Get Ready for Chapter 10

Diagnose Readiness You have two options for checking Prerequisite Skills.

Text Option

Take the Quick Check below. Refer to the Quick Review for help.

QuickCheck

Find each square root. If necessary, round to the nearest hundredth. (Lesson 0-2)

1. $\sqrt{82}$ **2.** $\sqrt{26}$

3. $\sqrt{15}$ **4.** $\sqrt{99}$

5. **SANDBOX** Isaac is making a square sandbox with an area of 100 square feet. How long is a side of the sandbox?

Simplify each expression. (Lesson 1-4)

6. $(21x + 15y) - (9x - 4y)$

7. $13x - 5y + 2y$

8. $(10a - 5b) + (6a + 5b)$

9. $6m + 5n + 4 - 3m - 2n + 6$

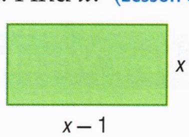
10. $x + y - 3x - 4y + 2x - 8y$

Solve each equation. (Lesson 8-4)

11. $2x^2 - 4x = 0$ **12.** $6x^2 - 5x - 4 = 0$

13. $x^2 - 7x + 10 = 0$ **14.** $2x^2 + 7x - 5 = -1$

15. **GEOMETRY** The area of the rectangle is 90 square feet. Find x. (Lesson 8-3)



Use cross products to determine whether each pair of ratios forms a proportion. Write *yes* or *no*. (Lesson 2-6)

16. $\frac{2}{3}$ and $\frac{4}{9}$ **17.** $\frac{3}{4}$ and $\frac{15}{20}$

18. **MAPS** On a map, 1 inch = 10 miles. If the distance between cities is 50 miles, how many inches will it be on the map? (Lesson 2-6)

QuickReview

EXAMPLE 1

Find the square root of $\sqrt{50}$. If necessary, round to the nearest hundredth.

$\sqrt{50} = 7.071067812\ldots$ **Use a calculator.**

To the nearest hundredth, $\sqrt{50} = 7.07$.

EXAMPLE 2

Simplify $3x + 7y - 4x - 8y$.

$3x + 7y - 4x - 8y$

$= (3x - 4x) + (7y - 8y)$ **Combine like terms.**

$= -x - y$ **Simplify.**

EXAMPLE 3

Solve $x^2 - 5x + 6 = 0$.

$x^2 - 5x + 6 = 0$ **Original equation**

$(x - 3)(x - 2) = 0$ **Factor.**

$x - 3 = 0$ or $x - 2 = 0$ **Zero Product Property**

$x = 3$ $\quad x = 2$ **Solve each equation.**

EXAMPLE 4

Use cross products to determine whether $\frac{2}{3}$ and $\frac{8}{12}$ form a proportion.

$\frac{2}{3} \stackrel{?}{=} \frac{8}{12}$ **Write the equation.**

$2(12) \stackrel{?}{=} 3(8)$ **Find the cross products.**

$24 = 24$ ✔ **Simplify.**

They form a proportion.

Online Option

Math Online Take a self-check Chapter Readiness Quiz at glencoe.com.

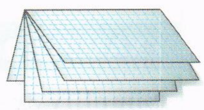
Get Started on Chapter 10

You will learn several new concepts, skills, and vocabulary terms as you study Chapter 10. To get ready, identify important terms and organize your resources. You may wish to refer to **Chapter 0** to review prerequisite skills.

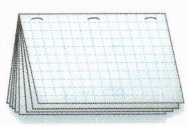
FOLDABLES® Study Organizer

Radical Functions and Geometry Make this Foldable to help you organize your Chapter 10 notes about radical functions and geometry. Begin with four sheets of grid paper.

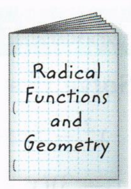
1. **Fold** in half along the width.



2. **Staple** along the fold.



3. **Turn** the fold to the left and write the title of the chapter on the front. On each left-hand page of the booklet, write the title of a lesson from the chapter.



New Vocabulary

English		Español
radicand	• p. 605 •	radicando
radical function	• p. 605 •	función radicales
conjugate	• p. 614 •	conjugado
radical equations	• p. 624 •	ecuaciones radicales
hypotenuse	• p. 630 •	hipotenusa
legs	• p. 630 •	catetos
converse	• p. 631 •	recíproco
midpoint	• p. 638 •	punto medio
similar triangles	• p. 642 •	semejantes
cosine	• p. 649 •	coseno
tangent	• p. 649 •	tangente
trigonometry	• p. 649 •	trigonometría
inverse cosine	• p. 651 •	coseno inverso
inverse sine	• p. 651 •	seno inverso
inverse tangent	• p. 651 •	tangente inverse

Review Vocabulary

FOIL method • p. 448 • metodo FOIL to multiply two binomials, find the sum of the products of the First terms, Outer terms, Inner terms, and Last terms

perfect square • p. P7 • cuadrado perfecto a number with a square root that is a rational number

proportion • p. 111 • proporcion an equation of the form $\frac{a}{b} = \frac{c}{d}$, $b \neq 0$, $d \neq 0$ stating that two ratios are equivalent

$$\frac{a}{b} \times \frac{c}{d}$$
$$ad = bc$$

Multilingual eGlossary glencoe.com

Math Online glencoe.com

- Study the chapter online
- Explore **Math in Motion**
- Get extra help from your own **Personal Tutor**
- Use **Extra Examples** for additional help
- Take a **Self-Check Quiz**
- **Review Vocabulary** in fun ways

10-1 Square Root Functions

Then

You solved quadratic equations by using the Quadratic Formula. (Lesson 9-5)

Now

- Graph and analyze dilations of radical functions.
- Graph and analyze reflections and translations of radical functions.

New Vocabulary

square root function
radical function
radicand

Math Online

glencoe.com

- Extra Examples
- Personal Tutor
- Self-Check Quiz
- Homework Help

Why?

Scientists use sounds of whales to track their movements. The distance to a whale can be found by relating time to the speed of sound in water.

The speed of sound in water can be described by the *square root function* $c = \sqrt{\frac{E}{d}}$, where E represents the bulk modulus elasticity of the water and d represents the density of the water.

Dilations of Radical Functions A **square root function** contains the square root of a variable. Square root functions are a type of **radical function**. The expression under the radical sign is called the **radicand**. For a square root to be a real number, the radicand cannot be negative. Values that make the radicand negative are not included in the domain.

Key Concept — Square Root Function

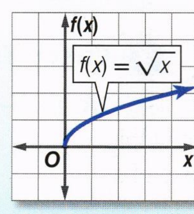
For Your FOLDABLE

Parent function: $f(x) = \sqrt{x}$

Type of graph: curve

Domain: $\{x \mid x \geq 0\}$

Range: $\{y \mid y \geq 0\}$



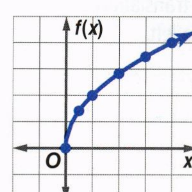
EXAMPLE 1 Dilation of the Square Root Function

Graph $f(x) = 2\sqrt{x}$. State the domain and range.

Step 1 Make a table.

x	0	0.5	1	2	3	4
f(x)	0	≈1.4	2	≈2.8	≈3.5	4

Step 2 Plot points. Draw a smooth curve.



The domain is $\{x \mid x \geq 0\}$, and the range is $\{y \mid y \geq 0\}$.

Check Your Progress

1A. $g(x) = 4\sqrt{x}$

1B. $h(x) = 6\sqrt{x}$

Personal Tutor glencoe.com

Reflections and Translations of Radical Functions Recall that when the value of a is negative in the quadratic function $f(x) = ax^2$, the graph of the parent function is reflected across the x-axis.

StudyTip

Graphing Radical Functions Choose perfect squares for x-values that will result in coordinates that are easy to plot.

Key Concept Graphing $y = a\sqrt{x + h} + c$

For Your FOLDABLE

Step 1 Draw the graph of $y = a\sqrt{x}$. The graph starts at the origin and passes through $(1, a)$. If $a > 0$, the graph is in quadrant I. If $a < 0$, the graph is reflected across the x-axis and is in quadrant IV.

Step 2 Translate the graph $|c|$ units up if $c > 0$ and down if $c < 0$.

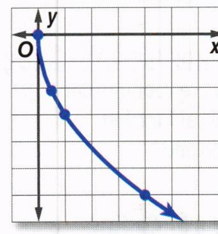
Step 3 Translate the graph $|h|$ units left if $h > 0$ and right if $h < 0$.

EXAMPLE 2 Reflection of the Square Root Function

Graph $y = -3\sqrt{x}$. Compare to the parent graph. State the domain and range.

Make a table of values. Then plot the points on a coordinate system and draw a smooth curve that connects them.

x	0	0.5	1	4
y	0	≈−2.1	−3	−6



Notice that the graph is in the 4th quadrant. It is obtained by stretching the graph of $y = \sqrt{x}$ vertically and then reflecting across the x-axis. The domain is $\{x \mid x \geq 0\}$, and the range is $\{y \mid y \leq 0\}$.

Check Your Progress

2A. $y = -2\sqrt{x}$

2B. $y = -4\sqrt{x}$

Personal Tutor glencoe.com

StudyTip

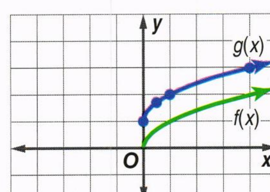
Translating Radical Functions If $c > 0$, a radical function $f(x) = \sqrt{x - c}$ is a horizontal translation c units to the right. $f(x) = \sqrt{x + c}$ is a horizontal translation c units to the left.

EXAMPLE 3 Translation of the Square Root Function

Graph each function. Compare to the parent graph. State the domain and range.

a. $g(x) = \sqrt{x} + 1$

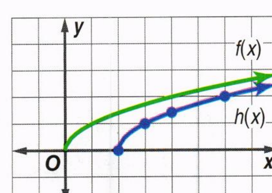
x	0	0.5	1	4	9
y	0	≈1.7	2	3	4



Notice that the values of $g(x)$ are 1 greater than those of $f(x) = \sqrt{x}$. This is a vertical translation 1 unit up from the parent function. The domain is $\{x \mid x \geq 0\}$, and the range is $\{y \mid y \geq 1\}$.

b. $h(x) = \sqrt{x - 2}$

x	2	3	4	6
y	0	1	≈1.4	2



This is a horizontal translation 2 units to the right of the parent function. The domain is $\{x \mid x \geq 2\}$, and the range is $\{y \mid y \geq 0\}$.

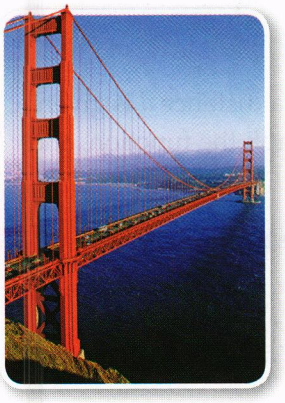
Check Your Progress

3A. $g(x) = \sqrt{x} - 4$

3B. $h(x) = \sqrt{x + 3}$

Personal Tutor glencoe.com

Physical phenomena such as motion can be modeled by radical functions. Often these functions are transformations of the parent square root function.



Real-World Link

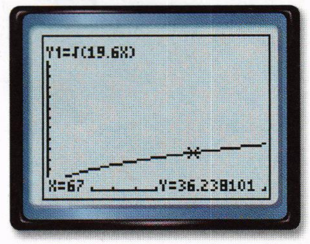
Approximately 39 million cars cross the Golden Gate Bridge in San Francisco each year.

Source: San Francisco Convention and Visitors Bureau

Real-World EXAMPLE 4 Analyze a Radical Function

BRIDGES The Golden Gate Bridge is about 67 meters above the water. The velocity v of a freely falling object that has fallen h meters is given by $v = \sqrt{2gh}$, where g is the constant 9.8 meters per second squared. Graph the function. If an object is dropped from the bridge, what is its velocity when it hits the water?

Use a graphing calculator to graph the function.



To find the velocity of the object, substitute 67 meters for h.

$v = \sqrt{2gh}$	**Original function**
$= \sqrt{2(9.8)(67)}$	**$g = 9.8$ and $h = 67$**
$= \sqrt{1313.2}$	**Simplify.**
≈ 36.2 m/s	**Use a calculator.**

The velocity of the object is about 36.2 meters per second after dropping 67 meters.

Check Your Progress

4. Use the graph above to estimate the initial height of an object if it is moving at 20 meters per second when it hits the water.

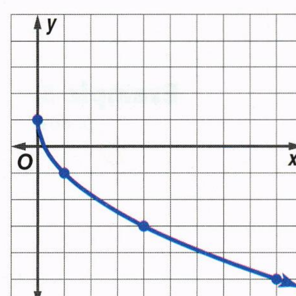
Personal Tutor glencoe.com

Transformations such as reflections, translations, and dilations can be combined in one equation.

EXAMPLE 5 Transformations of the Square Root Function

Graph $y = -2\sqrt{x} + 1$, and compare to the parent graph. State the domain and range.

x	0	1	4	9
y	1	−1	−3	−5



This graph is the result of a vertical stretch of the graph of $y = \sqrt{x}$ followed by a reflection across the x-axis, and then a translation 1 unit up. The domain is $\{x \mid x \geq 0\}$, and the range is $\{y \mid y \leq 1\}$.

Check Your Progress

5A. $y = \frac{1}{2}\sqrt{x} - 1$

5B. $y = -2\sqrt{x - 1}$

Personal Tutor glencoe.com

Check Your Understanding

Examples 1 and 3 pp. 605–606

Graph each function. Compare to the parent graph. State the domain and range.

1. $y = 3\sqrt{x}$
2. $y = -5\sqrt{x}$
3. $y = \frac{1}{3}\sqrt{x}$
4. $y = -\frac{1}{2}\sqrt{x}$
5. $y = \sqrt{x} + 3$
6. $y = \sqrt{x} - 2$
7. $y = \sqrt{x + 2}$
8. $y = \sqrt{x - 3}$

Example 4 p. 607

9. **FREE FALL** The time in seconds that it takes an object to fall a distance d is given by the function $t = \frac{1}{4}\sqrt{d}$ (assuming zero air resistance). Graph the function, and state the domain and range.

Example 5 p. 607

Graph each function, and compare to the parent graph. State the domain and range.

10. $y = \frac{1}{2}\sqrt{x} + 2$
11. $y = -\frac{1}{4}\sqrt{x} - 1$
12. $y = -2\sqrt{x + 1}$
13. $y = 3\sqrt{x - 2}$

Practice and Problem Solving

● = **Step-by-Step Solutions** begin on page R12.
Extra Practice begins on page 815.

Examples 1 and 3 pp. 605–606

Graph each function. Compare to the parent graph. State the domain and range.

14. $y = 5\sqrt{x}$
15. $y = \frac{1}{2}\sqrt{x}$
16. $y = -\frac{1}{3}\sqrt{x}$
17. $y = 7\sqrt{x}$
18. $y = -\frac{1}{4}\sqrt{x}$
19. $y = -\sqrt{x}$
20. $y = -\frac{1}{5}\sqrt{x}$
21. $y = -7\sqrt{x}$
22. $y = \sqrt{x} + 2$
23. $y = \sqrt{x} + 4$
24. $y = \sqrt{x} - 1$
25. $y = \sqrt{x} - 3$
26. $y = \sqrt{x} + 1.5$
27. $y = \sqrt{x} - 2.5$
28. $y = \sqrt{x + 4}$
29. $y = \sqrt{x - 4}$
30. $y = \sqrt{x + 1}$
31. $y = \sqrt{x - 0.5}$
32. $y = \sqrt{x + 5}$
33. $y = \sqrt{x - 1.5}$

Example 4 p. 607

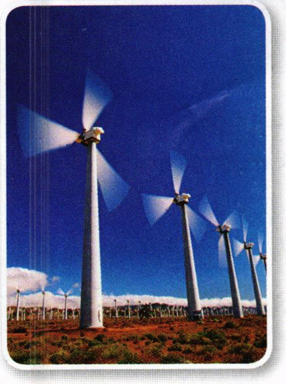
34. **GEOMETRY** The perimeter of a square is given by the function $P = 4\sqrt{A}$, where A is the area of the square.
    a. Graph the function.
    b. Determine the perimeter of a square with an area of 225 m^2.
    c. When will the perimeter and the area be the same value?

Example 5 p. 607

Graph each function, and compare to the parent graph. State the domain and range.

35. $y = -2\sqrt{x} + 2$
36. $y = -3\sqrt{x} - 3$
37. $y = \frac{1}{2}\sqrt{x + 2}$
38. $y = -\sqrt{x - 1}$
39. $y = \frac{1}{4}\sqrt{x - 1} + 2$
40. $y = \frac{1}{2}\sqrt{x - 2} + 1$

41. **ENERGY** An object has kinetic energy when it is in motion. The velocity in meters per second of an object of mass m kilograms with an energy of E joules is given by the function $v = \sqrt{\frac{2E}{m}}$. Use a graphing calculator to graph the function that represents the velocity of a basketball with a mass of 0.6 kilogram.

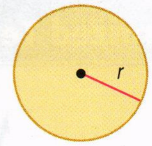


Real-World Link

Wind farms harness the kinetic energy of the wind and convert it to usable power. A turbine that is designed to power a house can have a rotor with a diameter of 50 feet.

Source: American Wind Energy Association

42. GEOMETRY The radius of a circle is given by $r = \sqrt{\frac{A}{\pi}}$, where A is the area of the circle.



a. Graph the function.

b. Use a graphing calculator to determine the radius of a circle that has an area of 27 in^2.

43. SPEED OF SOUND The speed of sound in air is determined by the temperature of the air. The speed c in meters per second is given by $c = 331.5\sqrt{1 + \frac{t}{273.15}}$, where t is the temperature of the air in degrees Celsius.

a. Use a graphing calculator to graph the function.

b. How fast does sound travel when the temperature is 55°C?

c. How is the speed of sound affected when the temperature increases by 10°?

44. MULTIPLE REPRESENTATIONS In this problem, you will explore the relationship between the graphs of square root functions and parabolas.

a. GRAPHICAL Graph $y = x^2$ on a coordinate system.

b. ALGEBRAIC Write a piecewise-defined function to describe the graph of $y^2 = x$ in each quadrant.

c. GRAPHICAL On the same coordinate system, graph $y = \sqrt{x}$ and $y = -\sqrt{x}$.

d. GRAPHICAL On the same coordinate system, graph $y = x$. Plot the points (2, 4), (4, 2), and (1, 1).

e. ANALYTICAL Compare the graph of the parabola to the graphs of the square root functions.

H.O.T. Problems

Use Higher-Order Thinking Skills

CHALLENGE Determine whether each statement is *true* or *false*. Provide an example or counterexample to support your answer.

45. Numbers in the domain of a radical function will always be nonnegative.

46. Numbers in the range of a radical function will always be nonnegative.

47. REASONING Write a radical function that translates $y = \sqrt{x}$ four units to the right. Graph the function.

48. CHALLENGE Write a radical function with a domain of all real numbers greater than or equal to 2 and a range of all real numbers less than or equal to 5.

49. WHICH DOES NOT BELONG? Identify the equation that does not belong. Explain.

$y = 3\sqrt{x}$	$y = 0.7\sqrt{x}$	$y = \sqrt{x} + 3$	$y = \frac{\sqrt{x}}{6}$

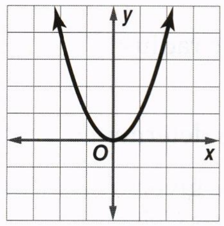
50. OPEN ENDED Write a function that is a reflection, translation, and a dilation of the parent graph $y = \sqrt{x}$.

51. REASONING If the range of the function $y = a\sqrt{x}$ is $\{y \mid y \leq 0\}$, what can you conclude about the value of a? Explain your reasoning.

52. WRITING IN MATH Compare and contrast the graphs of $f(x) = \sqrt{x} + 2$ and $g(x) = \sqrt{x + 2}$.

Standardized Test Practice

53.



Which function *best* represents the graph?

A $y = x^2$ C $y = \sqrt{x}$

B $y = 2^x$ D $y = x$

54. The statement "$x < 10$ and $3x - 2 \geq 7$" is true when x is equal to what?

F 0 H 8

G 2 J 12

55. Which of the following is the equation of a line parallel to $y = -\frac{1}{2}x + 3$ and passing through $(-2, -1)$?

A $y = \frac{1}{2}x$ C $y = -\frac{1}{2}x + 2$

B $y = 2x + 3$ D $y = -\frac{1}{2}x - 2$

56. SHORT RESPONSE A landscaper needs to mulch 6 rectangular flower beds that are 8 feet by 4 feet and 4 circular flower beds each with a radius of 3 feet. One bag of mulch covers 25 square feet. How many bags of mulch are needed to cover the flower beds?

Spiral Review

Graph each set of ordered pairs. Determine whether the ordered pairs represent a *linear* function, a *quadratic* function, or an *exponential* function. (Lesson 9-9)

57. $\{(-2, 5), (-1, 3), (0, 1), (1, -1), (2, -3)\}$

58. $\{(0, 0), (1, 3), (2, 4), (3, 3), (4, 0)\}$

59. $\left\{\left(-2, \frac{1}{4}\right), (0, 1), (1, 2), (2, 4), (3, 8)\right\}$

60. $\{(-4, 4), (-2, 1), (0, 0), (2, 1), (4, 4)\}$

Find the next three terms in each geometric sequence. (Lesson 9-8)

61. $5, 20, 80, 320, \ldots$

62. $-4, 2, -1, \frac{1}{2}, \ldots$

63. $\frac{1}{8}, \frac{1}{4}, \frac{1}{2}, 1, \ldots$

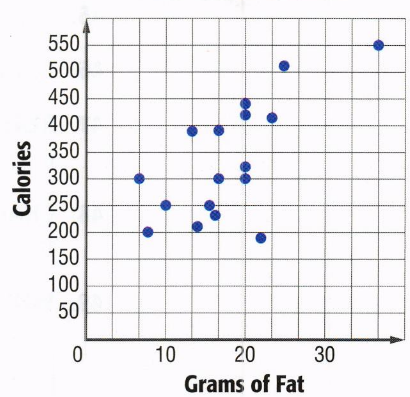
64. HEALTH Aida exercises every day by walking and jogging at least 3 miles. Aida walks at a rate of 4 miles per hour and jogs at a rate of 8 miles per hour. Suppose she has exactly one half-hour to exercise today. (Lesson 6-8)

a. Draw a graph showing the possible amounts of time she can spend walking and jogging.

b. List three possible solutions.

65. NUTRITION Determine whether the graph shows a *positive*, a *negative*, or *no* correlation. If there is a positive or negative correlation, describe its meaning in the situation. (Lesson 4-5)

Fast-Food Choices



Skills Review

Factor each monomial completely. (Lesson 8-1)

66. $28n^3$

67. $-33a^2b$

68. $150rt$

69. $-378nq^2r^2$

70. $225a^3b^2c$

71. $-160x^2y^4$

EXTEND
10-1 Graphing Technology Lab
Graphing Square Root Functions

Math Online glencoe.com
- Other Calculator Keystrokes
- Graphing Technology Personal Tutor

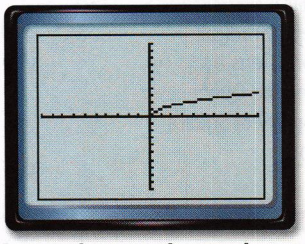
For a square root to be a real number, the radicand cannot be negative. When graphing a radical function, determine when the radicand would be negative and exclude those values from the domain.

ACTIVITY 1 Parent Function

Graph $y = \sqrt{x}$.

Enter the equation in the Y= list.

KEYSTROKES: Y= 2nd [√] X,T,θ,n) GRAPH



[−10, 10] scl: 1 by [−10, 10] scl: 1

1A. Examine the graph. What is the domain of the function?

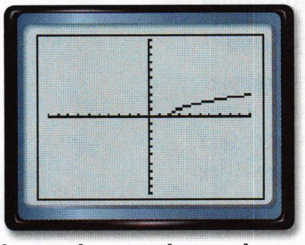
1B. What is the range of the function?

ACTIVITY 2 Translation of Parent Function

Graph $y = \sqrt{x - 2}$.

Enter the equation in the Y= list.

