Get Ready for the Chapter

Diagnose Readiness | **You have two options for checking prerequisite skills.**

1 Textbook Option

Take the Quick Check below. Refer to the Quick Review for help.

QuickCheck

Find the percent of the given number.

1. 26% of 500

2. 79% of 623

3. 19% of 82

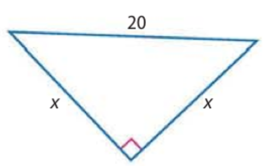
4. 10% of 180

5. 92% of 90

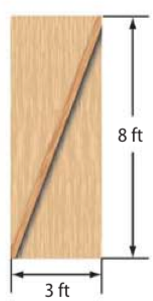
6. 65% of 360

7. **TIPPING** A couple ate dinner at an Italian restaurant where their bill was $32.50. If they want to leave an 18% tip, how much tip money should they leave?

8. Find x. Round to the nearest tenth.



9. **CONSTRUCTION** Jennifer is putting a brace in a board, as shown at the right. Find the length of the board used for a brace.



Solve each equation by using the Quadratic Formula. Round to the nearest tenth if necessary.

10. $5x^2 + 4x - 20 = 0$

11. $x^2 = x + 12$

12. **FIREWORKS** The Patriot Squad, a professional fireworks company, performed a show during a July 4th celebration. One of the rockets in the show followed the path modeled by $d = 80t - 16t^2$ where t is the time in seconds, but it failed to explode.

QuickReview

Example 1

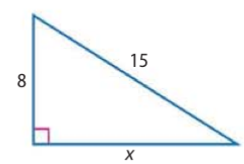
Find the percent of the given number.

15% of 35 = (0.15)(35)	Change the percent to a decimal.
= 5.25	Multiply.

So, 15% of 35 is 5.25.

Example 2

Find x. Round to the nearest tenth.



$a^2 + b^2 = c^2$	Pythagorean Theorem
$x^2 + 8^2 = 15^2$	Substitution
$x^2 + 64 = 225$	Simplify.
$x^2 = 161$	Subtract.
$x = \sqrt{161}$ or about 12.7	

Example 3

Solve $x^2 + 3x - 40 = 0$ by using the Quadratic Formula. Round to the nearest tenth.

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	Quadratic Formula
$= \dfrac{-3 \pm \sqrt{3^2 - 4(1)(-40)}}{2(1)}$	Substitution
$= \dfrac{-3 \pm \sqrt{169}}{2}$	Simplify.
$= 5$ or -8	Simplify.

2 Online Option

Take an online self-check Chapter Readiness Quiz at connectED.mcgraw-hill.com.

Get Started on the Chapter

You will learn several new concepts, skills, and vocabulary terms as you study Chapter 10. To get ready, identify important terms and organize your resources. You may wish to refer to Chapter 0 to review prerequisite skills.

FOLDABLES StudyOrganizer

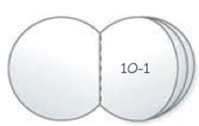
Circles Make this Foldable to help you organize your Chapter 10 notes on circles. Begin with nine sheets of paper.

1. **Trace** an 8-inch circle on each paper using a compass.

2. **Cut** out each of the circles.

3. **Staple** an inch from the left side of the papers.

4. **Label** as shown.



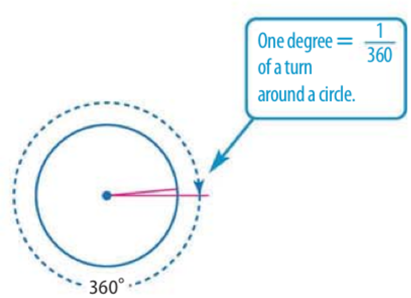
NewVocabulary

English		Español
circle	p. 697	círculo
center	p. 697	centro
radius	p. 697	radio
chord	p. 697	cuerda
diameter	p. 697	diámetro
circumference	p. 699	circunferencia
pi (π)	p. 699	pi (π)
inscribed	p. 700	inscrito
circumscribed	p. 700	circunscrito
central angle	p. 706	ángulo central
arc	p. 706	arco
tangent	p. 732	tangente
secant	p. 741	secante
chord segment	p. 750	segmento de cuerda

ReviewVocabulary

coplanar coplanar points that lie in the same plane

degree grado $\frac{1}{360}$ of the circular rotation about a point



LESSON 10-1 Circles and Circumference

Then

- You identified and used parts of parallelograms.

Now

1. Identify and use parts of circles.
2. Solve problems involving the circumference of a circle.

Why?

- The maxAir ride shown speeds back and forth and rotates counterclockwise. At times, the riders are upside down 140 feet above the ground experiencing "airtime"—a feeling of weightlessness. The ride's width, or *diameter,* is 44 feet. You can find the distance that a rider travels in one rotation by using this measure.

NewVocabulary

circle
center
radius
chord
diameter
concentric circles
circumference
pi (π)
inscribed
circumscribed

Common Core State Standards

Content Standards

G.CO.1 Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

G.C.1 Prove that all circles are similar.

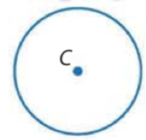
Mathematical Practices

4 Model with mathematics.

1 Make sense of problems and persevere in solving them.

1 Segments in Circles

A **circle** is the locus or set of all points in a plane equidistant from a given point called the **center** of the circle.



Circle C or $\odot C$

Segments that intersect a circle have special names.

KeyConcept Special Segments in a Circle

A **radius** (plural radii) is a segment with endpoints at the center and on the circle.

