation is already solved for y.
$3x + y = -9$

Step 2   Substitute $2x + 1$ for y in the second equation.

$$
\begin{array}{ll}
3x + y = -9 & \text{Second equation} \\
3x + 2x + 1 = -9 & \text{Substitute } 2x + 1 \text{ for } y. \\
5x + 1 = -9 & \text{Combine like terms.} \\
5x = -10 & \text{Subtract 1 from each side.} \\
x = -2 & \text{Divide each side by 5.}
\end{array}
$$

Step 3   Substitute -2 for x in either equation to find y.

$$
\begin{array}{ll}
y = 2x + 1 & \text{First equation} \\
= 2(-2) + 1 & \text{Substitute } -2 \text{ for } x. \\
= -3 & \text{Simplify.}
\end{array}
$$

The solution is $(-2, -3)$.

CHECK You can check your solution by graphing.
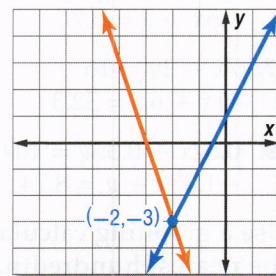
(−2,−3)

✓ Check Your Progress

1A. $y = 4x - 6$
$5x + 3y = -1$

1B. $2x + 5y = -1$
$y = 3x + 10$

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If a variable is not isolated in one of the equations in a system, solve an equation for a variable first. Then you can use substitution to solve the system.

EXAMPLE 2   Solve and then Substitute

Use substitution to solve the system of equations.

$x + 2y = 6$
$3x - 4y = 28$

Step 1   Solve the first equation for x since the coefficient is 1.

$x + 2y = 6$	**First equation**
$x + 2y - 2y = 6 - 2y$	**Subtract 2y from each side.**
$x = 6 - 2y$	**Simplify.**

Step 2   Substitute $6 - 2y$ for x in the second equation to find the value of y.

$3x - 4y = 28$	**Second equation**
$3(6 - 2y) - 4y = 28$	**Substitute 6 − 2y for x.**
$18 - 6y - 4y = 28$	**Distributive Property**
$18 - 10y = 28$	**Combine like terms.**
$18 - 10y - 18 = 28 - 18$	**Subtract 18 from each side.**
$-10y = 10$	**Simplify.**
$y = -1$	**Divide each side by −10.**

Step 3   Find the value of x.

$x + 2y = 6$	**First equation**
$x + 2(-1) = 6$	**Substitute −1 for y.**
$x - 2 = 6$	**Simplify.**
$x = 8$	**Add 2 to each side.**

The solution is $(8, -1)$.

✓ **Check Your Progress**

2A. $4x + 5y = 11$
       $y - 3x = -13$

2B. $x - 3y = -9$
       $5x - 2y = 7$

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Generally, if you solve a system of equations and the result is a false statement such as $3 = -2$, there is no solution. If the result is an identity, such as $3 = 3$, then there are an infinite number of solutions.

EXAMPLE 3   No Solution or Infinitely Many Solutions

Use substitution to solve the system of equations.

$y = 2x - 4$
$-6x + 3y = -12$

Substitute $2x - 4$ for y in the second equation.

$-6x + 3y = -12$	**Second equation**
$-6x + 3(2x - 4) = -12$	**Substitute 2x − 4 for y.**
$-6x + 6x - 12 = -12$	**Distributive Property**
$-12 = -12$	**Combine like terms.**

This statement is an identity. Thus, there are an infinite number of solutions.

Solve Real-World Problems You can use substitution to find the solution of a real-world problem involving a system of equations.
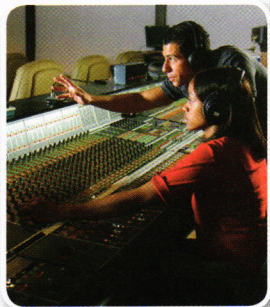
🌐 **Real-World EXAMPLE 4** **Write and Solve a System of Equations**

MUSIC A store sold a total of 125 car stereo systems and speakers in one week. The stereo systems sold for $104.95, and the speakers sold for $18.95. The sales from these two items totaled $6926.75. How many of each item were sold?

Let c = the number of car stereo systems sold, and let t = the number of speakers sold.

Number of Units Sold	c	t	125
Sales ($)	104.95c	18.95t	6926.75

So, the two equations are $c + t = 125$ and $104.95c + 18.95t = 6926.75$.

Step 1 Solve the first equation for c.

$c + t = 125$ **First equation**

$c + t - t = 125 - t$ **Subtract t from each side.**

$c = 125 - t$ **Simplify.**

Step 2 Substitute $125 - t$ for c in the second equation.

$104.95c + 18.95t = 6926.75$ **Second equation**

$104.95(125 - t) + 18.95t = 6926.75$ **Substitute $125 - t$ for c.**

$13{,}118.75 - 104.95t + 18.95t = 6926.75$ **Distributive Property**

$13{,}118.75 - 86t = 6926.75$ **Combine like terms.**

$-86t = -6192$ **Subtract 13118.75 from each side.**

$t = 72$ **Divide each side by −86.**

Step 3 Substitute 72 for t in either equation to find the value of c.

$c + t = 125$ **First equation**

$c + 72 = 125$ **Substitute 72 for t.**

$c = 53$ **Subtract 72 from each side.**

The store sold 53 car stereo systems and 72 speakers.

✓ **Check Your Progress**

4. BASEBALL As of 2007, the New York Yankees and the Cincinnati Reds together had won a total of 31 World Series. The Yankees had won 5.2 times as many as the Reds. How many World Series had each team won?

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Examples 1–3
pp. 342–343

Use substitution to solve each system of equations.

1. $y = x + 5$
   $3x + y = 25$

2. $x = y - 2$
   $4x + y = 2$

3. $3x + y = 6$
   $4x + 2y = 8$

4. $2x + 3y = 4$
   $4x + 6y = 9$

5. $x - y = 1$
   $3x = 3y + 3$

6. $2x - y = 6$
   $-3y = -6x + 18$

Example 4
p. 344

7. GEOMETRY The sum of the measures of angles X and Y is 180°. The measure of angle X is 24° greater than the measure of angle Y.

  a. Define the variables, and write equations for this situation.

  b. Find the measure of each angle.

Practice and Problem Solving

● = **Step-by-Step Solutions** begin on page R12.
Extra Practice begins on page 815.

Examples 1–3
pp. 342–343

Use substitution to solve each system of equations.

8. $y = 5x + 1$
   $4x + y = 10$

9 $y = 4x + 5$
   $2x + y = 17$

10. $y = 3x - 34$
   $y = 2x - 5$

11. $y = 3x - 2$
   $y = 2x - 5$

12. $2x + y = 3$
   $4x + 4y = 8$

13. $3x + 4y = -3$
   $x + 2y = -1$

14. $y = -3x + 4$
   $-6x - 2y = -8$

15. $-1 = 2x - y$
   $8x - 4y = -4$

16. $x = y - 1$
   $-x + y = -1$

17. $y = -4x + 11$
   $3x + y = 9$

18. $y = -3x + 1$
   $2x + y = 1$

19. $3x + y = -5$
   $6x + 2y = 10$

20. $5x - y = 5$
   $-x + 3y = 13$

21. $2x + y = 4$
   $-2x + y = -4$

22. $-5x + 4y = 20$
   $10x - 8y = -40$

Example 4
p. 344

23. ECONOMICS In 2000, the demand for nurses was 2,000,000, while the supply was only 1,890,000. The projected demand for nurses in 2010 is 2,820,000, while the supply is only projected to be 1,810,000.

  a. Define the variables, and write equations to represent these situations.

  b. Use substitution to determine during which year the supply of nurses was equal to the demand.

24. TOURISM The table shows the approximate number of tourists in two areas of the world during a recent year and the average rates of change in tourism.

Destination	Number of Tourists	Average Rates of Change in Tourists (millions per year)
South America and the Caribbean	40.3 million	increase of 0.8
Middle East	17.0 million	increase of 1.8

  a. Define the variables, and write an equation for each region's tourism rate.

  b. If the trends continue, in how many years would you expect the number of tourists in the regions to be equal?

25 SPORTS The table shows the winning times for the Triathlon World Championship.

Year	Men's	Women's
2000	1:51:39	1:54:43
2005	1:49:31	1:58:03

a. The times are in hours, minutes, and seconds. Rewrite the times rounded to the nearest minute.

b. Let the year 2000 be 0. Assume that the rate of change remains the same for years after 2000. Write an equation to represent each of the men's and women's winning times y in any year x.

c. If the trend continues, when would you expect the men's and women's winning times to be the same? Explain your reasoning.

26. CONCERT TICKETS Booker is buying tickets online for a concert. He finds tickets for himself and his friends for $65 each plus a one-time fee of $10. Paula is looking for tickets to the same concert. She finds them at another Web site for $69 and a one-time fee of $13.60.

a. Define the variables, and write equations to represent this situation.

b. Create a table of values for 1 to 5 tickets for each person's purchase.

c. Graph each of these equations.

d. Analyze the graph. How many solutions are there? Explain why.

H.O.T. Problems   Use **H**igher-**O**rder **T**hinking Skills

27. FIND THE ERROR In the system $a + b = 7$ and $1.29a + 0.49b = 6.63$, a represents pounds of apples and b represents pounds of bananas. Guillermo and Cara are finding and interpreting the solution. Is either of them correct? Explain.

Guillermo

$1.29a + 0.49b = 6.63$

$1.29a + 0.49(a + 7) = 6.63$

$1.29 + 0.49a + 3.43 = 6.63$

$0.49a = 3.2$

$a = 1.9$

$a + b = 7$, so $b = 5$. The solution (2, 5) means that 2 pounds of apples and 5 pounds of bananas were bought.

Cara

$1.29a + 0.49b = 6.63$

$1.29(7 - b) + 0.49b = 6.63$

$9.03 - 1.29b + 0.49b = 6.63$

$-0.8b = -2.4$

$b = 3$

The solution $b = 3$ means that 3 pounds of apples and 3 pounds of bananas were bought.

28. CHALLENGE A local charity has 60 volunteers. The ratio of boys to girls is 7:5. Find the number of boy and the number of girl volunteers.

29. REASONING Compare and contrast the solution of a system found by graphing and the solution of the same system found by substitution.

30. OPEN ENDED Create a system of equations that has one solution. Illustrate how the system could represent a real-world situation and describe the significance of the solution in the context of the situation.

31. WRITING IN MATH Explain how to determine what to substitute when using the substitution method of solving systems of equations.

32. The debate team plans to make and sell trail mix. They can spend $34.

Item	Cost Per Pound
sunflower seeds	$4.00
raisins	$1.50

The pounds of raisins in the mix are to be 3 times the pounds of sunflower seeds. Which system can be used to find r, the pounds of raisins, and p, pounds of sunflower seeds, they should buy?

A $3p = r$
$4p + 1.5r = 34$

C $3r = p$
$4p + 1.5r = 34$

B $3p = r$
$4r + 1.5p = 34$

D $3r = p$
$4r + 1.5p = 34$

33. GRIDDED RESPONSE The perimeters of two similar polygons are 250 centimeters and 300 centimeters, respectively. What is the scale factor between the two polygons?

34. Based on the graph, which statement is true?

Sports Drinks Supply
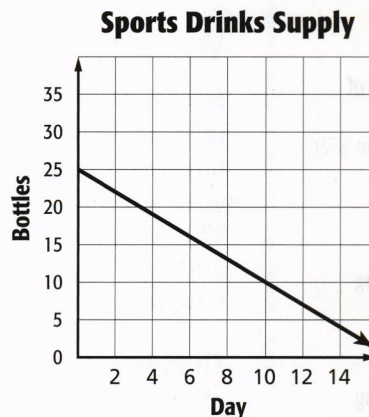
F Mary started with 30 bottles.

G On day 10, Mary will have 10 bottles left.

H Mary will be out of sports drinks on day 14.

J Mary drank 5 bottles the first two days.

35. If p is an integer, which of the following is the solution set for $2|p| = 16$?

A $\{0, 8\}$

C $\{-8, 8\}$

B $\{-8, 0\}$

D $\{-8, 0, 8\}$

Graph each system of equations. Then determine whether the system has *no* solution, *one* solution, or *infinitely many* solutions. If the system has one solution, name it. (Lesson 6-1)

36. $y = -5$
$3x + y = 1$

37. $x = 1$
$2x - y = 7$

38. $y = x + 5$
$y = x - 2$

39. $x + y = 1$
$3y + 3x = 3$

40. ENTERTAINMENT Coach Ross wants to take the soccer team out for pizza after their game. Her budget is at most $70. (Lesson 5-6)

   a. Using the sign, write an inequality that represents this situation.

   b. Are there any restrictions on the domain or range? Explain.

Welcome to Rini's Pizza

Large Pizza $12
Pitcher of Soft Drinks $2

Solve each inequality. Check your solution. (Lesson 5-3)

41. $6v + 1 \geq -11$

42. $24 > 18 + 2n$

43. $-11 \geq \frac{2}{5}q + 5$

44. $\frac{a}{8} - 10 > -3$

45. $-3t + 9 \leq 0$

46. $54 > -10 - 8n$

Rewrite each product using the Distributive Property. Then simplify. (Lesson 1-4)

47. $10b + 5(3 + 9b)$

48. $5(3t^2 + 4) - 8t$

49. $7h^2 + 4(3h + h^2)$

50. $-2(7a + 5b) + 5(2a - 7b)$

Elimination Using Addition and Subtraction

Why?

In Chicago, Illinois, there are two more months a when the mean high temperature is below 70°F than there are months b when it is above 70°F. The system of equations, $a + b = 12$ and $a - b = 2$, represents this situation.

Elimination Using Addition If you add these equations, the variable b will be eliminated. Using addition or subtraction to solve a system is called **elimination**.

Key Concept · Solving by Elimination · For Your FOLDABLE

Step 1 Write the system so like terms with the same or opposite coefficients are aligned.

Step 2 Add or subtract the equations, eliminating one variable. Then solve the equation.

Step 3 Substitute the value from Step 2 into one of the equations and solve for the other variable. Write the solution as an ordered pair.

EXAMPLE 1 · Elimination Using Addition

Use elimination to solve the system of equations.

$4x + 6y = 32$
$3x - 6y = 3$ ← **Step 1** $6y$ and $-6y$ have opposite coefficients.

Step 2 Add the equations.

$$\begin{array}{r} 4x + 6y = 32 \\ (+)\ 3x - 6y = 3 \\ \hline 7x = 35 \end{array}$$ **The variable y is eliminated.**

$\dfrac{7x}{7} = \dfrac{35}{7}$ **Divide each side by 7.**

$x = 5$ **Simplify.**

Step 3 Substitute 5 for x in either equation to find the value of y.

$4x + 6y = 32$ **First equation**

$4(5) + 6y = 32$ **Replace x with 5.**

$20 + 6y = 32$ **Multiply.**

$20 + 6y - 20 = 32 - 20$ **Subtract 20 from each side.**

$6y = 12$ **Simplify.**

$\dfrac{6y}{6} = \dfrac{12}{6}$ **Divide each side by 6.**

$y = 2$ **Simplify.**

The solution is (5, 2).

1A. $-4x + 3y = -3$
$4x - 5y = 5$

1B. $4y + 3x = 22$
$3x - 4y = 14$

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We can use elimination to find specific numbers that are described as being related to each other.

EXAMPLE 2 **Write and Solve a System of Equations**

Negative three times one number plus five times another number is −11. Three times the first number plus seven times the other number is −1. Find the numbers.

Negative three times one number	plus	five times another number	is	−11.
$-3x$	$+$	$5y$	$=$	-11

Three times the first number	plus	seven times the other number	is	−1.
$3x$	$+$	$7y$	$=$	-1

Steps 1 and 2 Write the equations vertically and add.

$$-3x + 5y = -11$$
$$\underline{(+)\ 3x + 7y = \ -1}$$

$12y = -12$ **The variable x is eliminated.**

$\dfrac{12y}{12} = \dfrac{-12}{12}$ **Divide each side by 12.**

$y = -1$ **Simplify.**

Step 3 Substitute −1 for y in either equation to find the value of x.

$3x + 7y = -1$	**Second equation**
$3x + 7(-1) = -1$	**Replace y with −1.**
$3x + (-7) = -1$	**Simplify.**
$3x + (-7) + 7 = -1 + 7$	**Add 7 to each side.**
$3x = 6$	**Simplify.**
$\dfrac{3x}{3} = \dfrac{6}{3}$	**Divide each side by 3.**
$x = 2$	**Simplify.**

The numbers are 2 and −1.

CHECK		
	$-3x + 5y = -11$	**First equation**
	$-3(2) + 5(-1) \stackrel{?}{=} -11$	**Substitute 2 for x and −1 for y.**
	$-11 = -11$ ✔	**Simplify.**
	$3x + 7y = -1$	**Second equation**
	$3(2) + 7(-1) \stackrel{?}{=} -1$	**Substitute 2 for x and −1 for y.**
	$-1 = -1$ ✔	**Simplify.**

✓ **Check Your Progress**

2. The sum of two numbers is −10. Negative three times the first number minus the second number equals 2. Find the numbers.

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Elimination Using Subtraction Sometimes we can eliminate a variable by subtracting one equation from another.

STANDARDIZED TEST EXAMPLE 3 | Elimination Using Subtraction

Solve the system of equations.

$$2t + 5r = 6$$
$$9r + 2t = 22$$

A $(-7, 15)$ **B** $\left(7, \frac{8}{9}\right)$ **C** $(4, -7)$ **D** $\left(4, -\frac{2}{5}\right)$

Read the Test Item

Since both equations contain $2t$, use elimination by subtraction.

Solve the Test Item

Step 1 Subtract the equations.

$$
\begin{array}{ll}
5r + 2t = 6 & \text{Write the system so like terms are aligned.}\\
\underline{(-)\ 9r + 2t = 22}\\
-4r = -16 & \text{The variable } t \text{ is eliminated.}\\
r = 4 & \text{Simplify.}
\end{array}
$$

Step 2 Substitute 4 for r in either equation to find the value of t.

$$
\begin{array}{ll}
5r + 2t = 6 & \text{First equation}\\
5(4) + 2t = 6 & r = 4\\
20 + 2t = 6 & \text{Simplify.}\\
20 + 2t - 20 = 6 - 20 & \text{Subtract 20 from each side.}\\
2t = -14 & \text{Simplify.}\\
t = -7 & \text{Simplify.}
\end{array}
$$

The solution is $(4, -7)$. The correct answer is C.