KEYSTROKES: Y= 2nd [√] X,T,θ,n − 2) GRAPH



[−10, 10] scl: 1 by [−10, 10] scl: 1

2A. What are the domain and range of the function?

2B. How does the graph of $y = \sqrt{x - 2}$ compare to the graph of the parent function $y = \sqrt{x}$?

Exercises

Graph each equation, and sketch the graph on your paper. State the domain and range. Describe how the graph differs from that of the parent function $y = \sqrt{x}$.

1. $y = \sqrt{x - 1}$
2. $y = \sqrt{x + 3}$
3. $y = \sqrt{x} - 2$
4. $y = \sqrt{-x}$
5. $y = -\sqrt{x}$
6. $y = \sqrt{2x}$
7. $y = \sqrt{2 - x}$
8. $y = \sqrt{x - 3} + 2$

9. Does $x = y^2$ represent a function? Explain your reasoning.

10. Does $x^2 + y^2 = 4$ determine y as a function of x? Explain.

11. Does $x^2 + y^2 = 2$ determine y as a function of x? Explain.

Write a function with a graph that translates $y = \sqrt{x}$ in each way.

12. Shifted 4 units to the left

13. Shifted up 7 units

14. Shifted down 6 units

15. Shifted 5 units to the right and up 3 units

10-2 Simplifying Radical Expressions

Then

You simplified radicals. (Lesson 0-2)

Now

- Simplify radical expressions by using the Product Property of Square Roots.
- Simplify radical expressions by using the Quotient Property of Square Roots.

New Vocabulary

radical expression
rationalizing the denominator
conjugate

Math Online

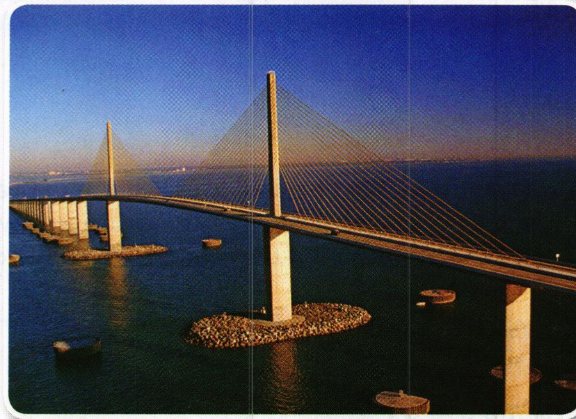
glencoe.com

- Extra Examples
- Personal Tutor
- Self-Check Quiz
- Homework Help

Why?

The Sunshine Skyway Bridge across Tampa Bay in Florida, is supported by 21 steel cables, each 9 inches in diameter.

To find the diameter a steel cable should have to support a given weight, you can use the equation $d = \sqrt{\frac{w}{8}}$, where d is the diameter of the cable in inches and w is the weight in tons.



Product Property of Square Roots A **radical expression** contains a radical, such as a square root. Recall the expression under the radical sign is called the radicand. A radicand is in simplest form if the following three conditions are true.

- No radicands have perfect square factors other than 1.
- No radicands contain fractions.
- No radicals appear in the denominator of a fraction.

The following property can be used to simplify square roots.

Key Concept — Product Property of Square Roots

For Your FOLDABLE

Words For any nonnegative real numbers a and b, the square root of ab is equal to the square root of a times the square root of b.

Symbols $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$, if $a \geq 0$ and $b \geq 0$

Examples $\sqrt{4 \cdot 9} = \sqrt{36}$ or 6 $\qquad \sqrt{4 \cdot 9} = \sqrt{4} \cdot \sqrt{9} = 2 \cdot 3$ or 6

EXAMPLE 1 Simplify Square Roots

Simplify $\sqrt{80}$.

$\sqrt{80} = \sqrt{2 \cdot 2 \cdot 2 \cdot 2 \cdot 5}$ **Prime factorization of 80**

$= \sqrt{2^2} \cdot \sqrt{2^2} \cdot \sqrt{5}$ **Product Property of Square Roots**

$= 2 \cdot 2 \cdot \sqrt{5}$ or $4\sqrt{5}$ **Simplify.**

Check Your Progress

1A. $\sqrt{54}$ $\qquad$ **1B.** $\sqrt{180}$

Personal Tutor glencoe.com

EXAMPLE 2 Multiply Square Roots

Simplify $\sqrt{2} \cdot \sqrt{14}$.

$$\sqrt{2} \cdot \sqrt{14} = \sqrt{2} \cdot \sqrt{2} \cdot \sqrt{7} \quad \text{Product Property of Square Roots}$$

$$= \sqrt{2^2} \cdot \sqrt{7} \text{ or } 2\sqrt{7} \quad \text{Product Property of Square Roots}$$

Check Your Progress

2A. $\sqrt{5} \cdot \sqrt{10}$

2B. $\sqrt{6} \cdot \sqrt{8}$

Personal Tutor glencoe.com

Consider the expression $\sqrt{x^2}$. It may seem that $x = \sqrt{x^2}$, but when finding the principal square root of an expression containing variables, you have to be sure that the result is not negative. Consider $x = -3$.

$$\sqrt{x^2} \stackrel{?}{=} x$$

$$\sqrt{(-3)^2} \stackrel{?}{=} -3 \quad \text{Replace } x \text{ with } -3.$$

$$\sqrt{9} \stackrel{?}{=} -3 \quad (-3)^2 = 9$$

$$3 \neq -3 \quad \sqrt{9} = 3$$

Notice in this case, if the right hand side of the equation were $|x|$, the equation would be true. For expressions where the exponent of the variable inside a radical is even and the simplified exponent is odd, you must use absolute value.

$$\sqrt{x^2} = |x| \qquad \sqrt{x^3} = x\sqrt{x} \qquad \sqrt{x^4} = x^2 \qquad \sqrt{x^6} = |x^3|$$

EXAMPLE 3 Simplify a Square Root with Variables

Simplify $\sqrt{90x^3y^4z^5}$.

$$\sqrt{90x^3y^4z^5} = \sqrt{2 \cdot 3^2 \cdot 5 \cdot x^3 \cdot y^4 \cdot z^5} \quad \text{Prime factorization}$$

$$= \sqrt{2} \cdot \sqrt{3^2} \cdot \sqrt{5} \cdot \sqrt{x^2} \cdot \sqrt{x} \cdot \sqrt{y^4} \cdot \sqrt{z^4} \cdot \sqrt{z} \quad \text{Product Property}$$

$$= \sqrt{2} \cdot 3 \cdot \sqrt{5} \cdot x \cdot \sqrt{x} \cdot y^2 \cdot z^2 \cdot \sqrt{z} \quad \text{Simplify.}$$

$$= 3y^2z^2\, x\sqrt{10xz} \quad \text{Simplify.}$$

Check Your Progress

3A. $\sqrt{32r^2k^4t^5}$

3B. $\sqrt{56xy^{10}z^5}$

Personal Tutor glencoe.com

Quotient Property of Square Roots To divide square roots and simplify radical expressions, you can use the Quotient Property of Square Roots.

ReadingMath

Fractions in the Radicand The expression $\sqrt{\frac{a}{b}}$ is read *the square root of a over b, or the square root of the quantity of a over b.*

Key Concept — Quotient Property of Square Roots

For Your FOLDABLE

Words For any real numbers a and b, where $a \geq 0$ and $b > 0$, the square root of $\frac{a}{b}$ is equal to the square root of a divided by the square root of b.

Symbols $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

You can use the properties of square roots to **rationalize the denominator** of a fraction with a radical. This involves multiplying the numerator and denominator by a factor that eliminates radicals in the denominator.

Test-Taking Tip

Simplify Look at the radicand to see if it can be simplified first. This may make your computations simpler.

STANDARDIZED TEST EXAMPLE 4

Which expression is equivalent to $\sqrt{\frac{35}{15}}$?

A $\frac{5\sqrt{21}}{15}$ B $\frac{\sqrt{21}}{3}$ C $\frac{\sqrt{525}}{15}$ D $\frac{\sqrt{35}}{15}$

Read the Test Item

The radical expression needs to be simplified.

Solve the Test Item

$$\sqrt{\frac{35}{15}} = \sqrt{\frac{7}{3}} \quad \text{Reduce } \tfrac{35}{15} \text{ to } \tfrac{7}{3}$$

$$= \frac{\sqrt{7}}{\sqrt{3}} \quad \text{Quotient Property of Square Roots}$$

$$= \frac{\sqrt{7}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} \quad \text{Multiply by } \tfrac{\sqrt{3}}{\sqrt{3}}.$$

$$= \frac{\sqrt{21}}{3} \quad \text{Product Property of Square Roots}$$

The correct choice is B.

Check Your Progress

4. Simplify $\frac{\sqrt{6y}}{\sqrt{12}}$.

Personal Tutor glencoe.com

Binomials of the form $a\sqrt{b} + c\sqrt{d}$ and $a\sqrt{b} - c\sqrt{d}$, where a, b, c, and d are rational numbers, are called **conjugates**. For example, $2 + \sqrt{7}$ and $2 - \sqrt{7}$ are conjugates. The product of two conjugates is a rational number and can be found using the pattern for the difference of squares.

EXAMPLE 5 Use Conjugates to Rationalize a Denominator

Simplify $\frac{3}{5 + \sqrt{2}}$.

$$\frac{3}{5+\sqrt{2}} = \frac{3}{5+\sqrt{2}} \cdot \frac{5-\sqrt{2}}{5-\sqrt{2}} \quad \text{The conjugate of } 5+\sqrt{2} \text{ is } 5-\sqrt{2}.$$

$$= \frac{3(5-\sqrt{2})}{5^2 - (\sqrt{2})^2} \quad (a-b)(a+b) = a^2 - b^2$$

$$= \frac{15 - 3\sqrt{2}}{25 - 2} \text{ or } \frac{15 - 3\sqrt{2}}{23} \quad (\sqrt{2})^2 = 2$$

Check Your Progress

Simplify each expression.

5A. $\frac{3}{2+\sqrt{2}}$ **5B.** $\frac{7}{3-\sqrt{7}}$

Personal Tutor glencoe.com

Check Your Understanding

Examples 1–3
pp. 612–613

Simplify each expression.

1. $\sqrt{24}$
2. $3\sqrt{16}$
3. $2\sqrt{25}$
4. $\sqrt{10} \cdot \sqrt{14}$
5. $\sqrt{3} \cdot \sqrt{18}$
6. $3\sqrt{10} \cdot 4\sqrt{10}$
7. $\sqrt{60x^4y^7}$
8. $\sqrt{88m^3p^2r^5}$
9. $\sqrt{99ab^5c^2}$

Example 4
p. 614

10. **MULTIPLE CHOICE** Which expression is equivalent to $\sqrt{\frac{45}{10}}$?

A $\frac{5\sqrt{2}}{10}$ B $\frac{\sqrt{45}}{10}$ C $\frac{\sqrt{50}}{10}$ D $\frac{3\sqrt{2}}{2}$

Example 5
p. 614

Simplify each expression.

11. $\frac{3}{3+\sqrt{5}}$
12. $\frac{5}{2-\sqrt{6}}$
13. $\frac{2}{1-\sqrt{10}}$
14. $\frac{1}{4+\sqrt{12}}$
15. $\frac{4}{6-\sqrt{7}}$
16. $\frac{6}{5+\sqrt{11}}$

Practice and Problem Solving

= **Step-by-Step Solutions** begin on page R12.
Extra Practice begins on page 815.

Examples 1 and 3
pp. 612–613

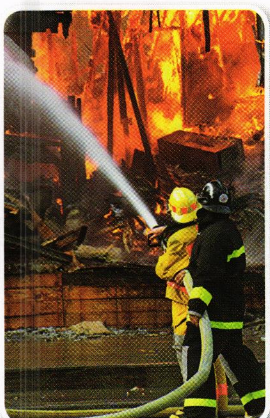
Simplify each expression.

17. $\sqrt{52}$
18. $\sqrt{56}$
19. $\sqrt{72}$
20. $3\sqrt{18}$
21. $\sqrt{243}$
22. $\sqrt{245}$
23. $\sqrt{5} \cdot \sqrt{10}$
24. $\sqrt{10} \cdot \sqrt{20}$
25. $3\sqrt{8} \cdot 2\sqrt{7}$
26. $4\sqrt{2} \cdot 5\sqrt{8}$
27. $3\sqrt{25t^2}$
28. $5\sqrt{81q^5}$
29. $\sqrt{28a^2b^3}$
30. $\sqrt{75qr^3}$
31. $7\sqrt{63m^3p}$
32. $4\sqrt{66g^2h^4}$
33. $\sqrt{2ab^2} \cdot \sqrt{10a^5b}$
34. $\sqrt{4c^3d^3} \cdot \sqrt{8c^3d}$

35. **ROLLER COASTER** The velocity v of a roller coaster in feet per second at the bottom of a hill can be approximated by $v = \sqrt{64h}$, where h is the height of the hill in feet.

a. Simplify the equation.

b. Determine the velocity of a roller coaster at the bottom of a 134-foot hill.



Real-World Link

In 1736, Benjamin Franklin founded the first volunteer fire organization, the Union Fire Company, in Philadelphia.

Source: *Firehouse Magazine*

36. **FIREFIGHTING** When fighting a fire, the velocity v of water being pumped into the air is modeled by the function $v = \sqrt{2hg}$, where h represents the maximum height of the water and g represents the acceleration due to gravity (32 ft/s^2).

a. Solve the function for h.

b. The Hollowville Fire Department needs a pump that will propel water 80 feet into the air. Will a pump advertised to project water with a velocity of 70 feet per second meet their needs? Explain.

c. The Jackson Fire Department must purchase a pump that will propel water 90 feet into the air. Will a pump that is advertised to project water with a velocity of 77 feet per second meet the fire department's need? Explain.

Examples 4 and 5
p. 614

Simplify each expression.

37. $\sqrt{\frac{32}{t^4}}$

38. $\sqrt{\frac{27}{m^5}}$

39. $\frac{\sqrt{68ac^3}}{\sqrt{27a^2}}$

40. $\frac{\sqrt{h^3}}{\sqrt{8}}$

41. $\sqrt{\frac{3}{16}} \cdot \sqrt{\frac{9}{5}}$

42. $\sqrt{\frac{7}{2}} \cdot \sqrt{\frac{5}{3}}$

43. $\frac{7}{5 + \sqrt{3}}$

44. $\frac{9}{6 - \sqrt{8}}$

45. $\frac{3\sqrt{3}}{-2 + \sqrt{6}}$

46. $\frac{3}{\sqrt{7} - \sqrt{2}}$

47. $\frac{5}{\sqrt{6} + \sqrt{3}}$

48. $\frac{2\sqrt{5}}{2\sqrt{7} + 3\sqrt{3}}$

Real-World Link

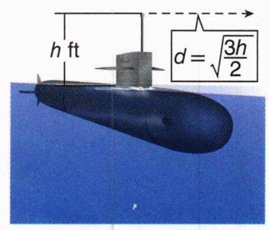
The first hand-held hair dryer was sold in 1925 and dried hair with 100 watts of heat. Modern hair dryers may have 2000 watts.

Source: *Enotes Encyclopedia*

49. **ELECTRICITY** The amount of current in amperes I that an appliance uses can be calculated using the formula $I = \sqrt{\frac{P}{R}}$, where P is the power in watts and R is the resistance in ohms.
   a. Simplify the formula.
   b. How much current does an appliance use if the power used is 75 watts and the resistance is 5 ohms?

50. **KINETIC ENERGY** The speed v of a ball can be determined by the equation $v = \sqrt{\frac{2k}{m}}$, where k is the kinetic energy and m is the mass of the ball.
   a. Simplify the formula if the mass of the ball is 3 kilograms.
   b. If the ball is traveling 7 meters per second, what is the kinetic energy of the ball in Joules?

51. **SUBMARINES** The greatest distance d in miles that a lookout can see on a clear day is modeled by the formula shown. Determine how high the submarine would have to raise its periscope to see a ship, if the submarine is the given distances away from the ship.



Distance	3	6	9	12	15
Height					

H.O.T. Problems
Use Higher-Order Thinking Skills

52. **REASONING** Explain how to solve $(3x - 2)^2 = (2x + 6)^2$.

53. **CHALLENGE** Solve $|y^3| = \frac{1}{3\sqrt{3}}$ for y.

54. **REASONING** Marge takes a number, subtracts 4, multiplies by 4, takes the square root, and takes the reciprocal to get $\frac{1}{2}$. What number did she start with? Write a formula to describe the process.

55. **OPEN ENDED** Write two binomials of the form $a\sqrt{b} + c\sqrt{f}$ and $a\sqrt{b} - c\sqrt{f}$. Then find their product.

56. **CHALLENGE** Use the Quotient Property of Square Roots to derive the Quadratic Formula by solving the quadratic equation $ax^2 + bx + c = 0$. (*Hint*: Begin by completing the square.)

57. **WRITING IN MATH** Summarize how to write a radical expression in simplest form.

Standardized Test Practice

58. Jerry's electric bill is \$23 less than his natural gas bill. The two bills are a total of \$109. Which of the following equations can be used to find the amount of his natural gas bill?

A $g + g = 109$
B $23 + 2g = 109$
C $g - 23 = 109$
D $2g - 23 = 109$

59. Solve $a^2 - 2a + 1 = 25$.

F $-4, -6$
G $4, -6$
H $-4, 6$
J $4, 6$

60. The expression $\sqrt{160x^2y^5}$ is equivalent to which of the following?

A $16|x|y^2\sqrt{10y}$
B $|x|y^2\sqrt{160y}$
C $4|x|y^2\sqrt{10y}$
D $10|x|y^2\sqrt{4y}$

61. **GRIDDED RESPONSE** Miki earns \$10 an hour and 10% commission on sales. If Miki worked 38 hours and had a total sales of \$1275 last week, how much did she make?

Spiral Review

Graph each function. Compare to the parent graph. State the domain and range. (Lesson 10-1)

62. $y = 2\sqrt{x} - 1$
63. $y = \frac{1}{2}\sqrt{x}$
64. $y = 2\sqrt{x+2}$
65. $y = -\sqrt{x+1}$
66. $y = -3\sqrt{x-3}$
67. $y = -2\sqrt{x} + 1$

Look for a pattern in each table of values to determine which kind of model best describes the data. (Lesson 9-9)

68.

x	0	1	2	3	4
y	1	3	9	27	81

69.

x	−3	−2	−1	0	1
y	18	8	2	0	2

70.

x	1	2	3	4	5
y	1	3	5	7	9

71. **POPULATION** The country of Latvia has been experiencing a 1.1% annual decrease in population. In 2005, its population was 2,290,237. If the trend continues, predict Latvia's population in 2015. (Lesson 9-7)

Solve each equation by using the Quadratic Formula. Round to the nearest tenth if necessary. (Lesson 9-5)

72. $x^2 - 25 = 0$
73. $r^2 + 25 = 0$
74. $4w^2 + 100 = 40w$
75. $2r^2 + r - 14 = 0$
76. $5v^2 - 7v = 1$
77. $11z^2 - z = 3$

Factor each polynomial, if possible. If the polynomial cannot be factored, write *prime*. (Lesson 8-5)

78. $n^2 - 81$
79. $4 - 9a^2$
80. $2x^5 - 98x^3$
81. $32x^4 - 2y^4$
82. $4t^2 - 27$
83. $x^3 - 3x^2 - 9x + 27$

84. **GARDENING** Cleveland is planting 120 jalapeno pepper plants in a rectangular arrangement in his garden. In what ways can he arrange them so that he has at least 4 rows of plants, the same number of plants in each row, and at least 6 plants in each row? (Lesson 8-1)

Skills Review

Write the prime factorization of each number. (Concepts and Skills Bank Lesson 6)

85. 24
86. 88
87. 180
88. 31
89. 60
90. 90

EXTEND 10-2

Graphing Technology Lab
Rational Exponents

Math Online glencoe.com
- Other Calculator Keystrokes
- Graphing Technology Personal Tutor

You have studied the properties of exponents that are whole numbers. Some exponents are rational numbers or fractions. You can use a calculator to explore the meaning of rational exponents.

ACTIVITY Rational Exponents

Step 1 Evaluate $16^{\frac{1}{2}}$ and $\sqrt{16}$.

KEYSTROKES: 16 [^] [(] 1 [÷] 2 [)] [ENTER]

KEYSTROKES: [2nd] [√] 16 [ENTER]

Record the results in a table like the one at the right.

Step 2 Use a calculator to evaluate each expression. Record each result in your table. To find a root other than a square root, choose the $\sqrt[x]{\ }$ function from the [MATH] menu.

Expression	Value	Expression	Value
$16^{\frac{1}{2}}$	4	$\sqrt{16}$	4
$25^{\frac{1}{2}}$		$\sqrt{25}$	
$64^{\frac{1}{3}}$		$\sqrt[3]{64}$	
$125^{\frac{1}{3}}$		$\sqrt[3]{125}$	
$64^{\frac{2}{3}}$		$\sqrt[3]{64^2}$	
$81^{\frac{3}{4}}$		$\sqrt[4]{81^3}$	

1A. Study the table. What do you observe about the value of an expression of the form $a^{\frac{1}{n}}$?

1B. What do you observe about the value of an expression of the form $a^{\frac{m}{n}}$?

Exercises

1. Recall the Power of a Power Property. For any number a and all integers m and n, $(a^m)^n = (a^{m \cdot n})$. Assume that fractional exponents behave as whole number exponents and find the value of $\left(b^{\frac{1}{2}}\right)^2$.

$$\left(b^{\frac{1}{2}}\right)^2 = b^{\frac{1}{2} \cdot 2} \quad \textbf{Power of a Power Property}$$

$$= b^1 \text{ or } b \quad \textbf{Simplify.}$$

Thus, $b^{\frac{1}{2}}$ is a number whose square equals b. So it makes sense to define $b^{\frac{1}{2}} = \sqrt{b}$. Use a similar process to define $b^{\frac{1}{n}}$.

2. Define $b^{\frac{m}{n}}$. Justify your answer.

Write each root as an expression using a fractional exponent. Then evaluate the expression.

3. $\sqrt{36}$
4. $\sqrt{121}$
5. $\sqrt[4]{256}$
6. $\sqrt[5]{32}$
7. $\sqrt[3]{8^2}$
8. $\sqrt[4]{1296}$
9. $\sqrt[4]{16^3}$
10. $\sqrt[3]{8^3}$

10-3 Operations with Radical Expressions

Then

You simplified radical expressions. (Lesson 10-2)

Now

- Add and subtract radical expressions.
- Multiply radical expressions.

Math Online

glencoe.com

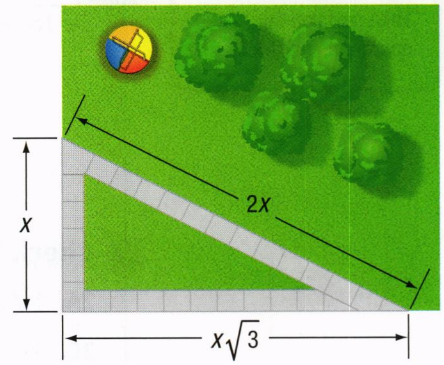
- Extra Examples
- Personal Tutor
- Self-Check Quiz
- Homework Help

Why?

Conchita is going to run in her neighborhood to get ready for the soccer season. She plans to run the course that she has laid out three times each day.

How far does Conchita have to run to complete the course that she laid out?

How far does she run every day?



Add or Subtract Radical Expressions To add or subtract radical expressions, the radicands must be alike in the same way that monomial terms must be alike to add or subtract.

Monomials	Radical Expressions
$4a + 2a = (4 + 2)a$	$4\sqrt{5} + 2\sqrt{5} = (4 + 2)\sqrt{5}$
$= 6a$	$= 6\sqrt{5}$
$9b - 2b = (9 - 2)b$	$9\sqrt{3} - 2\sqrt{3} = (9 - 2)\sqrt{3}$
$= 7b$	$= 7\sqrt{3}$

Notice that when adding and subtracting radical expressions, the radicand does not change. This is the same as when adding or subtracting monomials.