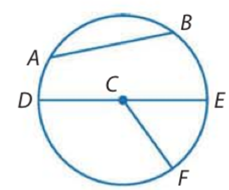
Examples $\overline{CD}$, $\overline{CE}$, and $\overline{CF}$ are radii of $\odot C$.

A **chord** is a segment with endpoints on the circle.

Examples $\overline{AB}$ and $\overline{DE}$ are chords of $\odot C$.

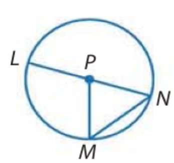
A **diameter** of a circle is a chord that passes through the center and is made up of collinear radii.

Example $\overline{DE}$ is a diameter of $\odot C$. Diameter $\overline{DE}$ is made up of collinear radii $\overline{CD}$ and $\overline{CE}$.



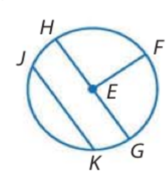
Example 1 Identify Segments in a Circle

a. Name the circle and identify a radius.



The circle has a center at P, so it is named circle P, or $\odot P$. Three radii are shown: $\overline{PL}$, $\overline{PN}$, and $\overline{PM}$.

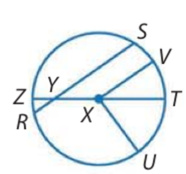
b. Identify a chord and a diameter of the circle.



Two chords are shown: $\overline{JK}$ and $\overline{HG}$. $\overline{HG}$ goes through the center, so $\overline{HG}$ is a diameter.

GuidedPractice

1. Name the circle, a radius, a chord, and a diameter of the circle.



Ilene MacDonald/Alamy

By definition, the distance from the center of a circle to any point on the circle is always the same. Therefore, all radii r of a circle are congruent. Since a diameter d is composed of two radii, all diameters of a circle are also congruent.

> **ReadingMath**
>
> **CCSS Precision** The words *radius* and *diameter* are used to describe lengths as well as segments. Since a circle has many different radii and diameters, the phrases *the radius* and *the diameter* refer to lengths rather than segments.

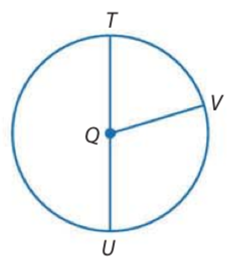
KeyConcept Radius and Diameter Relationships

If a circle has radius r and diameter d, the following relationships are true.

Radius Formula $r = \frac{d}{2}$ or $r = \frac{1}{2}d$ **Diameter Formula** $d = 2r$

Example 2 Find Radius and Diameter

If $QV = 8$ inches, what is the diameter of $\odot Q$?



$d = 2r$ Diameter Formula

$= 2(8)$ or 16 Substitute and simplify.

The diameter of $\odot Q$ is 16 inches.

GuidedPractice

2A. If $TU = 14$ feet, what is the radius of $\odot Q$?

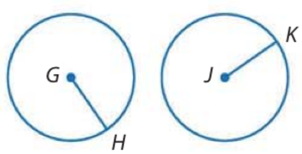
2B. If $QT = 11$ meters, what is QU?

As with other figures, pairs of circles can be congruent, similar, or share other special relationships.

> **ReviewVocabulary**
>
> **coplanar** points that lie in the same plane

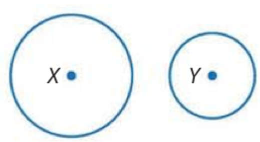
KeyConcept Circle Pairs

Two circles are congruent if and only if they have congruent radii.



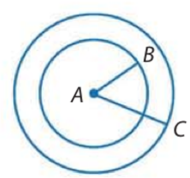
Example $\overline{GH} \cong \overline{JK}$, so $\odot G \cong \odot J$.

All circles are similar.



Example $\odot X \sim \odot Y$

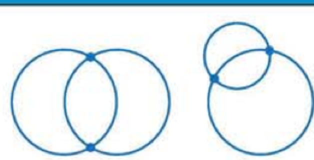
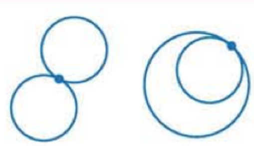
Concentric circles are coplanar circles that have the same center.



Example $\odot A$ with radius $\overline{AB}$ and $\odot A$ with radius $\overline{AC}$ are concentric.

You will prove that all circles are similar in Exercise 52.

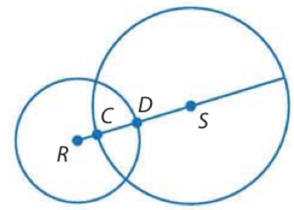
Two circles can intersect in two different ways.

2 Points of Intersection	1 Point of Intersection	No Points of Intersection
		

The segment connecting the centers of the two intersecting circles contains the radii of the two circles.

Example 3 Find Measures in Intersecting Circles

The diameter of ⊙S is 30 units, the diameter of ⊙R is 20 units, and $DS = 9$ units. Find CD.



Since the diameter of ⊙S is 30, $CS = 15$.
$\overline{CD}$ is part of radius $\overline{CS}$.

$CD + DS = CS$ — Segment Addition Postulate

$CD + 9 = 15$ — Substitution

$CD = 6$ — Subtract 9 from each side.

GuidedPractice

3. Use the diagram above to find RC.

2 Circumference

Circumference The **circumference** of a circle is the distance around the circle. By definition, the ratio $\frac{C}{d}$ is an irrational number called **pi (π)**. Two formulas for circumference can be derived by using this definition.