✓ Check Your Progress

3. Solve the system of equations.

$$8b + 3c = 11$$
$$8b + 7c = 7$$

F $(1.5, -1)$ **G** $(1.75, -1)$ **H** $(1.75, 1)$ **J** $(1.5, 1)$

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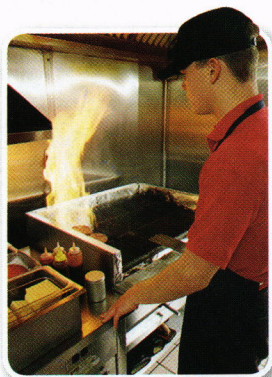
🌐 Real-World EXAMPLE 4 | Write and Solve a System of Equations

JOBS Cheryl and Jackie work at an ice cream shop. Cheryl earns $8.50 per hour and Jackie earns $7.50 per hour. During a typical week, Cheryl and Jackie earn $299.50 together. One week, Jackie doubles her work hours, and the girls earn $412. How many hours does each girl work during a typical week?

Understand You know how much Cheryl and Jackie each earn per hour and how much they earned together.

Plan Let c = Cheryl's hours and j = Jackie's hours.

Cheryl's pay	plus	Jackie's pay	equals	$299.50.
$8.50c$	$+$	$7.50j$	$=$	299.50

Cheryl's pay	plus	Jackie's pay	equals	$412.
$8.50c$	$+$	$7.50(2)j$	$=$	412

Solve Subtract the equations to eliminate one of the variables. Then solve for the other variable.

$$8.50c + 7.50j = 299.50$$ **Write the equations vertically.**
$$(-)\ 8.50c + 7.50(2)j = 412$$

$$8.50c + 7.50j = 299.50$$
$$(-)\ 8.50c + \ \ 15j = 412$$ **Simplify.**

$$-7.50j = -112.50$$ **Subtract. The variable c is eliminated.**

$$\frac{-7.50j}{-7.50} = \frac{-112.50}{-7.50}$$ **Divide each side by -7.50.**

$$j = 15$$ **Simplify.**

Now substitute 15 for j in either equation to find the value of c.

$$8.50c + 7.50j = 299.50$$ **First equation**

$$8.50c + 7.50(15) = 299.50$$ **Substitute 15 for j.**

$$8.50c + 112.50 = 299.50$$ **Simplify.**

$$8.50c = 187$$ **Subtract 112.50 from each side.**

$$c = 22$$ **Divide each side by 8.50.**

Check Substitute both values into the other equation to see if the equation holds true. If $c = 22$ and $j = 15$, then $8.50(22) + 15(15)$ or 412.

Cheryl works 22 hours, while Jackie works 15 hours during a typical week.

> **StudyTip**
>
> **Another Method**
> Instead of subtracting the equations, you could also multiply one equation by -1 and then add the equations.

✓ Check Your Progress

4. **PARTIES** Tamera and Adelina are throwing a birthday party for their friend. Tamera invited 5 fewer friends than Adelina. Together they invited 47 guests. How many guests did each girl invite?

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✓ Check Your Understanding

Examples 1 and 3
pp. 348 and 350

Use elimination to solve each system of equations.

1. $5m - p = 7$
   $7m - p = 11$

2. $8x + 5y = 38$
   $-8x + 2y = 4$

3. $7f + 3g = -6$
   $7f - 2g = -31$

4. $6a - 3b = 27$
   $2a - 3b = 11$

Example 2
p. 349

5. The sum of two numbers is 24. Five times the first number minus the second number is 12. What are the two numbers?

Example 4
pp. 350–351

6. **RECYCLING** The recycling and reuse industry employs approximately 1,025,000 more workers than the waste management industry. Together they provide 1,275,000 jobs. How many jobs does each industry provide?

Practice and Problem Solving

= **Step-by-Step Solutions** begin on page R12.
Extra Practice begins on page 815.

Examples 1 and 3
pp. 348 and 350

Use elimination to solve each system of equations.

7. $-v + w = 7$
$v + w = 1$

8. $y + z = 4$
$y - z = 8$

9. $-4x + 5y = 17$
$4x + 6y = -6$

10. $5m - 2p = 24$
$3m + 2p = 24$

11. $a + 4b = -4$
$a + 10b = -16$

12. $6r - 6t = 6$
$3r - 6t = 15$

13. $6c - 9d = 111$
$5c - 9d = 103$

14. $11f + 14g = 13$
$11f + 10g = 25$

15. $9x + 6y = 78$
$3x - 6y = -30$

16. $3j + 4k = 23.5$
$8j - 4k = 4$

17. $-3x - 8y = -24$
$3x - 5y = 4.5$

18. $6x - 2y = 1$
$10x - 2y = 5$

Example 2
p. 349

19. The sum of two numbers is 22, and their difference is 12. What are the numbers?

20. Find the two numbers with a sum of 41 and a difference of 9.

21 Three times a number minus another number is -3. The sum of the numbers is 11. Find the numbers.

22. A number minus twice another number is 4. Three times the first number plus two times the second number is 12. What are the numbers?

Example 4
pp. 350–351

23. TOURS The Blackwells and Joneses are going to Hershey's Really Big 3D Show in Pennsylvania. Find the adult price and the children's price of the show.

Family	Number of Adults	Number of Children	Total Cost
Blackwell	2	5	$31.65
Jones	2	3	$23.75

Use elimination to solve each system of equations.

24. $4(x + 2y) = 8$
$4x + 4y = 12$

25. $3x - 5y = 11$
$5(x + y) = 5$

26. $4x + 3y = 6$
$3x + 3y = 7$

27. $6x - 7y = -26$
$6x + 5y = 10$

28. $\frac{1}{2}x + \frac{2}{3}y = 2\frac{3}{4}$
$\frac{1}{4}x - \frac{2}{3}y = 6\frac{1}{4}$

29. $\frac{3}{5}x + \frac{1}{2}y = 8\frac{1}{3}$
$-\frac{3}{5}x + \frac{3}{4}y = 8\frac{1}{3}$

30. ARCHITECTURE The total height of an office building b and the granite statue that stands on top of it g is 326.6 feet. The difference in heights between the building and the statue is 295.4 feet.

a. How tall is the statue?

b. How tall is the building?

31. BIKE RACING Professional Mountain Bike Racing currently has 66 teams. The number of non-U.S. teams is 30 more than the number of U.S. teams.

a. Let x represent the number of non-U.S. teams and y represent the number of U.S. teams. Write a system of equations that represents the number of U.S. teams and non-U.S. teams.

b. Use elimination to find the solution of the system of equations.

c. Interpret the solution in the context of the situation.

d. Graph the system of equations to check your solution.

32. ONLINE CATALOGS Let x represent the number of years since 2004 and y represent the number of catalogs.

Catalogs	Number in 2004	Growth Rate (number per year)
online	7440	1293
print	3805	−1364

Source: MediaPost Publications

a. Write a system of equations to represent this situation.

b. Use elimination to find the solution to the system of equations.

c. Analyze the solution in terms of the situation. Determine the reasonableness of the solution.

33. MULTIPLE REPRESENTATIONS Collect 9 pennies and 9 paper clips. For this game, you may use a maximum of 9 objects to create a certain required number of points. Each paper clip is worth 1 point and each penny is worth 3 points. Let p represent a penny and c represent a paper clip.

9 points = 🪙 🪙 + 📎📎📎 = $2p + 3c$

a. **CONCRETE** You must have exactly 15 points using at least one of each piece. Compare your pattern to other students.

b. **ANALYTICAL** Write and solve a system of equations to find the number of paper clips and pennies used.

c. **TABULAR** Make a table showing the number of paper clips used and the total number of points when the number of pennies is 0, 1, 2, 3, 4, or 5.

d. **VERBAL** Does the result in the table match the results in part **b**? Explain.

H.O.T. Problems   Use **H**igher-**O**rder **T**hinking Skills

34. REASONING Describe the solution of a system of equations if after you added two equations the result was $0 = 0$.

35. REASONING What is the solution of a system of equations if the sum of the equations is $0 = 2$?

36. OPEN ENDED Create a system of equations that can be solved by using addition to eliminate one variable. Formulate a general rule for creating such systems.

37. REASONING The solution of a system of equations is $(-3, 2)$. One equation in the system is $x + 4y = 5$. Find a second equation for the system. Explain how you derived this equation.

38. CHALLENGE If a number is multiplied by 7, the result is 182. The sum of that number's two digits is 8. Define the variables and write the system of equations that you would use to find the number. Then solve the system and find the number.

39. WRITING IN MATH Describe when it would be most beneficial to use elimination to solve a system of equations.

40. SHORT RESPONSE Martina is on a train traveling at a speed of 188 mph between two cities 1128 miles apart. If the train has been traveling for an hour, how many more hours is her train ride?

41. GEOMETRY Ms. Miller wants to tile her rectangular kitchen floor. She knows the dimensions of the floor. Which formula should she use to find the area?

$\quad$ **A** $A = \ell w$ $\qquad$ **C** $P = 2\ell + 2w$

$\quad$ **B** $V = Bh$ $\qquad$ **D** $c^2 = a^2 + b^2$

42. If the pattern continues, what is the 8th number in the sequence?

$$2, 3, \frac{9}{2}, \frac{27}{4}, \frac{81}{8}, \ldots$$

$\quad$ **F** $\frac{2187}{64}$ $\quad$ **G** $\frac{2245}{64}$ $\quad$ **H** $\frac{2281}{64}$ $\quad$ **J** $\frac{2445}{64}$

43. What is the solution of this system of equations?

$$x + 4y = 1$$
$$2x - 3y = -9$$

$\quad$ **A** $(2, -8)$ $\qquad$ **C** no solution

$\quad$ **B** $(-3, 1)$ $\qquad$ **D** infinitely many solutions

Spiral Review

Use substitution to solve each system of equations. If the system does not have exactly one solution, state whether it has no solution or infinitely many solutions. (Lesson 6-2)

44. $y = 6x$
$\quad 2x + 3y = 40$

45. $x = 3y$
$\quad 2x + 3y = 45$

46. $x = 5y + 6$
$\quad x = 3y - 2$

47. $y = 3x + 2$
$\quad y = 4x - 1$

48. $3c = 4d + 2$
$\quad c = d - 1$

49. $z = v + 4$
$\quad 2z - v = 6$

50. FINANCIAL LITERACY Gregorio and Javier each want to buy a bicycle. Gregorio has already saved $35 and plans to save $10 per week. Javier has $26 and plans to save $13 per week. (Lesson 6-1)

$\quad$ **a.** In how many weeks will Gregorio and Javier have saved the same amount of money?

$\quad$ **b.** How much will each person have saved at that time?

51. GEOMETRY A *parallelogram* is a quadrilateral in which opposite sides are parallel. Determine whether *ABCD* is parallelogram. Explain your reasoning. (Lesson 4-4)

Solve each equation. Check your solution. (Lesson 2-2)

52. $6u = -48$

53. $75 = -15p$

54. $\frac{2}{3}a = 8$

55. $-\frac{3}{4}d = 15$
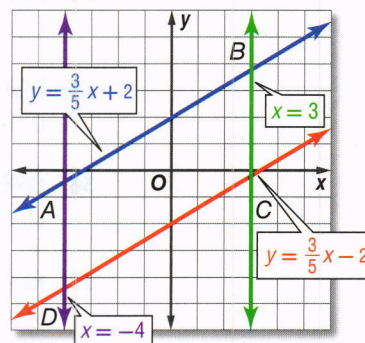
Skills Review

Simplify each expression. If not possible, write *simplified*. (Lesson 1-4)

56. $6q - 3 + 7q + 1$

57. $7w^2 - 9w + 4w^2$

58. $10(2 + r) + 3r$

59. $5y - 7(y + 5)$

Elimination Using Multiplication

Why?

The table shows the number of cars at Scott's Auto Repair Shop for each type of service.

Item	Repairs	Maintenance
body	3	4
engine	2	2

The manager has allotted 1110 minutes for body work and 570 minutes for engine work. The system $3r + 4m = 1110$ and $2r + 2m = 570$ can be used to find the average time for each service.

Elimination Using Multiplication In the system above, neither variable can be eliminated by adding or subtracting. You can use multiplication to solve.

Key Concept   Solving by Elimination   For Your **FOLDABLE**

Step 1 Multiply at least one equation by a constant to get two equations that contain opposite terms.

Step 2 Add the equations, eliminating one variable. Then solve the equation.

Step 3 Substitute the value from Step 2 into one of the equations and solve for the other variable. Write the solution as an ordered pair.

EXAMPLE 1   Multiply One Equation to Eliminate a Variable

Use elimination to solve the system of equations.
$5x + 6y = -8$
$2x + 3y = -5$

Steps 1 and 2
$5x + 6y = -8$
$2x + 3y = -5$ → Multiply each term by -2.

$\begin{array}{r} 5x + 6y = -8 \\ (+)\, -4x - 6y = 10 \\ \hline x \qquad = 2 \end{array}$ **Add.** y is eliminated.

Step 3
$2x + 3y = -5$ **Second equation**
$2(2) + 3y = -5$ **Substitution, $x = 2$**
$4 + 3y = -5$ **Simplify.**
$3y = -9$ **Subtract 4 from each side and simplify.**
$y = -3$ **Divide each side by 3 and simplify.**

The solution is $(2, -3)$.

✓ **Check Your Progress**

1A. $6x - 2y = 10$
       $3x - 7y = -19$

1B. $9r + q = 13$
       $3r + 2q = -4$

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Sometimes you have to multiply each equation by a different number in order to solve the system.

EXAMPLE 2 Multiply Both Equations to Eliminate a Variable

Use elimination to solve the system of equations.
$4x + 2y = 8$
$3x + 3y = 9$

Method 1 Eliminate x.

$4x + 2y = 8$ — Multiply by 3.
$3x + 3y = 9$ — Multiply by −4.

$$12x + 6y = 24$$
$$(+) \; -12x - 12y = -36$$ — Add equations.
$$-6y = -12$$ — x is eliminated.
$$\frac{-6y}{-6} = \frac{-12}{-6}$$ — Divide each side by −6.
$$y = 2$$ — Simplify.

Now substitute 2 for y in either equation to find the value of x.

$$3x + 3y = 9$$ — Second equation
$$3x + 3(2) = 9$$ — Substitute 2 for y.
$$3x + 6 = 9$$ — Simplify.
$$3x = 3$$ — Subtract 6 from each side and simplify.
$$\frac{3x}{3} = \frac{3}{3}$$ — Divide each side by 3.
$$x = 1$$ — The solution is (1, 2).

Method 2 Eliminate y.

$4x + 2y = 8$ — Multiply by 3.
$3x + 3y = 9$ — Multiply by −2.

$$12x + 6y = 24$$
$$(+) \; -6x - 6y = -18$$ — Add equations.
$$6x = 6$$ — y is eliminated.
$$\frac{6x}{6} = \frac{6}{6}$$ — Divide each side by 6.
$$x = 1$$ — Simplify.

Now substitute 1 for x in either equation to find the value of y.

$$3x + 3y = 9$$ — Second equation
$$3(1) + 3y = 9$$ — Substitute 1 for x.
$$3 + 3y = 9$$ — Simplify.
$$3y = 6$$ — Subtract 3 from each side and simplify.
$$\frac{3y}{3} = \frac{6}{3}$$ — Divide each side by 3.
$$y = 2$$ — Simplify.

The solution is (1, 2), which matches the result obtained with Method 1.

CHECK Substitute 1 for x and 2 for y in the first equation.

$$4x + 2y = 8$$ — Original equation
$$4(1) + 2(2) \stackrel{?}{=} 8$$ — Substitute (1, 2) for (x, y).
$$4 + 4 \stackrel{?}{=} 8$$ — Multiply.
$$8 = 8 \checkmark$$ — Add.

✓ **Check Your Progress**

2A. $5x - 3y = 6$
$2x + 5y = -10$

2B. $6a + 2b = 2$
$4a + 3b = 8$

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Solve Real-World Problems Sometimes it is necessary to use multiplication before elimination in real-world problem solving too.

Real-World EXAMPLE 3   Solve a System of Equations

FLIGHT A personal aircraft traveling with the wind flies 520 miles in 4 hours. On the return trip, the airplane takes 5 hours to travel the same distance. Find the speed of the airplane if the air is still.

You are asked to find the speed of the airplane in still air.

Let a = the rate of the airplane if the air is still.
Let w = the rate of the wind.

	r	t	d	$r \cdot t = d$
With the Wind	$a + w$	4	520	$(a + w)4 = 520$
Against the Wind	$a - w$	5	520	$(a - w)5 = 520$

So, our two equations are $4a + 4w = 520$ and $5a - 5w = 520$.

$4a + 4w = 520$ **Multiply by 5.**
$5a - 5w = 520$ **Multiply by 4.**

$$20a + 20w = 2600$$
$$(+)\ 20a - 20w = 2080$$
$$\overline{\quad 40a \qquad\quad = 4680\quad}$$ w **is eliminated.**

$$\frac{40a}{40} = \frac{4680}{40}$$ **Divide each side by 40.**

$$a = 117$$ **Simplify.**

The rate of the airplane in still air is 117 miles per hour.

✓ Check Your Progress

3. **CANOEING** A canoeist travels 4 miles downstream in 1 hour. The return trip takes the canoeist 1.5 hours. Find the rate of the boat in still water.

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✓ Check Your Understanding

Examples 1 and 2
pp. 355–356

Use elimination to solve each system of equations.

1. $2x - y = 4$
   $7x + 3y = 27$

2. $2x + 7y = 1$
   $x + 5y = 2$

3 $4x + 2y = -14$
   $5x + 3y = -17$

4. $9a - 2b = -8$
   $-7a + 3b = 12$

Example 3
p. 357

5. **KAYAKING** A kayaking group with a guide travels 16 miles downstream, stops for a meal, and then travels 16 miles upstream. The speed of the current remains constant throughout the trip. Find the speed of the kayak in still water.

Leave	10:00 A.M.
Stop for meal	12:00 noon
Return	1:00 P.M.
Finish	5:00 P.M.
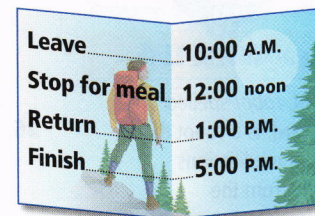
6. **PODCASTS** Steve subscribed to 10 podcasts for a total of 340 minutes. He used his two favorite tags, Hobbies and Recreation and Soliloquies. Each of the Hobbies and Recreation episodes lasted about 32 minutes. Each Soliloquies episode lasted 42 minutes. To how many of each tag did Steve subscribe?

Practice and Problem Solving

⬤ = **Step-by-Step Solutions** begin on page R12.
Extra Practice begins on page 815.

Examples 1 and 2
pp. 355–356

Use elimination to solve each system of equations.