EXAMPLE 1 Add and Subtract Expressions with Like Radicands

Simplify each expression.

a. $\mathbf{5\sqrt{2} + 7\sqrt{2} - 6\sqrt{2}}$

$5\sqrt{2} + 7\sqrt{2} - 6\sqrt{2} = (5 + 7 - 6)\sqrt{2}$ **Distributive Property**

$= 6\sqrt{2}$ **Simplify.**

b. $\mathbf{10\sqrt{7} + 5\sqrt{11} + 4\sqrt{7} - 6\sqrt{11}}$

$10\sqrt{7} + 5\sqrt{11} + 4\sqrt{7} - 6\sqrt{11} = (10 + 4)\sqrt{7} + (5 - 6)\sqrt{11}$ **Distributive Property**

$= 14\sqrt{7} - \sqrt{11}$ **Simplify.**

Check Your Progress

1A. $3\sqrt{2} - 5\sqrt{2} + 4\sqrt{2}$

1B. $6\sqrt{11} + 2\sqrt{11} - 9\sqrt{11}$

1C. $15\sqrt{3} - 14\sqrt{5} + 6\sqrt{5} - 11\sqrt{3}$

1D. $4\sqrt{3} + 3\sqrt{7} - 6\sqrt{3} + 3\sqrt{7}$

Personal Tutor glencoe.com

Not all radical expressions have like radicands. Simplifying the expressions may make it possible to have like radicands so that they can be added or subtracted.

StudyTip

Simplify First Simplify each radical term first. Then perform the operations needed.

EXAMPLE 2 Add and Subtract Expressions with Unlike Radicands

Simplify $2\sqrt{18} + 2\sqrt{32} + \sqrt{72}$.

$2\sqrt{18} + 2\sqrt{32} + \sqrt{72} = 2(\sqrt{3^2} \cdot \sqrt{2}) + 2(\sqrt{4^2} \cdot \sqrt{2}) + (\sqrt{6^2} \cdot \sqrt{2})$	Product Property
$= 2(3\sqrt{2}) + 2(4\sqrt{2}) + (6\sqrt{2})$	Simplify.
$= 6\sqrt{2} + 8\sqrt{2} + 6\sqrt{2}$	Multiply.
$= 20\sqrt{2}$	Simplify.

Check Your Progress

2A. $4\sqrt{54} + 2\sqrt{24}$

2B. $4\sqrt{12} - 6\sqrt{48}$

2C. $3\sqrt{45} + \sqrt{20} - \sqrt{245}$

2D. $\sqrt{24} - \sqrt{54} + \sqrt{96}$

Personal Tutor glencoe.com

Multiply Radical Expressions Multiplying radical expressions is similar to multiplying monomial algebraic expressions. Let $x \geq 0$.

Monomials	Radical Expressions
$(2x)(3x) = 2 \cdot 3 \cdot x \cdot x$	$(2\sqrt{x})(3\sqrt{x}) = 2 \cdot 3 \cdot \sqrt{x} \cdot \sqrt{x}$
$= 6x^2$	$= 6x$

You can also apply the Distributive Property to radical expressions.

Watch Out!

Multiplying Radicands Make sure that you multiply the radicands when multiplying radical expressions. A common mistake is to add the radicands rather than multiply.

EXAMPLE 3 Multiply Radical Expressions

Simplify each expression.

a. $3\sqrt{2} \cdot 2\sqrt{6}$

$3\sqrt{2} \cdot 2\sqrt{6} = (3 \cdot 2)(\sqrt{2} \cdot \sqrt{6})$	Associative Property
$= 6(\sqrt{12})$	Multiply.
$= 6(2\sqrt{3})$	Simplify.
$= 12\sqrt{3}$	Multiply.

b. $3\sqrt{5}(2\sqrt{5} + 5\sqrt{3})$

$3\sqrt{5}(2\sqrt{5} + 5\sqrt{3}) = (3\sqrt{5} \cdot 2\sqrt{5}) + (3\sqrt{5} \cdot 5\sqrt{3})$	Distributive Property
$= [(3 \cdot 2)(\sqrt{5} \cdot \sqrt{5})] + [(3 \cdot 5)(\sqrt{5} \cdot \sqrt{3})]$	Associative Property
$= [6(\sqrt{25})] + [15(\sqrt{15})]$	Multiply.
$= [6(5)] + [15(\sqrt{15})]$	Simplify.
$= 30 + 15\sqrt{15}$	Multiply.

Check Your Progress

3A. $2\sqrt{6} \cdot 7\sqrt{3}$

3B. $9\sqrt{5} \cdot 11\sqrt{15}$

3C. $3\sqrt{2}(4\sqrt{3} + 6\sqrt{2})$

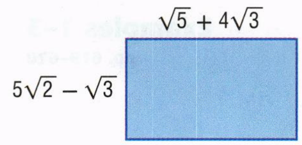
3D. $5\sqrt{3}(3\sqrt{2} - \sqrt{3})$

Personal Tutor glencoe.com

You can also multiply radical expressions with more than one term in each factor. This is similar to multiplying two algebraic binomials with variables.

Real-World EXAMPLE 4 Multiply Radical Expressions

GEOMETRY Find the area of the rectangle in simplest form.



$A = (5\sqrt{2} - \sqrt{3})(\sqrt{5} + 4\sqrt{3})$ $A = \ell \cdot w$

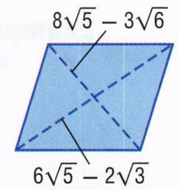
$$= \underbrace{(5\sqrt{2})(\sqrt{5})}_{\text{First Terms}} + \underbrace{(5\sqrt{2})(4\sqrt{3})}_{\text{Outer Terms}} + \underbrace{(-\sqrt{3})(\sqrt{5})}_{\text{Inner Terms}} + \underbrace{(-\sqrt{3})(4\sqrt{3})}_{\text{Last Terms}}$$

$= 5\sqrt{10} + 20\sqrt{6} - \sqrt{15} - 4\sqrt{9}$ **Multiply.**

$= 5\sqrt{10} + 20\sqrt{6} - \sqrt{15} - 12$ **Simplify.**

Check Your Progress

4. **GEOMETRY** The area A of a rhombus can be found using the equation $A = \frac{1}{2}d_1d_2$, where d_1 and d_2 are the lengths of the diagonals. What is the area of the rhombus at the right?



Personal Tutor glencoe.com

Review Vocabulary

FOIL Method Multiply two binomials by finding the sum of the products of the First terms, the Outer terms, the Inner terms, and the Last terms. (Lesson 7-7)

Concept Summary

For Your FOLDABLE

Operations with Radical Expressions

Operation	Symbols	Example
addition, $b \geq 0$	$a\sqrt{b} + c\sqrt{b} = (a + c)\sqrt{b}$ like radicands	$4\sqrt{3} + 6\sqrt{3} = (4 + 6)\sqrt{3} = 10\sqrt{3}$
subtraction, $b \geq 0$	$a\sqrt{b} - c\sqrt{b} = (a - c)\sqrt{b}$ like radicands	$12\sqrt{5} - 8\sqrt{5} = (12 - 8)\sqrt{5} = 4\sqrt{5}$
multiplication, $b \geq 0, g \geq 0$	$a\sqrt{b}(f\sqrt{g}) = af\sqrt{bg}$ Radicands do not have to be like radicands.	$3\sqrt{2}(5\sqrt{7}) = (3 \cdot 5)(\sqrt{2 \cdot 7}) = 15\sqrt{14}$

Check Your Understanding

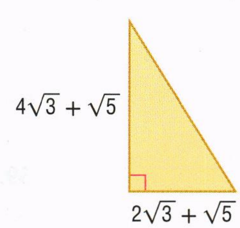
Examples 1–3 pp. 619–620

Simplify each expression.

1. $3\sqrt{5} + 6\sqrt{5}$
2. $8\sqrt{3} + 5\sqrt{3}$
3. $\sqrt{7} - 6\sqrt{7}$
4. $10\sqrt{2} - 6\sqrt{2}$
5. $4\sqrt{5} + 2\sqrt{20}$
6. $\sqrt{12} - \sqrt{3}$
7. $\sqrt{8} + \sqrt{12} + \sqrt{18}$
8. $\sqrt{27} + 2\sqrt{3} - \sqrt{12}$
9. $9\sqrt{2}(4\sqrt{6})$
10. $4\sqrt{3}(8\sqrt{3})$
11. $\sqrt{3}(\sqrt{7} + 3\sqrt{2})$
12. $\sqrt{5}(\sqrt{2} + 4\sqrt{2})$

Example 4 p. 621

13. **GEOMETRY** The area A of a triangle can be found by using the formula $A = \frac{1}{2}bh$, where b represents the base and h is the height. What is the area of the triangle at the right?



Practice and Problem Solving

= **Step-by-Step Solutions** begin on page R12. **Extra Practice** begins on page 815.

Examples 1–3 pp. 619–620

Simplify each expression.

14. $7\sqrt{5} + 4\sqrt{5}$

15. $2\sqrt{6} + 9\sqrt{6}$

16. $3\sqrt{5} - 2\sqrt{20}$

17. $3\sqrt{50} - 3\sqrt{32}$

18. $7\sqrt{3} - 2\sqrt{2} + 3\sqrt{2} + 5\sqrt{3}$

19. $\sqrt{5}(\sqrt{2} + 4\sqrt{2})$

20. $\sqrt{6}(2\sqrt{10} + 3\sqrt{2})$

21. $4\sqrt{5}(3\sqrt{5} + 8\sqrt{2})$

22. $5\sqrt{3}(6\sqrt{10} - 6\sqrt{3})$

23. $(\sqrt{3} - \sqrt{2})(\sqrt{15} + \sqrt{12})$

24. $(3\sqrt{11} + 3\sqrt{15})(3\sqrt{3} - 2\sqrt{2})$

25. $(5\sqrt{2} + 3\sqrt{5})(2\sqrt{10} - 5)$

Example 4 p. 621

26. GEOMETRY Find the perimeter and area of a rectangle with a width of $2\sqrt{7} - 2\sqrt{5}$ and a length of $3\sqrt{7} + 3\sqrt{5}$.

Simplify each expression.

27. $\sqrt{\frac{1}{5}} - \sqrt{5}$

28. $\sqrt{\frac{2}{3}} + \sqrt{6}$

29. $2\sqrt{\frac{1}{2}} + 2\sqrt{2} - \sqrt{8}$

30. $8\sqrt{\frac{5}{4}} + 3\sqrt{20} - 10\sqrt{\frac{1}{5}}$

31. $(3 - \sqrt{5})^2$

32. $(\sqrt{2} + \sqrt{3})^2$

33. ROLLER COASTERS The velocity v in feet per second of a roller coaster at the bottom of a hill is related to the vertical drop h in feet and the velocity v_0 of the coaster at the top of the hill by the formula $v_0 = \sqrt{v^2 - 64h}$.

a. What velocity must a coaster have at the top of a 225-foot hill to achieve a velocity of 120 feet per second at the bottom?

b. Explain why $v_0 = v - 8\sqrt{h}$ is not equivalent to the formula given.

Real-World Link

The Top Thrill Dragster roller coaster at Ohio's Cedar Point is powered by a 10,000 horsepower motor—equivalent to about 10 Formula One racecars.

Source: *National Geographic*

34. FINANCIAL LITERACY Tadi invests \$225 in a savings account. In two years, Tadi has \$270 in his account. You can use the formula $r = \sqrt{\frac{v_2}{v_0}} - 1$ to find the average annual interest rate r that the account has earned. The initial investment is v_0, and v_2 is the amount in two years. What was the average annual interest rate that Tadi's account earned?

35. ELECTRICITY Electricians can calculate the electrical current in amps A by using the formula $A = \frac{\sqrt{w}}{\sqrt{r}}$, where w is the power in watts and r the resistance in ohms. How much electrical current is running through a microwave oven that has 850 watts of power and 5 ohms of resistance? Write the number of amps in simplest radical form, and then estimate the amount of current to the nearest tenth.

H.O.T. Problems

Use Higher-Order Thinking Skills

36. CHALLENGE Determine whether the following statement is *true* or *false*. Provide an example or counterexample to support your answer.

$$x + y > \sqrt{x^2 + y^2} \text{ when } x > 0 \text{ and } y > 0$$

37. REASONING Let $a, b, c, d,$ and f be rational numbers. Show that if you multiply $a\sqrt{b} + c\sqrt{f}$ and $a\sqrt{b} - c\sqrt{f}$, the product has no radicals. Explain why this occurs.

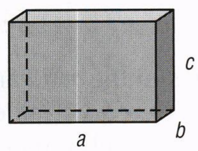
38. OPEN ENDED Write an equation that shows a sum of two radicals with different radicands. Explain how you could combine these terms.

39. WRITING IN MATH Describe step by step how to multiply two radical expressions, each with two terms. Write an example to demonstrate your description.

Standardized Test Practice

40. SHORT RESPONSE The population of a town is 13,000 and is increasing by about 250 people per year. This can be represented by the equation $p = 13{,}000 + 250y$, where y is the number of years from now and p represents the population. In how many years will the population of the town be 14,500?

41. GEOMETRY Which expression represents the sum of the lengths of the 12 edges on this rectangular solid?



A $2(a + b + c)$
B $3(a + b + c)$
C $4(a + b + c)$
D $12(a + b + c)$

42. Which of the following is equivalent to $8(3 - y) + 5(3 - y)$?

F $39 - y$
G $13(3 - y)$
H $40(30 - y)$
J $13(6 - 2y)$

43. The current I in a simple electrical circuit is given by the formula $I = \frac{V}{R}$, where V is the voltage and R is the resistance of the circuit. If the voltage remains unchanged, what effect will doubling the resistance of the circuit have on the current?

A The current will remain the same.
B The current will double its previous value.
C The current will be half its previous value.
D The current will be two units more than its previous value.

Spiral Review

Simplify. (Lesson 10-2)

44. $\sqrt{18}$ **45.** $\sqrt{24}$ **46.** $\sqrt{60}$

47. $\sqrt{50a^3b^5}$ **48.** $\sqrt{169x^4y^7}$ **49.** $\sqrt{63c^3d^4f^5}$

Graph each function. Compare to the parent graph. State the domain and range. (Lesson 10-1)

50. $y = 2\sqrt{x}$ **51.** $y = -3\sqrt{x}$ **52.** $y = \sqrt{x + 1}$

53. $y = \sqrt{x - 4}$ **54.** $y = \sqrt{x} + 3$ **55.** $y = \sqrt{x} - 2$

56. FINANCIAL LITERACY Determine the value of an investment if \$400 is invested at an interest rate of 7.25% compounded quarterly for 7 years. (Lesson 9-7)

Factor each trinomial. (Lesson 8-2)

57. $x^2 + 12x + 27$ **58.** $y^2 + 13y + 30$ **59.** $p^2 - 17p + 72$

60. $x^2 + 6x - 7$ **61.** $y^2 - y - 42$ **62.** $-72 + 6w + w^2$

95
+36
131
K79 -30
101
-24
77

Skills Review

Solve each equation. Round each solution to the nearest tenth, if necessary. (Lesson 2-3)

63. $-4c - 1.2 = 0.8$ **64.** $-2.6q - 33.7 = 84.1$ **65.** $0.3m + 4 = 9.6$

66. $-10 - \frac{n}{5} = 6$ **67.** $\frac{-4h - (-5)}{-7} = 13$ **68.** $3.6t + 6 - 2.5t = 8$

10-4 Radical Equations

Then

You added, subtracted, and multiplied radical expressions. (Lesson 10-3)

Now

- Solve radical equations.
- Solve radical equations with extraneous solutions.

New Vocabulary

radical equations
extraneous solutions

Math Online

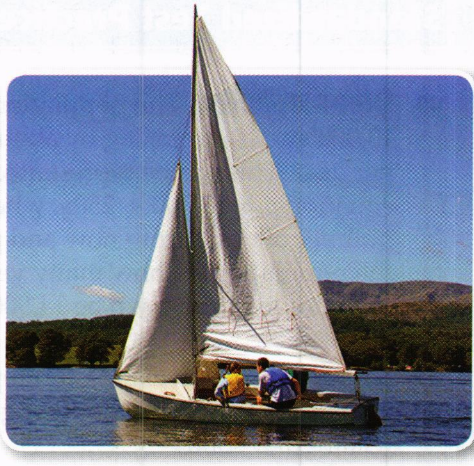
glencoe.com

- Extra Examples
- Personal Tutor
- Self-Check Quiz
- Homework Help

Why?

The waterline length of a sailboat is the length of the line made by the water's edge when the boat is full. A sailboat's hull speed is the fastest speed that it can travel.

You can estimate hull speed h by using the formula $h = 1.34\sqrt{\ell}$, where ℓ is the length of the sailboat's waterline.



Radical Equations Equations that contain variables in the radicand, like $h = 1.34\sqrt{\ell}$, are called **radical equations**. To solve, isolate the desired variable on one side of the equation first. Then square each side of the equation to eliminate the radical.

Key Concept — Power Property of Equality

For Your FOLDABLE

Words If you square both sides of a true equation, the resulting equation is still true.

Symbols If $a = b$, then $a^2 = b^2$.

Example If $\sqrt{x} = 4$, then $(\sqrt{x})^2 = 4^2$.

Real-World EXAMPLE 1 Variable as a Radicand

SAILING Idris and Sebastian are sailing in a friend's sailboat. They measure the hull speed at 9 nautical miles per hour. Find the length of the sailboat's waterline. Round to the nearest foot.

Understand You know how fast the boat will travel and that it relates to the length.

Plan The boat travels at 9 nautical miles per hour. The formula for hull speed is $h = 1.34\sqrt{\ell}$.

Solve

$h = 1.34\sqrt{\ell}$	Formula for hull speed
$9 = 1.34\sqrt{\ell}$	Substitute 9 for h.
$\frac{9}{1.34} = \frac{1.34\sqrt{\ell}}{1.34}$	Divide each side by 1.34.
$6.72 \approx \sqrt{\ell}$	Simplify.
$(6.72)^2 \approx (\sqrt{\ell})^2$	Square each side of the equation.
$45.16 \approx \ell$	Simplify.

The sailboat's waterline length is about 45 feet.

Check Check by substituting the estimate into the original formula.

$h = 1.34\sqrt{\ell}$	Formula for hull speed
$9 \stackrel{?}{=} 1.34\sqrt{45}$	$h = 9$ and $\ell = 45$
$9 \approx 8.98899327$ ✓	Multiply.

Check Your Progress

1. **DRIVING** The equation $v = \sqrt{2.5r}$ represents the maximum velocity that a car can travel safely on an unbanked curve when v is the maximum velocity in miles per hour and r is the radius of the turn in feet. If a road is designed for a maximum speed of 65 miles per hour, what is the radius of the turn?

Personal Tutor glencoe.com

To solve a radical equation, isolate the radical first. Then square both sides of the equation.

Watch Out!

Squaring Each Side Remember that when you square each side of the equation, you must square the entire side of the equation, even if there is more than one term on the side.

EXAMPLE 2 Expression as a Radicand

Solve $\sqrt{a+5} + 7 = 12$.

$\sqrt{a+5} + 7 = 12$	Original equation
$\sqrt{a+5} = 5$	Subtract 7 from each side.
$\left(\sqrt{a+5}\right)^2 = 5^2$	Square each side.
$a + 5 = 25$	Simplify.
$a = 20$	Subtract 5 from each side.

Check Your Progress Solve each equation.

2A. $\sqrt{c-3} - 2 = 4$

2B. $4 + \sqrt{h+1} = 14$

Personal Tutor glencoe.com

Extraneous Solutions Squaring each side of an equation sometimes produces a solution that is not a solution of the original equation. These are called **extraneous solutions**. Therefore, you must check all solutions in the original equation.

StudyTip

Extraneous Solutions When checking solutions for extraneous solutions, we are only interested in principal roots.

EXAMPLE 3 Variable on Each Side

Solve $\sqrt{k+1} = k - 1$. Check your solution.

$\sqrt{k+1} = k - 1$	Original equation
$\left(\sqrt{k+1}\right)^2 = (k-1)^2$	Square each side.
$k + 1 = k^2 - 2k + 1$ −k+1	Simplify.
$0 = k^2 - 3k$	Subtract k and 1 from each side.
$0 = k(k-3)$	Factor.
$k = 0$ or $k - 3 = 0$	Zero Product Property
$k = 3$	Solve.

CHECK $\sqrt{k+1} = k - 1$	Original equation	$\sqrt{k+1} = k - 1$	Original equation
$\sqrt{0+1} \stackrel{?}{=} 0 - 1$	$k = 0$	$\sqrt{3+1} \stackrel{?}{=} 3 - 1$	$k = 3$
$\sqrt{1} \stackrel{?}{=} -1$	Simplify.	$\sqrt{4} \stackrel{?}{=} 2$	Simplify.
$1 \neq -1$ ✗	False	$2 = 2$ ✓	True

Since 0 does not satisfy the original equation, 3 is the only solution.

Check Your Progress

Solve each equation. Check your solution.

3A. $\sqrt{t+5} = t + 3$

3B. $x - 3 = \sqrt{x-1}$

Personal Tutor glencoe.com

Check Your Understanding

Example 1 p. 624

1. **GEOMETRY** The surface area of a basketball is x square inches. What is the radius of the basketball if the formula for the surface area of a sphere is $SA = 4\pi r^2$?

Examples 2 and 3 p. 625

Solve each equation. Check your solution.

2. $\sqrt{10h} + 1 = 21$
3. $\sqrt{7r + 2} + 3 = 7$
4. $5 + \sqrt{g - 3} = 6$
5. $\sqrt{3x - 5} = x - 5$
6. $\sqrt{2n + 3} = n$
7. $\sqrt{a - 2} + 4 = a$

Practice and Problem Solving

= **Step-by-Step Solutions** begin on page R12.
Extra Practice begins on page 815.

Example 1 p. 624

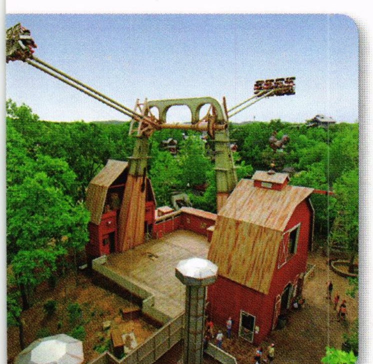
8. **EXERCISE** Suppose the function $S = \pi\sqrt{\frac{9.8\ell}{7}}$, where S represents speed in meters per second and ℓ is the leg length of a person in meters, can approximate the maximum speed that a person can run.
   a. What is the maximum running speed of a person with a leg length of 1.1 meters to the nearest tenth of a meter?
   b. What is the leg length of a person with a running speed of 2.7 meters per second to the nearest tenth of a meter?
   c. As leg length increases, does maximum speed increase or decrease? Explain.

Examples 2 and 3 p. 625

Solve each equation. Check your solution.

9. $\sqrt{a} + 11 = 21$
10. $\sqrt{t} - 4 = 7$
11. $\sqrt{n - 3} = 6$
12. $\sqrt{c + 10} = 4$
13. $\sqrt{h - 5} = 2\sqrt{3}$
14. $\sqrt{k + 7} = 3\sqrt{2}$
15. $y = \sqrt{12 - y}$
16. $\sqrt{u + 6} = u$
17. $\sqrt{r + 3} = r - 3$
18. $\sqrt{1 - 2t} = 1 + t$
19. $5\sqrt{a - 3} + 4 = 14$
20. $2\sqrt{x - 11} - 8 = 4$

21. **RIDES** The amount of time t, in seconds, that it takes a simple pendulum to complete a full swing is called the *period*. It is given by $t = 2\pi\sqrt{\frac{\ell}{32}}$, where ℓ is the length of the pendulum, in feet.
   a. The Giant Swing completes a period in about 8 seconds. About how long is the pendulum's arm? Round to the nearest foot.
   b. Does increasing the length of the pendulum increase or decrease the period? Explain.



Real-World Link

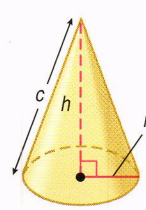
The Giant Swing at Silver Dollar City in Branson, Missouri, swings riders at 45 miles per hour and reaches a height of 7 stories.