$\frac{C}{d} = \pi$ — Definition of pi

$C = \pi d$ — Multiply each side by d.

$C = \pi(2r)$ — $d = 2r$

$C = 2\pi r$ — Simplify.

KeyConcept Circumference

Words If a circle has diameter d or radius r, the circumference C equals the diameter times pi or twice the radius times pi.

Symbols $C = \pi d$ or $C = 2\pi r$

David Cannon/Getty Images Sport/Getty Images

Real-WorldLink

In 2005, Roger Federer and Andre Agassi played tennis on the helipad of the Burj Al Arab hotel in the United Arab Emirates. The helipad has a diameter of 79 feet and is nearly 700 feet high.

Source: Burj Al Arab, Emporis Buildings

Real-World Example 4 Find Circumference

TENNIS Find the circumference of the helipad described at the left.

$C = \pi d$ — Circumference formula

$= \pi(79)$ — Substitution

$= 79\pi$ — Simplify.

≈ 248.19 — Use a calculator.

The circumference of the helipad is 79π feet or about 248.19 feet.

GuidedPractice

Find the circumference of each circle described. Round to the nearest hundredth.

4A. radius = 2.5 centimeters

4B. diameter = 16 feet

These circumference formulas can also be used to determine the diameter and radius of a circle when the circumference is given.

Example 5 Find Diameter and Radius

Find the diameter and radius of a circle to the nearest hundredth if the circumference of the circle is 106.4 millimeters.

$C = \pi d$	Circumference Formula	$r = \frac{1}{2}d$	Radius Formula
$106.4 = \pi d$	Substitution	$\approx \frac{1}{2}(33.87)$	$d \approx 33.87$
$\frac{106.4}{\pi} = d$	Divide each side by π.	≈ 16.94 mm	Use a calculator.
$33.87 \text{ mm} \approx d$	Use a calculator.		

Guided Practice

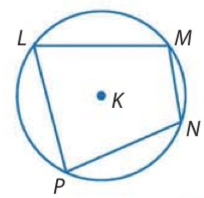
5. Find the diameter and radius of a circle to the nearest hundredth if the circumference of the circle is 77.8 centimeters.

StudyTip

Levels of Accuracy Since π is irrational, its value cannot be given as a terminating decimal. Using a value of 3 for π provides a quick estimate in calculations. Using a value of 3.14 or $\frac{22}{7}$ provides a closer approximation. For the most accurate approximation, use the π key on a calculator. Unless stated otherwise, assume that in this text, a calculator with a π key was used to generate answers.

A polygon is **inscribed** in a circle if all of its vertices lie on the circle. A circle is **circumscribed** about a polygon if it contains all the vertices of the polygon.

- Quadrilateral *LMNP* is *inscribed in* ⊙*K*.
- Circle *K* is *circumscribed about* quadrilateral *LMNP*.



Standardized Test Example 6 Circumference of Circumscribed Polygon

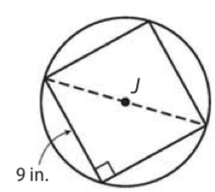
SHORT RESPONSE A square with side length of 9 inches is inscribed in ⊙*J*. Find the exact circumference of ⊙*J*.

Read the Test Item

You need to find the diameter of the circle and use it to calculate the circumference.

Solve the Test Item

First, draw a diagram. The diagonal of the square is the diameter of the circle and the hypotenuse of a right triangle.



$a^2 + b^2 = c^2$	Pythagorean Theorem
$9^2 + 9^2 = c^2$	Substitution
$162 = c^2$	Simplify.
$9\sqrt{2} = c$	Take the positive square root of each side.

The diameter of the circle is $9\sqrt{2}$ inches.

Find the circumference in terms of π by substituting $9\sqrt{2}$ for d in $C = \pi d$. The exact circumference is $9\pi\sqrt{2}$ inches.

Guided Practice

Find the exact circumference of each circle by using the given polygon.

6A. inscribed right triangle with legs 7 meters and 3 meters long

6B. circumscribed square with side 10 feet long

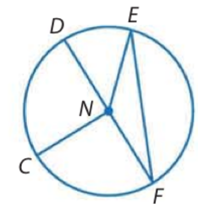
StudyTip

Circumcircle A *circumcircle* is a circle that passes through all of the vertices of a polygon.

Check Your Understanding

= Step-by-Step Solutions begin on page R14.

Examples 1–2 **For Exercises 1–4, refer to ⊙*N*.**



1. Name the circle.

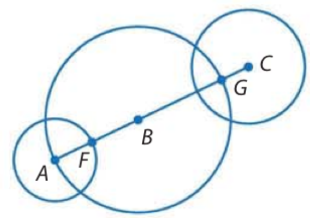
2. Identify each.

a. a chord **b.** a diameter **c.** a radius

3. If $CN = 8$ centimeters, find DN.

4. If $EN = 13$ feet, what is the diameter of the circle?

Example 3 **The diameters of ⊙*A*, ⊙*B*, and ⊙*C* are 8 inches, 18 inches, and 11 inches, respectively. Find each measure.**



5. FG

6. FB

Example 4

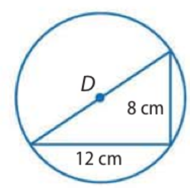
7. RIDES The circular ride described at the beginning of the lesson has a diameter of 44 feet. What are the radius and circumference of the ride? Round to the nearest hundredth, if necessary.

Example 5

8. CCSS MODELING The circumference of the circular swimming pool shown is about 56.5 feet. What are the diameter and radius of the pool? Round to the nearest hundredth.