7. $x + y = 2$
$-3x + 4y = 15$

8. $x - y = -8$
$7x + 5y = 16$

9. $x + 5y = 17$
$-4x + 3y = 24$

10. $6x + y = -39$
$3x + 2y = -15$

11. $2x + 5y = 11$
$4x + 3y = 1$

12. $3x - 3y = -6$
$-5x + 6y = 12$

13. $3x + 4y = 29$
$6x + 5y = 43$

14. $8x + 3y = 4$
$-7x + 5y = -34$

15. $8x + 3y = -7$
$7x + 2y = -3$

16. $4x + 7y = -80$
$3x + 5y = -58$

17. $12x - 3y = -3$
$6x + y = 1$

18. $-4x + 2y = 0$
$10x + 3y = 8$

Example 3
p. 357

19 **NUMBER THEORY** Seven times a number plus three times another number equals negative one. The sum of the two numbers is negative three. What are the numbers?

20. **FOOTBALL** A field goal is 3 points and the extra point after a touchdown is 1 point. In a recent post-season, Adam Vinatieri of the Indianapolis Colts made a total of 21 field goals and extra point kicks for 49 points. Find the number of field goals and extra points that he made.

Use elimination to solve each system of equations.

21. $2.2x + 3y = 15.25$
$4.6x + 2.1y = 18.325$

22. $-0.4x + 0.25y = -2.175$
$2x + y = 7.5$

23. $\frac{1}{4}x + 4y = 2\frac{3}{4}$
$3x + \frac{1}{2}y = 9\frac{1}{4}$

24. $\frac{2}{5}x + 6y = 24\frac{1}{5}$
$3x + \frac{1}{2}y = 3\frac{1}{2}$

25. **ROBOTS** TOBOR saves 120 minutes of a nurse's time n and 180 minutes of support staff time s each day. Another robot that aids stroke patients' limbs is estimated to save 90 minutes of nursing time and 120 minutes of support staff time each day.

a. To be cost effective, TOBOR must save a total of 1500 minutes per day. Write an equation that represents this relationship.

b. To make the stroke assistant cost effective, it must save a total of 1050 minutes per day. Write an equation that represents this relationship.

c. Solve the system of equations, and interpret the solution in the context of the situation.

Real-World Link

TOBOR, *robot* spelled backward, is a robot that delivers medications directly from the pharmacy to a patient's room. TOBOR can talk, detect when someone is blocking its path, and interface with an elevator.

Source: *U.S. Medicine Magazine*

26. **GEOMETRY** The graphs of $x + 2y = 6$ and $2x + y = 9$ contain two of the sides of a triangle. A vertex of the triangle is at the intersection of the graphs.

a. What are the coordinates of the vertex?

b. Draw the graph of the two lines. Identify the vertex of the triangle.

c. The line that forms the third side of the triangle is the line $x - y = -3$. Draw this line on the previous graph.

d. Name the other two vertices of the triangle.

27. ENTERTAINMENT At an entertainment center, two groups of people bought batting tokens and miniature golf games, as shown in the table.

Group	Number of Batting Tokens	Number of Miniature Golf Games	Total Cost
A	16	3	$30
B	22	5	$43

a. Define the variables, and write a system of linear equations from this situation.

b. Solve the system of equations, and explain what the solution represents.

28. TESTS Mrs. Henderson discovered that she had accidentally reversed the digits of a test score and did not give a student 36 points. Mrs. Henderson told the student that the sum of the digits was 14 and agreed to give the student his correct score plus extra credit if he could determine his actual score. What was his correct score?

H.O.T. Problems   Use **H**igher-**O**rder **T**hinking Skills

29. REASONING Explain how you could recognize a system of linear equations with infinitely many solutions.

30. FIND THE ERROR Jason and Daniela are solving a system of equations. Is either of them correct? Explain your reasoning.

Jason

$2r + 7t = 11$

$r - 9t = -7$

————————

$2r + 7t = 11$

$(-) 2r - 18t = -14$

————————

$25t = 25$

$t = 1$

$2r + 7t = 11$

$2r + 7(1) = 11$

$2r + 7 = 11$

$2r = 4$

$\dfrac{2r}{2} = \dfrac{4}{2}$

$r = 2$

The solution is (2, 1).

Daniela

$2r + 7t = 11$

$(-) r - 9t = -7$

————————

$r = 18$

$2r + 7t = 11$

$2(18) + 7t = 11$

$36 + 7t = 11$

$7t = -25$

$\dfrac{7t}{7} = -\dfrac{25}{7}$

$t = -3.6$

The solution is (18, -3.6).

31. OPEN ENDED Write a system of equations that can be solved by multiplying one equation by −3 and then adding the two equations together.

32. CHALLENGE The solution of the system $4x + 5y = 2$ and $6x - 2y = b$ is $(3, a)$. Find the values of a and b. Discuss the steps that you used.

33. WRITING IN MATH Explain how to decide which variable to eliminate when using multiplication.

Real-World Link

In addition to batting cages and miniature golf, many entertainment centers offer go-karts, bumper boats, and a video arcade.

Source: Camelot Park

34. What is the solution of this system of equations?

$2x - 3y = -9$
$-x + 3y = 6$

A $(3, 3)$ **C** $(-3, 1)$

B $(-3, 3)$ **D** $(1, -3)$

35. A buffet has one price for adults and another for children. The Taylor family has two adults and three children, and their bill was $40.50. The Wong family has three adults and one child. Their bill was $38. Which system of equations could be used to determine the price for an adult and for a child?

F $x + y = 40.50$ **H** $2x + 3y = 40.50$
    $x + y = 38$ $x + 3y = 38$

G $2x + 3y = 40.50$ **J** $2x + 2y = 40.50$
    $3x + y = 38$ $3x + y = 38$

36. SHORT RESPONSE A customer at the paint store has ordered 3 gallons of ivy green paint. Melissa mixes the paint in a ratio of 3 parts blue to one part yellow. How many quarts of blue paint does she use?

37. PROBABILITY The table shows the results of a number cube being rolled. What is the experimental probability of rolling a 3?

Outcome	Frequency
1	4
2	8
3	2
4	0
5	5
6	1

A $\frac{2}{3}$ **B** $\frac{1}{3}$ **C** 0.2   **D** 0.1

Use elimination to solve each system of equations. (Lesson 6-3)

38. $f + g = -3$
    $f - g = 1$

39. $6g + h = -7$
    $6g + 3h = -9$

40. $5j + 3k = -9$
    $3j + 3k = -3$

41. $2x - 4z = 6$
    $x - 4z = -3$

42. $-5c - 3v = 9$
    $5c + 2v = -6$

43. $4b - 6n = -36$
    $3b - 6n = -36$

44. JOBS Brandy and Adriana work at an after-school child care center. Together they cared for 32 children this week. Brandy cared for 0.6 times as many children as Adriana. How many children did each girl care for? (Lesson 6-2)

Solve each inequality. Then graph the solution set. (Lesson 5-5)

45. $|m - 5| \le 8$ **46.** $|q + 11| < 5$ **47.** $|2w + 9| > 11$ **48.** $|2r + 1| \ge 9$

Translate each sentence into a formula. (Lesson 2-1)

49. The area A of a triangle equals one half times the base b times the height h.

50. The circumference C of a circle equals the product of 2, π, and the radius r.

51. The volume V of a rectangular box is the length ℓ times the width w multiplied by the height h.

52. The volume of a cylinder V is the same as the product of π and the radius r to the second power multiplied by the height h.

53. The area of a circle A equals the product of π and the radius r squared.

54. Acceleration A equals the increase in speed s divided by time t in seconds.

Use the graph to determine whether each system is *consistent* or *inconsistent* and if it is *independent* or *dependent*.
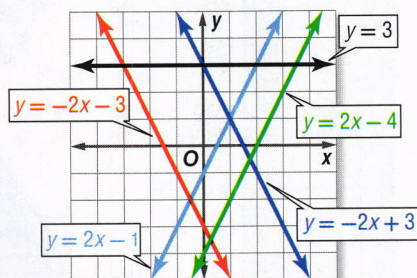
1. $y = 2x - 1$
$y = -2x + 3$

2. $y = -2x + 3$
$y = -2x - 3$

Graph each system and determine the number of solutions that it has. If it has one solution, name it.

3. $y = 2x - 3$
$y = x + 4$

4. $x + y = 6$
$x - y = 4$

5. $x + y = 8$
$3x + 3y = 24$

6. $x - 4y = -6$
$y = -1$

7. $3x + 2y = 12$
$3x + 2y = 6$

8. $2x + y = -4$
$5x + 3y = -6$

Use substitution to solve each system of equations.

9. $y = x + 4$
$2x + y = 16$

10. $y = -2x - 3$
$x + y = 9$

11. $x + y = 6$
$x - y = 8$

12. $y = -4x$
$6x - y = 30$

13. FOOD The cost of two meals at a restaurant is shown in the table below.

Meal	Total Cost
3 tacos, 2 burritos	$7.40
4 tacos, 1 burrito	$6.45

a. Define variables to represent the cost of a taco and the cost of a burrito.

b. Write a system of equations to find the cost of a single taco and a single burrito.

c. Solve the systems of equations, and explain what the solution means.

d. How much would a customer pay for 2 tacos and 2 burritos?

14. AMUSEMENT PARKS The cost of two groups going to an amusement park is shown in the table.

Group	Total Cost
4 adults, 2 children	$184
4 adults, 3 children	$200

a. Define variables to represent the cost of an adult ticket and the cost of a child ticket.

b. Write a system of equations to find the cost of an adult ticket and a child ticket.

c. Solve the system of equations, and explain what the solution means.

d. How much will a group of 3 adults and 5 children be charged for admission?

15. MULTIPLE CHOICE Angelina needs to buy 12 pieces of candy to take to a meeting. She has $16. Each chocolate bar costs $2, and each lollipop costs $1. Determine how many of each she can buy.

A 6 chocolate bars, 6 lollipops

B 4 chocolate bars, 8 lollipops

C 7 chocolate bars, 5 lollipops

D 3 chocolate bars, 9 lollipops

Use elimination to solve each system of equations.

16. $x + y = 9$
$x - y = -3$

17. $x + 3y = 11$
$x + 7y = 19$

18. $9x - 24y = -6$
$3x + 4y = 10$

19. $-5x + 2y = -11$
$5x - 7y = 1$

20. MULTIPLE CHOICE The Blue Mountain High School Drama Club is selling tickets to their spring musical. Adult tickets are $4 and student tickets are $1. A total of 285 tickets are sold for $765. How many of each type of ticket are sold?

F 145 adult, 140 student

G 120 adult, 165 student

H 180 adult, 105 student

J 160 adult, 125 student
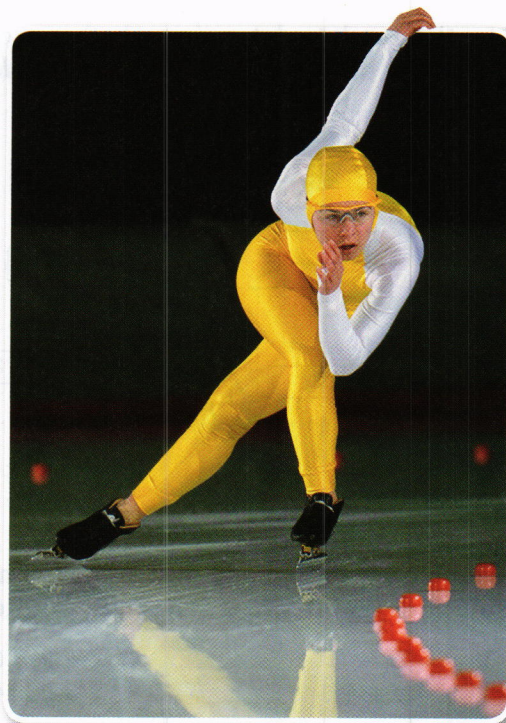
Applying Systems of Linear Equations

Why?

In speed skating, competitors race two at a time on a double track. Indoor speed skating rinks have two track sizes for race events: an official track and a short track.

Speed Skating Tracks	
official track	x
short track	y

The total length of the two tracks is 511 meters. The official track is 44 meters less than four times the short track. The total length is represented by $x + y = 511$. The length of the official track is represented by $x = 4y - 44$.

You can solve the system of equations to find the length of each track.

Determine the Best Method You have learned five methods for solving systems of linear equations. The table summarizes the methods and the types of systems for which each method works best.

Concept Summary	Solving Systems of Equations · For Your FOLDABLE
Method	**The Best Time to Use**
Graphing	To estimate solutions, since graphing usually does not give an exact solution.
Substitution	If one of the variables in either equation has a coefficient of 1 or −1.
Elimination Using Addition	If one of the variables has opposite coefficients in the two equations.
Elimination Using Subtraction	If one of the variables has the same coefficient in the two equations.
Elimination Using Multiplication	If none of the coefficients are 1 or −1 and neither of the variables can be eliminated by simply adding or subtracting the equations.

Substitution and elimination are algebraic methods for solving systems of equations. An algebraic method is best for an exact solution. Graphing, with or without technology, is a good way to estimate a solution.

EXAMPLE 1   Choose the Best Method

Determine the best method to solve the system of equations. Then solve the system.

$$4x - 4y = 8$$
$$-8x + y = 19$$

Understand   To determine the best method to solve the system of equations, look closely at the coefficients of each term.

Plan   Neither the coefficients of x nor y are the same or additive inverses, so you cannot add or subtract to eliminate a variable. Since the coefficient of y in the second equation is 1, you can use substitution.

Solve   First, solve the second equation for y.

$-8x + y = 19$	**Second equation**
$-8x + y + 8x = 19 + 8x$	**Add $8x$ to each side.**
$y = 19 + 8x$	**Simplify.**

Next, substitute $19 + 8x$ for y in the first equation.

$4x - 4y = 8$	**First equation**
$4x - 4(19 + 8x) = 8$	**Substitution**
$4x - 76 - 32x = 8$	**Distributive Property**
$-28x - 76 = 8$	**Simplify.**
$-28x - 76 + 76 = 8 + 76$	**Add 76 to each side.**
$-28x = 84$	**Simplify.**
$\dfrac{-28x}{-28} = \dfrac{84}{-28}$	**Divide each side by -28.**
$x = -3$	**Simplify.**

Last, substitute -3 for x in the second equation.

$-8x + y = 19$	**Second equation**
$-8(-3) + y = 19$	**$x = -3$**
$y = -5$	**Simplify.**

The solution of the system of equations is $(-3, -5)$.

Check   Use a graphing calculator to check your solution. If your algebraic solution is correct, then the graphs will intersect at $(-3, -5)$.
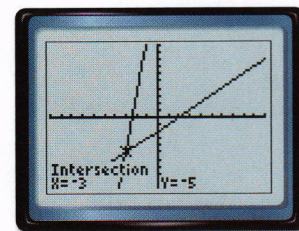
$[-10, 10]$ scl: 1 $[-10, 10]$ scl: 1

✔ **Check Your Progress**

1A. $5x + 7y = 2$
  $-2x + 7y = 9$

1B. $3x - 4y = -10$
  $5x + 8y = -2$

1C. $x - y = 9$
  $7x + y = 7$

1D. $5x - y = 17$
  $3x + 2y = 5$

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Apply Systems of Linear Equations When applying systems of linear equations to problems, it is important to analyze each solution in the context of the situation.

🌐 Real-World EXAMPLE 2   Apply Systems of Linear Equations

PENGUINS Of the 17 species of penguins in the world, the largest species is the emperor penguin. One of the smallest is the Galapagos penguin. The total height of the two penguins is 169 centimeters. The emperor penguin is 22 centimeters more than twice the height of the Galapagos penguin. Find the height of each penguin.

The total height of the two species can be represented by $p + g = 169$, where p represents the height of the emperor penguin and g the height of the Galapagos penguin. Next write an equation to represent the height of the emperor penguin.

Words	The emperor penguin	is	22 centimeters	more than	twice the height of the Galapagos penguin.
Variables	Let p = the height of the emperor penguin and g = the height of the Galapagos penguin.				
Equation	p	=	22	+	$2g$

First rewrite the second equation.

$$p = 22 + 2g \quad \text{Second equation}$$
$$p - 2g = 22 \quad \text{Subtract } 2g \text{ from each side.}$$

You can use elimination by subtraction to solve this system of equations.

$$
\begin{array}{rl}
p + g = 169 & \text{First equation} \\
(-) \; p - 2g = 22 & \text{Subtract the second equation.} \\
\hline
3g = 147 & \text{Eliminate } p. \\
\dfrac{3g}{3} = \dfrac{147}{3} & \text{Divide each side by 3.} \\
g = 49 & \text{Simplify.}
\end{array}
$$

Next substitute 49 for g in one of the equations.

$$
\begin{array}{rl}
p = 22 + 2g & \text{Second equation} \\
 = 22 + 2(49) & g = 49 \\
 = 120 & \text{Simplify.}
\end{array}
$$

The height of the emperor penguin is 120 centimeters, and the height of the Galapagos penguin is 49 centimeters.

Does the solution make sense in the context of the problem?

Check by verifying the given information. The penguins' heights added together would be $120 + 49$ or 169 centimeters and $22 + 2(49)$ is 120 centimeters.

✓ Check Your Progress

2. **VOLUNTEERING** Jared has volunteered 50 hours and plans to volunteer 3 hours in each coming week. Clementine is a new volunteer who plans to volunteer 5 hours each week. Write and solve a system of equations to find how long it will be before they will have volunteered the same number of hours.

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Example 1
p. 363

Determine the best method to solve each system of equations. Then solve the system.

1. $2x + 3y = -11$
$-8x - 5y = 9$

2. $3x + 4y = 11$
$2x + y = -1$

3. $3x - 4y = -5$
$-3x + 2y = 3$

4. $3x + 7y = 4$
$5x - 7y = -12$

Example 2
p. 364

5. **SHOPPING** At a sale, Salazar bought 4 T-shirts and 3 pairs of jeans for $181. At the same store, Jenna bought 1 T-shirt and 2 pairs of jeans for $94. The T-shirts were all the same price, and the jeans were all the same price.

   a. Write a system of equations that can be used to represent this situation.

   b. Determine the best method to solve the system of equations.

   c. Solve the system.

Practice and Problem Solving

> ● = **Step-by-Step Solutions** begin on page R12.
> **Extra Practice** begins on page 815.

Example 1
p. 363

Determine the best method to solve each system of equations. Then solve the system.