Source: Silver Dollar City Amusement Park

Solve each equation. Check your solution.

22. $\sqrt{6a - 6} = a + 1$
23. $\sqrt{x^2 + 9x + 15} = x + 5$
24. $6\sqrt{\frac{5k}{4}} - 3 = 0$
25. $\sqrt{\frac{5y}{6}} - 10 = 4$
26. $\sqrt{2a^2 - 121} = a$
27. $\sqrt{5x^2 - 9} = 2x$

28. **GEOMETRY** The formula for the slant height c of a cone is $c = \sqrt{h^2 + r^2}$, where h is the height of the cone and r is the radius of its base. Find the height of the cone if the slant height is 4 and the radius is 2. Round to the nearest tenth.



29. **MULTIPLE REPRESENTATIONS** Consider $\sqrt{2x-7} = x - 7$.

a. **GRAPHICAL** Clear the Y= list. Enter the left side of the equation as Y1 $= \sqrt{2x-7}$. Enter the right side of the equation as Y2 $= x - 7$. Press GRAPH.

b. **GRAPHICAL** Sketch what is shown on the screen.

c. **ANALYTICAL** Use the **intersect** feature on the CALC menu to find the point of intersection.

d. **ANALYTICAL** Solve the radical equation algebraically. How does your solution compare to the solution from the graph?

30. **PACKAGING** A cylindrical container of chocolate drink mix has a volume of 162 cubic inches. The radius r of the container can be found by using the formula $r = \sqrt{\frac{V}{\pi h}}$, where V is the volume of the container and h is the height.

a. If the radius is 2.5 inches, find the height of the container. Round to the nearest hundredth.

b. If the height of the container is 10 inches, find the radius. Round to the nearest hundredth.

Real-World Link

Packaging has several objectives, including physical protection, information transmission, marketing, convenience, security, and portion control.

Source: *Packaging World*

H.O.T. Problems

Use Higher-Order Thinking Skills

31. **FIND THE ERROR** Jada and Fina solved $\sqrt{6-b} = \sqrt{b+10}$. Is either of them correct? Explain.

Jada	Fina
$\sqrt{6-b} = \sqrt{b+10}$	$\sqrt{6-b} = \sqrt{b+10}$
$(\sqrt{6-b})^2 = (\sqrt{b+10})^2$	$(\sqrt{6-b})^2 = (\sqrt{b+10})^2$
$6 - b = b + 10$	$6 - b = b + 10$
$-2b = 4$	$2b = 4$
$b = -2$	$b = 2$
Check $\sqrt{6-(-2)} \stackrel{?}{=} \sqrt{(-2)+10}$	Check $\sqrt{6-(2)} \stackrel{?}{=} \sqrt{(2)+10}$
$\sqrt{8} = \sqrt{8}$ ✓	$\sqrt{4} \neq \sqrt{12}$ X
	no solution

32. **REASONING** Which equation has the same solution set as $\sqrt{4} = \sqrt{x+2}$? Explain.

A. $\sqrt{4} = \sqrt{x} + \sqrt{2}$ **B.** $4 = x + 2$ **C.** $2 - \sqrt{2} = \sqrt{x}$

33. **REASONING** Explain how solving $5 = \sqrt{x} + 1$ is different from solving $5 = \sqrt{x+1}$.

34. **OPEN ENDED** Write a radical equation with a variable on each side. Then solve the equation.

35. **REASONING** Is the following equation *sometimes*, *always* or *never* true? Explain.

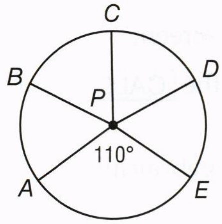
$$\sqrt{(x-2)^2} = x - 2$$

36. **CHALLENGE** Solve $\sqrt{x+9} = \sqrt{3} + \sqrt{x}$.

37. **WRITING IN MATH** Write some general rules about how to solve radical equations. Demonstrate your rules by solving a radical equation.

Standardized Test Practice

38. SHORT RESPONSE Zack needs to drill a hole at A, B, C, D, and E on circle P.



If Zack drills holes so that $m\angle APE = 110°$ and the other four angles are congruent, what is $m\angle CPD$?

39. Which expression is undefined when $w = 3$?

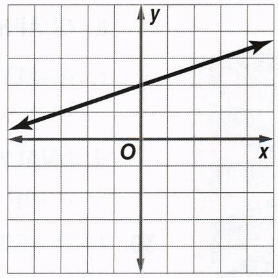
A $\frac{w-3}{w+1}$

B $\frac{w^2-3w}{3w}$

C $\frac{w+1}{w^2-3w}$

D $\frac{3w}{3w^2}$

40. What is the slope of a line that is parallel to the line?



F -3

G $-\frac{1}{3}$

H 3

J $\frac{1}{3}$

41. What are the solutions of $\sqrt{x+3} - 1 = x - 4$?

A $1, 6$

B $-1, -6$

C 1

D 6

Spiral Review

42. ELECTRICITY The voltage V required for a circuit is given by $V = \sqrt{PR}$, where P is the power in watts and R is the resistance in ohms. How many more volts are needed to light a 100-watt light bulb than a 75-watt light bulb if the resistance of both is 110 ohms? (Lesson 10-3)

Simplify each expression. (Lesson 10-2)

43. $\sqrt{6} \cdot \sqrt{8}$ **44.** $\sqrt{3} \cdot \sqrt{6}$ **45.** $7\sqrt{3} \cdot 2\sqrt{6}$

46. $\sqrt{\frac{27}{a^2}}$ **47.** $\sqrt{\frac{5c^5}{4d^5}}$ **48.** $\frac{\sqrt{9x^3y}}{\sqrt{16x^2y^2}}$

49. PHYSICAL SCIENCE A projectile is shot straight up from ground level. Its height h, in feet, after t seconds is given by $h = 96t - 16t^2$. Find the value(s) of t when h is 96 feet. (Lesson 9-5)

Factor each trinomial, if possible. If the trinomial cannot be factored using integers, write *prime*. (Lesson 8-4)

50. $2x^2 + 7x + 5$ **51.** $6p^2 + 5p - 6$ **52.** $5d^2 + 6d - 8$

53. $8k^2 - 19k + 9$ **54.** $9g^2 - 12g + 4$ **55.** $2a^2 - 9a - 18$

Determine whether each expression is a monomial. Write *yes* or *no*. Explain. (Lesson 7-1)

56. 12 **57.** $4x^3$ **58.** $a - 2b$ **59.** $4n + 5p$ **60.** $\frac{x}{y^2}$ **61.** $\frac{1}{5}abc^{14}$

Skills Review

Simplify. (Lesson 1-1)

62. 9^2 **63.** 10^6 **64.** 4^5 **65.** $(8v)^2$ **66.** $\left(\frac{w^3}{9}\right)^2$ **67.** $(10y^2)^3$

CHAPTER 10

Mid-Chapter Quiz

Lessons 10-1 through 10-4

Math Online glencoe.com
Math *in Motion*, Animation

Graph each function. Compare to the parent graph. State the domain and range. (Lesson 10-1)

1. $y = 2\sqrt{x}$

2. $y = -4\sqrt{x}$

3. $y = \frac{1}{2}\sqrt{x}$

4. $y = \sqrt{x} - 3$

5. $y = \sqrt{x - 1}$

6. $y = 2\sqrt{x - 2}$

7. **GEOMETRY** The length of the side of a square is given by the function $s = \sqrt{A}$, where A is the area of the square. What is the length of the side of a square that has an area of 121 square inches? (Lesson 10-1)

A 121 inches
B 11 inches
C 44 inches
D 10 inches

Simplify each expression. (Lesson 10-2)

8. $2\sqrt{25}$

9. $\sqrt{12} \cdot \sqrt{8}$

10. $\sqrt{72xy^5z^6}$

11. $\frac{3}{1 + \sqrt{5}}$

12. $\frac{1}{5 - \sqrt{7}}$

13. **SATELLITES** A satellite is launched into orbit 200 kilometers above Earth. The orbital velocity of a satellite is given by the formula $v = \sqrt{\frac{Gm_E}{r}}$. v is velocity in meters per second, G is a given constant, m_E is the mass of Earth, and r is the radius of the satellite's orbit in meters. (Lesson 10-2)

a. The radius of Earth is 6,380,000 meters. What is the radius of the satellite's orbit in meters?

b. The mass of Earth is 5.97×10^{24} kilograms, and the constant G is $6.67 \times 10^{-11}\ \text{N} \cdot \frac{\text{m}^2}{\text{kg}^2}$ where N is in Newtons. Use the formula to find the orbital velocity of the satellite in meters per second.

14. Which expression is equivalent to $\sqrt{\frac{16}{32}}$? (Lesson 10-2)

F $\frac{1}{2}$

G 2

H $\frac{\sqrt{2}}{2}$

J 4

Simplify each expression. (Lesson 10-3)

15. $3\sqrt{2} + 5\sqrt{2}$

16. $\sqrt{11} - 3\sqrt{11}$

17. $6\sqrt{2} + 4\sqrt{50}$

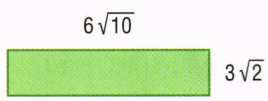
18. $\sqrt{27} - \sqrt{48}$

19. $4\sqrt{3}(2\sqrt{6})$

20. $3\sqrt{20}(2\sqrt{5})$

21. $(\sqrt{5} + \sqrt{7})(\sqrt{20} + \sqrt{3})$

22. **GEOMETRY** Find the area of the rectangle. (Lesson 10-3)



Solve each equation. Check your solution. (Lesson 10-4)

23. $\sqrt{5x} - 1 = 4$

24. $\sqrt{a - 2} = 6$

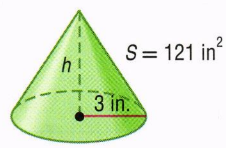
25. $\sqrt{15 - x} = 4$

26. $\sqrt{3x^2 - 32} = x$

27. $\sqrt{2x - 1} = 2x - 7$

28. $\sqrt{x + 1} + 2 = 4$

29. **GEOMETRY** The lateral surface area S of a cone can be found by using the formula $S = \pi r\sqrt{r^2 + h^2}$, where r is the radius of the base and h is the height of the cone. Find the height of the cone. (Lesson 10-4)



10-5 The Pythagorean Theorem

Then

You solved quadratic equations by using the Square Root Property. (Lesson 8-6)

Now

- Solve problems by using the Pythagorean Theorem.
- Determine whether a triangle is a right triangle.

New Vocabulary

hypotenuse
legs
converse
Pythagorean triple

Math Online

glencoe.com

- Extra Examples
- Personal Tutor
- Self-Check Quiz
- Homework Help

Why?

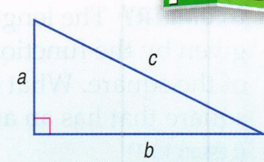
The designer television shown is made of black and white leather just like a real soccer ball. Televisions are measured along the diagonal of the screen. If the height and width of the screen is known, the Pythagorean Theorem can be used to find the measure of the diagonal.

The Pythagorean Theorem In a right triangle, the side opposite the right angle is the **hypotenuse**. This side is always the longest. The other two sides are the **legs**.

Key Concept — The Pythagorean Theorem

For Your FOLDABLE

Words If a triangle is a right triangle, then the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs.

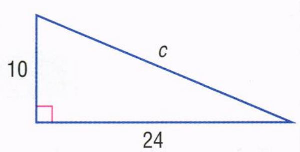


Symbols $c^2 = a^2 + b^2$

EXAMPLE 1 Find the Length of a Side

Find each missing length. If necessary, round to the nearest hundredth.

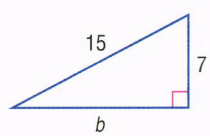
a.



$c^2 = a^2 + b^2$	Pythagorean Theorem
$c^2 = 10^2 + 24^2$	$a = 10$ and $b = 24$
$c^2 = 100 + 576$	Evaluate squares.
$c^2 = 676$	Simplify.
$c = \pm\sqrt{676}$	Take the square root of each side.
$c = \pm 26$	$(\pm 26)^2 = 676$

A length cannot be negative. The missing length is 26 units.

b.

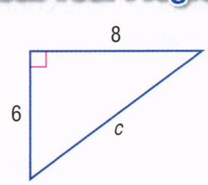


$c^2 = a^2 + b^2$	Pythagorean Theorem
$15^2 = 7^2 + b^2$	$a = 7$ and $c = 15$
$225 = 49 + b^2$	Evaluate squares.
$176 = b^2$	Subtract 49 from each side.
$\pm\sqrt{176} = b$	Take the square root of each side.
$\pm 13.27 \approx b$	Use a calculator to evaluate $\sqrt{176}$.

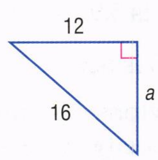
The missing length is 13.27 units.

Check Your Progress

1A.



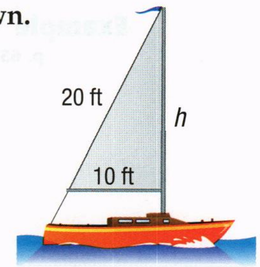
1B.



Personal Tutor glencoe.com

Real-World EXAMPLE 2 Find the Length of a Side

SAILING The sail of a keelboat forms a right triangle as shown. Find the height of the sail.



$20^2 = h^2 + 10^2$	Pythagorean Theorem
$400 = h^2 + 100$	Evaluate squares.
$300 = h^2$	Subtract 100 from each side.
$\pm 17.32 \approx h$	Take the square root of each side.
$17.32 \approx h$	Use the positive value.

The sail is approximately 17.32 feet high.

Check Your Progress

2. Suppose the longest side of the sail is 30 feet long and the shortest side is 14 feet long. Find the height of the sail.

Personal Tutor glencoe.com

Real-World Link

A keelboat is a sailboat with a weighted keel, a vertical fin at the bottom of the boat. Keels are 20 to 30 inches in length.

Source: United States Sailing Association

Right Triangles If you exchange the hypothesis and conclusion of an if-then statement, the result is the **converse** of the statement. The converse of the Pythagorean Theorem can be used to determine whether a triangle is a right triangle.

Key Concept — Converse of the Pythagorean Theorem

For Your FOLDABLE

If a triangle has side lengths a, b, and c such that $c^2 = a^2 + b^2$, then the triangle is a right triangle. If $c^2 \neq a^2 + b^2$, then the triangle is not a right triangle.

A **Pythagorean triple** is a group of three counting numbers that satisfy the equation $c^2 = a^2 + b^2$, where c is the greatest number. Examples include (3, 4, 5) and (5, 12, 13). Multiples of Pythagorean triples also satisfy the converse of the Pythagorean Theorem, so (6, 8, 10) is also a Pythagorean triple.

EXAMPLE 3 Check for Right Triangles

Determine whether 9, 12, and 16 can be the lengths of the sides of a right triangle.

Since the measure of the longest side is 16, let $c = 16$, $a = 9$, and $b = 12$.

$c^2 = a^2 + b^2$	Pythagorean Theorem
$16^2 \stackrel{?}{=} 9^2 + 12^2$	$a = 9$, $b = 12$, and $c = 16$
$256 \stackrel{?}{=} 81 + 144$	Evaluate squares.
$256 \neq 225$	Add.

Since $c^2 \neq a^2 + b^2$, segments with these measures cannot form a right triangle.

Check Your Progress

Determine whether each set of measures can be the lengths of the sides of a right triangle.

3A. 30, 40, 50

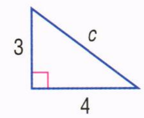
3B. 6, 12, 18

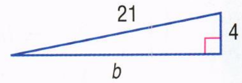
Personal Tutor glencoe.com

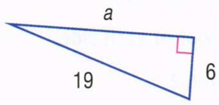
Check Your Understanding

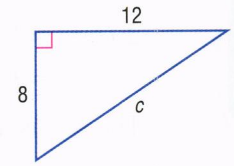
Example 1 p. 630

Find each missing length. If necessary, round to the nearest hundredth.

1.


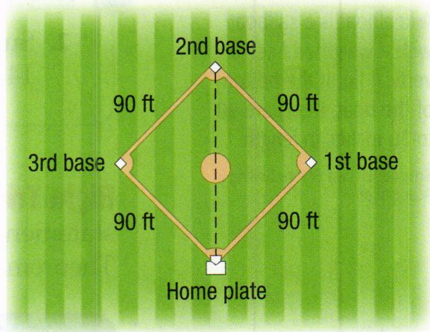
2.


3.


4.


Example 2 p. 631

5. BASEBALL A baseball diamond is a square. The distance between consecutive bases is 90 feet.



a. How far does a catcher have to throw the ball from home plate to second base?

b. How far does a third baseman have to throw the ball to the first baseman?

c. If the catcher is five feet behind home plate, how far does he have to throw the ball to second base?

Example 3 p. 631

Determine whether each set of measures can be the lengths of the sides of a right triangle.

6. 8, 12, 16

7. 28, 45, 53

8. 7, 24, 25

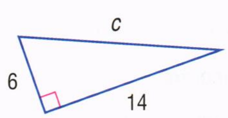
9. 15, 25, 45

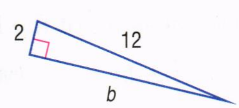
Practice and Problem Solving

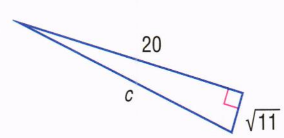
= **Step-by-Step Solutions** begin on page R12.
Extra Practice begins on page 815.

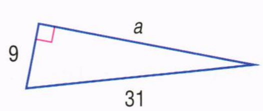
Example 1 p. 630

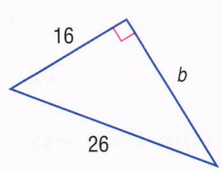
Find each missing length. If necessary, round to the nearest hundredth.

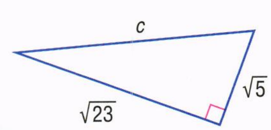
10.


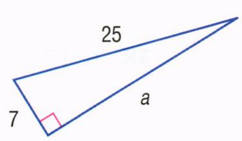
11.


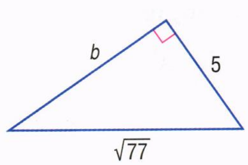
12.


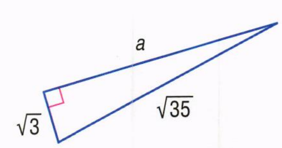
13.


14.


15.


16.


17.


18.


Example 2 p. 631

19. **TELEVISION** Larry is buying an entertainment stand for his television. The diagonal of his television is 27 inches. The space for the television measures 20 inches by 26 inches. Will Larry's television fit? Explain.

Example 3 p. 631

Determine whether each set of measures can be the lengths of the sides of a right triangle. Then determine whether they form a Pythagorean triple.

20. 9, 40, 41 **21.** $3, 2\sqrt{10}, \sqrt{41}$ **22.** $4, \sqrt{26}, 12$

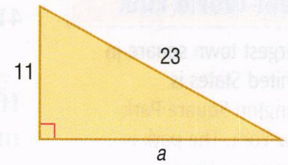
23. $\sqrt{5}, 7, 14$ **24.** 8, 31.5, 32.5 **25.** $\sqrt{65}, 6\sqrt{2}, \sqrt{97}$

26. 18, 24, 30 **27.** 36, 77, 85 **28.** 17, 33, 98

29. **GEOMETRY** Refer to the triangle at the right.

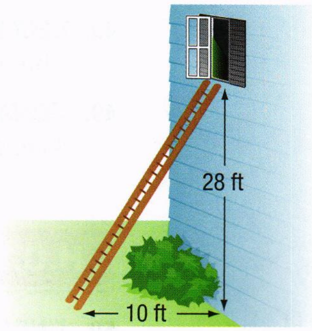
a. What is a?

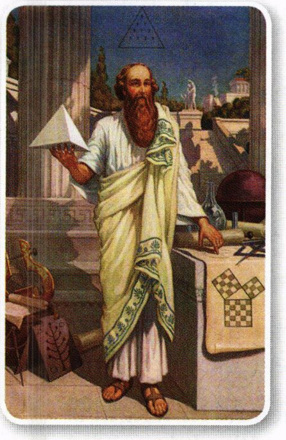
b. Find the area of the triangle.



30. **GARDENING** Khaliah wants to plant flowers in a triangular plot. She has three lengths of plastic garden edging that measure 8 feet, 15 feet, and 17 feet. Determine whether these pieces form a right triangle. Explain.

31. **LADDER** Mr. Takeo is locked out of his house. The only open window is on the second floor. There is a bush along the edge of the house, so he places the neighbor's ladder 10 feet from the house. To the nearest foot, what length of ladder does he need to reach the window?



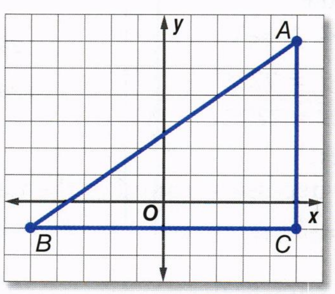


Math History Link

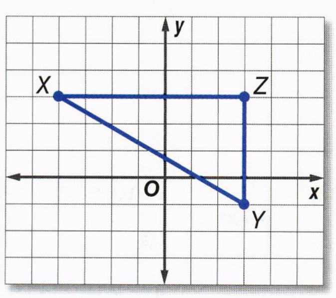
Pythagoras
(580–500 B.C.)
Greek mathematician Pythagoras founded the famous Pythagorean school for study of philosophy, mathematics, and natural science.

Find the length of the hypotenuse. Round to the nearest hundredth.

32.



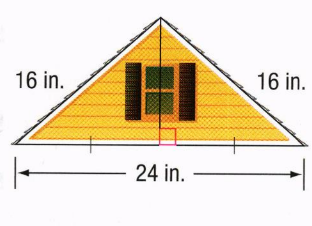
33.



34. **GEOMETRY** A rectangle has a base of 5 feet and a height of 12 feet. What is the length of the diagonal?

35. **GEOMETRY** A square has a diagonal with length of 6 meters. Find the length of the sides of the square.

36. **DOLLHOUSE** Alonso is building a dollhouse for his sister's birthday. The house is 24 inches across and the slanted side is 16 inches long as shown. Find the height of the roof to the nearest tenth of an inch.



37. **GEOMETRY** Each side of a cube is 5 inches long. Find the length of a diagonal of the cube.

Real-World Link

The largest town square in the United States is Washington Square Park in New York. The park is 9.75 acres or 39,000 square meters.

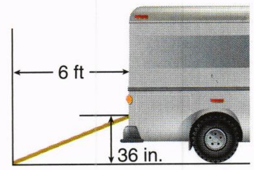
Source: *The New York Times*

38. **TOWN SQUARES** The largest town square in the world is Tiananmen Square in Beijing, China, covering 98 acres.

    a. One square mile is 640 acres. Assuming that Tiananmen Square is a square, how many feet long is a side to the nearest foot?

    b. To the nearest foot, what is the diagonal distance across Tiananmen Square?

39. **TRUCKS** Violeta needs to construct a ramp to roll a cart of moving boxes from her garage into the back of her truck. How long does the ramp have to be?



40. **GEOMETRY** A square has an area of 242 square inches. Find the length of a diagonal.

41. **GEOMETRY** A rectangle has a width that is twice as long as its length and an area of 722 square inches. Find the length of a diagonal.

If c is the measure of the hypotenuse of a right triangle, find each missing measure. If necessary, round to the nearest hundredth.

42. $a = x, b = x + 41, c = 85$
43. $a = 8, b = x, c = x + 2$
44. $a = 12, b = x - 2, c = x$
45. $a = x, b = x + 7, c = 97$
46. $a = x - 47, b = x, c = x + 2$
47. $a = x - 32, b = x - 1, c = x$

48. **GEOMETRY** A right triangle has one leg that is 8 inches shorter than the other leg. The hypotenuse is 30 inches long. Find the length of each leg.

49. **GEOMETRY** A rectangle has a diagonal with length 8 centimeters. Its length is 4 centimeters greater than the width. Find the length and width of the rectangle.

