### **Important Concepts**

# **Addition and Subtraction of Decimals**

**DECIMALS AS FRACTIONS** Write decimals as fractions, find common denominators, add or subtract the fractions, and express the answers as decimals. This confirms that when adding or subtracting, one must compute with digits of the same place value.

PLACE-VALUE INTERPRETATION Students consider the place value of digits and what that means when adding or subtracting numbers.

#### **Multiplication of Decimals**

**DECIMALS AS FRACTIONS** Write decimals as fractions, multiply, write the answer as a decimal, and relate the number of decimal places in the factors to the answer.

PLACE-VALUE INTERPRETATION Students see why counting decimal points make sense and use the short-cut algorithm: multiply the decimals as whole numbers and adjust the place of the decimal in the product.

#### **Division of Decimals**

**DECIMALS AS FRACTIONS** Write decimals as fractions with common denominators and divide the numerators.

PLACE-VALUE INTERPRETATION Write an equivalent problem by multiplying both the dividend and the divisor by the same power of ten until both are whole numbers.

#### **Decimal Forms of Rational Numbers**

FINITE (OR TERMINATING) DECIMALS are decimals that "end." The simplified fraction has prime factors of only 2s or 5s in the denominator.

**FNFINITE REPEATING DECIMALS** are decimals that "go on forever" but show a repeating pattern. These fractions have prime factors other than 2 or 5 in the simplest denominator form.

#### **Using Percents**

PERCENT OF A PRICE "A CD costs \$7.50. The sales tax is 6%. How much is the tax?"

ON WHAT AMOUNT THE PERCENT WAS FIGURED "Customers left Jill \$2.50 as a tip. The tip was 20% of the total. How much was the bill?"

WHAT PERCENT ONE NUMBER IS OF ANOTHER NUMBER "Sam got a \$12 discount off a \$48 purchase. What percent discount did he get?"

## Examples

Zeke buys cider for \$1.97 and donuts for \$0.89. The clerk said the bill was \$10.87. What did the clerk do wrong?

The cider is \$1.97 =  $\frac{197}{100}$  and the donuts are \$0.89 =  $\frac{89}{100}$ 

So the cost is  $\frac{197}{100} + \frac{89}{100} = \frac{286}{100} = 2.86$ . In 1.97 + 0.89, we add hundredths to hundredths (1.97 + 0.89), tenths to tenths (1.97 + 0.89), and ones to ones (1.97 + 0.89).

The clerk incorrectly added dollars and pennies (ones and tenths, tenths and hundredths).

We can look at a problem using equivalent fractions.

$$0.3 \times 2.3 = \frac{3}{10} \times 2\frac{3}{10} = \frac{3}{10} \times \frac{23}{10}$$

The product as a fraction is  $\frac{69}{100}$ , as a decimal 0.69.

The 100 in the denominator shows that there should be two decimal places (hundredths) in the answer. The denominator of the fraction tells us the place value needed in the decimal.

Using the fact that  $25 \times 31 = 775$  students reason about a related product:  $2.5 \times 0.31$  (2.5 is a tenth of 25, 0.31 is a hundredth of 31, so the product is a thousandth of 775) = 0.775.

$$3.25 \div 0.5 = \frac{325}{100} \div \frac{5}{10} = \frac{325}{100} \div \frac{50}{100} = 325 \div 50 = 6.5$$

$$37.5 \div 0.015 = \frac{375}{10} \div \frac{15}{1,000} = \frac{37,500}{1,000} \div \frac{15}{1,000} = 37,500 \div 15 = 2,500$$

This makes a whole number problem with the same quotient as the original decimal problem.

The fraction approach explains why moving decimal places works.

$$0.015\overline{)37.5} = 0.015 \times 1,000\overline{)37.5 \times 1,000} = 15\overline{)37500}$$

# $\frac{1}{2} = 0.5, \ \frac{1}{8} = 0.125, \frac{12}{75} = 0.16. \frac{4}{25} = \frac{16}{100} = 0.16$

In simplified fraction form  $\frac{12}{75} = \frac{4}{25}$  has only factors of five  $\left(\frac{4}{5\times5}\right)$  in the denominator.

$$\frac{1}{3} = 0.333..., \frac{2}{3} = 0.666..., \frac{8}{15} = 0.533..., \frac{3}{7} = 0.42857142...$$

In simplified fraction form  $\frac{26}{150} = \frac{13}{75} = \frac{13}{3 \times 5 \times 5} = 0.1733333...$ 

 $6\% \text{ of } \$7.50 = \cos t \text{ of } \tan x$ 

1% of \$7.50 =  $\frac{1}{100}$  of \$7.50 = \$7.50 ÷ 100 = 0.075 6 of the 1%'s will give me 6%. So, 6% of \$7.50 = \$0.45.

20% of some number equals \$2.50

Find how many 20%s it takes to make 100%. In this case we need five. So,  $5 \times \$2.50$  gives us \$12.50.

Find what % 12 is of 48. Students can solve this by computing how many 12s in 48. It takes four, so the percent is  $\frac{1}{4}$  of 100% or 25%.

On the CMP Parent Web Site, you can learn more about the mathematical goals of each unit, see the glossary, and examine worked-out examples of ACE problems. http://PHSchool.com/cmp2parents