Example 6

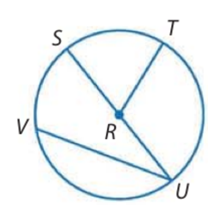
9. SHORT RESPONSE The right triangle shown is inscribed in ⊙*D*. Find the exact circumference of ⊙*D*.



Practice and Problem Solving

Extra Practice is on page R10.

Examples 1–2 **For Exercises 10–13, refer to ⊙*R*.**



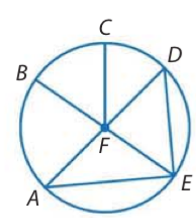
10. Name the center of the circle.

11. Identify a chord that is also a diameter.

12. Is $\overline{VU}$ a radius? Explain.

13. If $SU = 16.2$ centimeters, what is RT?

For Exercises 14–17, refer to ⊙*F*.



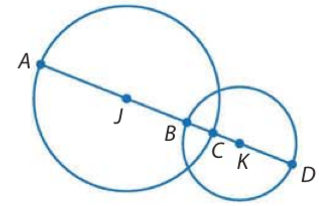
14. Identify a chord that is not a diameter.

15 If $CF = 14$ inches, what is the diameter of the circle?

16. Is $\overline{AF} \cong \overline{EF}$? Explain.

17. If $DA = 7.4$ centimeters, what is EF?

Example 3 **Circle J has a radius of 10 units, $\odot K$ has a radius of 8 units, and $BC = 5.4$ units. Find each measure.**



18. CK

19. AB

20. JK

21. AD

Example 4 **22. PIZZA** Find the radius and circumference of the pizza shown. Round to the nearest hundredth, if necessary.

23. BICYCLES A bicycle has tires with a diameter of 26 inches. Find the radius and circumference of a tire. Round to the nearest hundredth, if necessary.

Example 5 **Find the diameter and radius of a circle with the given circumference. Round to the nearest hundredth.**

24. $C = 18$ in.

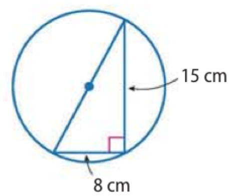
25. $C = 124$ ft

26. $C = 375.3$ cm

27. $C = 2608.25$ m

Example 6 **CCSS SENSE-MAKING** **Find the exact circumference of each circle by using the given inscribed or circumscribed polygon.**

28.



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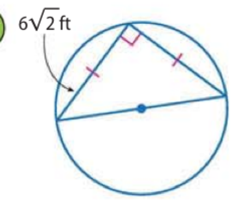

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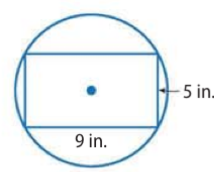
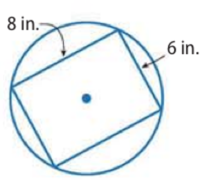
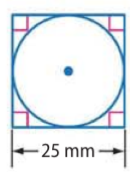


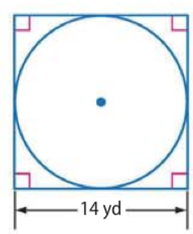
31.



32.

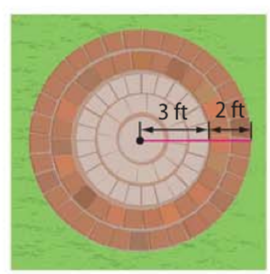


33.



34. DISC GOLF Disc golf is similar to regular golf, except that a flying disc is used instead of a ball and clubs. For professional competitions, the maximum weight of a disc in grams is 8.3 times the diameter in centimeters. What is the maximum allowable weight for a disc with circumference 66.92 centimeters? Round to the nearest tenth.

35. PATIOS Mr. Martinez is going to build the patio shown.



a. What is the patio's approximate circumference?

b. If Mr. Martinez changes the plans so that the inner circle has a circumference of approximately 25 feet, what should the radius of the circle be to the nearest foot?

The radius, diameter, or circumference of a circle is given. Find each missing measure to the nearest hundredth.

36. $d = 8\frac{1}{2}$ in., $r =$ __?__, $C =$ __?__

37. $r = 11\frac{2}{5}$ ft, $d =$ __?__, $C =$ __?__

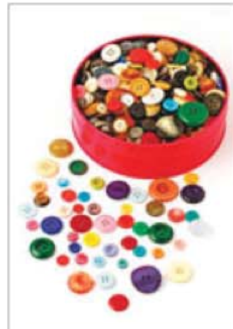
38. $C = 35x$ cm, $d =$ __?__, $r =$ __?__

39. $r = \frac{x}{8}$, $d =$ _____, $C =$ __?__

Determine whether the circles in the figures below appear to be *congruent, concentric,* or *neither.*

40.

41.



42.

43 **HISTORY** The *Indian Shell Ring* on Hilton Head Island approximates a circle. If each unit on the coordinate grid represents 25 feet, how far would someone have to walk to go completely around the ring? Round to the nearest tenth.

44. **CCSS MODELING** A brick path is being installed around a circular pond. The pond has a circumference of 68 feet. The outer edge of the path is going to be 4 feet from the pond all the way around. What is the approximate circumference of the path? Round to the nearest hundredth.

45. **MULTIPLE REPRESENTATIONS** In this problem, you will explore changing dimensions in circles.

a. Geometric Use a compass to draw three circles in which the scale factor from each circle to the next larger circle is 1 : 2.

b. Tabular Calculate the radius (to the nearest tenth) and circumference (to the nearest hundredth) of each circle. Record your results in a table.