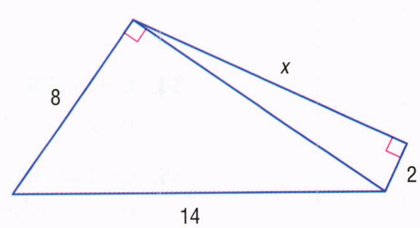
H.O.T. Problems

Use Higher-Order Thinking Skills

50. **FIND THE ERROR** Wyatt and Dario are determining whether 36, 77, and 85 form a Pythagorean triple. Is either of them correct? Explain your reasoning.

Wyatt	Dario
$36^2 + 77^2 \stackrel{?}{=} 85^2$	$36^2 + 85^2 \stackrel{?}{=} 77^2$
$1296 + 5929 \stackrel{?}{=} 7225$	$1296 + 7725 \stackrel{?}{=} 5929$
$7225 = 7225$	$9021 \neq 5929$
yes	no

51. **CHALLENGE** Find the value of x in the figure.



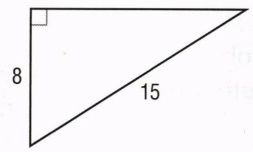
52. **REASONING** Provide a counterexample to the statement. *Any two right triangles with the same hypotenuse have the same area.*

53. **OPEN ENDED** Draw a right triangle that has a hypotenuse of $\sqrt{72}$ units.

54. **WRITING IN MATH** Explain how to determine whether segments in three lengths could form a right triangle.

Standardized Test Practice

55. GEOMETRY Find the missing length.



A -17

B $-\sqrt{161}$

C $\sqrt{161}$

D 17

56. What is a solution of this equation?

$$x + 1 = \sqrt{x + 1}$$

F 0, 3

G 3

H 0

J no solutions

57. SHORT RESPONSE A plumber charges \$40 for the first hour of each house call plus \$8 for each additional half hour. If the plumber works for 4 hours, how much does he charge?

58. Find the next term in the geometric sequence $4, 3, \frac{9}{4}, \frac{27}{16}, \ldots$.

A $\frac{4}{3}$ B $\frac{81}{64}$ C $\frac{64}{81}$ D $\frac{243}{64}$

Spiral Review

Solve each equation. Check your solution. (Lesson 10-4)

59. $\sqrt{x} = 16$ **60.** $\sqrt{4x} = 64$ **61.** $\sqrt{10x} = 10$

62. $\sqrt{8x} + 1 = 65$ **63.** $\sqrt{x+1} + 2 = 4$ **64.** $\sqrt{x - 15} = 3 - \sqrt{x}$

Simplify each expression. (Lesson 10-3)

65. $2\sqrt{3} + 5\sqrt{3}$ **66.** $4\sqrt{5} - 2\sqrt{5}$ **67.** $6\sqrt{7} + 2\sqrt{28}$

68. $\sqrt{18} - 4\sqrt{2}$ **69.** $3\sqrt{5} - 5\sqrt{3} + 9\sqrt{5}$ **70.** $4\sqrt{3} + 6\sqrt{12}$

71. BUSINESS The amount of money spent at West Outlet Mall continues to increase. The total $T(x)$ in millions of dollars can be estimated by the function $T(x) = 12(1.12)^x$, where x is the number of years after it opened in 2005. Find the amount of sales in 2015, 2016, and 2017. (Lesson 9-6)

Describe how the graph of each function is related to the graph of $f(x) = x^2$. (Lesson 9-3)

72. $g(x) = x^2 - 8$ **73.** $h(x) = \frac{1}{4}x^2$ **74.** $h(x) = -x^2 + 5$

75. $g(x) = x^2 + 10$ **76.** $g(x) = -2x^2$ **77.** $h(x) = -x^2 - \frac{4}{3}$

78. ROCK CLIMBING While rock climbing, Damaris launches a grappling hook from a height of 6 feet with an initial upward velocity of 56 feet per second. The hook just misses the stone ledge that she wants to scale. As it falls, the hook anchors on a ledge 30 feet above the ground. How long was the hook in the air? (Lesson 8-4)

Find each product. (Lesson 7-7)

79. $(b + 8)(b + 2)$ **80.** $(x - 4)(x - 9)$ **81.** $(y + 4)(y - 8)$

82. $(p + 2)(p - 10)$ **83.** $(2w - 5)(w + 7)$ **84.** $(8d + 3)(5d + 2)$

Skills Review

Solve each proportion. (Lesson 2-6)

85. $\frac{x}{5} = \frac{12}{3}$ **86.** $\frac{12}{x} = \frac{3}{4}$ **87.** $\frac{5}{4} = \frac{10}{x}$ **88.** $\frac{3}{5} = \frac{12}{x + 8}$

10-6 The Distance and Midpoint Formulas

Then

You used the Pythagorean Theorem. (Lesson 10-5)

Now

- Find the distance between two points on a coordinate plane.
- Find the midpoint between two points on a coordinate plane.

New Vocabulary

Distance Formula
midpoint
Midpoint Formula

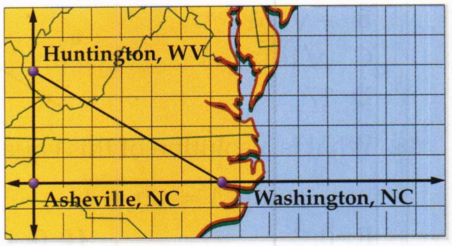
Math Online

glencoe.com

- Extra Examples
- Personal Tutor
- Self-Check Quiz
- Homework Help

Why?

Rescue helicopters use electronic Global Positioning Systems (GPS) to compute direct distances between two locations.



A rescue helicopter can fly 450 miles before it needs to refuel. A person needs to be flown from Washington, North Carolina, to Huntington, West Virginia. Sides of the grid squares are 50 miles long. Asheville, North Carolina, is at the origin, Huntington is at (0, 196), and Washington is at (310, 0). Can the helicopter make the trip without refueling?

Distance Formula The GPS system calculates direct distances by using the **Distance Formula**, which is based on the Pythagorean Theorem.

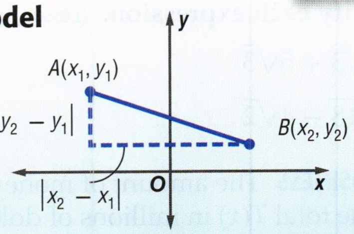
Key Concept — The Distance Formula

For Your FOLDABLE

Words The distance d between any two points with coordinates (x_1, y_1) and (x_2, y_2) is given by the following formula.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Model



You can use the Distance Formula to find the distance between any two points on a coordinate plane.

EXAMPLE 1 Distance Between Two Points

Find the distance between points at (5, 3) and (1, −2).

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	Distance Formula
$= \sqrt{(1 - 5)^2 + (-2 - 3)^2}$	$(x_1, y_1) = (5, 3)$ and $(x_2, y_2) = (1, -2)$
$= \sqrt{(-4)^2 + (-5)^2}$	Simplify.
$= \sqrt{16 + 25}$	Evaluate squares.
$= \sqrt{41}$ or about 6.4 units	Simplify.

Check Your Progress

Find the distance between points with the given coordinates.

1A. (4, 2) and (−3, −1).

1B. (−7, −2) and (−5, −8)

Personal Tutor glencoe.com

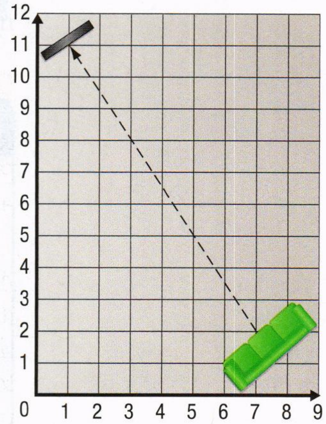
Real-World Link

About 22% of U.S. residents have a home theater. Technology installations is a \$9.6 billion per year business.

Source: *Realty Times*

Real-World EXAMPLE 2 Use the Distance Formula

ENTERTAINMENT The Vaccaro Family is having a home theater system installed. The TV and the seating will be placed in opposite corners of the room. The manufacturer of the TV recommends that for the size of TV that they want, the seating should be placed at least 13 feet away. If each grid square is 1 foot long, is the Vaccaro's room large enough for the TV?



The front of the TV screen is located at (1, 11), and the front of the sofa is located at (7, 2).

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad \text{Distance Formula}$$

$$= \sqrt{(7 - 1)^2 + (2 - 11)^2} \quad (x_1, y_1) = (1, 11) \text{ and } (x_2, y_2) = (7, 2)$$

$$= \sqrt{6^2 + (-9)^2} \quad \text{Simplify.}$$

$$= \sqrt{117} \text{ or about } 10.8 \text{ feet}$$

No, the room is not large enough for the TV.

Check Your Progress

2. The manufacturer of the speakers recommends that they be placed at least 8 feet from the seating. If one of the speakers is being placed at (0, 9), is the Vaccaros' family room large enough for the speakers? Explain.

Personal Tutor glencoe.com

When we know the distance and one of the points, we can use the Distance Formula to find the coordinates of the other point.

StudyTip

Two Possibilities When finding a missing coordinate, there will usually be two possibilities because the point could be the same distance in different directions.

EXAMPLE 3 Find a Missing Coordinate

Find the possible values for a if the distance between points at (4, 7) and (a, 3) are 5 units apart.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad \text{Distance Formula}$$

$$5 = \sqrt{(a - 4)^2 + (3 - 7)^2} \quad (x_1, y_1) = (4, 7) \text{ and } (x_2, y_2) = (a, 3), \text{ and } d = 5$$

$$5 = \sqrt{(a - 4)^2 + (-4)^2} \quad \text{Simplify.}$$

$$5 = \sqrt{a^2 - 8a + 32} \quad \text{Evaluate squares and simplify.}$$

$$25 = a^2 - 8a + 32 \quad \text{Square each side.}$$

$$0 = a^2 - 8a + 7 \quad \text{Subtract 25 from each side.}$$

$$0 = (a - 1)(a - 7) \quad \text{Factor.}$$

$$a - 1 = 0 \quad \text{or} \quad a - 7 = 0 \quad \text{Zero Product Property}$$

$$a = 1 \qquad a = 7 \quad \text{Solve each equation.}$$

Check Your Progress

3. Find the possible values of a if the distance between points at (2, a) and (−6, 2) is 10 units.

Personal Tutor glencoe.com

Midpoint Formula The point on the segment that joins two points and is equidistant from the endpoints is called the **midpoint**. You can find the coordinates of the midpoint by using the **Midpoint Formula**.

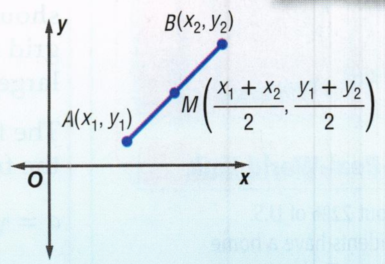
Key Concept — The Midpoint Formula

For Your FOLDABLE

Words The midpoint M of a line segment with endpoints at (x_1, y_1) and (x_2, y_2) is given by

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right).$$

Model



Watch Out!

Midpoint Formula Be careful to add, and not subtract, when using the Midpoint Formula.

EXAMPLE 4 Find the Midpoint

Find the coordinates of the midpoint of the segment with endpoints at (−1, −2) and (3, −4).

$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ **Midpoint Formula**

$= \left(\frac{-1 + 3}{2}, \frac{-2 + (-4)}{2}\right)$ **$(x_1, y_1) = (-1, -2)$ and $(x_2, y_2) = (3, -4)$**

$= \left(\frac{2}{2}, \frac{-6}{2}\right)$ **Simplify the numerators.**

$= (1, -3)$ **Simplify.**

Check Your Progress

Find the coordinates of the midpoint of the segment with the given endpoints.

4A. (12, 3), (−8, 3) **4B.** (0, 0), (5, 12) **4C.** (6, 8), (3, 4)

Personal Tutor glencoe.com

Check Your Understanding

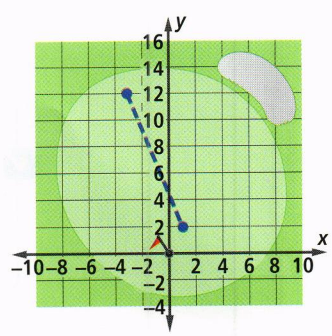
Example 1 p. 636

Find the distance between points with the given coordinates.

1. (6, −2), (12, 8)
2. (4, 8), (−3, −6)
3. (3, 0), (6, −2)
4. (−2, −4), (−5, −3)

Example 2 p. 637

5. **GOLF** Addison hit a golf ball from a tee to the point at (−3, 12), 12 feet past the hole and 3 feet to the left. The hole is located at the point at (0, 0). Her first putt traveled to the point at (1, 2), 2 feet above the hole and 1 foot to the right.



   a. How far did the ball travel on her first putt?

   b. How far was her first putt from the cup?

Example 3 p. 637

Find the possible values for a if the points with the given coordinates are the indicated distance apart.

6. $(-5, a), (3, 1); d = \sqrt{89}$
7. $(6, a), (5, 0); d = \sqrt{17}$
8. $(5, 8), (a, 2); d = 3\sqrt{5}$
9. $(a, 6), (-6, 2); d = 4\sqrt{10}$

Example 4 p. 638

Find the coordinates of the midpoint of the segment with the given endpoints.

10. $(5, -10), (5, 8)$
11. $(2, -2), (6, 2)$
12. $(5, 0), (0, 3)$
13. $(-4, 1), (3, -1)$
14. $(3, -17), (2, -8)$
15. $(-2, 2), (4, 10)$
16. $(3, 10), (3, 3)$
17. $(-17, 8), (-2, 20)$

Practice and Problem Solving

= **Step-by-Step Solutions** begin on page R12.
Extra Practice begins on page 815.

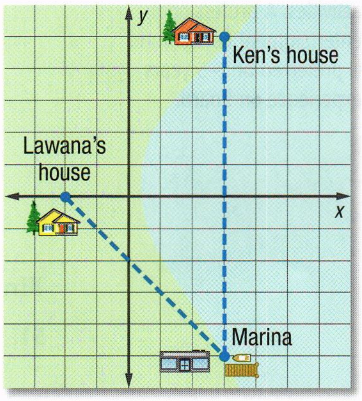
Example 1 p. 636

Find the distance between points with the given coordinates.

18. $(5, 8), (5, 7)$
19. $(6, -9), (9, -9)$
20. $(3, -3), (7, 2)$
21. $(5, 1), (0, 4)$
22. $(-5, 2), (4, -2)$
23. $(3, 5), (-6, 0)$
24. $(-7, 8), (3, 10)$
25. $(-11, 9), (3, -4)$
26. $(8, 6), (-13, -2)$
27. $(5, 2), (3, -3)$
28. $(4, 2), (5, 5)$
29. $(-3, 5), (5, -3)$

Example 2 p. 637

30. **NAVIGATION** Lawana and Ken are meeting at a restaurant in a marina. Ken takes his boat, while Lawana is driving her car. The sides of each grid square on the map represent 1 mile.
   a. How far did Ken travel?
   b. How far did Lawana travel?
   c. How many times as great is the distance that Ken traveled as the distance that Lawana traveled?



Example 3 p. 637

Find the possible values for a if the points with the given coordinates are the indicated distance apart.

31. $(-9, -2), (a, 5); d = 7$
32. $(a, -6), (-5, 2); d = 10$
33. $(a, 0), (3, 1); d = \sqrt{2}$
34. $(4, a), (8, 4); d = 2\sqrt{5}$
35. $(7, 5), (-9, a); d = 2\sqrt{65}$
36. $(-2, a), (6, 1); d = 4\sqrt{5}$

Example 4 p. 638

Find the coordinates of the midpoint of the segment with the given endpoints.

37. $(0, 2), (7, 3)$
38. $(5, -2), (3, -6)$
39. $(-4, 0), (0, 14)$
40. $(10, -3), (-8, -5)$
41. $(-5, 5), (3, -3)$
42. $(-16, -7), (-4, -3)$

Find the distance between points with the given coordinates.

43. $(4, 2), \left(6, -\frac{2}{3}\right)$
44. $\left(\frac{4}{5}, -1\right), \left(2, -\frac{1}{2}\right)$
45. $(4\sqrt{5}, 7), (6\sqrt{5}, 1)$
46. **GEOMETRY** Triangle ABC has vertices $A(1, 3)$, $B(-2, 5)$, and $C(8, 8)$. Find the perimeter of the triangle. Use a calculator to estimate the perimeter to the nearest tenth.
47. **GEOMETRY** Quadrilateral $JKLM$ has vertices $J(-3, -4)$, $K(-1, 4)$, $L(4, 5)$, and $M(6, -5)$. Find the perimeter of the quadrilateral to the nearest tenth.

48. **TEMPERATURE** The temperature dropped from 25°F to −8°F over a 12-hour period beginning at 12:00 noon as shown.

Time	12:00 noon	4:00	8:00	12:00 midnight
Temperature (°F)	25°F	14°F	3°F	−8°F

a. Plot these points on a coordinate plane with time on the x-axis and temperature on the y-axis. Let x represent the number of hours, and let 12:00 noon correspond to $x = 0$.

b. Draw a segment to connect the points. Find the midpoint of this segment. Interpret the meaning of the midpoint in this situation.

Real-World Career

Cruise Director
The cruise director is in charge of all onboard entertainment and activities. A professional entertainment background is preferred or 2–5 years experience on board.

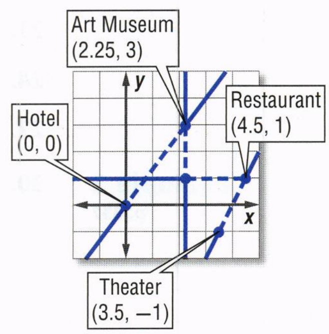
49. **NAVIGATION** Two cruise ships are leaving St. Lucia Island at the same time. One travels 10 miles due east and then 8 miles north. The second ship travels 12 miles due north and then 6 miles west.

a. If St. Lucia is at the origin, how far is the first ship from St. Lucia?

b. How far is the second ship from St. Lucia?

c. How far apart are the ships?

50. **TOURING** Sasha is using the GPS system in her car to go from her hotel to the art museum, to a restaurant, and then to the theater. Sides of grid squares represent 500 feet. Round your answers to the nearest hundredth.



a. How far must she travel from the hotel to the art museum?

b. What is the distance from the art museum to the restaurant?

c. How far is it from the restaurant to the theater?

d. If Sasha gets a direct distance reading from the theater to her hotel, how far is it?

Find the coordinates of the midpoint of the segment with the given endpoints.

51. (4.25, 2.5), (2.5, −3) 52. $\left(5, -\frac{1}{2}\right), \left(-3, \frac{5}{2}\right)$ 53. $\left(\frac{2}{5}, -\frac{1}{5}\right), \left(\frac{1}{3}, \frac{5}{2}\right)$

H.O.T. Problems

Use **H**igher-**O**rder **T**hinking Skills

54. **CHALLENGE** $A(-7, 3)$, $B(4, 0)$, and $C(-4, 4)$ are the vertices of a triangle. Discuss two different ways to determine whether $\triangle ABC$ is a right triangle.

55. **REASONING** Explain why there are usually two possible values when looking for a missing coordinate when you are given two sets of coordinates and the distance between the two points.

56. **REASONING** Is the following statement *true* or *false*? Explain your reasoning.

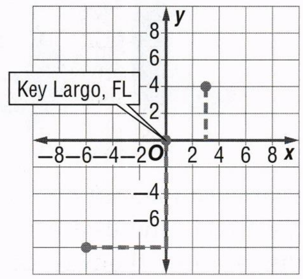
It matters which ordered pair is first when using the Distance Formula.

57. **OPEN ENDED** Plot two points on a coordinate plane and draw the segment between them. Find the coordinates of the midpoint.

58. **WRITING IN MATH** Explain how the Midpoint Formula is related to finding the mean.

Standardized Test Practice

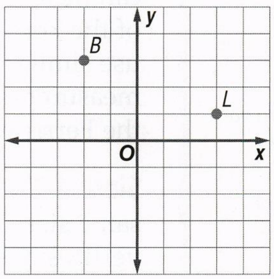
59. SHORT RESPONSE Two sailboats leave Key Largo, Florida, at the same time. One travels east and then north. The other travels south and then west. How far apart are the boats?



60. While in Tokyo, Callie spent 560 yen for a strand of pearls. The cost of the pearls was equivalent to \$35 in U.S. currency. At the time of Callie's purchase, how many yen were equivalent to \$20 in U.S. currency?

A 109 yen
B 320 yen
C 980 yen
D 2350 yen

61. SHORT RESPONSE L represents a lighthouse and B represents a buoy. A ship is at the midpoint between L and B. Which coordinates best represent the ship's position?



F $\left(2, \frac{1}{2}\right)$ **H** $\left(\frac{1}{2}, 2\right)$

G $\left(1, \frac{1}{2}\right)$ **J** $\left(\frac{1}{2}, 5\right)$

62. At a family reunion, Guido cut a slice of cheesecake that was about one sixteenth of the cake. If the entire cheesecake contained 4480 Calories, which is the closest to the number of Calories in Guido's slice?

A 280 **C** 498
B 373 **D** 560

Spiral Review

If c is the measure of the hypotenuse of a right triangle, find each missing measure. If necessary, round to the nearest hundredth. (Lesson 10-5)

63. $a = 16, b = 63, c = ?$

64. $b = 3, a = \sqrt{112}, c = ?$

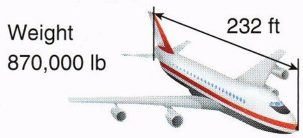
65. $c = 14, a = 9, b = ?$

66. $a = 6, b = 3, c = ?$

67. $b = \sqrt{77}, c = 12, a = ?$

68. $a = 4, b = \sqrt{11}, c = ?$

69. AVIATION The relationship between a plane's length L in feet and the pounds P its wings can lift is described by $L = \sqrt{kP}$, where k is the constant of proportionality. Find k for this plane to the nearest hundredth. (Lesson 10-4)



Skills Review

Solve each proportion. If necessary, round to the nearest hundredth. (Lesson 2-6)

70. $\frac{4}{d} = \frac{2}{10}$

71. $\frac{6}{5} = \frac{f}{15}$

72. $\frac{20}{28} = \frac{h}{21}$

73. $\frac{6}{7} = \frac{7}{j}$

74. $\frac{16}{7} = \frac{9}{m}$

75. $\frac{p}{2} = \frac{45}{68}$

10-7 Similar Triangles

Then

You solved proportions. (Lesson 2-6)

Now

- Determine whether two triangles are similar.
- Find the unknown measures of sides of two similar triangles.

New Vocabulary

similar triangles

Math Online

glencoe.com

- Extra Examples
- Personal Tutor
- Self-Check Quiz
- Homework Help
- Math in Motion

Why?

Simona needs to measure the height of a Ferris wheel for a class project. Simona can measure her shadow and the shadow of the Ferris wheel. She can then use similar triangles and indirect measurement to find the height of the Ferris wheel.

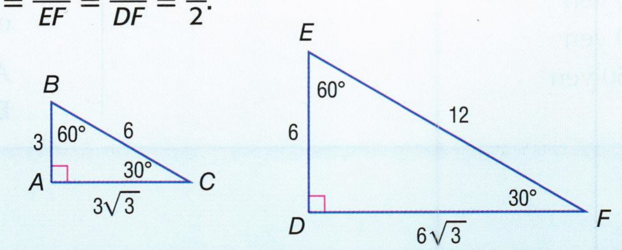
Similar Triangles **Similar triangles** have the same shape, but not necessarily the same size. The symbol ~ is used to denote that two triangles are similar. The vertices of similar triangles are written in order to show the corresponding parts.

Key Concept — Similar Triangles

For Your FOLDABLE

Words If two triangles are similar, then the measures of their corresponding angles are equal, and the measures of their corresponding sides are proportional.

Example If $\triangle ABC \sim \triangle DEF$, then $m\angle A = m\angle D$, $m\angle B = m\angle E$, $m\angle C = m\angle F$, and $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} = \frac{1}{2}$.

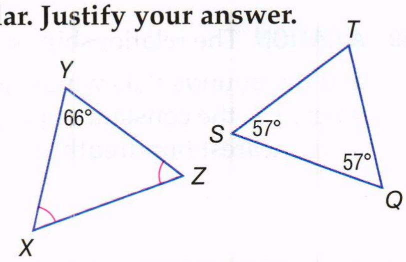


Math *in Motion*, Animation glencoe.com

You can determine whether two triangles are similar by analyzing the corresponding angles. If the corresponding angles have the same measure, then the triangles are similar.