6. $-3x + y = -3$
$4x + 2y = 14$

7. $2x + 6y = -8$
$x - 3y = 8$

8. $3x - 4y = -5$
$-3x - 6y = -5$

9. $5x + 8y = 1$
$-2x + 8y = -6$

10. $y + 4x = 3$
$y = -4x - 1$

⑪ $-5x + 4y = 7$
$-5x - 3y = -14$

Example 2
p. 364

12. **FINANCIAL LITERACY** For a Future Teachers of America fundraiser, Denzell sold food as shown in the table. He sold 11 more subs than pizzas and earned a total of $233. Write and solve a system of equations to represent this situation. Then describe what the solution means.

Item	Selling Price
pizza	$5.00
sub	$3.00

13. **DVDs** Manuela has a total of 40 DVDs of movies and television shows. The number of movies is 4 less than 3 times the number of television shows. Write and solve a system of equations to find the numbers of movies and television shows that she has on DVD.

14. **CAVES** The Caverns of Sonora have two different tours: the Crystal Palace tour and the Horseshoe Lake tour. The total length of both tours is 3.25 miles. The Crystal Palace tour is a half-mile less than twice the distance of the Horseshoe Lake tour. Determine the length of each tour.

15. **YEARBOOKS** The *break-even point* is the point at which income equals expenses. Ridgemont High School is paying $13,200 for the writing and research of their yearbook plus a printing fee of $25 per book. If they sell the books for $40 each, how many will they have to sell to break even? Explain.

16. **PAINTBALL** Clara and her friends are planning a trip to a paintball park. Find the cost of lunch and the cost of each paintball. What would be the cost for 400 paintballs and lunch?
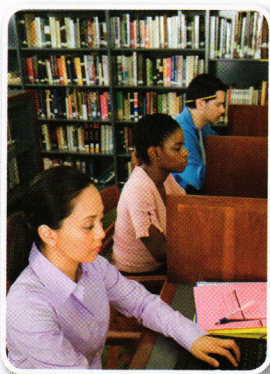
PAINTBALL IN THE PARK
• $25 for 500 paintballs
• $15 for 200 paintballs
Lunch is included

17 **RECYCLING** Mara and Ling each recycled aluminum cans and newspaper, as shown in the table. Mara earned $3.77, and Ling earned $4.65.

Materials	Pounds Recycled	
	Mara	Ling
aluminum cans	9	9
newspaper	26	114

   a. Define variables and write a system of linear equations from this situation.

   b. What was the price per pound of aluminum? Determine the reasonableness of your solution.

18. **BOOKS** The library is having a book sale. Hardcover books sell for $4 each, and paperback books are $2 each. If Connie spends $26 for 8 books, how many hardcover books did she buy?

19. **MUSIC** An online music club offers individual songs for one price or entire albums for another. Kendrick pays $14.90 to download 5 individual songs and 1 album. Geoffrey pays $21.75 to download 3 individual songs and 2 albums.

   a. How much does the music club charge to download a song?

   b. How much does the music club charge to download an entire album?

20. **DRIVING** Malik drove his car for 45 miles at an average speed of r miles per hour. On the return trip, traffic has increased, and Malik's average speed is $\frac{3}{4}r$. The round trip took a total of 1 hour and 45 minutes. Find the average speed for each portion of the trip.

H.O.T. Problems   Use **H**igher-**O**rder **T**hinking Skills

21. **OPEN ENDED** Formulate a system of equations that represents a situation in your school. Describe the method that you would use to solve the system. Then solve the system and explain what the solution means.

22. **REASONING** In a system of equations, x represents the time spent riding a bike, and y represents the distance traveled. You determine the solution to be $(-1, 7)$. Use this problem to discuss the importance of analyzing solutions in the context of real-world problems.

23. **CHALLENGE** Solve the following system of equations by using three different methods. Show your work.

$$4x + y = 13$$
$$6x - y = 7$$

24. **WRITE A QUESTION** A classmate says that elimination is the best way to solve a system of equations. Write a question to challenge his conjecture.

25. **WHICH ONE DOESN'T BELONG?** Which system is different? Explain.

|			
$x - y = 3$ $x + \frac{1}{2}y = 1$	$-x + y = 0$ $5x = 2y$	$y = x - 4$ $y = \frac{2}{x}$	$y = x + 1$ $y = 3x$

26. **WRITING IN MATH** Explain when graphing would be the best method of solving a system of equations. When would solving a system of equations algebraically be the best method?

27. If $5x + 3y = 12$ and $4x - 5y = 17$, what is y?

　　A -1　　**B** 3　　**C** $(-1, 3)$　　**D** $(3, -1)$

28. STATISTICS The scatter plot shows the number of hay bales used on the Bostwick farm during the last year.

Hay Bales Used
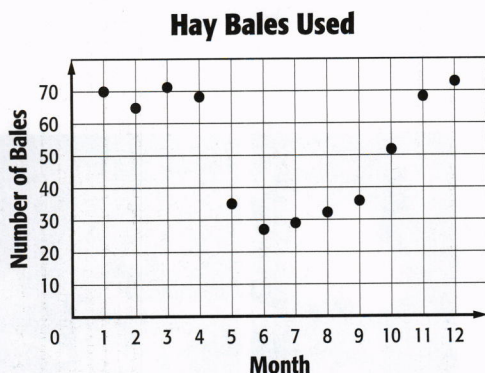
Which is an invalid conclusion?

F The Bostwicks used less hay in the summer than they did in the winter.

G The Bostwicks used about 629 bales of hay during the year.

H On average, the Bostwicks used about 52 bales each month.

J The Bostwicks used the most hay in February.

29. SHORT RESPONSE At noon, Cesar cast a shadow 0.15 foot long. Next to him a streetlight cast a shadow 0.25 foot long. If Cesar is 6 feet tall, how tall is the streetlight?

30. The graph shows the solution to which of the following systems of equations?
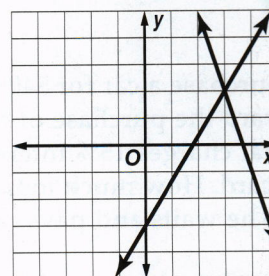
A $y = -3x + 11$
　　$3y = 5x - 9$

B $y = 5x - 15$
　　$2y = x + 7$

C $y = -3x + 11$
　　$2y = 4x - 5$

D $y = 5x - 15$
　　$3y = 2x + 18$

Spiral Review

Use elimination to solve each system of equations. (Lesson 6-4)

31. $x + y = 3$
　　$3x - 4y = -12$

32. $-4x + 2y = 0$
　　$2x - 3y = 16$

33. $4x + 2y = 10$
　　$5x - 3y = 7$

34. TRAVELING A youth group is traveling in two vans to visit an aquarium. The number of people in each van and the cost of admission for that van are shown. What are the adult and student prices? (Lesson 6-3)

Van	Number of Adults	Number of Students	Total Cost
A	2	5	$77
B	2	7	$95

Graph each inequality. (Lesson 5-6)

35. $y < 4$　　　　**36.** $x \geq 3$　　　　**37.** $7x + 12y > 0$　　　　**38.** $y - 3x \leq 4$

Skills Review

Find each sum or difference. (Lesson 0-4)

39. $(-3.81) + (-8.5)$　　　　**40.** $12.625 + (-5.23)$　　　　**41.** $21.65 + (-15.05)$

42. $(-4.27) + 1.77$　　　　**43.** $(-78.94) - 14.25$　　　　**44.** $(-97.623) - (-25.14)$

You can use a spreadsheet to compare the advantages and disadvantages of using cash versus using a credit card for a purchase.

ACTIVITY

Jun wants to purchase a car for $4000. He can save $350 per month toward the purchase of the car. Or he can use a credit card that charges 15% interest and pay $200 a month on the card. How much money will Jun save on his purchase if he waits and pays cash?

Part 1 To find out how long it will take Jun to pay cash for the car, divide $4000 by $350. This is about 11.4 months. So Jun would need to save for 12 months to pay cash for the car.

Part 2 If Jun uses his credit card to pay for the car, he would have the car right away, but he would have to pay interest. You can use a spreadsheet to find the costs by month.

Column A: List months from 0–30.

Column B: List each of the $200 payments.

Column C: Place the remaining balance. Begin with 4000 in C2.

Column D: Find the amount of interest paid each month by using the formula =C2·0.15/12.

Column E: Find the principal paid by subtracting the interest from 200 using the formula =200−D2.

Spreadsheet.xls

	A	B	C	D	E
1	Month	Payment	Remaining Balance	Interest Paid	Principal Paid
2	0	200	4000	50	150
3	1	200	3850.00	48.13	151.875
4	2	200	3698.13	46.23	153.77
5	3	200	3544.35	44.30	155.70
6	4	200	3388.66	42.36	157.64
21	19	200	805.48	10.07	189.93
22	20	200	615.55	7.69	192.31
23	21	200	423.25		
24	22	200			

Sheet 1 / Sheet 2 / Sheet 3

Analyze the Results

1. How long will it take Jun to pay for his car using his credit card?
2. What is the amount of Jun's last payment?
3. How can you find how much Jun pays in interest as he pays back his credit card?
4. How much total interest did Jun pay?
5. What are the benefits of using cash to pay for the car instead of using a credit card?

Organizing Data Using Matrices

Why?

The table shows high school participation in various sports.

Sport	Girls		Boys	
	Schools	Participation	Schools	Participation
basketball	17,175	456,543	17,482	545,497
cross country	12,345	170,450	12,727	201,719
lacrosse	1270	48,086	1334	59,993
tennis	9646	169,292	9426	148,530

Source: The National Federation of State High School Associations

These data can be organized into two matrices with the figures for girls and boys.

Organize Data Using Matrices A **matrix** is a rectangular arrangement of numbers in rows and columns enclosed in brackets. Each number in a matrix is called an **element**. A matrix is usually named using an uppercase letter. A matrix can be described by its **dimensions** or the number of rows and columns in the matrix. A matrix with m rows and n columns is an $m \times n$ matrix (read "m by n").

$$A = \begin{bmatrix} 7 & -9 & 5 & 3 \\ -1 & 3 & -3 & 6 \\ 0 & -4 & 8 & 2 \end{bmatrix} \right\} \text{3 rows}$$

The element -1 is in Row 2, Column 1.

The element 2 is in Row 3, Column 4.

4 columns

Matrix A above is a 3×4 matrix because it has 3 rows and 4 columns.

EXAMPLE 1 Dimensions of a Matrix

State the dimensions of each matrix. Then identify the position of the circled element in each matrix.

a. $A = \begin{bmatrix} 1 & \boxed{7} \\ -4 & 0 \\ 2 & -5 \end{bmatrix}$

b. $B = \begin{bmatrix} -5 & 10 & \boxed{2} & -3 \end{bmatrix}$

$\begin{bmatrix} 1 & 7 \\ -4 & 0 \\ 2 & -5 \end{bmatrix} \right\} \text{3 rows}$

2 columns

Matrix A has 3 rows and 2 columns. Therefore, it is a 3×2 matrix. The circled element is in the first row and the second column.

$\begin{bmatrix} -5 & 10 & 2 & -3 \end{bmatrix} \right\} \text{1 row}$
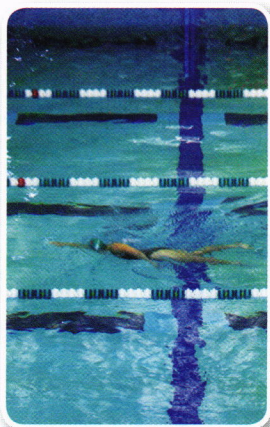
4 columns

Matrix B has 1 row and 4 columns. Therefore, it is a 1×4 matrix. The circled element is in the first row and the third column.

✓ Check Your Progress

1A. $C = \begin{bmatrix} -1 & 7 & 12 \\ 4 & \boxed{-5} & -2 \end{bmatrix}$

1B. $D = \begin{bmatrix} -3 & 6 \\ 4 & \boxed{-8} \end{bmatrix}$

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🌐 Real-World EXAMPLE 2   Organize Data into a Matrix

SWIMMING At a meet, 10 points were awarded for each first-place finish, 8 points for each second-place finish, and 5 points for each third-place finish. Use a matrix to organize each team's points. Which school had the most first-place finishes?

School	Freestyle	Backstroke	Breaststroke	Butterfly
North	10	8	8	10
South	5	5	10	8
Jefferson	8	10	5	5

Organize the points awarded into labeled columns and rows.

$$\begin{array}{c} \text{North} \\ \text{South} \\ \text{Jefferson} \end{array} \begin{bmatrix} \overset{\text{Freestyle}}{10} & \overset{\text{Backstroke}}{8} & \overset{\text{Breaststroke}}{8} & \overset{\text{Butterfly}}{10} \\ 5 & 5 & 10 & 8 \\ 8 & 10 & 5 & 5 \end{bmatrix}$$

North High School earned 10 points in both the freestyle event and the butterfly event, so they had the most first-place finishes.

✔ Check Your Progress

2. MOVIES For a matinee, a movie theater charges $5.25 for an adult and $4.50 for a child. Evening admission is $8.75 for an adult and $5.75 for a child. Organize the prices into a matrix. What are the dimensions of the matrix?

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Matrix Operations If two matrices have the same dimensions, they can be added together. You add matrices by adding the corresponding elements of the matrices.

EXAMPLE 3   Add Matrices

Find each sum for $A = \begin{bmatrix} 12 & 2 \\ -9 & 15 \end{bmatrix}$, $B = \begin{bmatrix} -4 & 4 \\ -3 & -10 \end{bmatrix}$, and $C = \begin{bmatrix} -2 \\ 8 \end{bmatrix}$.

a. $A + B$

$$A + B = \begin{bmatrix} 12 & 2 \\ -9 & 15 \end{bmatrix} + \begin{bmatrix} -4 & 4 \\ -3 & -10 \end{bmatrix} \qquad \text{Substitution}$$

$$= \begin{bmatrix} 12 + (-4) & 2 + 4 \\ -9 + (-3) & 15 + (-10) \end{bmatrix} \text{ or } \begin{bmatrix} 8 & 6 \\ -12 & 5 \end{bmatrix} \qquad \text{Simplify.}$$

b. $B + C$

$$B + C = \begin{bmatrix} -4 & 4 \\ -3 & -10 \end{bmatrix} + \begin{bmatrix} -2 \\ 8 \end{bmatrix} \qquad \text{Substitution}$$

Matrix B is a 2×2 matrix, and matrix C is a 2×1 matrix. Since the matrices do not have the same dimensions, it is not possible to add these matrices.

✔ Check Your Progress

3A. $\begin{bmatrix} 7 & -2 & 11 \\ -14 & 8 & 1 \end{bmatrix} + \begin{bmatrix} -3 & 4 & 5 \\ -4 & 6 & -1 \end{bmatrix}$

3B. $\begin{bmatrix} 12 & 8 \\ -3 & -7 \\ -6 & 9 \end{bmatrix} + \begin{bmatrix} -2 & 4 \\ 1 & -1 \\ -9 & 5 \end{bmatrix}$

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If two matrices have the same dimensions, then they can also be subtracted. You subtract matrices by subtracting the corresponding elements of the matrices.

EXAMPLE 4 Subtract Matrices

Find each difference for $A = \begin{bmatrix} 4 & -2 & 8 \\ -17 & 10 & 6 \end{bmatrix}$, $B = \begin{bmatrix} 6 & -14 & 2 \end{bmatrix}$, and

$C = \begin{bmatrix} 1 & -3 & 5 \\ -10 & 8 & 7 \end{bmatrix}$. If the difference does not exist, write *impossible*.

a. $B - A$

$B - A = \begin{bmatrix} 6 & -14 & 2 \end{bmatrix} - \begin{bmatrix} 4 & -2 & 8 \\ -7 & 10 & 6 \end{bmatrix}$ **Substitution**

Matrix B is a 1×3 matrix, while matrix A is a 2×3 matrix. Since the dimensions are not the same, it is not possible to subtract these matrices.

b. $A - C$

$A - C = \begin{bmatrix} 4 & -2 & 8 \\ -17 & 10 & 6 \end{bmatrix} - \begin{bmatrix} 1 & -3 & 5 \\ -10 & 8 & 7 \end{bmatrix}$ **Substitution**

$= \begin{bmatrix} 4 - 1 & -2 - (-3) & 8 - 5 \\ -17 - (-10) & 10 - 8 & 6 - 7 \end{bmatrix}$ or $\begin{bmatrix} 3 & 1 & 3 \\ -7 & 2 & -1 \end{bmatrix}$ **Simplify.**

✔ **Check Your Progress**

4A. $\begin{bmatrix} 2 & -5 \\ 8 & 12 \end{bmatrix} - \begin{bmatrix} 6 & -3 \\ -9 & 1 \end{bmatrix}$

4B. $\begin{bmatrix} 16 & -6 & 1 \\ -2 & 5 & -1 \end{bmatrix} - \begin{bmatrix} 21 & 3 & -6 \\ -12 & -2 & 1 \end{bmatrix}$

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You can multiply any matrix by a constant called a **scalar**. This operation is called **scalar multiplication** and is done by multiplying each element of the matrix by the scalar.

EXAMPLE 5 Multiply a Matrix by a Scalar

If $A = \begin{bmatrix} 2 & -4 & -7 & 9 \\ 1 & -10 & 8 & 6 \end{bmatrix}$, find $4A$.

$4A = 4\begin{bmatrix} 2 & -4 & -7 & 9 \\ 1 & -10 & 8 & 6 \end{bmatrix}$ **Substitution**

$= \begin{bmatrix} 4(2) & 4(-4) & 4(-7) & 4(9) \\ 4(1) & 4(-10) & 4(8) & 4(6) \end{bmatrix}$ **Definition of scalar multiplication**

$= \begin{bmatrix} 8 & -16 & -28 & 36 \\ 4 & -40 & 32 & 24 \end{bmatrix}$ **Simplify.**

✔ **Check Your Progress**

5A. If $A = \begin{bmatrix} -9 & 3 \\ 5 & -11 \\ -2 & 7 \end{bmatrix}$, find $-3A$.

5B. If $B = \begin{bmatrix} -12 & 8 \\ -3 & 0 \end{bmatrix}$, find $2B$.

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Example 1
p. 369

State the dimensions of each matrix. Then identify the position of the circled element in each matrix.

1. $\begin{bmatrix} 8 & -2 & 1 & -3 \\ ⓪ & 5 & 7 & -11 \end{bmatrix}$

2. $\begin{bmatrix} 6 \\ -7 \\ 2 \\ ① \end{bmatrix}$

3. $\begin{bmatrix} 9 & -12 & ⑥ & 2 \end{bmatrix}$

4. $\begin{bmatrix} 6 & -8 & 12 & 9 & -1 \\ 2 & 9 & 7 & ⑪ & -5 \\ 5 & 0 & 1 & 3 & 4 \end{bmatrix}$

Example 2
p. 370

5. **HOTELS** The costs for an overnight stay at a hotel are listed in the table at the right.