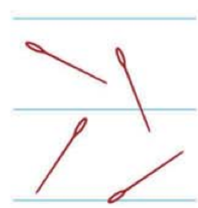
c. Verbal Explain why these three circles are geometrically similar.

d. Verbal Make a conjecture about the ratio between the circumferences of two circles when the ratio between their radii is 2.

e. Analytical The scale factor from $\odot A$ to $\odot B$ is $\frac{b}{a}$. Write an equation relating the circumference (C_A) of $\odot A$ to the circumference (C_B) of $\odot B$.

f. Numerical If the scale factor from $\odot A$ to $\odot B$ is $\frac{1}{3}$, and the circumference of $\odot A$ is 12 inches, what is the circumference of $\odot B$?

46. **BUFFON'S NEEDLE** Measure the length ℓ of a needle (or toothpick) in centimeters. Next, draw a set of horizontal lines that are ℓ centimeters apart on a sheet of plain white paper.

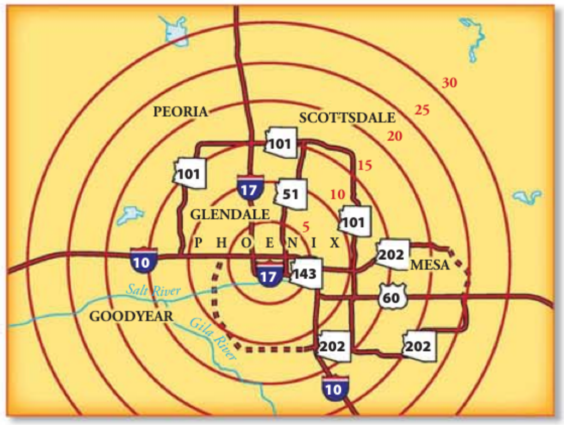


a. Drop the needle onto the paper. When the needle lands, record whether it touches one of the lines as a hit. Record the number of hits after 25, 50, and 100 drops.

b. Calculate the ratio of two times the total number of drops to the number of hits after 25, 50, and 100 drops.

c. How are the values you found in part **b** related to π?

47 **MAPS** The concentric circles on the map below show the areas that are 5, 10, 15, 20, 25, and 30 miles from downtown Phoenix.



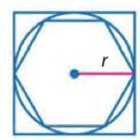
a. How much greater is the circumference of the outermost circle than the circumference of the center circle?

b. As the radii of the circles increase by 5 miles, by how much does the circumference increase?

H.O.T. Problems Use Higher-Order Thinking Skills

48. **WRITING IN MATH** How can we describe the relationships that exist between circles and lines?

49. **REASONING** In the figure, a circle with radius r is inscribed in a regular polygon and circumscribed about another.



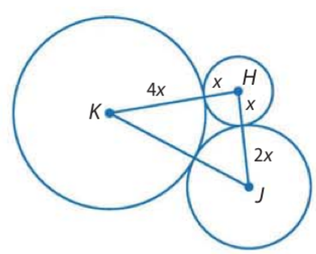
a. What are the perimeters of the circumscribed and inscribed polygons in terms of r? Explain.

b. Is the circumference C of the circle greater or less than the perimeter of the circumscribed polygon? the inscribed polygon? Write a compound inequality comparing C to these perimeters.

c. Rewrite the inequality from part **b** in terms of the diameter d of the circle and interpret its meaning.

d. As the number of sides of both the circumscribed and inscribed polygons increase, what will happen to the upper and lower limits of the inequality from part **c**, and what does this imply?

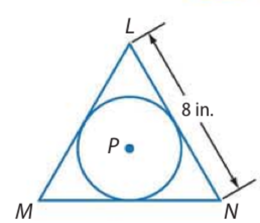
50. **CHALLENGE** The sum of the circumferences of circles H, J, and K shown at the right is 56π units. Find KJ.



51. **REASONING** Is the distance from the center of a circle to a point in the interior of a circle *sometimes*, *always*, or *never* less than the radius of the circle? Explain.

52. **CCSS ARGUMENTS** Use the locus definition of a circle and dilations to prove that all circles are similar.

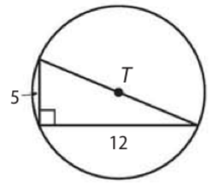
53. **CHALLENGE** In the figure, $\odot P$ is inscribed in equilateral triangle LMN. What is the circumference of $\odot P$?



54. **WRITING IN MATH** Research and write about the history of pi and its importance to the study of geometry.

Standardized Test Practice

55. GRIDDED RESPONSE What is the circumference of ⊙T? Round to the nearest tenth.



56. What is the radius of a table with a circumference of 10 feet?

A 1.6 ft
B 2.5 ft
C 3.2 ft
D 5 ft

57. ALGEBRA Bill is planning a circular vegetable garden with a fence around the border. If he can use up to 50 feet of fence, what radius can he use for the garden?

F 10 **G** 9 **H** 8 **J** 7

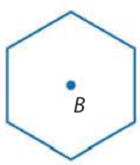
58. SAT/ACT What is the radius of a circle with an area of $\frac{\pi}{4}$ square units?