EXAMPLE 1 Determine Whether Two Triangles are Similar

Determine whether the pair of triangles is similar. Justify your answer.



The measure of $\angle T$ is $180 - (57 + 57)$ or $66°$.
In $\triangle XYZ$, $\angle X$ and $\angle Z$ have the same measure.
Let x = the measure of $\angle X$ and $\angle Z$.

$x + x + 66 = 180$	Sum of the angle measures is 180.
$2x = 114$	Subtract 66 from each side.
$x = 57$	Divide each side by 2.

So, $m\angle X = 57°$ and $m\angle Z = 57°$. Since the corresponding angles have equal measures, $\triangle XYZ \sim \triangle STQ$.

Check Your Progress

1. Determine whether $\triangle ABC$ with $m\angle A = 68°$ and $m\angle B = m\angle C$ is similar to $\triangle DEF$ with $m\angle E = m\angle F = 54°$. Justify your answer.

Personal Tutor glencoe.com

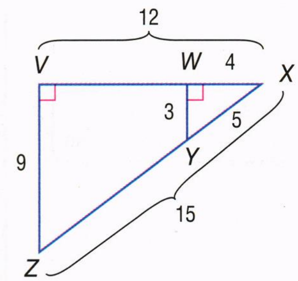
The ratios of the lengths of the corresponding sides can also be compared to show that two triangles are similar.

ReadingMath

Angle Measures The $m\angle A$ is read as *the measure of angle A.*

EXAMPLE 2 Determine Whether Two Triangles are Similar

Determine whether the pair of triangles is similar. Justify your answer.



If $\triangle VXZ$ and $\triangle WXY$ are similar, then the measures of their corresponding sides are proportional.

$\frac{VX}{WX} = \frac{12}{4} = 3 \qquad \frac{XZ}{XY} = \frac{15}{5} = 3 \qquad \frac{VZ}{WY} = \frac{9}{3} = 3$

Since the corresponding sides are proportional, $\triangle VXZ \sim \triangle WXY$.

Check Your Progress

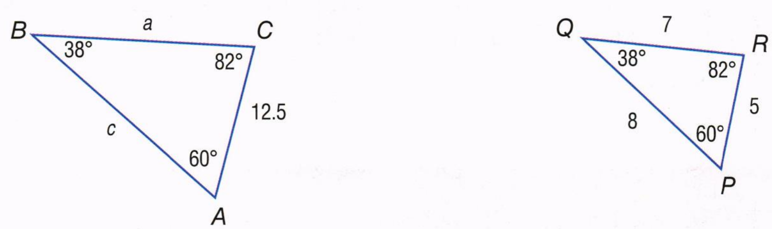
2. Determine whether $\triangle ABC$ with $AB = 6$, $BC = 16$, and $AC = 20$ is similar to $\triangle JKL$ with $JK = 3$, $KL = 8$, and $JL = 9$. Justify your answer.

Personal Tutor glencoe.com

Find Unknown Measures When some of the measurements of the sides of similar triangles are known, proportions can be used to find the missing measures.

EXAMPLE 3 Find Missing Measures

Find the missing measures for the pair of similar triangles.



$\frac{AB}{PQ} = \frac{AC}{PR}$	**Corresponding sides of similar triangles are proportional.**	$\frac{BC}{QR} = \frac{AC}{PR}$	**Corresponding sides of similar triangles are proportional.**
$\frac{c}{8} = \frac{12.5}{5}$	***AB* = *c*, *AC* = 12.5, *PQ* = 8, *PR* = 5**	$\frac{a}{7} = \frac{12.5}{5}$	***BC* = *a*, *AC* = 12.5, *QR* = 7, *PR* = 5**
$5c = 100$	**Find the cross products.**	$5a = 87.5$	**Find the cross products.**
$c = 20$	**Divide each side by 5.**	$a = 17.5$	**Divide each side by 5.**

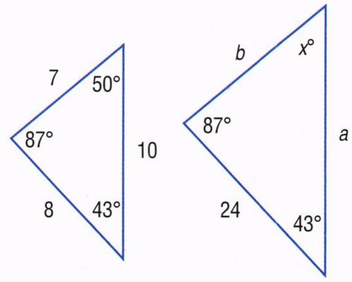
The missing measures are 20 and 17.5.

StudyTip

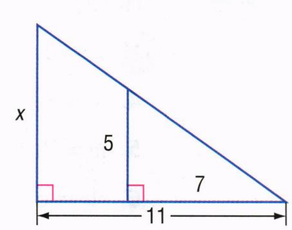
Overlapping Triangles If two triangles overlap, you may want to draw each triangle separately. Make sure the corresponding parts are in the same position and label the corresponding angles and/or sides.

Check Your Progress

3A.



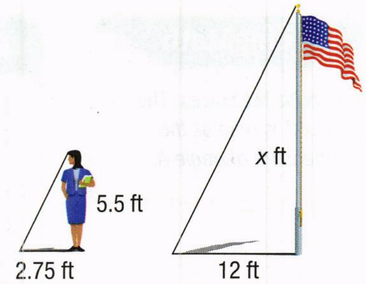
3B.



Personal Tutor glencoe.com

Real-World EXAMPLE 4 Indirect Measurement

SHADOWS Tori is 5 feet 6 inches tall, and her shadow is 2 feet 9 inches long. She is standing next to a flagpole. If the length of the shadow of the flagpole is 12 feet long, how tall is the flagpole?



Math in Motion, BrainPOP® glencoe.com

Understand Find the height of the flagpole.

Plan Make a sketch of the situation.

Solve The Sun's rays form similar triangles. Write a proportion that compares the heights of the objects and the lengths of their shadows.

Let x = the height of the flagpole.

height of flagpole → / Tori's height → $\frac{x}{5.5} = \frac{12}{2.75}$ ← length of the flagpole's shadow / ← Tori's shadow

$$2.75x = 66$$

$$x = 24$$ The height of the flagpole is 24 feet.

Check $\frac{24}{5.5} \stackrel{?}{=} \frac{12}{2.75}$ Substitute 24 for x.

$4.36 = 4.36$ ✓

Check Your Progress

4. **TENTS** The directions for pitching a tent include a scale drawing in which 1 inch represents 4.5 feet. In the drawing, the tent is $1\frac{3}{4}$ inches tall. How tall should the actual tent be?

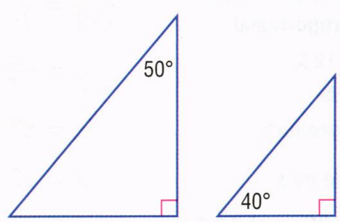
Personal Tutor glencoe.com

Check Your Understanding

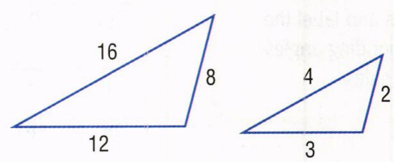
Examples 1 and 2 pp. 642–643

Determine whether each pair of triangles is similar. Justify your answer.

1.

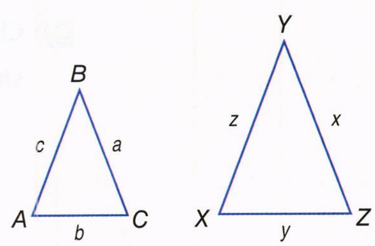


2.



Example 3 p. 643

Find the missing measures for the pair of similar triangles if $\triangle ABC \sim \triangle XYZ$.



3. $a = 4, b = 6, c = 8, x = 6$

4. $x = 9, y = 15, z = 21, c = 7$

5. $a = 2, b = 5, x = 10, z = 30$

6. $b = 6, c = 10, x = 30, y = 15$

Example 4 p. 644

7. **TREES** Marla wants to know the height of the tree in her backyard. The tree casts a shadow 8 feet 6 inches long. Marla is 5 feet tall, and her shadow is 2 feet 6 inches long. How tall is the tree?

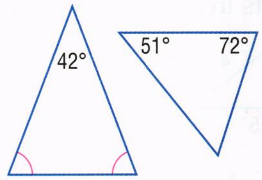
Practice and Problem Solving

= **Step-by-Step Solutions** begin on page R12.
Extra Practice begins on page 815.

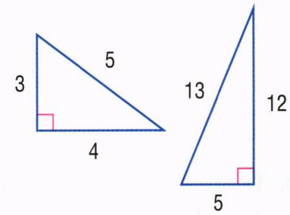
Examples 1 and 2
pp. 642–643

Determine whether each pair of triangles is similar. Justify your answer.

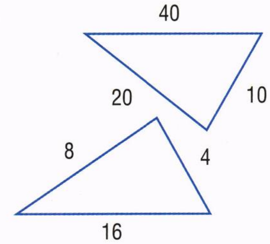
8.



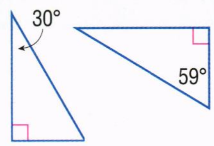
9.



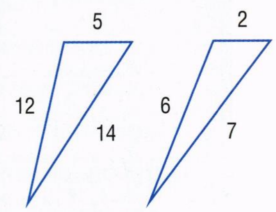
10.



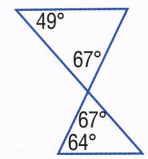
11.



12.

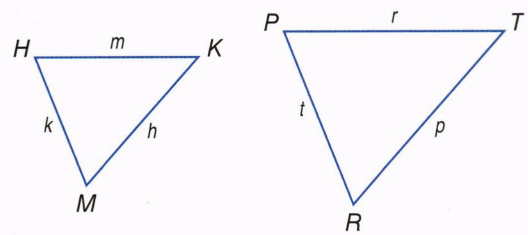


13.



Example 3
p. 643

Find the missing measures for the pair of similar triangles if $\triangle HKM \sim \triangle PTR$.



14. $m = 2, k = 7, h = 6, r = 4$

15. $r = 7.5, p = 15, t = 20, h = 6$

16. $m = 3.5, k = 9, t = 13.5, p = 9.75$

17. $m = 1.4, h = 2.8, p = 0.56, t = 0.84$

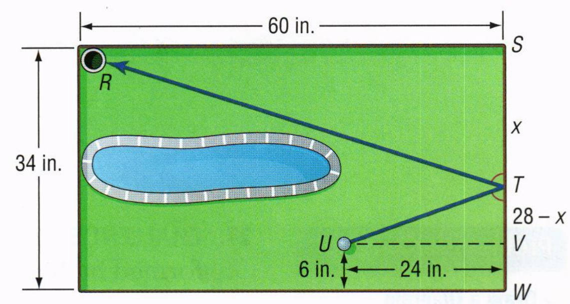
18. $m = \sqrt{7}, h = 2\sqrt{2}, t = 4\sqrt{3}, r = \sqrt{21}$

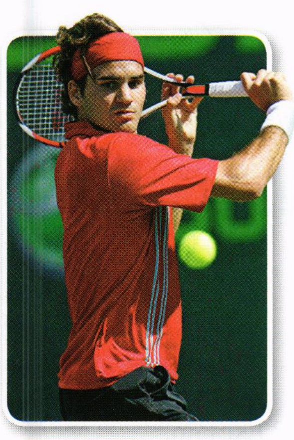
19. $m = \sqrt{2}, k = \sqrt{7}, t = \sqrt{14}, p = \sqrt{10}$

Example 4
p. 644

20. **TOYS** Diecast model cars use a scale of 1 inch : 2 feet of the real vehicle. The original vehicle has a window shaped like a right triangle. If the height of the window on the actual vehicle is 2.5 feet, what will the height of the window be on the model?

21. **GOLF** Beatriz is playing miniature golf on a hole like the one shown at the right. She wants to putt her ball U so that it will bank at T and travel into the hole at R. Use similar triangles to find where Beatriz's ball should strike the wall.





Real-World Link

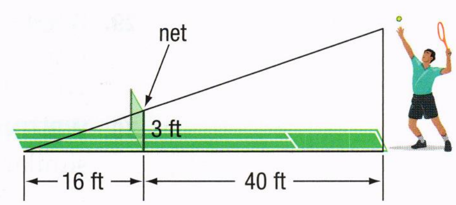
Roger Federer was the number 1 ranked tennis player for four consecutive years beginning in 2004. He was the first player to reach all four Grand Slam finals in back-to-back years.

Source: ATP Tennis

22. **MAPS** The scale on an Ohio map shows that 2.5 centimeters represents 100 miles. The distance on the map from Cleveland to Cincinnati is 5.5 centimeters. About how many miles apart are the two cities?

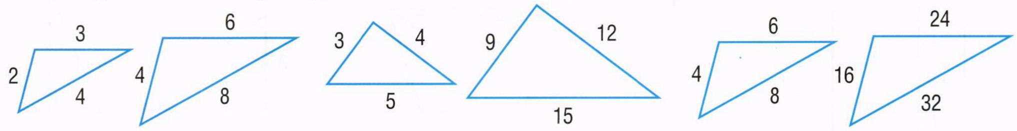
23. **SCHOOL PROJECT** For extra credit in his history class, Marquez plans to make a model of the Statue of Liberty in the scale 1 inch : 10 feet. If the height of the actual Statue of Liberty is 151 feet, what will be the height of the model?

24. **TENNIS** Andy wants to hit the ball just over the net so it will land 16 feet away from the base of the net. If Andy hits the ball 40 feet away from the net, how high does he have to hit the ball?



25 **MULTIPLE REPRESENTATIONS** In this problem, you will compare the ratios of corresponding sides and perimeters of similar triangles.

a. ALGEBRAIC What is the ratio of the corresponding sides of each pair of similar triangles? Record your results in a table like the one below.



b. TABULAR Find the perimeter of each triangle. Then find the ratio of the perimeters for each pair of triangles. Record your results in your table.

Similar Triangles		Ratios of Sides	Perimeters	Ratios of Perimeters
Pair 1	smaller triangle			
	larger triangle			
Pair 2	smaller triangle			
	larger triangle			
Pair 3	smaller triangle			
	larger triangle			

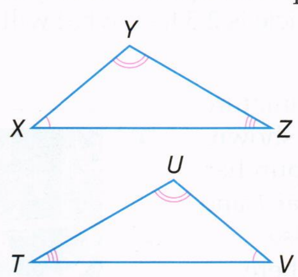
c. ANALYTICAL How is the ratio of the perimeters related to the ratio of the lengths of corresponding sides for each pair of triangles?

d. ANALYTICAL If the ratio of the lengths of corresponding sides of two similar triangles is 1:6, what would be the ratio of their perimeters?

H.O.T. Problems

Use **H**igher-**O**rder **T**hinking Skills

26. FIND THE ERROR Kwam and Rosalinda are comparing the similar triangles. Is either of them correct? Explain.

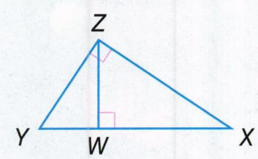


Kwam	Rosalinda
$m\angle X = m\angle T$	$m\angle X = m\angle U$
$m\angle Y = m\angle U$	$m\angle Y = m\angle V$
$m\angle Z = m\angle V$	$m\angle Z = m\angle T$
$\triangle XYZ, \triangle TUV$	$\triangle XYZ, \triangle UVT$

Problem-Solving Tip

Draw a Diagram When a problem involves spatial reasoning, or geometric figures, draw a diagram. For example, in Exercise 27, draw each triangle separately to help determine the answer.

27. CHALLENGE Triangle XYZ is similar to the two triangles formed by the line segment from Z perpendicular to $\overline{XY}$, and these two triangles are similar to each other. Write three similarity statements about these triangles. Why are the triangles similar to each other?



28. REASONING Is the statement *sometimes, always,* or *never* true? Explain.

If the measures of the sides of a triangle are multiplied by 3, then the measures of the angles of the enlarged triangle will have the same measures as the angles of the original triangle.

29. OPEN ENDED Draw and label a triangle ABC. Then draw and label a similar triangle PQR so that the area of $\triangle PQR$ is four times the area of $\triangle ABC$. Explain your strategy.

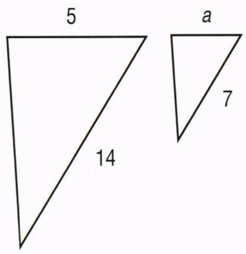
30. WRITING IN MATH Summarize how to determine whether two triangles are similar to each other and how to find missing measures of similar triangles.

Standardized Test Practice

31. Find the distance between the points at (2, −4) and (−5, 8).

A 5
B 7
C $\sqrt{95}$
D $\sqrt{193}$

32. GEOMETRY Find the value of a if the two triangles are similar.

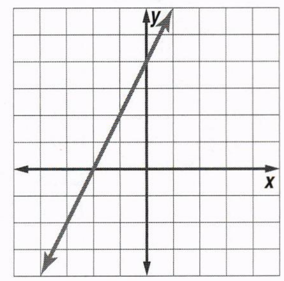


F 2.5 G 10 H 19 J 21

33. Which equation represents a line with a y-intercept of −4 and a slope of 6?

A $y = 6x - 4$
B $y = -4x + 6$
C $y = -6x - 4$
D $y = 6x + 4$

34. SHORT RESPONSE What are the x- and y-intercepts of the function graphed below?



Spiral Review

Find the distance between the points with the given coordinates. (Lesson 10-6)

35. (0, 3), (1, 9)
36. (−2, 4), (5, 13)
37. (1, −5), (−1, −5)
38. (7, −2), (−2, 4)
39. (−6, −3), (−1, 2)
40. (−4, −3), (−7, −8)

Determine whether the measures can be the lengths of the sides of a right triangle. (Lesson 10-5)

41. 3, 4, 5
42. 8, 10, 12
43. 10, 24, 26
44. 5, 12, 13
45. 6, 9, 14
46. 4, 5, 6

47. NUTRITION In the function $y = 0.059x^2 - 7.423x + 362.1$, y represents the consumption of bread and cereal in pounds per person in the United States, and x represents the number of years since 1900. If this trend continues, in what future year will the average American consume 300 pounds of bread and cereal? (Lesson 9-4)

Factor each polynomial, if possible. If the polynomial cannot be factored, write *prime*. (Lesson 8-6)

48. $4k^2 - 100$
49. $4a^2 - 36b^2$
50. $x^2 + 6x - 9$
51. $50g^2 + 40g + 8$
52. $9t^3 + 66t^2 - 48t$
53. $20n^2 + 34n + 6$

54. DRIVING Average speed is calculated by dividing distance by time. If the speed limit on an interstate is 65 miles per hour, how far can a person travel legally in $1\frac{1}{2}$ hours? (Lesson 5-2)

Skills Review

Evaluate if $a = 3$, $b = -2$, and $c = 6$. (Lesson 1-2)

55. $\frac{b}{c}$
56. $\frac{2ab}{c}$
57. $\frac{ac}{-4b}$
58. $\frac{-3ac}{2b}$
59. $\frac{-2bc}{a}$

EXPLORE **10-8**

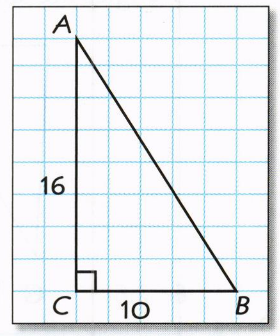
Algebra Lab
Investigating Trigonometric Ratios

Math Online glencoe.com
Math *in Motion,* Animation

Objective
Investigate trigonometric ratios.

You can use paper triangles to investigate the ratios of the lengths of sides of right triangles.

Collect the Data



Step 1 Use a ruler and grid paper to draw several right triangles with legs in a ratio of 5:8. Include right triangles with the side lengths listed in the table below and several more right triangles similar to these three. Label the vertices of each triangle as *A*, *B*, and *C*, where *C* is at the right angle, *B* is opposite the longest leg, and *A* is opposite the shortest leg.

Step 2 Copy the table below. Complete the first three columns by measuring the hypotenuse (side $\overline{AB}$) in each right triangle you created and recording its length to the nearest tenth.

Step 3 Calculate and record the ratios in the middle two columns. Round to the nearest hundredth.

Step 4 Use a protractor to carefully measure angles *A* and *B* to the nearest degree in each right triangle. Record the angle measures in the table.

Side Lengths			Ratios		Angle Measures		
side *BC*	side *AC*	side *AB*	$\frac{BC}{AC}$	$\frac{BC}{AB}$	angle *A*	angle *B*	angle *C*
2.5	4						90°
5	8						90°
10	16						90°
							90°
							90°
							90°

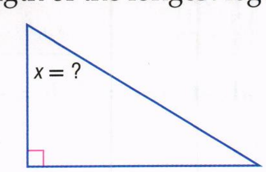
Analyze the Results

1. Examine the measures and ratios in the table. What do you notice? Write a sentence or two to describe any patterns you see.

Make a Conjecture

2. For any right triangle similar to the ones you have drawn here, what will be the value of the ratio of the length of the shortest leg to the length of the longest leg?

3. If you draw a right triangle and calculate the ratio of the length of the shortest leg to the length of the hypotenuse to be approximately 0.53, what will be the measure of the larger acute angle in the right triangle?



10-8 Trigonometric Ratios

Then

You used the Pythagorean Theorem (Lesson 10-5)

Now

- Find trigonometric ratios of angles.
- Use trigonometry to solve triangles.

New Vocabulary

trigonometry
trigonometric ratio
sine
cosine
tangent
solving the triangle

Math Online

glencoe.com

- Extra Examples
- Personal Tutor
- Self-Check Quiz
- Homework Help

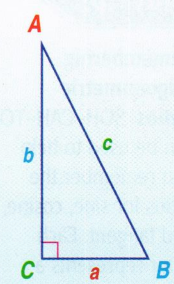
Why?

If a road has a percent grade of 8%, this means the road rises or falls 8 feet over a horizontal distance of 100 feet. Trigonometric ratios can be used to determine the angle that the road rises or falls.

Trigonometric Ratios **Trigonometry** is the study of relationships among the angles and sides of triangles. A **trigonometric ratio** is a ratio that compares the side lengths of two sides of a right triangle. The three most common trigonometric ratios, **sine**, **cosine**, and **tangent**, are described below.

Key Concept — Trigonometric Ratios

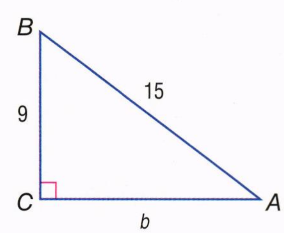
For Your FOLDABLE

Words	Symbols	Model
sine of $\angle A = \dfrac{\text{leg opposite } \angle A}{\text{hypotenuse}}$	$\sin A = \dfrac{a}{c}$	
cosine of $\angle A = \dfrac{\text{leg adjacent to } \angle A}{\text{hypotenuse}}$	$\cos A = \dfrac{b}{c}$	
tangent of $\angle A = \dfrac{\text{leg opposite } \angle A}{\text{leg adjacent to } \angle A}$	$\tan A = \dfrac{a}{b}$	

Opposite, adjacent, and hypotenuse are abbreviated *opp*, *adj*, and *hyp*, respectively.

EXAMPLE 1 Find Sine, Cosine, and Tangent Ratios

Find the values of the three trigonometric ratios for angle A.



Step 1 Use the Pythagorean Theorem to find AC.

$a^2 + b^2 = c^2$ **Pythagorean Theorem**

$9^2 + b^2 = 15^2$ **$a = 9$ and $c = 15$**

$81 + b^2 = 225$ **Simplify.**

$b^2 = 144$ **Subtract 81 from each side.**

$b = 12$ **Take the square root of each side.**

Step 2 Use the side lengths to write the trigonometric ratios.

$$\sin A = \frac{\text{opp}}{\text{hyp}} = \frac{9}{15} = \frac{3}{5} \qquad \cos A = \frac{\text{adj}}{\text{hyp}} = \frac{12}{15} = \frac{4}{5} \qquad \tan A = \frac{\text{opp}}{\text{adj}} = \frac{9}{12} = \frac{3}{4}$$

Check Your Progress

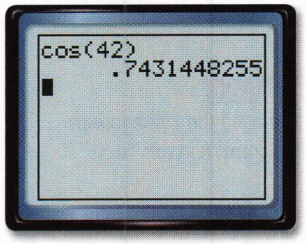
1. Find the values of the three trigonometric ratios for angle B.