Room	Weekday	Weekend
single	$69	$89
double	$79	$109
suite	$99	$139

   a. Write a matrix to organize the costs of an overnight stay at the hotel.

   b. What are the dimensions of the matrix?

   c. Which room and night is the most expensive? least expensive?

Examples 3–5
pp. 370–371

Perform the indicated matrix operations. If the matrix does not exist, write *impossible*.

6. $\begin{bmatrix} 5 & -2 \\ 7 & -6 \end{bmatrix} + \begin{bmatrix} -5 & -8 \\ 3 & 1 \end{bmatrix}$

7. $\begin{bmatrix} 8 & 11 & 5 \\ -3 & 7 & 8 \\ -1 & -2 & 0 \end{bmatrix} - \begin{bmatrix} -4 & 10 & -9 \\ 6 & -12 & -1 \\ 7 & 5 & 3 \end{bmatrix}$

8. $-2\begin{bmatrix} 7 & -2 & 0 & 1 \\ -8 & 11 & -9 & 3 \\ -4 & -7 & 5 & 6 \end{bmatrix}$

9. $\begin{bmatrix} 15 \\ -8 \\ 4 \end{bmatrix} - \begin{bmatrix} -3 & -2 & 7 \end{bmatrix}$

Practice and Problem Solving

● = **Step-by-Step Solutions** begin on page R12.
Extra Practice begins on page 815.

Example 1
p. 369

State the dimensions of each matrix. Then identify the position of the circled element in each matrix.

10. $\begin{bmatrix} 6 & 8 & -2 & 3 \\ -7 & -12 & 58 & 1 \\ 86 & ⑫ & 7 & -9 \\ 0 & -6 & 21 & 79 \end{bmatrix}$

11. $\begin{bmatrix} 2 & 9 \\ ⓷ & -5 \\ 7 & -8 \\ 1 & -1 \\ -2 & 3 \end{bmatrix}$

12. $\begin{bmatrix} 8 & -10 & 4 & 6 & -2 \\ ③ & 7 & 9 & 5 & -1 \end{bmatrix}$

13. $\begin{bmatrix} 8 & 2 & -1 & 4 & 3 & -7 \\ 9 & 10 & -17 & 0 & 1 & -8 \\ -1 & 5 & -2 & ⑦ & -3 & 0 \\ -9 & 7 & 5 & 3 & -6 & 6 \end{bmatrix}$

14. $\begin{bmatrix} 1 & -20 & -16 \\ -5 & 0 & 7 \\ 9 & -13 & 12 \\ 8 & ⓷ & 2 \\ -3 & 5 & 10 \\ 6 & -14 & 25 \end{bmatrix}$

15. $\begin{bmatrix} 3 & 2 & 7 & 0 \\ 4 & 9 & 10 & ④ \\ -1 & 7 & 6 & 5 \\ 0 & -3 & 12 & -5 \\ 8 & -5 & -10 & -8 \\ -2 & 4 & 11 & -2 \end{bmatrix}$

16. $\begin{bmatrix} 8 & 6 & -4 & 2 & 1 & 3 \\ -8 & -4 & 0 & ⑨ & -5 & 6 \\ 2 & 3 & -1 & 7 & -9 & 0 \end{bmatrix}$

Example 2
p. 370

17 **GEOGRAPHY** The land area in square miles and the number of people per square mile in 2000 are shown.

State	Land Area	People per Square Mile
Ohio	40,948	277.3
Florida	53,926	296.4
New York	47,213	401.9
North Carolina	48,710	165.2

a. Write a matrix to organize the given data.

b. What are the dimensions of the matrix?

c. Which state has the most people per square mile? the fewest people per square mile?

Perform the indicated matrix operations. If the matrix does not exist, write *impossible*.

18. $\begin{bmatrix} 8 & -5 & 1 \\ 3 & -7 & -4 \end{bmatrix} - \begin{bmatrix} 6 & -2 & -7 & 9 \\ 10 & -3 & 1 & -4 \end{bmatrix}$ 19. $\begin{bmatrix} -9 & 5 & 1 \\ 14 & -6 & 7 \end{bmatrix} + \begin{bmatrix} 3 & -4 & -1 \\ -7 & -2 & 8 \end{bmatrix}$

20. $-3 \begin{bmatrix} 6 & -8 & 9 & 1 & -3 \\ 0 & 7 & -2 & -4 & 5 \end{bmatrix}$ 21. $5 \begin{bmatrix} 2 & -1 & 0 \\ 1 & -3 & 5 \\ 7 & 10 & -11 \\ 8 & -9 & -4 \end{bmatrix}$

22. $\begin{bmatrix} 17 & 10 \\ -5 & 1 \\ 7 & 6 \\ -8 & -2 \\ 3 & 8 \end{bmatrix} - \begin{bmatrix} 20 & 6 \\ -4 & -5 \\ -9 & 0 \\ -1 & 9 \\ 6 & -12 \end{bmatrix}$ 23. $\begin{bmatrix} 6 & 8 & -4 & -2 \end{bmatrix} - \begin{bmatrix} 9 & -4 & 7 & 8 \end{bmatrix}$

24. **VOTING** The results of a recent poll are organized in the matrix shown at the right.

$\begin{array}{c} \\ \text{Propositon 1} \\ \text{Propositon 2} \\ \text{Propositon 3} \end{array} \begin{array}{cc} \text{For} & \text{Against} \\ \begin{bmatrix} 562 & 1025 \\ 789 & 921 \\ 1255 & 301 \end{bmatrix} \end{array}$

a. How many people voted for Proposition 1?

b. How many more people voted against Proposition 2 than for Proposition 2?

c. How many votes were cast against the propositions?

25. **SALES** The manager of The Donut Delight Shop keeps records of the types of donuts sold each day. Two days of sales are shown.

| Day	Store	Sales of Each Type of Donut ($)			
		Chocolate	Glazed	Powdered	Lemon Filled
Saturday	Main St.	95	205	70	51
	Elm St.	105	245	79	49
Sunday	Main St.	167	295	99	79
	Elm St.	159	289	107	88

a. Describe what 245 represents.

b. Write a matrix that represents the sales for each day.

c. How much did each store make in sales over the two days for each type of donut?

d. Which donut made the company the most money?
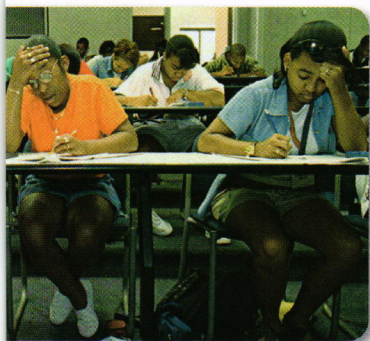
26. SCORES The average SAT scores for males and females are shown.

Year	Verbal Score		Mathematical Score	
	Male	Female	Male	Female
1998	509	502	531	496
2000	507	504	533	498
2002	507	502	534	500
2004	512	504	537	501
2005	513	505	538	504

a. Organize the verbal scores and mathematical scores into two matrices.

b. Find the total score that males and females earned on the SATs each year.

c. Express the difference between the verbal and mathematical scores in a matrix.

Perform the indicated matrix operations. If an operation cannot be performed, write *impossible*.

27. $2\begin{bmatrix} -5 & 8 & 2 \end{bmatrix} + \begin{bmatrix} -6 & 9 & 5 \end{bmatrix}$

28. $\begin{bmatrix} 9 & -5 \\ -3 & 4 \end{bmatrix} + (-9)\begin{bmatrix} 2 & -1 \\ 0 & -7 \end{bmatrix}$

29. $-3\begin{bmatrix} 7 \\ -4 \\ -2 \\ 1 \end{bmatrix} - 2\begin{bmatrix} 7 \\ -5 \\ -3 \end{bmatrix}$

30. $-1\begin{bmatrix} 5 & -8 & 14 \\ 12 & -7 & -3 \\ 1 & 0 & 8 \end{bmatrix} + \begin{bmatrix} 7 & 8 & 2 \\ -7 & -2 & 6 \\ -6 & 3 & -1 \end{bmatrix}$

31 $\begin{bmatrix} -5 & 2 \\ 12 & -11 \\ 9 & 0 \\ -1 & 7 \\ 6 & 5 \\ -4 & 2 \end{bmatrix} + 4\begin{bmatrix} 10 & 4 \\ -1 & -3 \\ 5 & -8 \\ -9 & 0 \\ 1 & 4 \\ -3 & 2 \end{bmatrix}$

32. $2\begin{bmatrix} 4 & 0 \\ -1 & 7 \\ -3 & 2 \end{bmatrix} - 3\begin{bmatrix} 1 & -4 \\ -8 & 9 \\ 10 & 7 \end{bmatrix} + \begin{bmatrix} 6 & 3 \\ 8 & -4 \\ -1 & -2 \end{bmatrix}$

H.O.T. Problems Use **H**igher-**O**rder **T**hinking Skills

33. OPEN ENDED Write two matrices with a difference of $\begin{bmatrix} 5 & -2 & 7 \\ -3 & 1 & 0 \end{bmatrix}$.

34. CHALLENGE Write three matrices with a sum of $\begin{bmatrix} 5 & -1 \\ 9 & 7 \\ -5 & 8 \end{bmatrix}$ if at least one of the addends is shown as the product of a scalar and a matrix.

35. REASONING For matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, the *transpose* of A is $A^T = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$. Write a matrix B that is equal to its transpose B^T.

36. REASONING Is it possible to add a 3×2 matrix and a 2×3 matrix? Explain. Include an example or counterexample to support your answer.

37. OPEN ENDED Describe a real-world situation that can be modeled by using a matrix. Then write a matrix to model the situation.

38. WRITING IN MATH Summarize how to perform matrix operations on matrices.

39. If $A = \begin{bmatrix} 5 & -8 & 1 \\ 7 & -3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 9 & -5 & 2 \\ -1 & -7 & 6 \end{bmatrix}$, find $A + B$.

A $\begin{bmatrix} 14 & -13 & 3 \\ 7 & -3 & 4 \end{bmatrix}$ **C** $\begin{bmatrix} 14 & -13 & 3 \\ 6 & -10 & 10 \end{bmatrix}$

B $\begin{bmatrix} 5 & -8 & 1 \\ 6 & -10 & 10 \end{bmatrix}$ **D** $\begin{bmatrix} -4 & -3 & -1 \\ 8 & 4 & -2 \end{bmatrix}$

40. SHORT RESPONSE The difference between the length and width of a rectangle is 9 inches. Find the dimensions of the rectangle if its perimeter is 52 inches.

41. At a movie theater, the costs for various amounts of popcorn and hot dogs are shown.

Hot Dogs	Boxes of Popcorn	Total Cost
1	1	$8.50
2	4	$21.60

Which pair of equations can be used to find p, the cost of a box of popcorn, and h, the cost of a hot dog?

F $p + h = 8.5$
$p + 2h = 10.8$

G $p + h = 8.5$
$2h + 4p = 21.6$

H $p + h = 8.5$
$2p + 4h = 21.6$

J $p + h = 8.5$
$2p + 2h = 21.6$

42. What is the solution set for $9 + x \geq 3$?

A $\{x \mid x \geq -6\}$ **C** $\{x \mid x \leq -6\}$

B $\{x \mid x \geq 6\}$ **D** $\{x \mid x \leq 6\}$

43. CHEMISTRY Orion Labs needs to make 500 gallons of 34% acid solution. The only solutions available are a 25% acid solution and a 50% acid solution. Write and solve a system of equations to find the number of gallons of each solution that should be mixed to make the 34% solution. (Lesson 6-5)

Use elimination to solve each system of equations. (Lesson 6-4)

44. $x + y = 7$
$2x + y = 11$

45. $a - b = 9$
$7a + b = 7$

46. $q + 4r = -8$
$3q + 2r = 6$

47. SALES Marissa wants to make at least $75 selling caramel apples at the school carnival. She plans to sell each apple for $1.50. Write and solve an inequality to find the number of apples a she needs to make and sell to reach her goal if it costs her $0.30 per apple. (Lesson 5-3)

Find the next three terms of each arithmetic sequence. (Lesson 3-5)

48. 4, 7, 10, 13, . . .

49. 18, 24, 30, 36, . . .

50. $-66, -70, -74, -78, . . .$

51. CRAFTS Mandy makes baby blankets and stuffed rabbits to sell at craft fairs. She sells blankets for $28 and rabbits for $18. Write and evaluate an expression to find her total amount of sales if she sells 25 blankets and 25 rabbits. (Lesson 1-4)

Solve each equation. (Lesson 2-3)

52. $5 = 4t - 7$

53. $-3x + 10 = 19$

54. $\frac{c}{-4} - 2 = -36$

55. $6 + \frac{y}{3} = -45$

56. $9 = \frac{d + 5}{8}$

57. $\frac{r + 1}{3} = 8$

Using Matrices to Solve Systems of Equations

Why?

The 30 members of the Washington High School's Ski Club went on a one-day ski trip. Members can rent skis for $22 per day or snowboards for $24 per day. The club paid a total of $700 for rental equipment.

The resort can use this information to find how many members rented each type of equipment.

Augmented Matrices You can use a matrix called an **augmented matrix** to solve a system of equations. An augmented matrix consists of the coefficients and the constant terms of a system of equations. The coefficients and constant terms are usually separated by a dashed line.

Linear System	Augmented Matrix
$x - 3y = 8$	$\begin{bmatrix} 1 & -3 & \vdots & 8 \\ -9 & 2 & \vdots & -4 \end{bmatrix}$
$-9x + 2y = -4$	

Make sure that the coefficients of the x-terms are listed in one column, the coefficients of the y-terms are in another column, and the constant terms are in a third column.

EXAMPLE 1 **Write an Augmented Matrix**

Write an augmented matrix for each system of equations.

a. $-2x + 7y = 11$
$\quad\;\; 6x - 4y = 2$

Place the coefficients of the equations and the constant terms into a matrix.

$$\begin{aligned} -2x + 7y &= 11 \\ 6x - 4y &= 2 \end{aligned} \quad \longrightarrow \quad \begin{bmatrix} -2 & 7 & \vdots & 11 \\ 6 & -4 & \vdots & 2 \end{bmatrix}$$

b. $x - 2y = 5$
$\qquad\quad y = -4$

$$\begin{aligned} x - 2y &= 5 \\ y &= -4 \end{aligned} \quad \longrightarrow \quad \begin{bmatrix} 1 & -2 & \vdots & 5 \\ 0 & 1 & \vdots & -4 \end{bmatrix}$$

✓ Check Your Progress

1A. $6x - 8y = -10$
$\quad\;\; -5x = -20$

1B. $3x - 2y = 6$
$\quad\;\; 2x + 3y = 12$

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Solve Systems of Equations You can solve a system of equations by using an augmented matrix. By performing row operations, you can change the form of the matrix.

Key Concept — Elementary Row Operations

The following operations can be performed on an augmented matrix.

- Interchange any two rows.
- Multiply all elements in a row by a nonzero constant.
- Replace one row with the sum of that row and a multiple of another row.

Row reduction is the process of performing elementary row operations on an augmented matrix to solve a system. The goal is to get the coefficients portion of the matrix to have the form $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, also known as the **identity matrix**.

The first row will give you the solution for x, because the coefficient of y is 0 and the coefficient of x is 1. The second row will give you the solution for y, because the coefficient of x is 0 and the coefficient of y is 1.

EXAMPLE 2 — Use Row Operations to Solve a System

Use an augmented matrix to solve the system of equations.
$$-5x + 3y = 6$$
$$x - y = 4$$

Step 1 Write the augmented matrix: $\left[\begin{array}{cc|c} -5 & 3 & 6 \\ 1 & -1 & 4 \end{array}\right]$.

Step 2 Notice that the first element in the second row is 1. Interchange the rows so 1 can be in the upper left-hand corner.

$\left[\begin{array}{cc|c} -5 & 3 & 6 \\ 1 & -1 & 4 \end{array}\right]$ **Interchange R_1 and R_2.** $\left[\begin{array}{cc|c} 1 & -1 & 4 \\ -5 & 3 & 6 \end{array}\right]$

Step 3 To make the first element in the second row a 0, multiply the first row by 5 and add the result to row 2.

$\left[\begin{array}{cc|c} 1 & -1 & 4 \\ -5 & 3 & 6 \end{array}\right]$ **$5R_1 + R_2$** $\left[\begin{array}{cc|c} 1 & -1 & 4 \\ 0 & -2 & 26 \end{array}\right]$ **The result is placed in row 2.**

Step 4 To make the second element in the second row a 1, multiply the second row by $-\frac{1}{2}$.

$\left[\begin{array}{cc|c} 1 & -1 & 4 \\ 0 & -2 & 26 \end{array}\right]$ **$-\frac{1}{2}R_2$** $\left[\begin{array}{cc|c} 1 & -1 & 4 \\ 0 & 1 & -13 \end{array}\right]$ **The result is placed in row 2.**

Step 5 To make the second element in the first row a 0, add the rows together.

$\left[\begin{array}{cc|c} 1 & -1 & 4 \\ 0 & 1 & -13 \end{array}\right]$ **$R_1 + R_2$** $\left[\begin{array}{cc|c} 1 & 0 & -9 \\ 0 & 1 & -13 \end{array}\right]$ **The result is placed in row 1.**

The solution is $(-9, -13)$.

Check Your Progress

2A. $x + 2y = 6$
$2x + y = 9$

2B. $2x - 3y = 3$
$x + y = 14$

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StudyTip

Alternate Method
Row operations can be performed in different orders to arrive at the same result. In Example 2, you could have started by multiplying the first row, R_1, by $-\frac{1}{5}$ instead of interchanging the rows.

Matrices are useful for solving real-world problems. First, model the situation with a system of equations and then write the augmented matrix.

PARKS A youth group traveling in two vans visited Yellowstone National Park. The number of people in each van and the park fees are shown in the table.

Van	Number of Adults	Number of Students	Total Cost
A	2	6	$102
B	2	7	$114

Real-World Link

Yellowstone National Park is America's first national park. It is located in Wyoming, Montana, and Idaho. The park was established in 1872. The most popular geyser in the world, Old Faithful, is located there.

Source: National Park Service

a. Write a system of linear equations to model the situation. Then write the augmented matrix.

Let a represent the adult fee, and let s represent the student fee.

$$\begin{aligned} 2a + 6s &= 102 \\ 2a + 7s &= 114 \end{aligned} \longrightarrow \begin{bmatrix} 2 & 6 & | & 102 \\ 2 & 7 & | & 114 \end{bmatrix}$$

b. Find the entrance fee for an adult and a student.