A 0.4 units
B 0.5 units
C 2 units
D 4 units
E 16 units

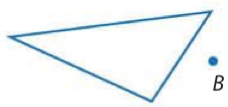
Spiral Review

Copy each figure and point *B*. Then use a ruler to draw the image of the figure under a dilation with center *B* and the scale factor *r* indicated. (Lesson 9-6)

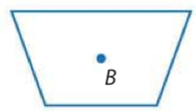
59. $r = \frac{1}{5}$



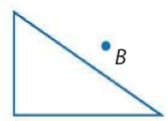
60. $r = \frac{2}{5}$



61. $r = 2$



62. $r = 3$



State whether each figure has rotational symmetry. If so, copy the figure, locate the center of symmetry, and state the order and magnitude of symmetry. (Lesson 9-5)

63.

64.

65.

66.

Determine the truth value of the following statement for each set of conditions. Explain your reasoning. (Lesson 2-2)

If you are over 18 years old, then you vote in all elections.

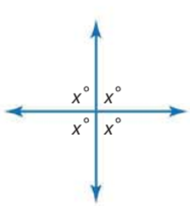
67. You are 19 years old and you vote.

68. You are 21 years old and do not vote.

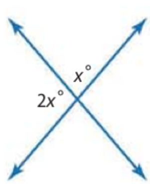
Skills Review

Find *x*.

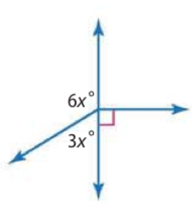
69.



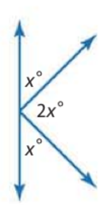
70.



71.



72.



LESSON 10-2

Measuring Angles and Arcs

Then

- You measured angles and identified congruent angles.

Now

1. Identify central angles, major arcs, minor arcs, and semicircles, and find their measures.
2. Find arc lengths.

Why?

- The thirteen stars of the Betsy Ross flag are arranged equidistant from each other and from a fixed point. The distance between consecutive stars varies depending on the size of the flag, but the measure of the central angle formed by the center of the circle and any two consecutive stars is always the same.

NewVocabulary

central angle
arc
minor arc
major arc
semicircle
congruent arcs
adjacent arcs
arc length

Common Core State Standards

Content Standards

G.C.2 Identify and describe relationships among inscribed angles, radii, and chords.

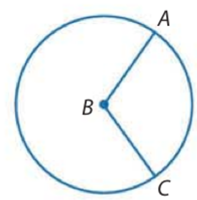
G.C.5 Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.

Mathematical Practices

6 Attend to precision.
4 Model with mathematics.

1 Angles and Arcs

A **central angle** of a circle is an angle with a vertex in the center of the circle. Its sides contain two radii of the circle. $\angle ABC$ is a central angle of $\odot B$.

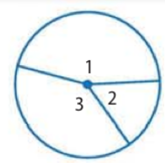


Recall from Lesson 1-4 that a *degree* is $\frac{1}{360}$ of the circular rotation about a point. This leads to the following relationship.

KeyConcept Sum of Central Angles

Words The sum of the measures of the central angles of a circle with no interior points in common is 360.

Example $m\angle 1 + m\angle 2 + m\angle 3 = 360$



Example 1 Find Measures of Central Angles

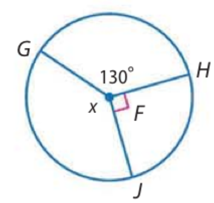
Find the value of x.

$m\angle GFH + m\angle HFJ + m\angle GFJ = 360$ Sum of Central Angles

$130 + 90 + m\angle GFJ = 360$ Substitution

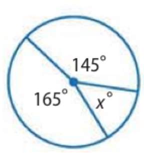
$220 + m\angle GFJ = 360$ Simplify.

$m\angle GFJ = 140$ Subtract 220 from each side.

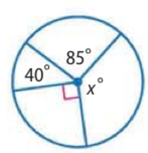


GuidedPractice

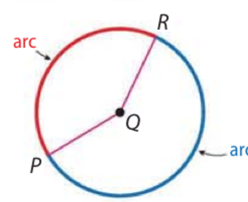
1A.



1B.



An **arc** is a portion of a circle defined by two endpoints. A central angle separates the circle into two arcs with measures related to the measure of the central angle.

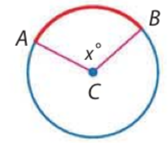
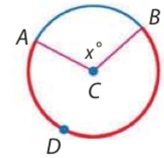
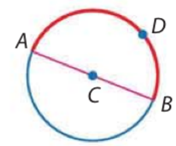


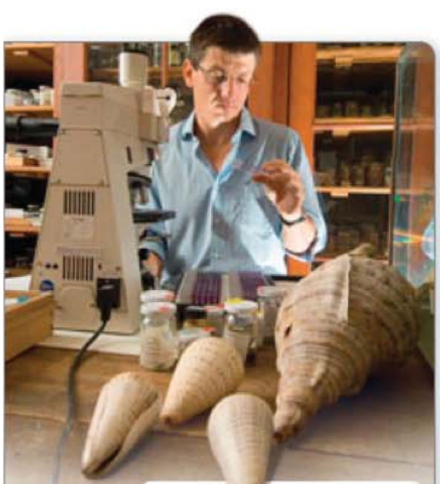
Photodisc/Getty Images

StudyTip

Naming Arcs Minor arcs are named by their endpoints. Major arcs and semicircles are named by their endpoints and another point on the arc that lies between these endpoints.