Personal Tutor glencoe.com

EXAMPLE 2 Use a Calculator to Evaluate Expressions

Use a calculator to find cos 42° to the nearest ten-thousandth.

KEYSTROKES: [COS] 42 [ )] [ENTER]



Rounded to the nearest ten-thousandth, $\cos 42° \approx 0.7431$.

Check Your Progress

2A. $\sin 31°$ **2B.** $\tan 76°$ **2C.** $\cos 55°$

Personal Tutor glencoe.com

Use Trigonometric Ratios When you find all unknown measures of the sides and angles of a right triangle, you are **solving the triangle**. You can find the missing measures if you know the measure of two sides of the triangle or the measure of one side and the measure of one acute angle.

StudyTip

Remembering Trigonometric Ratios SOH–CAH–TOA can be used to help you remember the ratios for sine, cosine, and tangent. Each letter represents a word.

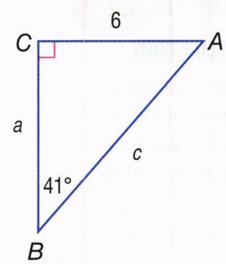
$\sin A = \frac{\text{opp}}{\text{hyp}}$

$\cos A = \frac{\text{adj}}{\text{hyp}}$

$\tan A = \frac{\text{opp}}{\text{adj}}$

EXAMPLE 3 Solve a Triangle

Solve the right triangle. Round each side length to the nearest tenth.



Step 1 Find the measure of $\angle A$. $180° - (90° + 41°) = 49°$
The measure of $\angle A = 49°$.

Step 2 Find a. Since you are given the measure of the side opposite $\angle B$ and are finding the measure of the side adjacent to $\angle B$, use the tangent ratio.

$\tan 41° = \frac{6}{a}$ **Definition of tangent**

$a \tan 41° = 6$ **Multiply each side by a.**

$a = \frac{6}{\tan 41°}$ or about 6.9 **Divide each side by tan 41°. Use a calculator.**

So the measure of a or $\overline{BC}$ is about 6.9.

Step 3 Find c. Since you are given the measure of the side opposite $\angle B$ and are finding the measure of the hypotenuse, use the sine ratio.

$\sin 41° = \frac{6}{c}$ **Definition of sine**

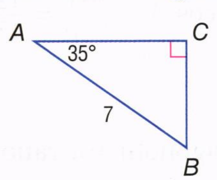
$c \sin 41° = 6$ **Multiply each side by c.**

$c = \frac{6}{\sin 41°}$ or about 9.1 **Divide each side by sin 41°. Use a calculator.**

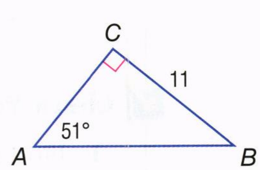
So the measure of c or $\overline{AB}$ is about 9.1.

Check Your Progress

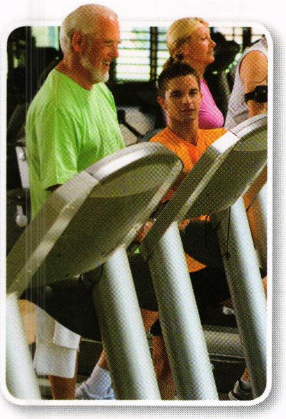
3A.



3B.

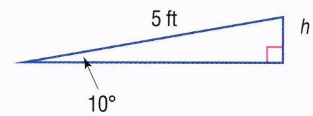


Personal Tutor glencoe.com



Real-World EXAMPLE 4 Find a Missing Side Length

EXERCISE A trainer sets the incline on a treadmill to 10°. The walking surface of the treadmill is 5 feet long. About how many inches is the end of the treadmill from the floor?



$\sin 10° = \frac{h}{5}$ **Definition of sine**

$5 \cdot \sin 10° = h$ **Multiply each side by 5.**

$0.87 \approx h$ **Use a calculator.**

The value of h is in feet. Multiply 0.87 by 12 to convert feet to inches. The trainer raised the treadmill about 10.4 inches.

Real-World Link

For optimum health, all adults ages 18–65 should get at least 30 minutes of moderately intense activity five days per week.

Source: American Heart Association

Check Your Progress

4. **SKATEBOARDING** The angle that a skateboarding ramp forms with the ground is 25° and the height of the ramp is 6 feet. Determine the length of the ramp.

Personal Tutor glencoe.com

A trigonometric function has a rule given by a trigonometric ratio. If you know the sine, cosine, or tangent of an acute angle, you can use the *inverse* of the trigonometric function to find the measure of the angle.

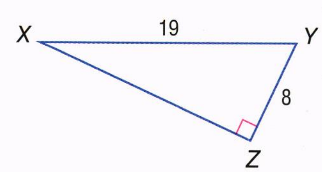
Key Concept Inverse Trigonometric Functions

For Your FOLDABLE

Words	If $\angle A$ is an acute angle and the sine of A is x, then the **inverse sine** of x is the measure of $\angle A$.
Symbols	If $\sin A = x$, then $\sin^{-1} x = m\angle A$.
Words	If $\angle A$ is an acute angle and the cosine of A is x, then the **inverse cosine** of x is the measure of $\angle A$.
Symbols	If $\cos A = x$, then $\cos^{-1} x = m\angle A$.
Words	If $\angle A$ is an acute angle and the tangent of A is x, then the **inverse tangent** of x is the measure of $\angle A$.
Symbols	If $\tan A = x$, then $\tan^{-1} x = m\angle A$.

EXAMPLE 5 Find a Missing Angle Measure

Find $m\angle Y$ to the nearest degree.



You know the measure of the side adjacent to $\angle Y$ and the measure of the hypotenuse. Use the cosine ratio.

$\cos Y = \frac{8}{19}$ **Definition of cosine**

Use a calculator and the [$\cos^{-1}$] function to find the measure of the angle.

KEYSTROKES: 2nd [$\cos^{-1}$] 8 ÷ 19) ENTER 65.098937 So, $m\angle Y = 65°$.

Check Your Progress

5. Find $m\angle X$ to the nearest degree if $XY = 14$ and $YZ = 5$.

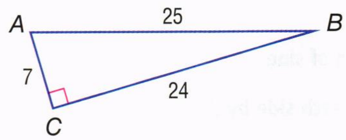
Personal Tutor glencoe.com

Check Your Understanding

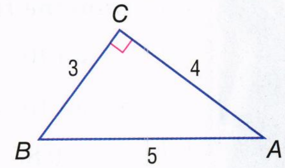
Example 1 p. 649

Find the values of the three trigonometric ratios for angle A.

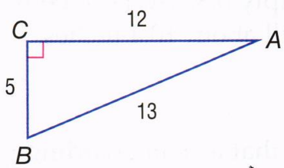
1.



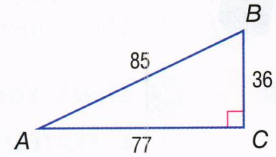
2.



3.



4.



Example 2 p. 650

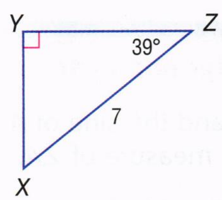
Use a calculator to find the value of each trigonometric ratio to the nearest ten-thousandth.

5. sin 37° **6.** cos 23° **7.** tan 14° **8.** cos 82°

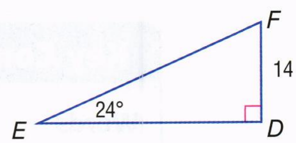
Example 3 p. 650

Solve each right triangle. Round each side length to the nearest tenth.

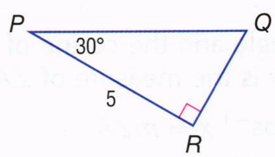
9.



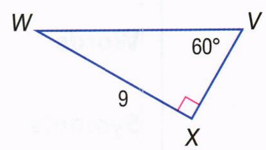
10.



11.

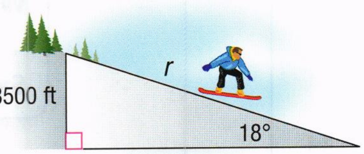


12.



Example 4 p. 651

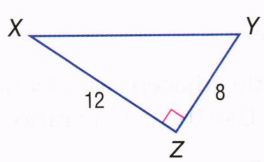
13. **SNOWBOARDING** A hill used for snowboarding has a vertical drop of 3500 feet. The angle the run makes with the ground is 18°. Estimate the length of r.



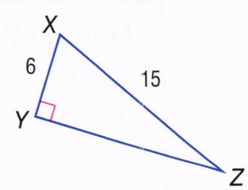
Example 5 p. 651

Find m∠X for each right triangle to the nearest degree.

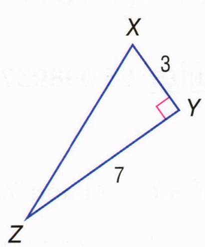
14.



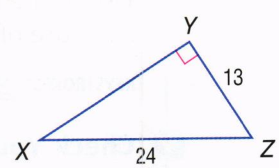
15.



16.



17.



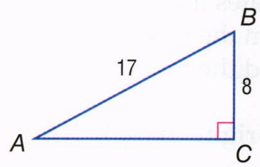
Practice and Problem Solving

● = **Step-by-Step Solutions** begin on page R12.
Extra Practice begins on page 815.

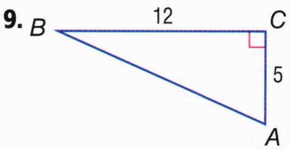
Example 1 p. 649

Find the values of the three trigonometric ratios for angle B.

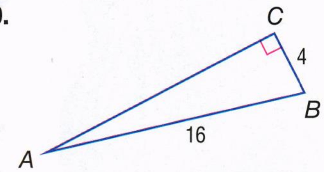
18.



19.



20.



Example 2 p. 650

Use a calculator to find the value of each trigonometric ratio to the nearest ten-thousandth.

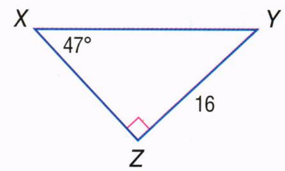
21. tan 2° **22.** sin 89° **23.** cos 44° **24.** tan 45°

25. sin 73° **26.** cos 90° **27.** sin 30° **28.** tan 60°

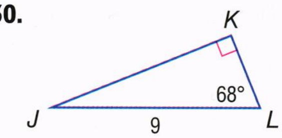
Example 3 p. 650

Solve each right triangle. Round each side length to the nearest tenth.

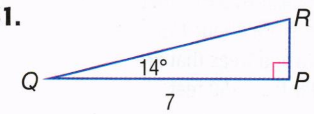
29.



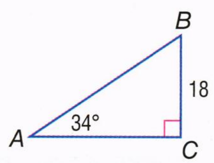
30.



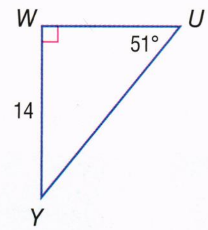
31.



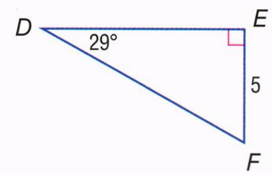
32.



33.

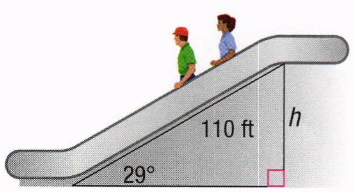


34.



Example 4 p. 651

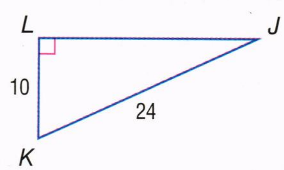
35. ESCALATORS At a local mall, an escalator is 110 feet long. The angle the escalator makes with the ground is 29°. Find the height reached by the escalator.



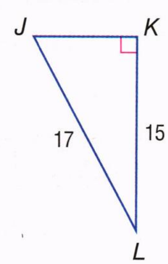
Example 5 p. 651

Find $m\angle J$ for each right triangle to the nearest degree.

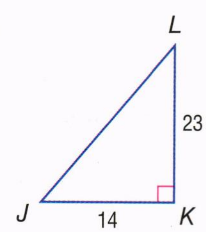
36.



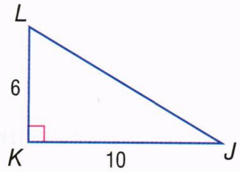
37.



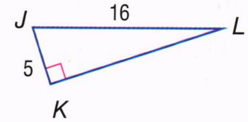
38.



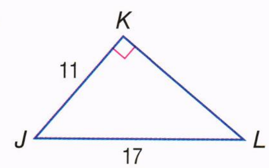
39.



40.



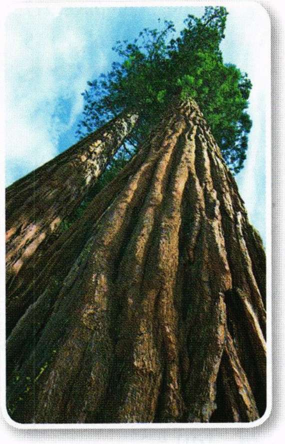
41.



42. MONUMENTS The Lincoln Memorial building measures 204 feet long, 134 feet wide, and 99 feet tall. Chloe is looking at the top of the monument at an angle of 55°. How far away is she standing from the monument?

43 **AIRPLANES** Ella looks down at a city from an airplane window. The airplane is 5000 feet in the air, and she looks down at an angle of 8°. Determine the horizontal distance to the city.

44. **FORESTS** A forest ranger estimates the height of a tree is about 175 feet. If the forest ranger is standing 100 feet from the base of the tree, what is the measure of the angle formed by the ranger and the top of the tree?



Real-World Link

The tallest living tree is a redwood called Hyperion. It measures 378.1 feet. There are about 135 redwood trees that are taller than 350 feet.

Source: SFGate

Suppose $\angle A$ is an acute angle of right triangle ABC.

45. Find $\sin A$ and $\tan A$ if $\cos A = \frac{3}{4}$.

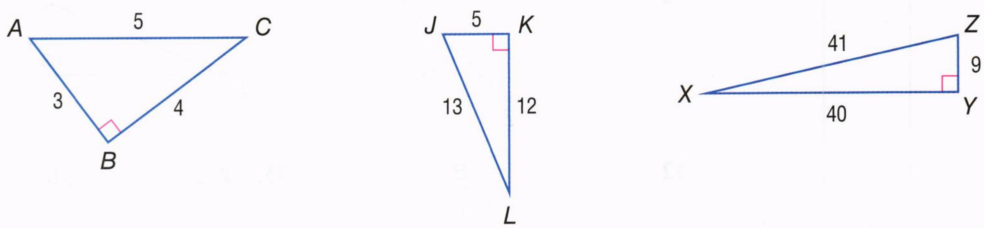
46. Find $\tan A$ and $\cos A$ if $\sin A = \frac{2}{7}$.

47. Find $\cos A$ and $\tan A$ if $\sin A = \frac{1}{4}$.

48. Find $\sin A$ and $\cos A$ if $\tan A = \frac{5}{3}$.

49. **SUBMARINES** A submarine descends into the ocean at an angle of 10° below the water line and travels 3 miles diagonally. How far beneath the surface of the water has the submarine reached?

50. **MULTIPLE REPRESENTATIONS** In this problem, you will explore a relationship between the sine and cosine functions.



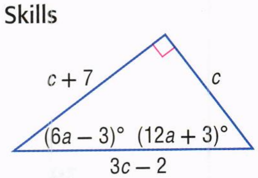
a. **TABULAR** Copy and complete the table using the triangles shown above.

Triangle	Trigonometric Ratios		$\sin^2$	$\cos^2$	$\sin^2 + \cos^2 =$
ABC	$\sin A =$	$\cos A =$	$\sin^2 A =$	$\cos^2 A =$	
	$\sin C =$	$\cos C =$	$\sin^2 C =$	$\cos^2 C =$	
JKL	$\sin J =$	$\cos J =$	$\sin^2 J =$	$\cos^2 J =$	
	$\sin L =$	$\cos L =$	$\sin^2 L =$	$\cos^2 L =$	
XYZ	$\sin X =$	$\cos X =$	$\sin^2 X =$	$\cos^2 X =$	
	$\sin Z =$	$\cos Z =$	$\sin^2 Z =$	$\cos^2 Z =$	

b. **VERBAL** Make a conjecture about the sum of the squares of the sine and cosine functions of an acute angle in a right triangle.

H.O.T. Problems

Use Higher-Order Thinking Skills



51. **CHALLENGE** Solve the triangle shown.

52. **REASONING** Use the definitions of the sine and cosine ratios to define the tangent ratio.

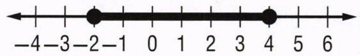
53. **OPEN ENDED** Write a problem that uses the cosine ratio to find the measure of an unknown angle in a triangle. Then solve the problem.

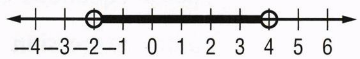
54. **REASONING** The sine and cosine of an acute angle in a right triangle are equal. What can you conclude about the triangle?

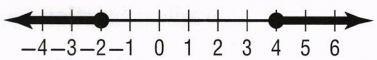
55. **WRITING IN MATH** Explain how to use trigonometric ratios to find the missing length of a side of a right triangle given the measure of one acute angle and the length of one side.

Standardized Test Practice

56. Which graph below represents the solution set for $-2 \leq x \leq 4$?

A 

B 

C 

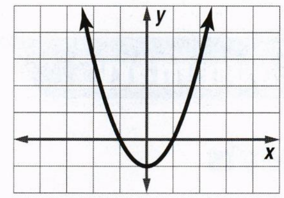
D 

57. PROBABILITY Suppose one chip is chosen from a bin with the chips shown. To the nearest tenth, what is the probability that a green chip is chosen?

Color	Number
yellow	7
blue	9
orange	3
green	5
red	6

F 0.2 H 0.6
G 0.5 J 0.8

58. In the graph, for what value(s) of x is $y = 0$?



A 0 C 1
B –1 D 1 and –1

59. EXTENDED RESPONSE A 16-foot ladder is placed against the side of a house so that the bottom of the ladder is 8 feet from the base of the house.

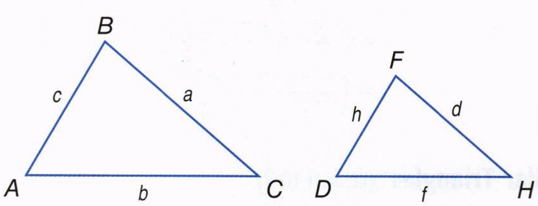
a. If the bottom of the ladder is moved closer to the base of the house, does the height reached by the ladder increase or decrease?

b. What conclusion can you make about the distance between the bottom of the ladder and the base of the house and the height reached by the ladder?

c. How high does the ladder reach if the ladder is 3 feet from the base of the house?

Spiral Review

For each set of measures given, find the measures of the missing sides if $\triangle ABC \sim \triangle DFH$. (Lesson 10-7)



60. $a = 16, b = 12, c = 8, f = 6$

61. $d = 9, f = 6, h = 4, b = 18$

62. $a = 36, b = 21, h = 11, f = 14$

63. $c = 22.5, b = 20, h = 9, d = 2$

Find the coordinates of the midpoint of the segment with the given endpoints. (Lesson 10-6)

64. $(5, 3), (11, 9)$ **65.** $(8, 2), (6, 4)$ **66.** $(-1, 7), (13, -3)$

67. FINANCIAL LITERACY A salesperson is paid $32,000 a year plus 5% of the amount in sales made. What is the amount of sales needed to have an annual income greater than $45,000? (Lesson 5-3)

Skills Review

Solve each proportion. (Lesson 2-6)

68. $\frac{8}{9} = \frac{6}{z}$ **69.** $\frac{p}{6} = \frac{4}{3}$ **70.** $\frac{0.3}{r} = \frac{0.9}{1.7}$ **71.** $\frac{0.6}{1.1} = \frac{y}{8.47}$

Study Guide and Review

Math Online glencoe.com
• STUDY TO GO
• Vocabulary Review

Chapter Summary

Key Concepts

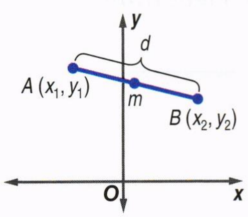
Simplifying Radical Expressions (Lesson 10-2)

- A radical expression is in simplest form when
  - no radicands have perfect square factors other than 1,
  - no radicals contain fractions,
  - and no radicals appear in the denominator of a fraction.

Operations with Radical Expressions and Equations (Lessons 10-3 and 10-4)

- Radical expressions with like radicals can be added or subtracted.
- Use the FOIL method to multiply radical expressions.

Pythagorean Theorem, Distance Formula, and Midpoint Formula (Lessons 10-5 and 10-6)



$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$m = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

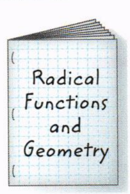
Similar Triangles (Lesson 10-7)

- Similar triangles have congruent corresponding angles and proportional corresponding sides.

If $\triangle ABC \sim \triangle DEF$, then $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$.

FOLDABLES® Study Organizer

Be sure the Key Concepts are noted in your Foldable.



Key Vocabulary

Vocabulary Check

State whether each sentence is *true* or *false*. If *false*, replace the underlined word, phrase, expression, or number to make a true sentence.

1. A triangle with sides having measures of 3, 4, and 6 is a right triangle.
2. Two triangles are congruent if corresponding angles are congruent.
3. The expressions $2 + \sqrt{5}$ and $2 - \sqrt{5}$ are conjugates.
4. In the expression $-5\sqrt{2}$, the radicand is 2.
5. The shortest side of a right triangle is the hypotenuse.
6. The cosine of an angle is found by dividing the measure of the side opposite the angle by the hypotenuse.
7. The domain of the function $y = \sqrt{x}$ is $\{x \mid x \leq 0\}$.
8. After the first step in solving $\sqrt{2x + 4} = x + 5$, you would have $2x + 4 = x^2 + 10x + 25$
9. The converse of the Pythagorean Theorem is true.
10. The range of the function $y = \sqrt{x}$ is $\{y \mid > 0\}$.

Lesson-by-Lesson Review

10-1 Radical Functions (pp. 605–610)

Graph each function. Compare to the parent graph. State the domain and range.

11. $y = \sqrt{x} - 3$

12. $y = \sqrt{x} + 2$

13. $y = -5\sqrt{x}$

14. $y = \sqrt{x} - 6$

15. $y = \sqrt{x - 1}$

16. $y = \sqrt{x} + 5$

17. **GEOMETRY** The function $s = \sqrt{A}$ can be used to find the length of a side of a square given its area. Use this function to determine the length of a side of a square with an area of 90 square inches. Round to the nearest tenth if necessary.

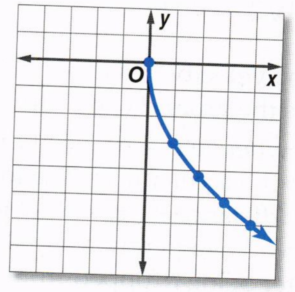
EXAMPLE 1

Graph $y = -3\sqrt{x}$. Compare to the parent graph. State the domain and range.

Make a table. Choose nonnegative values for x.

x	0	1	2	3	4
y	0	−3	≈−4.2	≈−5.2	−6

Plot points and draw a smooth curve.



The graph of $y = \sqrt{x}$ is stretched vertically and is reflected across the x-axis.

The domain is $\{x \mid x \geq 0\}$.

The range is $\{y \mid y \leq 0\}$.

10-2 Simplifying Radical Expressions (pp. 612–617)

Simplify.

18. $\sqrt{36x^2y^7}$

19. $\sqrt{20ab^3}$

20. $\sqrt{3} \cdot \sqrt{6}$

21. $2\sqrt{3} \cdot 3\sqrt{12}$

22. $(4 - \sqrt{5})^2$

23. $(1 + \sqrt{2})^2$

24. $\sqrt{\frac{50}{a^2}}$

25. $\sqrt{\frac{2}{5}} \cdot \sqrt{\frac{3}{4}}$

26. $\frac{3}{2 - \sqrt{5}}$

27. $\frac{5}{\sqrt{7} + 6}$

28. **WEATHER** To estimate how long a thunderstorm will last, use $t = \sqrt{\frac{d^3}{216}}$, where t is the time in hours and d is the diameter of the storm in miles. A storm is 10 miles in diameter. How long will it last?