Step 1 To make the first element in the first row a 1, multiply the first row by $\frac{1}{2}$.

$$\begin{bmatrix} 2 & 6 & | & 102 \\ 2 & 7 & | & 114 \end{bmatrix} \quad \frac{1}{2}R_1 \Longrightarrow \quad \begin{bmatrix} 1 & 3 & | & 51 \\ 2 & 7 & | & 114 \end{bmatrix}$$

Step 2 To make the first element in the second row a 0, multiply the first row by -2 and add the result to row 2.

$$\begin{bmatrix} 1 & 3 & | & 51 \\ 2 & 7 & | & 114 \end{bmatrix} \quad -2R_1 + R_2 \Longrightarrow \quad \begin{bmatrix} 1 & 3 & | & 51 \\ 0 & 1 & | & 12 \end{bmatrix}$$

Step 3 To make the second element in the first row a 0, multiply the second row by -3 and add the result to row 1.

$$\begin{bmatrix} 1 & 3 & | & 51 \\ 0 & 1 & | & 12 \end{bmatrix} \quad -3R_2 + R_1 \Longrightarrow \quad \begin{bmatrix} 1 & 0 & | & 15 \\ 0 & 1 & | & 12 \end{bmatrix}$$

The solution is (15, 12). The adult fee is $15, and the student fee is $12.

Check Your Progress

3. CARNIVAL At a carnival, 44 tickets are required for 4 meals and 8 rides, and 58 tickets are required for 6 meals and 10 rides. How many tickets are required for each item?

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Check Your Understanding

Example 1
p. 376

Write an augmented matrix for each system of equations.

1. $\begin{aligned} -x + 3y &= -10 \\ 5x - 2y &= 7 \end{aligned}$

2. $\begin{aligned} x - 4y &= 5 \\ -2x + 8y &= 1 \end{aligned}$

3. $\begin{aligned} x + 2y &= -1 \\ 2x - 2y &= -9 \end{aligned}$

4. $\begin{aligned} -4x + 6y &= 2 \\ -x - 8y &= 0 \end{aligned}$

5 $\begin{aligned} 3x + 4y &= -5 \\ 2x - y &= 6 \end{aligned}$

6. $\begin{aligned} -x + 3y &= 8 \\ 6x - 3y &= -3 \end{aligned}$

Example 2
p. 377

Use an augmented matrix to solve each system of equations.

7. $x + y = -3$
$x - y = 1$

8. $x - y = -2$
$2x + 2y = 12$

9. $3x - 4y = -27$
$x + 2y = 11$

10. $x + 4y = -6$
$2x - 5y = 1$

Example 3
p. 378

11. SHOPPING Darnell and Sandra went shopping for graphic novels. The store charges one price for all new books and another for all old books.

a. Write a system of linear equations to model the situation. Let n represent new books, and let b represent old books.

b. Write the augmented matrix.

c. What is the price for each type of book?

Darnell
3 new
4 old
$22.25

Sandra
3 new
10 old
$29.75

Practice and Problem Solving

Example 1
p. 376

Write an augmented matrix for each system of equations.

12. $x + 2y = -3$
$3x - y = 2$

13. $-4x - 3y = -8$
$x + y = -12$

14. $2x + y = 1$
$x + 4y = -5$

15. $-6x + y = -15$
$x - 2y = 13$

16. $3x + y = -6$
$x - 5y = -7$

17. $x - y = 7$
$9x - 5y = 23$

Example 2
p. 377

Use an augmented matrix to solve each system of equations.

18. $x - 3y = -2$
$4x + y = 31$

19. $x + 2y = 3$
$-3x + 3y = 27$

20. $2x - 3y = -20$
$x + 2y = 11$

21. $x - y = 2$
$3x - 2y = 2$

22. $x - 2y = -15$
$2x + 5y = 78$

23. $4x + 3y = -9$
$x + 4y = -25$

24. $-2x - 2y = 8$
$5x + 2y = -2$

25. $3x - 6y = 36$
$2x + 4y = -40$

26. $2x + 3y = 11$
$3x - y = -11$

27. $4x - 3y = 24$
$2x + 5y = -14$

28. $3x + 2y = 6$
$x + 2y = 8$

29. $4x - 3y = 5$
$2x + 9y = 6$

Example 3
p. 378

30. CHEERLEADING If the Franklin High School cheerleaders replace 8 uniforms and 6 poms, the cost is $378. If they replace 6 uniforms and 9 poms, the cost is $333.

a. Write a system of linear equations to model the situation. Let u represent uniforms, and let p represent poms.

b. Write the augmented matrix.

c. What is the cost of each uniform and each pom?

Write a system of equations for each augmented matrix.

31. $\begin{bmatrix} 1 & 0 & | & 16 \\ 0 & 1 & | & -2 \end{bmatrix}$

32. $\begin{bmatrix} 1 & 7 & | & 3 \\ -5 & 10 & | & 4 \end{bmatrix}$

33. $\begin{bmatrix} 3 & 2 & | & 7 \\ -1 & -4 & | & 5 \end{bmatrix}$

34. $\begin{bmatrix} 1 & 0 & | & -10 \\ 0 & 4 & | & 8 \end{bmatrix}$

35. $\begin{bmatrix} -1 & 9 & | & 12 \\ 2 & 3 & | & -7 \end{bmatrix}$

36. $\begin{bmatrix} 6 & -6 & | & 5 \\ -8 & 11 & | & 0 \end{bmatrix}$

37 **SCHOOL STORE** Nari is checking items being shipped to the school store. The shipment contains notebooks that cost $22 per box and mugs that cost $40 per box. She counts 16 boxes, and the invoice states that the order totals $460. How many boxes of each item were received?

38. PARTIES Mel is having a few friends over, and she is buying subs and cans of sodas for them. Mel bought 28 items. If Mel spent $56.70, how many subs did she buy? How many sodas did she buy?

Subs $4.95
Soda $0.40
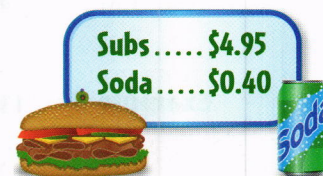
39. RENTALS Makya and his three sisters rented 2 items each at the video store. Members can rent movies for $4 and video games for $4.50. If Makya and his sisters spent $33.50, how many movies did they rent? How many games did they rent?

Use an augmented matrix to solve each system of equations.

40. $2x + y = -1$
$-2x + y = -4$

41. $x + 2y = 3$
$2x + 4y = 6$

42. $3x - y = 1$
$-12x + 4y = 3$

43. $3x - 9y = 12$
$-2x + 6y = 9$

44. $4x - 3y = 1$
$-8x + 6y = -2$

45. $6x - 2y = -4$
$-3x + y = 2$

46. **MULTIPLE REPRESENTATIONS** In this problem, you will investigate the different representations of the following problem and their impact on the solution.

Paloma exercises every morning for 40 minutes. She does a combination of aerobics, which burns about 11 Calories per minute, and stretching, which burns about 4 Calories per minute. Her goal is to burn 335 Calories during her routine. How long should she do each activity to burn 335 Calories?

a. VERBAL List the representations that would be appropriate to solve the problem.

b. ALGEBRAIC Select a representation and solve the problem.

c. ALGEBRAIC Select a different representation and solve the problem.

d. VERBAL Write about the relationship between the representations of the problem. How did each affect your solution?

H.O.T. Problems   Use **H**igher-**O**rder **T**hinking Skills

47. REASONING Explain why the system represented by $\begin{bmatrix} 6 & 2 & | & 3 \\ 6 & 2 & | & -5 \end{bmatrix}$ has no solution.

48. WRITING IN MATH Describe the advantages and disadvantages of using an augmented matrix to solve a system of equations.

49. CHALLENGE For $a \neq 0$, what is the solution of the system represented by $\begin{bmatrix} a & 2 & | & 4 \\ a & -3 & | & -6 \end{bmatrix}$?

50. OPEN ENDED Write a word problem for the system represented by $\begin{bmatrix} 3 & 1 & | & 13 \\ 2 & 1 & | & 9.5 \end{bmatrix}$. Solve the system, and explain its meaning in this situation.

51. WRITING IN MATH Summarize how to write and use an augmented matrix to solve a system of linear equations.

52. SHORT RESPONSE Tonisha paid $25.75 for 3 games of miniature golf and 2 rides on go-karts. Trevor paid $35.75 for 4 games of miniature golf and 3 rides on go-karts. How much did each activity cost?

53. What is the solution of this system of equations?

$$\begin{cases} 0.5x - 2y = 17 \\ 2x + y = 104 \end{cases}$$

A $(50, 4)$

B $(4, 50)$

C no solution

D infinitely many solutions

54. PROBABILITY Lexis scored 88, 95, 77, and 93 on her first four tests. What grade must she get on her fifth test to earn an average of 90 for all five tests?

F 85   **H** 96

G 90   **J** 97

55. Pablo's Pizza Place estimates that 42% of their annual sales go toward paying employees. If the pizza place makes $4156.50 on Friday, approximately how much went for paying employees?

A $98.96   **C** $174.57

B $1745.73   **D** $17457.30

Spiral Review

Perform the indicated matrix operations. If an operation cannot be performed, write *impossible*. (Lesson 6-6)

56. $\begin{bmatrix} 5 & -2 & 4 \\ -3 & 7 & 9 \\ 12 & -1 & 8 \end{bmatrix} + \begin{bmatrix} -6 & 7 & -12 \\ 15 & -8 & 1 \\ 9 & 3 & -5 \end{bmatrix}$

57. $\begin{bmatrix} 8 & -13 \\ 4 & 2 \\ -4 & -6 \end{bmatrix} - \begin{bmatrix} -3 & 5 \\ 10 & -16 \\ 8 & -2 \end{bmatrix}$

58. $-3 \begin{bmatrix} -7 & -1 & 3 & 0 \\ -1 & 5 & 7 & 9 \end{bmatrix}$

59. $\begin{bmatrix} 5 \\ -8 \\ 3 \end{bmatrix} + \begin{bmatrix} 9 & -4 & 2 \end{bmatrix}$

60. SPORTS In the 2006 Winter Olympic Games, the total number of gold and silver medals won by the U.S. was 18. The total points scored for gold and silver medals was 45. Write and solve a system of equations to find how many gold and silver medals were won by the U.S. (Lesson 6-5)

Gold 3 points

Silver 2 points

61. DRIVING Tires should be kept within 2 pounds per square inch (psi) of the manufacturer's recommended tire pressure. If the recommendation for a tire is 30 psi, what is the range of acceptable pressures? (Lesson 5-5)

Write an equation in slope-intercept form for the line that passes through the given point and is parallel to the graph of each equation. (Lesson 4-4)

62. $(-3, 2); y = x - 6$

63. $(2, -1); y = 2x + 2$

64. $(-5, -4); y = \frac{1}{2}x + 1$

65. $(3, 3); y = \frac{2}{3}x - 1$

66. $(-4, -3); y = -\frac{1}{3}x + 3$

67. $(-1, 2); y = -\frac{1}{2}x - 4$

Skills Review

Simplify each expression. (Lesson 1-4)

68. $2(7p + 4)$

69. $-3(3 - 8x)$

70. $-3y + 5(4y)$

71. $-2(4m - 6 + 8m)$

72. $5g - 8g + 2(-4g)$

73. $12(4c + 3b)$

Systems of Inequalities

Why?

Jacui is beginning an exercise program that involves an intense cardiovascular workout. Her trainer recommends that for a person her age, her heart rate should stay within the following range as she exercises.

- It should be higher than 102 beats per minute.
- It should not exceed 174 beats per minute.

The graph shows the maximum and minimum target heart rate for people ages 0 to 30 as they exercise. If the preferred range is in light green, how old do you think Jacui is?
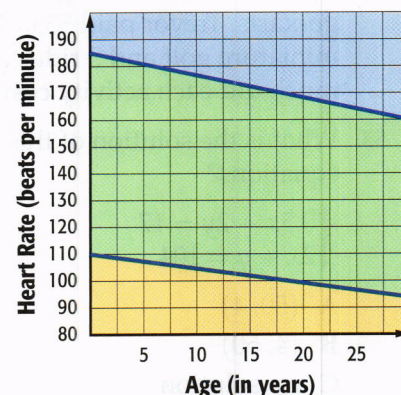
Systems of Inequalities The graph above is a graph of two inequalities. A set of two or more inequalities with the same variables is called a **system of inequalities**.

The solution of a system of inequalities with two variables is the set of ordered pairs that satisfy all of the inequalities in the system. The solution set is represented by the overlap, or intersection, of the graphs of the inequalities.

EXAMPLE 1 Solve by Graphing

Solve the system of inequalities by graphing.

$y > -2x + 1$
$y \leq x + 3$

The graph of $y = -2x + 1$ is dashed and is not included in the graph of the solution. The graph of $y = x + 3$ is solid and is included in the graph of the solution.

The solution of the system is the set of ordered pairs in the intersection of the graphs of $y > -2x + 1$ and $y \leq x + 3$. This region is shaded in green.

When graphing more than one region, it is helpful to use two different colored pencils or two different patterns for each region. This will make it easier to see where the regions intersect and find possible solutions.
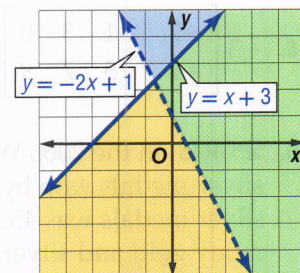
✔ Check Your Progress

1A. $y \leq 3$
$x + y \geq 1$

1B. $2x + y \geq 2$
$2x + y < 4$

1C. $y \geq -4$
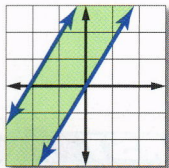$3x + y \leq 2$

1D. $x + y > 2$
$-4x + 2y < 8$

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Sometimes the regions never intersect. When this happens, there is no solution because there are no points in common.

EXAMPLE 2 No Solution

Solve the system of inequalities by graphing.

$3x - y \geq 2$
$3x - y < -5$

The graphs of $3x - y = 2$ and $3x - y = -5$ are parallel lines. The two regions do not intersect at any point, so the system has no solution.
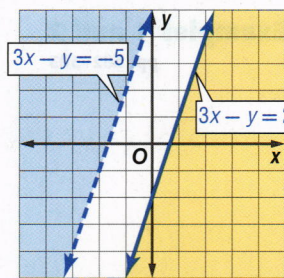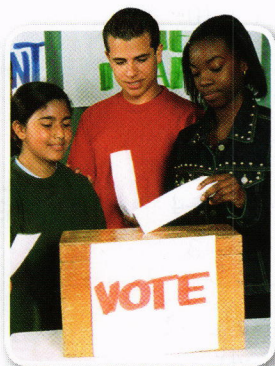
✓ **Check Your Progress**

2A. $y > 3$
$\quad\;\; y < 1$

2B. $x + 6y \leq 2$
$\quad\;\; y \geq -\frac{1}{6}x + 7$

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When using a system of inequalities in a real-world problem, sometimes only whole-number solutions will make sense.

🌐 **Real-World EXAMPLE 3** Whole-Number Solutions

ELECTIONS Monifa is running for student council. The election rules say that for the election to be valid, at least 80% of the 900 students must vote. Monifa knows that she needs more than 330 votes to win.

a. **Define the variables, and write a system of inequalities to represent this situation. Then graph the system.**

Let r = the number of votes required by the election rules; 80% of 900 students is 720 students. So $r \geq 720$.

Let v = the number of votes that Monifa needs to win. So $v > 330$.

The system of inequalities is $r \geq 720$ and $v > 330$.

b. **Name one possible solution.**

Only whole-number solutions make sense in this problem. One possible solution is (800, 400); 800 students voted and Monifa received 400 votes.
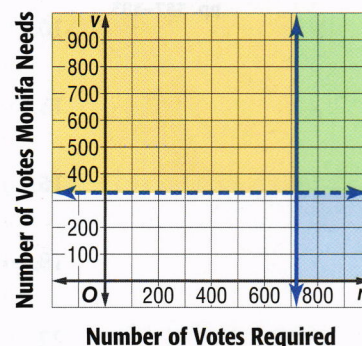
✓ **Check Your Progress**

3. FUNDRAISING The Theater Club is selling shirts. They have only enough supplies to print 120 shirts. They will sell sweatshirts for $22 and T-shirts for $15, with a goal of at least $2000 in sales.

   A. Define the variables, and write a system of inequalities to represent this situation.

   B. Then graph the system.

   C. Name one possible solution.

   D. Is (45, 30) a solution? Explain.

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Examples 1 and 2
pp. 382–383

Solve each system of inequalities by graphing.

1. $x \geq 4$
$y \leq x - 3$

2. $y > -2$
$y \leq x + 9$

3. $y < 3x + 8$
$y \geq 4x$

4. $3x - y \geq -1$
$2x + y \geq 5$

5. $y \leq 2x - 7$
$y \geq 2x + 7$

6. $y > -2x + 5$
$y \geq -2x + 10$

7. $2x + y \leq 5$
$2x + y \leq 7$

8. $5x - y < -2$
$5x - y > 6$

Example 3
p. 383

9. AUTO RACING At a racecar driving school there are safety requirements.

   a. Define the variables, and write a system of inequalities to represent the height and weight requirements in this situation. Then graph the system.

   b. Name one possible solution.

   c. Is (50, 180) a solution? Explain.

UR **FAST**
DRIVING SCHOOL
RULES TO QUALIFY
18 years of age or older
Good physical condition
Under 6 ft 7 in. tall
Under 295 lb

Practice and Problem Solving

● = **Step-by-Step Solutions** begin on page R12.
Extra Practice begins on page 815.

Examples 1 and 2
pp. 382–383

Solve each system of inequalities by graphing.

10. $y < 6$
$y > x + 3$

11. $y \geq 0$
$y \leq x - 5$

12. $y \leq x + 10$
$y > 6x + 2$

13. $y < 5x - 2$
$y > -6x + 2$

14. $2x - y \leq 6$
$x - y \geq -1$

15. $3x - y > -5$
$5x - y < 9$

16. $y \geq x + 10$
$y \leq x - 3$

17. $y < 5x - 5$
$y > 5x + 9$

18. $y \geq 3x - 5$
$3x - y > -4$

19. $4x + y > -1$
$y < -4x + 1$

20. $3x - y \geq -2$
$y < 3x + 4$

21. $y > 2x - 3$
$2x - y \geq 1$

22. $5x - y < -6$
$3x - y \geq 4$

23. $x - y \leq 8$
$y < 3x$

24. $4x + y < -2$
$y > -4x$

Example 3
p. 383

25. ICE RINKS Ice resurfacers are used for rinks of at least 1000 square feet and up to 17,000 square feet. The price ranges from as little as $10,000 to as much as $150,000.

   a. Define the variables, and write a system of inequalities to represent this situation. Then graph the system.

   b. Name one possible solution.

   c. Is (15,000, 30,000) a solution? Explain.