KeyConcept Arcs and Arc Measure

Arc	Measure	
A **minor arc** is the shortest arc connecting two endpoints on a circle.	The measure of a minor arc is less than 180 and equal to the measure of its related central angle. $m\overset{\frown}{AB} = m\angle ACB = x$	
A **major arc** is the longest arc connecting two endpoints on a circle.	The measure of a major arc is greater than 180, and equal to 360 minus the measure of the minor arc with the same endpoints. $m\overset{\frown}{ADB} = 360 - m\overset{\frown}{AB} = 360 - x$	
A **semicircle** is an arc with endpoints that lie on a diameter.	The measure of a semicircle is 180. $m\overset{\frown}{ADB} = 180$	

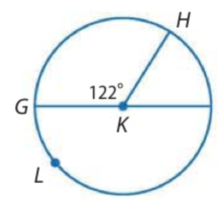


imagebroker/Alamy

Real-World Career

Historical Researcher Research in museums includes authentication, verification, and description of artifacts. Employment as a historical researcher requires a minimum of a bachelor's degree in history. Refer to Exercises 42–43.

Example 2 Classify Arcs and Find Arc Measures



$\overline{GJ}$ is a diameter of ⊙K. Identify each arc as a *major arc*, *minor arc*, or *semicircle*. Then find its measure.

a. $m\overset{\frown}{GH}$

$\overset{\frown}{GH}$ is a minor arc, so $m\overset{\frown}{GH} = m\angle GKH$ or 122.

b. $m\overset{\frown}{GLH}$

$\overset{\frown}{GLH}$ is a major arc that shares the same endpoints as minor arc $\overset{\frown}{GH}$.

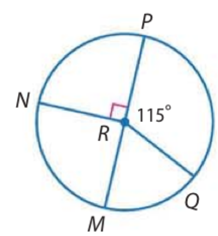
$m\overset{\frown}{GHL} = 360 - m\overset{\frown}{GH}$

$= 360 - 122$ or 238

c. $m\overset{\frown}{GLJ}$

$\overset{\frown}{GLJ}$ is a semicircle, so $m\overset{\frown}{GLJ} = 180$.

GuidedPractice



$\overline{PM}$ is a diameter of ⊙R. Identify each arc as a *major arc*, *minor arc*, or *semicircle*. Then find its measure.

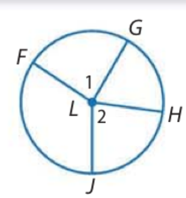
2A. $\overset{\frown}{MQ}$ **2B.** $\overset{\frown}{MNP}$ **2C.** $\overset{\frown}{MNQ}$

Congruent arcs are arcs in the same or congruent circles that have the same measure.

Theorem 10.1

Words In the same circle or in congruent circles, two minor arcs are congruent if and only if their central angles are congruent.

Example If $\angle 1 \cong \angle 2$, then $\overset{\frown}{FG} \cong \overset{\frown}{HJ}$.
If $\overset{\frown}{FG} \cong \overset{\frown}{HJ}$, then $\angle 1 \cong \angle 2$.



You will prove Theorem 10.1 in Exercise 52.

Real-World Example 3 Find Arc Measures in Circle Graphs

SPORTS Refer to the circle graph. Find each measure.

a. $m\widehat{CD}$

$\widehat{CD}$ is a minor arc. $m\widehat{CD} = m\angle CSD$

$\angle CSD$ represents 18% of the whole, or 18% of the circle.

$m\angle CSD = 0.18(360)$ Find 18% of 360.

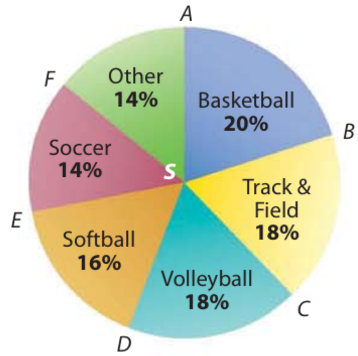
$= 64.8$ Simplify.

b. $m\widehat{BC}$

The percents for volleyball and track and field are equal, so the central angles are congruent and the corresponding arcs are congruent.

$m\widehat{BC} = m\widehat{CD} = 64.8$

Female Participation in Sports

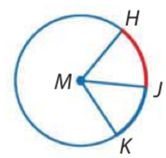


GuidedPractice

3A. $m\widehat{EF}$

3B. $m\widehat{FA}$

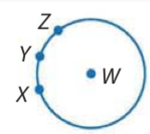
Adjacent arcs are arcs in a circle that have exactly one point in common. In $\odot M$, $\widehat{HJ}$ and $\widehat{JK}$ are adjacent arcs. As with adjacent angles, you can add the measures of adjacent arcs.



Postulate 10.1 Arc Addition Postulate

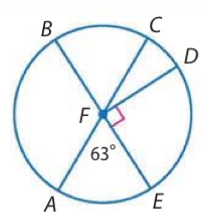
Words The measure of an arc formed by two adjacent arcs is the sum of the measures of the two arcs.

Example $m\widehat{XYZ} = m\widehat{XY} + m\widehat{YZ}$



Example 4 Use Arc Addition to Find Measures of Arcs

Find each measure in $\odot F$.



a. $m\widehat{AED}$

$m\widehat{AED} = m\widehat{AE} + m\widehat{ED}$ Arc Addition Postulate

$= m\angle AFE + m\angle EFD$ $m\widehat{AE} = m\angle AFE$, $m\widehat{ED} = m\angle EFD$

$= 63 + 90$ or 153 Substitution

b. $m\widehat{ADB}$

$m\widehat{ADB} = m\widehat{AE} + m\widehat{EDB}$ Arc Addition Postulate

$= 63 + 180$ or 243 $\widehat{EDB}$ is a semicircle, so $m\widehat{EDB} = 180$.