EXAMPLE 2

Simplify $\frac{2}{4 + \sqrt{3}}$.

$$\frac{2}{4 + \sqrt{3}}$$

$$= \frac{2}{4 + \sqrt{3}} \cdot \frac{4 - \sqrt{3}}{4 - \sqrt{3}} \quad \text{Rationalize the denominator.}$$

$$= \frac{2(4) - 2\sqrt{3}}{4^2 - (\sqrt{3})^2} \quad (a - b)(a + b) = a^2 - b^2$$

$$= \frac{8 - 2\sqrt{3}}{16 - 3} \quad (\sqrt{3})^2 = 3$$

$$= \frac{8 - 2\sqrt{3}}{13} \quad \text{Simplify.}$$

CHAPTER 10

Study Guide and Review

10-3 Operations with Radical Expressions (pp. 619–623)

Simplify each expression.

29. $\sqrt{6} - \sqrt{54} + 3\sqrt{12} + 5\sqrt{3}$

30. $2\sqrt{6} - \sqrt{48}$

31. $4\sqrt{3x} - 3\sqrt{3x} + 3\sqrt{3x}$

32. $\sqrt{50} + \sqrt{75}$

33. $\sqrt{2}(5 + 3\sqrt{3})$

34. $(2\sqrt{3} - \sqrt{5})(\sqrt{10} + 4\sqrt{6})$

35. $(6\sqrt{5} + 2)(4\sqrt{2} + \sqrt{3})$

36. **MOTION** The velocity of a dropped object when it hits the ground can be found using $v = \sqrt{2gd}$, where v is the velocity in feet per second, g is the acceleration due to gravity, and d is the distance in feet the object drops. Find the speed of a penny when it hits the ground, after being dropped from 984 feet. Use 32 feet per second squared for g.

EXAMPLE 3

Simplify $2\sqrt{6} - \sqrt{24}$.

$2\sqrt{6} - \sqrt{24} = 2\sqrt{6} - \sqrt{4 \cdot 6}$	Product Property
$= 2\sqrt{6} - 2\sqrt{6}$	Simplify.
$= 0$	Simplify.

EXAMPLE 4

Simplify $(\sqrt{3} - \sqrt{2})(\sqrt{3} + 2\sqrt{2})$.

$$(\sqrt{3} - \sqrt{2})(\sqrt{3} + 2\sqrt{2})$$
$$= (\sqrt{3})(\sqrt{3}) + (\sqrt{3})(2\sqrt{2}) + (-\sqrt{2})(\sqrt{3}) + (\sqrt{2})(2\sqrt{2})$$
$$= 3 + 2\sqrt{6} - \sqrt{6} + 4$$
$$= 7 + \sqrt{6}$$

10-4 Radical Equations (pp. 624–628)

Solve each equation. Check your solution.

37. $10 + 2\sqrt{x} = 0$

38. $\sqrt{5 - 4x} - 6 = 7$

39. $\sqrt{a + 4} = 6$

40. $\sqrt{3x} = 2$

41. $\sqrt{x + 4} = x - 8$

42. $\sqrt{3x - 14} + x = 6$

43. **FREE FALL** Assuming no air resistance, the time t in seconds that it takes an object to fall h feet can be determined by $t = \frac{\sqrt{h}}{4}$. If a skydiver jumps from an airplane and free falls for 10 seconds before opening the parachute, how many feet does she free fall?

EXAMPLE 5

Solve $\sqrt{7x + 4} - 18 = 5$.

$\sqrt{7x + 4} - 18 = 5$	Original equation
$\sqrt{7x + 4} = 23$	Add 18 to each side.
$(\sqrt{7x + 4})^2 = 23^2$	Square each side.
$7x + 4 = 529$	Simplify.
$7x = 525$	Subtract 4 from each side.
$x = 75$	Divide each side by 7.

CHECK

$\sqrt{7x + 4} - 18 = 5$	Original equation
$\sqrt{7(75) + 4} - 18 \stackrel{?}{=} 5$	$x = 75$
$\sqrt{525 + 4} - 18 \stackrel{?}{=} 5$	Multiply.
$\sqrt{529} - 18 \stackrel{?}{=} 5$	Add.
$23 - 18 \stackrel{?}{=} 5$	Simplify.
$5 = 5$ ✓	True.

MIXED PROBLEM SOLVING
For mixed problem-solving practice, see page 845.

10-5 The Pythagorean Theorem (pp. 630–635)

Determine whether each set of measures can be the lengths of the sides of a right triangle.

44. 6, 8, 10
45. 3, 4, 5
46. 12, 16, 21
47. 10, 12, 15
48. 2, 3, 4
49. 7, 24, 25
50. 5, 12, 13
51. 15, 19, 23

52. LADDER A ladder is leaning on a building. The base of the ladder is 10 feet from the building, and the ladder reaches up 15 feet on the building. How long is the ladder?

EXAMPLE 6

Determine whether the set of measures 12, 16, and 20 can be the lengths of the sides of a right triangle.

$a^2 + b^2 = c^2$	Pythagorean Theorem
$12^2 + 16^2 \stackrel{?}{=} 20^2$	$a = 12, b = 16$, and $c = 20$
$144 + 256 \stackrel{?}{=} 400$	Multiply.
$400 = 400$ ✓	Add.

The measures can be the lengths of the sides of a right triangle.

10-6 The Distance and Midpoint formulas (pp. 636–641)

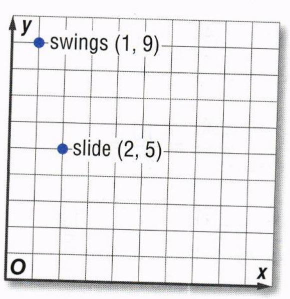
Find the distance between points with the given coordinates and the midpoint of the segment with the given endpoints. Round to the nearest hundredth if necessary.

53. (2, 4), (−3, 4)
54. (−1, −3), (3, 5)
55. (−6, 7), (0, 0)
56. (1, 5), (−4, −5)

Find the possible values for a if the points with the given coordinates are the indicated distance apart.

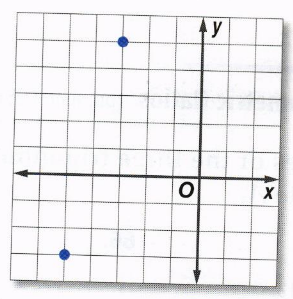
57. $(5, -2), (a, -3); d = \sqrt{170}$
58. $(1, a), (-3, 2); d = 5$

59. PLAYGROUND How far apart in feet are the swings from the slide?



EXAMPLE 7

Find the distance between (−3, 5) and (−5, −3) and the midpoint of the segment with those endpoints. Round to the nearest tenth if necessary.



$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	Distance Formula
$= \sqrt{[-3 - (-5)]^2 + [5 - (-3)]^2}$	$(x_1, y_1) = (-5, -3)$, $(x_2, y_2) = (-3, 5)$
$= \sqrt{2^2 + 8^2}$	Simplify.
$= \sqrt{4 + 64}$	Evaluate squares.
≈ 8.2	Simplify.
$M = \left(\frac{-5 + (-3)}{2}, \frac{-3 + 5}{2}\right)$	Midpoint Formula
$= (-4, 1)$	Simplify.

CHAPTER 10 Study Guide and Review

10-7 Similar Triangles (pp. 642–647)

Find the missing measures for the pair of similar triangles if $\triangle ABC \sim \triangle DEF$.

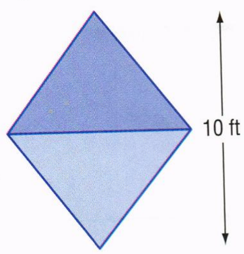
60. $a = 3, b = 4, c = 5, d = 12$

61. $a = 3, b = 4, c = 5, d = 4.5$

62. $a = 4, b = 8, c = 11, e = 4$

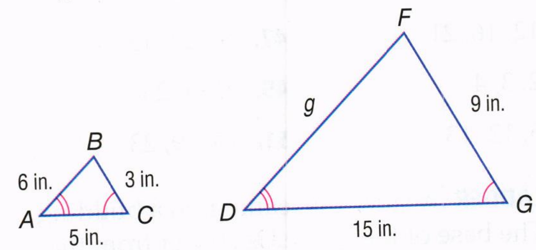
63. $a = 5, b = 7, c = 9, f = 18$

64. **MODELS** Kristin is making a model of the artwork shown in the scale of 1 inch = 2 feet. If the height of the artwork is 10 feet, what will the height of the model be?



EXAMPLE 8

Find the missing length for the pair of triangles if $\triangle ABC \sim \triangle DFG$.



$\frac{AB}{DF} = \frac{BC}{FG}$ Corresponding sides of similar triangles are proportional.

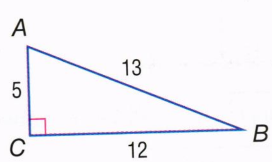
$\frac{6}{g} = \frac{3}{9}$ $AB = 6$, $BC = 3$, $DF = g$, and $FG = 9$

$18 = g$ Find the cross products and simplify.

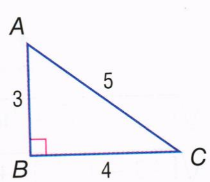
10-8 Trigonometric Ratios (pp. 649–655)

Find the values of the three trigonometric ratios for angle A.

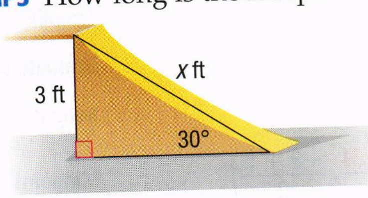
65.



66.

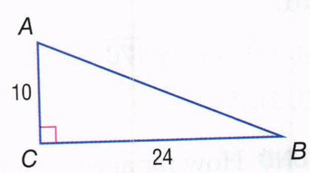


67. **RAMPS** How long is the ramp?



EXAMPLE 9

Find the values of the three trigonometric ratios for angle A.



Find the hypotenuse: $c^2 = 10^2 + 24^2$, so $c = 26$.

$$\sin A = \frac{\text{leg opposite } \angle A}{\text{hypotenuse}} = \frac{24}{26} = \frac{12}{13}$$

$$\cos A = \frac{\text{leg adjacent } \angle A}{\text{hypotenuse}} = \frac{10}{26} = \frac{5}{13}$$

$$\tan A = \frac{\text{leg opposite } \angle A}{\text{leg adjacent } \angle A} = \frac{24}{10} = \frac{12}{5}$$

CHAPTER 10 Practice Test

Math Online glencoe.com
Chapter Test

Graph each function, and compare to the parent graph. State the domain and range.

1. $y = -\sqrt{x}$

2. $y = \frac{1}{4}\sqrt{x}$

3. $y = \sqrt{x} + 5$

4. $y = \sqrt{x + 4}$

5. GEOMETRY The length of the side of a square is given by the function $s = \sqrt{A}$, where A is the area of the square. What is the perimeter of a square that has an area of 64 square inches?

A 64 inches

B 8 inches

C 32 inches

D 16 inches

Simplify each expression.

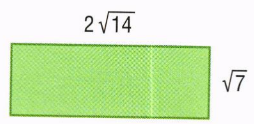
6. $5\sqrt{36}$

7. $\frac{3}{1 - \sqrt{2}}$

8. $2\sqrt{3} + 7\sqrt{3}$

9. $3\sqrt{6}(5\sqrt{2})$

10. GEOMETRY Find the area of the rectangle.



F $14\sqrt{2}$

G 14

H $98\sqrt{2}$

J $7\sqrt{2}$

Solve each equation. Check your solution.

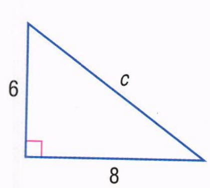
11. $\sqrt{10x} = 20$

12. $\sqrt{4x - 3} = 6 - x$

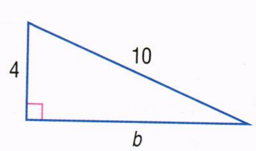
13. PACKAGING A cylindrical container of chocolate drink mix has a volume of about 162 in^3. The radius of the container can be found by using the formula $r = \sqrt{\frac{V}{\pi h}}$, where r is the radius and h is the height. If the height is 8.25 inches, find the radius of the container.

Find each missing length. If necessary, round to the nearest tenth.

14.



15.



Find the distance between the points with the given coordinates.

16. (2, 3), (3, 5)

17. (−3, 4), (−2, −3)

18. (−1, −1), (3, 2)

19. (−4, −6), (−7, 1)

Find the coordinates of the midpoint of the segment with the given endpoints.

20. (2, 3), (3, 5)

21. (−3, 4), (−2, −3)

22. (−1, −1), (3, 2)

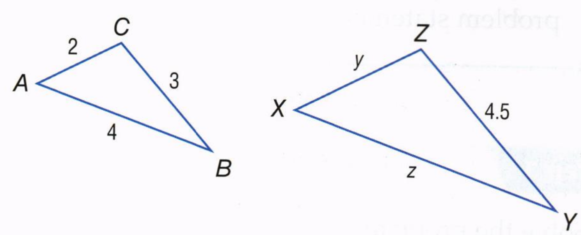
23. (−4, −8), (10, −6)

24. PIZZA DELIVERY The Pizza Place delivers to any location within a radius of 5 miles from the store for free. A delivery person drives 32 blocks north and then 45 blocks east to deliver a pizza. In this city, there are about 6 blocks per half mile.

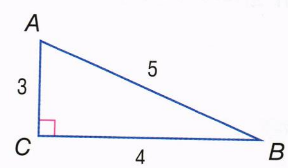
a. Should there be a charge for delivery? Explain.

b. Describe two delivery situations that would result in about 5 miles.

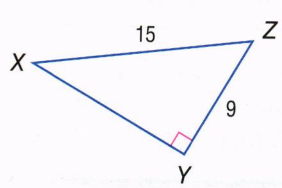
25. Find the missing lengths if $\triangle ABC \sim \triangle XYZ$.



26. Find the values of the three trigonometric ratios for angle A.



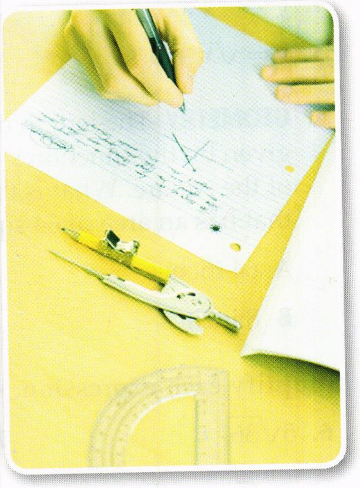
27. Find $m\angle X$ to the nearest degree.



CHAPTER 10

Preparing for Standardized Tests

Draw a Picture



Sometimes it is easier to visualize how to solve a problem if you draw a picture first. You can sketch your picture on scrap paper or in your test booklet (if allowed). Be careful not make any marks on your answer sheet other than your answers.

Strategies for Drawing a Picture

Step 1

Read the problem statement carefully.

Ask yourself:

- What am I being asked to solve?
- What information is given in the problem?
- What is the unknown quantity for which I need to solve?

Step 2

Sketch and label your picture.

- Draw your picture as clearly and accurately as possible.
- Label the picture carefully. Be sure to include all of the information given in the problem statement.

Step 3

Solve the problem.

- Use your picture to help you model the problem situation with an equation. Then solve the equation.
- Check your answer to make sure it is reasonable.

EXAMPLE

Read the problem. Identify what you need to know. Then use the information in the problem to solve. Show your work.

One sunny day, a church steeple casts a shadow that is 24 feet 3 inches long. At the same time, Nicole casts a shadow that is 1 foot 9 inches long. If Nicole is 5 feet 3 inches tall, what is the height of the steeple?

Read the problem statement carefully. You know Nicole's height, her shadow length, and the shadow length of the steeple. You need to find the height of the steeple.

Example of a 2-point response:

Scoring Rubric	
Criteria	**Score**
Full Credit: The answer is correct and a full explanation is provided that shows each step.	2
Partial Credit: • The answer is correct, but the explanation is incomplete. • The answer is incorrect, but the explanation is correct.	1
No Credit: Either an answer is not provided or the answer does not make sense.	0

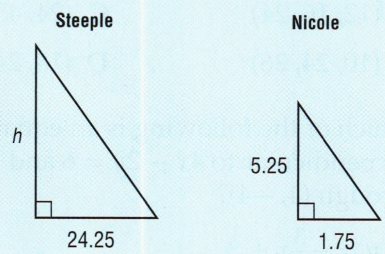
First convert all measurements to feet.

24 feet 3 inches $= 24\frac{3}{12}$ or 24.25 feet

1 foot 9 inches $= 1\frac{9}{12}$ or 1.75 feet

5 feet 3 inches $= 5\frac{3}{12}$ or 5.25 feet

Use similar triangles to find the height of the church steeple. Draw and label two triangles to represent the situation.



Use the similar triangles to set up and solve a proportion.

$$\frac{h}{24.25} = \frac{5.25}{1.75}$$

$$1.75h = (24.25)(5.25)$$

$$1.75h = 127.3125$$

$$h = 72.75$$

The height of the church steeple is 72.75 feet or 72 feet 9 inches.

Exercises

Read each problem. Identify what you need to know. Then use the information in the problem to solve. Show your work.

1. A building casts a 15-foot shadow, while a billboard casts a 4.5-foot shadow. If the billboard is 26 feet high, what is the height of the building? Round to the nearest tenth if necessary.

2. Jamey places a mirror on the ground at a distance of 56 feet from the base of a water tower. When he stands at a distance of 6 feet from the mirror, he can see the top of the water tower in the mirror's reflection and forms a pair of similar triangles. If Jamey is 5 feet 6 inches tall, what is the height of the water tower? Express your answer in feet and inches.

CHAPTER 10

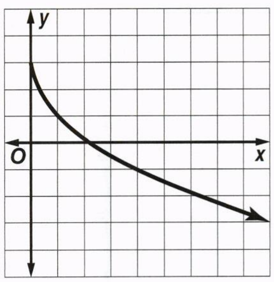
Standardized Test Practice

Cumulative, Chapters 1 through 10

Multiple Choice

Read each question. Then fill in the correct answer on the answer document provided by your teacher on a sheet of paper.

1. What is the equation of the square root function graphed below?



A $y = -2\sqrt{x} + 1$

B $y = -2\sqrt{x} + 3$

C $y = 2\sqrt{x} + 3$

D $y = 2\sqrt{x} + 1$

2. Simplify $\frac{1}{4 + \sqrt{2}}$.

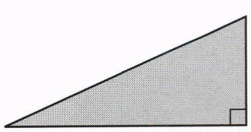
F $\frac{4 + \sqrt{2}}{14}$

G $\frac{2 - \sqrt{2}}{7}$

H $\frac{4 - \sqrt{2}}{14}$

J $\frac{2 + \sqrt{2}}{7}$

3. What is the area of the triangle below?



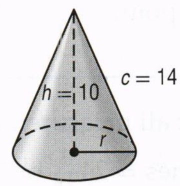
A $3\sqrt{2} + 10\sqrt{5}$

B $17 + 5\sqrt{10}$

C $12\sqrt{2} + 8\sqrt{5}$

D $8.5 + 2.5\sqrt{10}$

4. The formula for the slant height c of a cone is $c = \sqrt{h^2 + r^2}$, where h is the height of the cone and r is the radius of its base. What is the radius of the cone below? Round to the nearest tenth.



F 4.9

G 6.3

H 9.8

J 10.2

5. Which of the following sets of measures could not be the sides of a right triangle?

A (12, 16, 24)

B (10, 24, 26)

C (24, 45, 51)

D (18, 24, 30)

6. Which of the following is an equation of the line perpendicular to $4x - 2y = 6$ and passing through $(4, -4)$?

F $y = -\frac{3}{4}x + 3$

G $y = -\frac{3}{4}x - 1$

H $y = -\frac{1}{2}x - 4$

J $y = -\frac{1}{2}x - 2$

7. The scale on a map shows that 1.5 centimeters is equivalent to 40 miles. If the distance on the map between two cities is 8 centimeters, about how many miles apart are the cities?

A 178 miles

B 213 miles

C 224 miles

D 275 miles

Test-Taking Tip

Question 4 Substitute for c and h in the formula. Then solve for r.

Short Response/Gridded Response

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

8. **GRIDDED RESPONSE** How many times does the graph of $y = x^2 - 4x + 10$ cross the x-axis?

9. Factor $2x^4 - 32$ completely.

10. **GRIDDED RESPONSE** In football, a field goal is worth 3 points, and the extra point after a touchdown is worth 1 point. During the 2006 season, John Kasay of the Carolina Panthers scored a total of 100 points for his team by making a total of 52 field goals and extra points. How many field goals did he make?

11. Shannon bought a satellite radio and a subscription to satellite radio. What is the total cost for his first year of service?

Item	Cost
radio	$39.99
subscription	$11.99 per month

12. **GRIDDED RESPONSE** The distance required for a car to stop is directly proportional to the square of its velocity. If a car can stop in 242 meters at 22 kilometers per hour, how many meters are needed to stop at 30 kilometers per hour?

13. The highest point in Kentucky is at an elevation of 4145 feet above sea level. The lowest point in the state is at an elevation of 257 feet above sea level. Write an inequality to describe the possible elevations in Kentucky.

14. Simplify the expression below. Show your work.

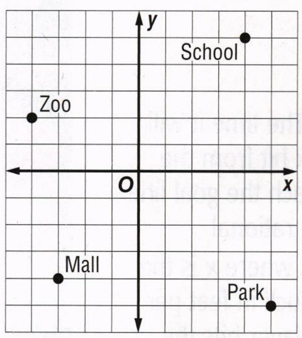
$$\left(\frac{-2r^{-2}q^5t^2}{5r^4q^2t^{-3}}\right)^{-2}$$

15. **GRIDDED RESPONSE** For the first home basketball game, 652 tickets were sold for a total revenue of $5216. If each ticket costs the same, how much is the cost per ticket? State your answer in dollars.

Extended Response

Record your answers on a sheet of paper. Show your work.

16. Karen is making a map of her hometown using a coordinate grid. The scale of her map is 1 unit = 2.5 miles.



a. What is the actual distance between Karen's school and the park? Round to the nearest tenth of a mile if necessary.

b. Suppose Karen's house is located midway between the mall and the school. What coordinates represent her house? Show your work.

Need Extra Help?												
If you missed Question...	1	2	3	4	5	6	7	8	9	10	11	12
Go to Lesson or Page...	10-7	7-5	8-2	10-5	10-6	9-4	9-6	10-2	10-4	4-4	10-8	4-3

CHAPTER 11

Rational Functions and Equations

Then

In Chapter 7, you simplified expressions involving monomials and polynomials.

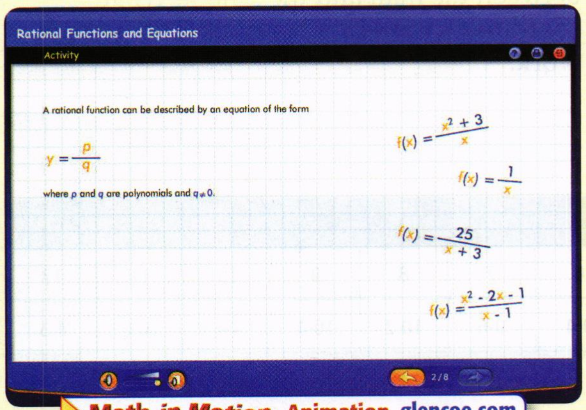
Now

In Chapter 11, you will:

- Identify and graph inverse variations.
- Identify excluded values of rational functions.
- Multiply, divide, and add rational expressions.
- Divide polynomials.
- Solve rational equations.

Why?

HOCKEY The time it will take for a puck hit from the blue line to reach the goal line is given by the rational expression $\frac{64}{x}$, where x is the speed of the puck in feet per seconds. If a player hits the puck at 100 miles per hour, the puck will reach the goal line in 0.34 second.



Math *in Motion*, Animation glencoe.com