26. PIZZERIA Josefina works between 10 and 30 hours per week at a pizzeria. She earns $6.50 an hour, but can earn tips when she delivers pizzas.

   a. Write a system of inequalities to represent the dollars d she could earn for working h hours in a week.

   b. Graph this system.

   c. If Josefina received $17.50 in tips and earned a total of $180 for the week, how many hours did she work?

Solve each system of inequalities by graphing.

27. $x + y \geq 1$
$x + y \leq 2$

28. $3x - y < -2$
$3x - y < 1$

29. $2x - y \leq -11$
$3x - y \geq 12$

30. $y < 4x + 13$
$4x - y \geq 1$

31. $4x - y < -3$
$y \geq 4x - 6$

32. $y \leq 2x + 7$
$y < 2x - 3$

33. $y > -12x + 1$
$y \leq 9x + 2$

34. $2y \geq x$
$x - 3y > -6$

35. $x - 5y > -15$
$5y \geq x - 5$

36. **CLASS PROJECT** An economics class formed a company to sell school supplies. They would like to sell at least 20 notebooks and 50 pens per week, with a goal of earning at least $60 per week.

a. Define the variables, and write a system of inequalities to represent this situation.

b. Graph the system.

c. Name one possible solution.

37 **FINANCIAL LITERACY** Opal makes $15 per hour working for a photographer. She also coaches a competitive soccer team for $10 per hour. Opal needs to earn at least $90 per week, but she does not want to work more than 20 hours per week.

a. Define the variables, and write a system of inequalities to represent this situation.

b. Graph this system.

c. Give two possible solutions to describe how Opal can meet her goals.

d. Is (2, 2) a solution? Explain.

Real-World Link

Junior Achievement is a worldwide organization of volunteers that teaches students about entrepreneurship, work readiness, and finance through hands-on programs.

Source: *Junior Achievement*

H.O.T. Problems *Use **H**igher-**O**rder **T**hinking Skills*

38. **CHALLENGE** Create a system of inequalities equivalent to $|x| \leq 4$.

39. **REASONING** State whether the following statement is *sometimes*, *always*, or *never* true. Explain your answer with an example or counterexample.

Systems of inequalities with parallel boundaries have no solutions.

40. **REASONING** Describe the graph of the solution of this system without graphing.
$6x - 3y \leq -5$
$6x - 3y \geq -5$

41. **OPEN ENDED** One inequality in a system is $3x - y > 4$. Write a second inequality so that the system will have no solution.

42. **CHALLENGE** Graph the system of inequalities. Estimate the area of the solution.
$y \geq 1$
$y \leq x + 4$
$y \leq -x + 4$

43. **WRITING IN MATH** Refer to the beginning of the lesson. Explain what each colored region of the graph represents. Explain how shading in various colors can help to clearly show the solution set of a system of inequalities.

44. EXTENDED RESPONSE To apply for a scholarship, you must have a minimum of 20 hours of community service and a grade-point average of at least 3.75. Another scholarship requires at least 40 hours of community service and a minimum grade-point average of 3.0.

  a. Write a system of inequalities to represent the grade point average g and community service hours c you must have to apply.

  b. Graph the system of inequalities.

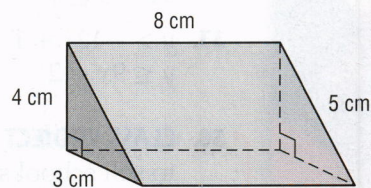 **c.** If you are eligible for both scholarships, give one possible solution.

45. GEOMETRY What is the measure of $\angle 1$?

25° 1 62°
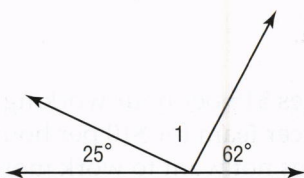
  A 83°   **C** 90°
  B 87°   **D** 93°

46. GEOMETRY What is the volume of the triangular prism?

8 cm

4 cm   5 cm

3 cm

  F 120 cm³   **H** 48 cm³
  G 96 cm³   **J** 30 cm³

47. Ten pounds of fresh tomatoes make about 15 cups of cooked tomatoes. How many cups of cooked tomatoes does one pound of fresh tomatoes make?

  A $1\frac{1}{2}$ cups
  B 3 cups
  C 4 cups
  D 5 cups

Spiral Review

Solve each system by using matrices. (Lesson 6-7)

48. $3x + 2y = 12$
$-2x - 2y = -10$

49. $-2x + 5y = -9$
$2x - 4y = 6$

50. $2x + y = 2$
$5x + 2y = 7$

51. $-3x - 6y = -42$
$x + 4y = 30$

52. $-5x + 6y = 41$
$3x - 4y = -27$

53. $-3x - 4y = -46$
$7x - 6y = 0$

If $A = \begin{bmatrix} 4 & 5 & 6 \\ 0 & -1 & -3 \\ 2 & -3 & 7 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 9 & 1 \\ -3 & 5 & -3 \\ 1 & 2 & 6 \end{bmatrix}$, **find each sum, difference, or product.** (Lesson 6-6)
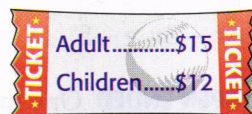
54. $A + B$ **55.** $A - B$ **56.** $3A$

57. $B + A$ **58.** $B - A$ **59.** $-2B$

60. ENTERTAINMENT A group of 11 adults and children bought tickets for the baseball game. If the total cost was $156, how many of each type of ticket did they buy? (Lesson 6-4)

Adult............$15
Children......$12

Graph each inequality. (Lesson 5-6)

61. $4x - 2 \geq 2y$ **62.** $9x - 3y < 0$ **63.** $2y \leq -4x - 6$

Skills Review

Evaluate each expression. (Lesson 1-1)

64. 3^3 **65.** 2^4 **66.** $(-4)^3$

You can use a TI-Nspire™ or TI-Nspire™ CAS technology to explore systems of inequalities. To prepare your calculator, select New Document from the Home screen. Then select **Add Graphs & Geometry**.

ACTIVITY **Graph Systems of Inequalities**

Mr. Jackson owns a car washing and detailing business. It takes 20 minutes to wash a car and 60 minutes to detail a car. He works at most 8 hours per day and does at most 4 details per day. Write a system of linear inequalities to represent this situation.

First, write a linear inequality that represents the time it takes for car washing and car detailing. Let x represent the number of car washes, and let y represent the number of car details. Then $20x + 60y \leq 480$.

To graph this using a graphing calculator, solve for y.

$$20x + 60y \leq 480$$ **Original inequality**
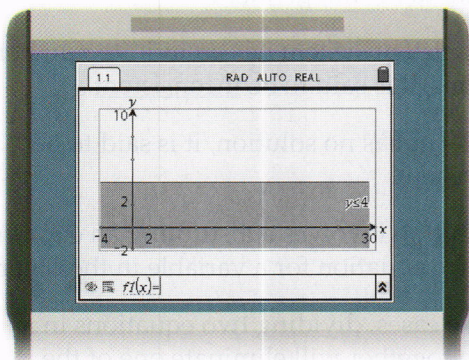
$$60y \leq -20x + 480$$ **Subtract 20x from each side and simplify.**
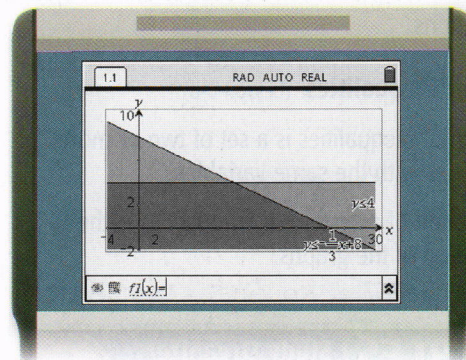
$$y \leq -\frac{1}{3}x + 8$$ **Divide each side by 60 and simplify.**

Mr. Jackson does at most 4 details per day. This means that $y \leq 4$.

Step 1 Graph $y \leq 4$. Press (menu) Window; Window Settings (enter) −4, 30, −2, 10 (enter). Press **clear** to delete = and then type (<) (=) 4 (enter).

Step 2 Graph $y \leq -\frac{1}{3}x + 8$. Press **clear** once, delete =, and then type (<)(=)(ans) (1 ÷ 3) $x + 8$ (enter).

The darkest shaded half-plane of the graph represents the solutions.

Analyze the Results

1. If Mr. Jackson charges $75 for each car he details and $25 for each car wash, what is the maximum amount of money he could earn in one day?

2. What is the greatest number of car washes that Mr. Jackson could do in a day? Explain your reasoning.

Chapter Summary

Key Concepts

Systems of Equations (Lessons 6-1 through 6-4)

- A system with a graph of two intersecting lines has one solution and is *consistent and independent.*

- Graphing a system of equations can only provide approximate solutions. For exact solutions, you must use algebraic methods.

- In the substitution method, one equation is solved for a variable and the expression substituted to find the value of another variable.

- In the elimination method, one variable is eliminated by adding or subtracting the equations. Sometimes multiplying one equation by a constant makes it easier to use the elimination method.

Matrices (Lessons 6-6 and 6-7)

- Matrices can be added or subtracted only if they have the same dimensions. Add or subtract corresponding elements.

- To multiply a matrix by a scalar *k*, multiply each element in the matrix by *k*.

- An augmented matrix can be used to solve a system of equations.

Systems of Inequalities (Lesson 6-8)

- A system of inequalities is a set of two or more inequalities with the same variables.

- The solution of a system of inequalities is the intersection of the graphs.

FOLDABLES® **Study Organizer**

6–1 Graphing Systems of Equations
6–2 Substitution
6–3 Elimination Using Addition and Subtraction
6–4 Elimination Using Multiplication
6–5 Applying Systems of Linear Equations
6–6 Organizing Data Using Matrices
6–7 Using Matrices to Solve Systems of Equations
6–8 System of Inequalities

Be sure the Key Concepts are noted in your Foldable.

Key Vocabulary

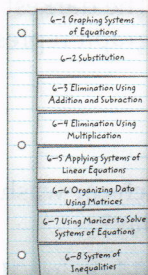|	
augmented matrix (p. 376)	**independent** (p. 333)
consistent (p. 333)	**matrix** (p. 369)
dependent (p. 333)	**scalar** (p. 371)
dimension (p. 369)	**scalar multiplication** (p. 371)
element (p. 369)	**substitution** (p. 342)
elimination (p. 348)	**system of equations** (p. 333)
inconsistent (p. 333)	**system of inequalities** (p. 382)

Vocabulary Check

State whether each sentence is *true* or *false*. If *false*, replace the underlined term to make a true sentence.

1. If a system has at least one solution, it is said to be <u>consistent</u>.

2. If a consistent system has exactly <u>two</u> solution(s), it is said to be independent.

3. If a consistent system has an infinite number of solutions, it is said to be <u>inconsistent</u>.

4. If a system has no solution, it is said to be <u>inconsistent</u>.

5. <u>Substitution</u> involves substituting an expression from one equation for a variable in the other.

6. In some cases, <u>dividing</u> two equations in a system together will eliminate one of the variables. This process is called elimination.

7. Each number in a matrix is called a(n) <u>dimension</u>.

8. A constant by which you multiply a matrix is called a <u>scalar</u>.

9. An <u>augmented matrix</u> consists of the coefficients and the constant terms of a system of equations.

10. A set of two or more inequalities with the same variables is called a <u>system of equations</u>.

Lesson-by-Lesson Review

6-1 Graphing Systems of Equations (pp. 333–339)

Graph each system and determine the number of solutions that it has. If it has one solution, name it.

11. $x - y = 1$
$x + y = 5$

12. $y = 2x - 4$
$4x + y = 2$

13. $2x - 3y = -6$
$y = -3x + 2$

14. $-3x + y = -3$
$y = x - 3$

15. $x + 2y = 6$
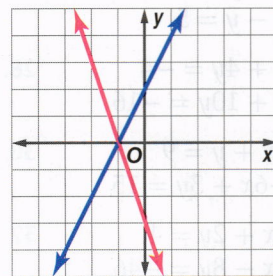$3x + 6y = 8$

16. $3x + y = 5$
$6x = 10 - 2y$

17. MAGIC NUMBERS Sean is trying to find two numbers with a sum of 14 and a difference of 4. Define two variables, write a system of equations, and solve by graphing.

EXAMPLE 1

Graph the system and determine the number of solutions it has. If it has one solution, name it.

$y = 2x + 2$
$y = -3x - 3$

The lines appear to intersect at the point $(-1, 0)$. You can check this by substituting -1 for x and 0 for y.

CHECK

$y = 2x + 2$		**Original equation**
$0 \stackrel{?}{=} 2(-1) + 2$		**Substitution**
$0 \stackrel{?}{=} -2 + 2$		**Multiply.**
$0 = 0$ ✔		
$y = -3x - 3$		**Original equation**
$0 \stackrel{?}{=} -3(-1) - 3$		**Substitution**
$0 \stackrel{?}{=} 3 - 3$		**Multiply.**
$0 = 0$ ✔		

The solution is $(-1, 0)$.

6-2 Substitution (pp. 342–347)

Use substitution to solve each system of equations.

18. $x + y = 3$
$x = 2y$

19. $x + 3y = -28$
$y = -5x$

20. $3x + 2y = 16$
$x = 3y - 2$

21. $x - y = 8$
$y = -3x$

22. $y = 5x - 3$
$x + 2y = 27$

23. $x + 3y = 9$
$x + y = 1$

24. GEOMETRY The perimeter of a rectangle is 48 inches. The length is 6 inches greater than the width. Define the variables, and write equations to represent this situation. Solve the system by using substitution.

EXAMPLE 2

Use substitution to solve the system.

$3x - y = 18$
$y = x - 4$

$3x - y = 18$	**First equation**
$3x - (x - 4) = 18$	**Substitute $x - 4$ for y.**
$2x + 4 = 18$	**Simplify.**
$2x = 14$	**Subtract 4 from each side.**
$x = 7$	**Divide each side by 2.**

Use the value of x and either equation to find the value for y.

$y = x - 4$	**Second equation**
$= 7 - 4$ or 3	**Substitute and simplify.**

The solution is $(7, 3)$.

6-3 Elimination Using Addition and Subtraction (pp. 348–354)

Use elimination to solve each system of equations.

25. $x + y = 13$
$x - y = 5$

26. $-3x + 4y = 21$
$3x + 3y = 14$

27. $x + 4y = -4$
$x + 10y = -16$

28. $2x + y = -5$
$x - y = 2$

29. $6x + y = 9$
$-6x + 3y = 15$

30. $x - 4y = 2$
$3x + 4y = 38$

31. $2x + 2y = 4$
$2x - 8y = -46$

32. $3x + 2y = 8$
$x + 2y = 2$

33. BASEBALL CARDS Cristiano bought 24 baseball cards for $50. One type cost $1 per card, and the other cost $3 per card. Define the variables, and write equations to find the number of each type of card he bought. Solve by using elimination.

EXAMPLE 3

Use elimination to solve the system of equations.

$3x - 5y = 11$
$x + 5y = -3$

$$\begin{array}{r} 3x - 5y = 11 \\ (+)\quad x + 5y = -3 \\ \hline 4x \qquad = 8 \\ x = 2 \end{array}$$

The variable y is eliminated.
Divide each side by 4.

Now, substitute 2 for x in either equation to find the value of y.

$3x - 5y = 11$	First equation
$3(2) - 5y = 11$	Substitute.
$6 - 5y = 11$	Multiply.
$-5y = 5$	Subtract 6 from each side.
$y = -1$	Divide each side by −5.

The solution is $(2, -1)$.

6-4 Elimination Using Multiplication (pp. 355–360)

Use elimination to solve each system of equations.

34. $x + y = 4$
$-2x + 3y = 7$

35. $x - y = -2$
$2x + 4y = 38$

36. $3x + 4y = 1$
$5x + 2y = 11$

37. $-9x + 3y = -3$
$3x - 2y = -4$

38. $8x - 3y = -35$
$3x + 4y = 33$

39. $2x + 9y = 3$
$5x + 4y = 26$

40. $-7x + 3y = 12$
$2x - 8y = -32$

41. $8x - 5y = 18$
$6x + 6y = -6$

42. BAKE SALE On the first day, a total of 40 items were sold for $356. Define the variables, and write a system of equations to find the number of cakes and pies sold. Solve by using elimination.

> **MONARCH MIDDLE SCHOOL**
>
> Bake Sale
>
> Pies $10
> Cakes $8

EXAMPLE 4

Use elimination to solve the system of equations.

$3x + 6y = 6$
$2x + 3y = 5$

Notice that if you multiply the second equation by −2, the coefficients of the y-terms are additive inverses.

$3x + 6y = 6$

$2x + 3y = 5$ **Multiply by −2.**

$$\begin{array}{r} 3x + 6y = 6 \\ (+) -4x - 6y = -10 \\ \hline -x \qquad = -4 \\ x = 4 \end{array}$$

Now, substitute 4 for x in either equation to find the value of y.

$2x + 3y = 5$	Second equation
$2(4) + 3y = 5$	Substitution
$8 + 3y = 5$	Multiply.
$3y = -3$	Subtract 8 from both sides.
$y = -1$	Divide each side by 3.

The solution is $(4, -1)$.

6-5 Applying Systems of Linear Equations (pp. 362–367)

Determine the best method to solve each system of equations. Then solve the system.

43. $y = x - 8$
$y = -3x$

44. $y = -x$
$y = 2x$

45. $x + 3y = 12$
$x = -6y$

46. $x + y = 10$
$x - y = 18$

47. $3x + 2y = -4$
$5x + 2y = -8$

48. $6x + 5y = 9$
$-2x + 4y = 14$

49. $3x + 4y = 26$
$2x + 3y = 19$

50. $11x - 6y = 3$
$5x - 8y = -25$

51. COINS Tionna has saved dimes and quarters in her piggy bank. Define the variables, and write a system of equations to determine the number of dimes and quarters. Then solve the system using the best method for the situation.

$4.00
25 coins

EXAMPLE 5

Determine the best method to solve the system of equations. Then solve the system.

$3x + 5y = 4$
$4x + y = -6$

Solve the second equation for y.

$4x + y = -6$	**Second equation**
$y = -6 - 4x$	**Subtract 4x from each side.**

Substitute $-6 - 4x$ for y in the first equation.

$3x + 5(-6 - 4x) = 4$	**Substitute.**
$3x - 30 - 20x = 4$	**Distributive Property**
$-17x - 30 = 4$	**Simplify.**
$-17x = 34$	**Add 30 to each side.**
$x = -2$	**Divide by −17.**

Last, substitute -2 for x in either equation to find y.

$4x + y = -6$	**Second equation**
$4(-2) + y = -6$	**Substitute.**
$-8 + y = -6$	**Multiply.**
$y = 2$	**Add 8 to each side.**

The solution is $(-2, 2)$.

6-6 Organizing Data Using Matrices (pp. 369–375)

Perform the indicated matrix operation.

52. $\begin{bmatrix} 5 & -6 \\ -9 & 4 \end{bmatrix} + \begin{bmatrix} 2 & -3 \\ -4 & 2 \end{bmatrix}$

53. $\begin{bmatrix} 5 & -8 & 4 \\ -6 & 2 & -9 \end{bmatrix} - \begin{bmatrix} 6 & -4 & -3 \\ 8 & 2 & 1 \end{bmatrix}$

54. $3\begin{bmatrix} 5 & -3 & -1 & 4 \\ 7 & -6 & 8 & 1 \end{bmatrix}$

55. POLLS The results of a poll are shown. How many votes were cast against both tax levies?

	For	Against
Tax Levy 1	68	105
Tax Levy 2	69	71

EXAMPLE 6

Find $\begin{bmatrix} 3 & 9 & -6 \\ 7 & 2 & 1 \end{bmatrix} + \begin{bmatrix} -2 & -1 & 6 \\ -6 & 4 & -5 \end{bmatrix}$.

To add the 2×3 matrices, add the corresponding elements.

$\begin{bmatrix} 3 & 9 & -6 \\ 7 & 2 & 1 \end{bmatrix} + \begin{bmatrix} -2 & -1 & 6 \\ -6 & 4 & -5 \end{bmatrix}$

$= \begin{bmatrix} 3 + (-2) & 9 + (-1) & -6 + 6 \\ 7 + (-6) & 2 + 4 & 1 + (-5) \end{bmatrix}$

$= \begin{bmatrix} 1 & 8 & 0 \\ 1 & 6 & -4 \end{bmatrix}$

6-7 Using Matrices to Solve Systems of Equations (pp. 376–381)

Use an augmented matrix to solve each system of equations.

56. $x - 2y = -5$
$2x + 3y = 4$

57. $x - y = 5$
$3x + 3y = 3$

58. $2x + 3y = 8$
$x - 5y = -22$

59. $2x + 4y = -16$
$x - 2y = 0$

60. $2x - 4y = 10$
$x + 2y = 5$

61. $x - 4y = 11$
$3x + 5y = -1$

62. $x - 2y = -26$
$2x - 3y = -42$

63. $x - 3y = 24$
$5x - 2y = 29$

64. $x - 2y = -16$
$3x + 4y = 72$

65. $x + 3y = 12$
$2x - 4y = 14$

66. ART Mateo spent a total of $36 on 10 art supply items. How many bottles of paint and paint brushes did Mateo buy?

Acrylic Paint $5
BLUE GREEN YELLOW
Paint Brushes $1.50

EXAMPLE 7

Use an augmented matrix to solve the system of equations.

$x + 3y = 11$
$3x + 4y = 18$

Step 1 Write the augmented matrix.

$$\begin{bmatrix} 1 & 3 & | & 11 \\ 3 & 4 & | & 18 \end{bmatrix}$$

Step 2 Multiply the first row by -3 and add the result to row 2.

$$\begin{bmatrix} 1 & 3 & | & 11 \\ 3 & 4 & | & 18 \end{bmatrix} \xrightarrow{-3R_1 + R_2} \begin{bmatrix} 1 & 3 & | & 11 \\ 0 & -5 & | & -15 \end{bmatrix}$$

Step 3 Multiply the second row by $-\frac{1}{5}$.

$$\begin{bmatrix} 1 & 3 & | & 11 \\ 0 & -5 & | & -15 \end{bmatrix} \xrightarrow{-\frac{1}{5}R_2} \begin{bmatrix} 1 & 3 & | & 11 \\ 0 & 1 & | & 3 \end{bmatrix}$$

Step 4 Multiply the second row by -3 and add the result to row 1.

$$\begin{bmatrix} 1 & 3 & | & 11 \\ 0 & 1 & | & 3 \end{bmatrix} \xrightarrow{-3R_2 + R_1} \begin{bmatrix} 1 & 0 & | & 2 \\ 0 & 1 & | & 3 \end{bmatrix}$$

The solution is $(2, 3)$.

6-8 Systems of Inequalities (pp. 382–387)

Solve each system of inequalities by graphing.

67. $x > 3$
$y < x + 2$

68. $y \leq 5$
$y > x - 4$

69. $y < 3x - 1$
$y \geq -2x + 4$

70. $y \leq -x - 3$
$y \geq 3x - 2$

71. JOBS Kishi makes $7 an hour working at the grocery store and $10 an hour delivering newspapers. She cannot work more than 20 hours per week. Graph two inequalities that Kishi can use to determine how many hours she needs to work at each job if she wants to earn at least $90 per week.
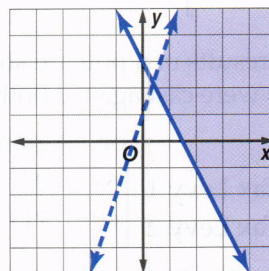
EXAMPLE 8

Solve the system of inequalities by graphing.

$y < 3x + 1$
$y \geq -2x + 3$

The solution set of the system is the set of ordered pairs in the intersection of the two graphs. This portion is shaded in the graph below.

Graph each system and determine the number of solutions that it has. If it has one solution, name it.

1. $y = 2x$
$y = 6 - x$

2. $y = x - 3$
$y = -2x + 9$

3. $x - y = 4$
$x + y = 10$

4. $2x + 3y = 4$
$2x + 3y = -1$

Use substitution to solve each system of equations.

5. $y = x + 8$
$2x + y = -10$

6. $x = -4y - 3$
$3x - 2y = 5$

7. GARDENING Corey has 42 feet of fencing around his garden. The garden is rectangular in shape, and its length is equal to twice the width minus 3 feet. Define the variables, and write a system of equations to find the length and width of the garden. Solve the system by using substitution.
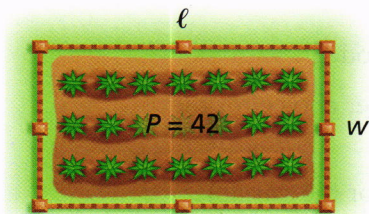
8. MULTIPLE CHOICE Use elimination to solve the system.

$$6x - 4y = 6$$
$$-6x + 3y = 0$$

A $(5, 6)$

B $(-3, -6)$

C $(1, 0)$

D $(4, -8)$

9. SHOPPING Shelly has $175 to shop for jeans and sweaters. Each pair of jeans costs $25, each sweater costs $20, and she buys 8 items. Determine the number of pairs of jeans and sweaters Shelly bought.

Use elimination to solve each system of equations.

10. $x + y = 13$
$x - y = 5$

11. $3x + 7y = 2$
$3x - 4y = 13$

12. $x + y = 8$
$x - 3y = -4$

13. $2x + 6y = 18$
$3x + 2y = 13$

14. MAGAZINES Julie subscribes to a sports magazine and a fashion magazine. She received 24 issues this year. The number of fashion issues is 6 less than twice the number of sports issues. Define the variables, and write a system of equations to find the number of issues of each magazine.

Perform the indicated matrix operations. If an operation cannot be performed, write *impossible*.

15. $\begin{bmatrix} 3 & -5 & 2 \end{bmatrix} + \begin{bmatrix} 12 & -7 & -6 \end{bmatrix}$

16. $\begin{bmatrix} 1 & 8 & -4 \\ -2 & 6 & 7 \\ 0 & 9 & -6 \end{bmatrix} - \begin{bmatrix} -4 & 2 & 0 \\ -5 & 3 & 8 \\ -9 & -7 & 1 \end{bmatrix}$

17. $-2 \begin{bmatrix} -2 & 7 \\ 6 & -4 \\ 8 & 5 \end{bmatrix}$

Use an augmented matrix to solve each system of equations.

18. $y = 3x$
$x + 2y = 21$

19. $x + y = 12$
$y = x - 4$

20. $x + y = 15$
$x - y = 9$

21. $3x + 5y = 7$
$2x - 3y = 11$

Solve each system of inequalities by graphing.

22. $x > 2$
$y < 4$

23. $x + y \leq 5$
$y \geq x + 2$

24. $3x - y > 9$
$y > -2x$

25. $y \geq 2x + 3$
$-4x - 3y > 12$

Guess and Check

It is very important to pace yourself and keep track of how much time you have when taking a standardized test. If time is running short, or if you are unsure how to solve a problem, the guess and check strategy may help you determine the correct answer quickly.
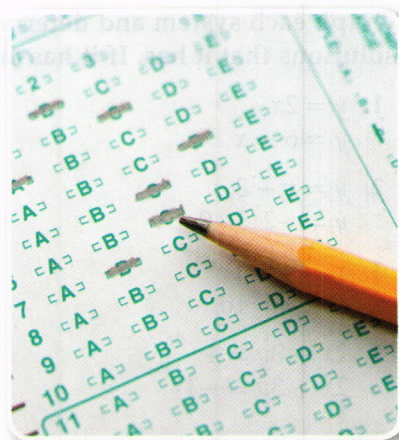
Strategies for Guessing and Checking

Step 1

Carefully look over each possible answer choice, and evaluate for reasonableness. Eliminate unreasonable answers.

Ask yourself:

- Are there any answer choices that are clearly incorrect?

- Are there any answer choices that are not in the proper format?

- Are there any answer choices that do not have the proper units for the correct answer?

Step 2

For the remaining answer choices, use the guess and check method.

- **Equations:** If you are solving an equation, substitute the answer choice for the variable and see if this results in a true number sentence.

- **Inequalities:** Likewise, you can substitute the answer choice for the variable and see if it satisfies the inequality.

- **System of Equations:** Find the answer choice that satisfies both equations of the system.

Step 3

Choose an answer choice and see if it satisfies the constraints of the problem statement. Identify the correct answer.

- If the answer choice you are testing does not satisfy the problem, move on to the next reasonable guess and check it.

- When you find the correct answer choice, stop. You do not have to check the other answer choices.

Read the problem. Identify what you need to know. Then use the information in the problem to solve.

Solve $\begin{cases} 4x - 8y = 20 \\ -3x + 5y = -14 \end{cases}$.

A $(5, 0)$ **C** $(3, -1)$

B $(4, -2)$ **D** $(-6, -5)$

The solution of a system of equations is an ordered pair, (x, y). Since all four answer choices are of this form, they are all possible correct answers and must be checked. Begin with the first answer choice and substitute it in each equation. Continue until you find the ordered pair that satisfies both equations of the system.

	First Equation	Second Equation
Guess: (5, 0)	$4x - 8y = 20$ $4(5) - 8(0) = 20$ ✓	$-3x + 5y = -14$ $-3(5) + 5(0) \neq -14$ ✗

	First Equation	Second Equation
Guess: (4, -2)	$4x - 8y = 20$ $4(4) - 8(-2) \neq 20$ ✗	$-3x + 5y = -14$ $-3(4) + 5(-2) \neq -14$ ✗

	First Equation	Second Equation
Guess: (3, -1)	$4x - 8y = 20$ $4(3) - 8(-1) = 20$ ✓	$-3x + 5y = -14$ $-3(3) + 5(-1) = -14$ ✓

The ordered pair $(3, -1)$ satisfies both equations of the system. So, the correct answer is C.

Exercises

Read each problem. Eliminate any unreasonable answers. Then use the information in the problem to solve.

1. Gina bought 5 hot dogs and 3 soft drinks at the ball game for $11.50. Renaldo bought 4 hot dogs and 2 soft drinks for $8.50. How much does a single hot dog and a single drink cost?

   A hot dogs: $1.25   **C** hot dogs: $1.50
   soft drinks: $1.50   soft drinks: $1.25

   B hot dogs: $1.25   **D** hot dogs: $1.50
   soft drinks: $1.75   soft drinks: $1.75

2. The bookstore hopes to sell at least 30 binders and calculators each week. The store also hopes to have sales revenue of at least $200 in binders and calculators. How many binders and calculators could be sold to meet both of these sales goals?

Store Prices	
Item	**Price**
binders	$3.65
calculators	$14.80

   F 25 binders,   **H** 22 binders,
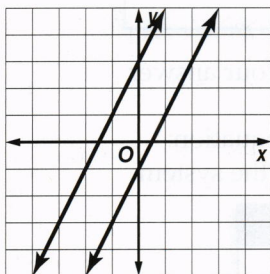   5 calculators   9 calculators

   G 12 binders,   **J** 28 binders,
   15 calculators   6 calculators

Multiple Choice

Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.

1. Which of the following terms *best* describes the system of equations shown in the graph?

A consistent

B consistent and dependent

C consistent and independent

D inconsistent

2. Use substitution to solve the system of equations below.

$$\begin{cases} y = 4x - 7 \\ 3x - 2y = -1 \end{cases}$$

F $(3, 5)$ H $(5, -2)$

G $(4, -1)$ J $(-6, 2)$

3. Which ordered pair is the solution of the system of linear equations shown below?

$$\begin{cases} 3x - 8y = -50 \\ 3x - 5y = -38 \end{cases}$$

A $\left(\dfrac{5}{8}, \dfrac{3}{2}\right)$ C $\left(-\dfrac{2}{7}, \dfrac{4}{9}\right)$

B $(4, -9)$ D $(-6, 4)$

4. A home goods store received $881 from the sale of 4 table saws and 9 electric drills. If the receipts from the saws exceeded the receipts from the drills by $71, what is the price of an electric drill?

F $45   H $108

G $59   J $119
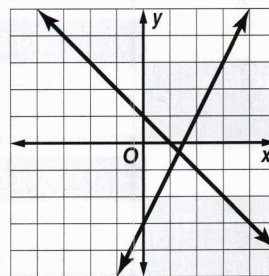
5. A region is defined by this system.

$$y > -\frac{1}{2}x - 1$$
$$y > -x + 3$$

In which quadrant(s) of the coordinate plane is the region located?

A I and IV only   C I, II, and IV only

B III only   D II and III only

6. Which of the following terms *best* describes the system of equations shown in the graph?

F consistent

G consistent and independent

H consistent and dependent

J inconsistent

7. Use elimination to solve the system of equations below.

$$3x + 2y = -2$$
$$2x - 2y = -18$$

A $(1, 3)$ C $(-2, -3)$

B $(7, -4)$ D $(-4, 5)$

8. What is the solution of the following system of equations?

$$\begin{cases} y = 6x - 1 \\ y = 6x + 4 \end{cases}$$

F $(2, 11)$ H $(7, 5)$

G $(-3, -14)$ J no solution

Test-TakingTip

Question 8 You can subtract the second equation from the first equation to eliminate the *x*-variable. Then solve for *y*.

Short Response/Gridded Response

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

9. **GRIDDED RESPONSE** Angie and her sister have $15 to spend on pizza. A medium pizza costs $11.50 plus $0.75 per topping. What is the maximum number of toppings Angie and her sister can get on their pizza?

10. Write an inequality for the graph below.
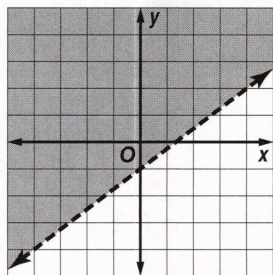
11. **GRIDDED RESPONSE** Christy is taking a road trip. After she drives 12 more miles, she will have driven at least half of the 108-mile trip. What is the least number of miles she has driven so far?

12 Write an equation in slope-intercept form with a slope of $-\frac{2}{3}$ and a y-intercept of 6.

13. A rental company charges $9.50 per hour for a scooter plus a $15 fee. Write an equation in slope-intercept form for the total rental cost C of renting a scooter for h hours.

14. **GRIDDED RESPONSE** A computer supplies store is having a storewide sale this weekend. An inkjet printer that normally sells for $179.00 is on sale for $143.20. What is the percent discount of the sale price?

15. In 1980, the population of Kentucky was about 3.66 million people. By 2000, this number had grown to about 4.04 million people. What was the annual rate of change in population from 1980 to 2000?

16. Joseph's cell phone service charges him $0.15 per each text message sent. Write an equation that represents the cost C of his cell phone service for text messages t sent each month.

17. A store is offering a $15 mail-in-rebate on all printers. If Mark is looking at printers that range from $45 to $89, how much can he expect to pay?

Extended Response

Record your answers on a sheet of paper. Show your work.

18. The table shows how many canned goods were collected during the first day of a charity food drive.

Food Drive Day 1 Results	
Class	**Number Collected**
10th graders	78
11th graders	80
12th graders	92

a. Estimate how many canned goods will be collected during the 5-day food drive. Explain your answer.

b. Is this estimate a reasonable expectation? Explain.

Need Extra Help?																		
If you missed Question...	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Go to Lesson or Page...	6-1	6-2	6-3	6-3	6-8	6-1	6-3	6-3	5-3	5-6	5-3	4-2	4-2	3-3	2-7	2-1	5-4	1-4

Polynomials

Then

In Chapter 1, you performed operations on expressions with exponents.

Now

In Chapter 7, you will:

- Simplify expressions involving monomials.
- Use scientific notation.
- Find degrees of polynomials, write polynomials in standard form, and add, subtract, and multiply polynomials.

Why?

🌐 **SPACE** The Very Large Array is an arrangement of 27 radio antennas in a Y pattern. The data the antennas collect is used by astronomers around the world to study the planets and stars. Astrophysicists use and apply properties of exponents to model the distance and orbit of celestial bodies.

Polynomials

Introduction

Space

The Very Large Array is an arrangement of 27 radio antennas in a Y pattern. The data the antennas collect is used by astronomers around the world to study the planets and stars. Astrophysicists use and apply properties of exponents to model the distance and orbit of celestial bodies.

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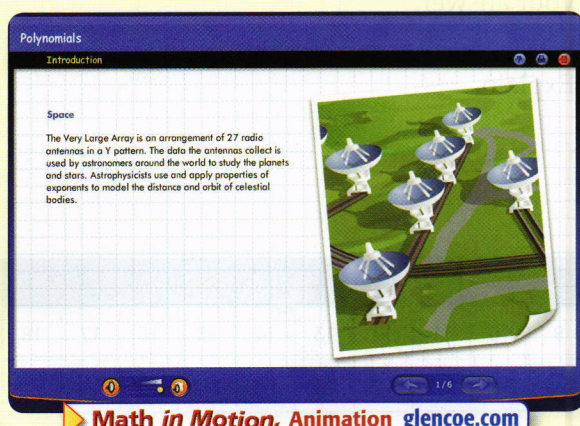
▶ **Math** *in Motion*, **Animation** glencoe.com