GuidedPractice

4A. $m\widehat{CE}$

4B. $m\widehat{ABD}$

Math HistoryLink

Euclid (c. 325–265 B.C.) The 13 books of Euclid's *Elements* are influential works of science. In them, geometry and other branches of mathematics are logically developed. Book 3 of *Elements* is devoted to circles, arcs, and angles.

Hulton Archive/Getty Images

2 Arc Length

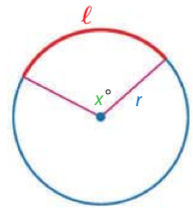
Arc length is the distance between the endpoints along an arc measured in linear units. Since an arc is a portion of a circle, its length is a fraction of the circumference.

KeyConcept Arc Length

Words The ratio of the length of an arc ℓ to the circumference of the circle is equal to the ratio of the degree measure of the arc to 360.

Proportion $\frac{\ell}{2\pi r} = \frac{x}{360}$ or

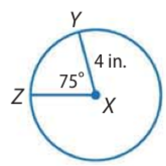
Equation $\ell = \frac{x}{360} \cdot 2\pi r$



Example 5 Find Arc Length

Find the length of $\widehat{ZY}$. Round to the nearest hundredth.

a.

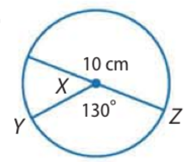


$\ell = \frac{x}{360} \cdot 2\pi r$ Arc Length Equation

$= \frac{75}{360} \cdot 2\pi(4)$ Substitution

≈ 5.24 in. Use a calculator.

b.

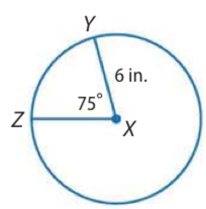


$\ell = \frac{x}{360} \cdot 2\pi r$ Arc Length Equation

$= \frac{130}{360} \cdot 2\pi(5)$ Substitution

≈ 11.34 cm Use a calculator.

c.



$\ell = \frac{x}{360} \cdot 2\pi r$ Arc Length Equation

$= \frac{75}{360} \cdot 2\pi(6)$ Substitution

≈ 7.85 in. Use a calculator.

Notice that $\widehat{ZY}$ has the same measure, 75, in both Examples 5a and 5c. The arc lengths, however, are different. This is because they are in circles that have different radii.

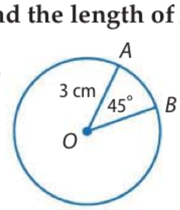
StudyTip

Alternate Method The arc lengths in Examples 5a, 5b, and 5c could also have been calculated using the arc length proportion $\frac{\ell}{2\pi r} = \frac{x}{360}$.

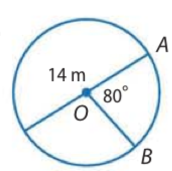
GuidedPractice

Find the length of $\widehat{AB}$. Round to the nearest hundredth.

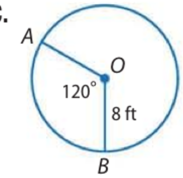
5A.



5B.



5C.

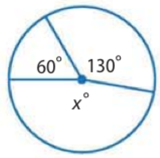


Check Your Understanding

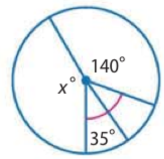
● = Step-by-Step Solutions begin on page R14.

Example 1 **Find the value of x.**

1.

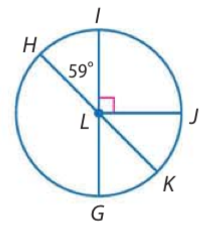


2.



Example 2 **CCSS PRECISION** **$\overline{HK}$ and $\overline{IG}$ are diameters of ⊙L. Identify each arc as a *major arc, minor arc,* or *semicircle*. Then find its measure.**

3. $m\widehat{IHJ}$ **4.** $m\widehat{HI}$ **5.** $m\widehat{HGK}$

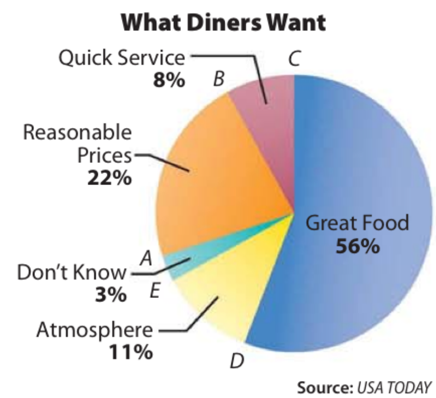


Example 3 **6. RESTAURANTS** The graph shows the results of a survey taken by diners relating what is most important about the restaurants where they eat.

a. Find $m\widehat{AB}$.

b. Find $m\widehat{BC}$.

c. Describe the type of arc that the category Great Food represents.

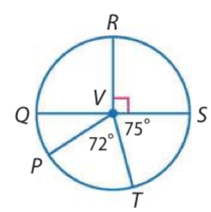


Example 4 **$\overline{QS}$ is a diameter of ⊙V. Find each measure.**

7. $m\widehat{STP}$

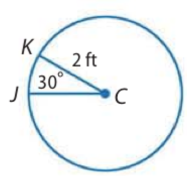
8. $m\widehat{QRT}$

9. $m\widehat{PQR}$

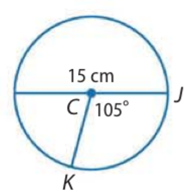


Example 5 **Find the length of $\widehat{JK}$. Round to the nearest hundredth.**

10.



11.

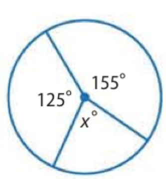


Practice and Problem Solving

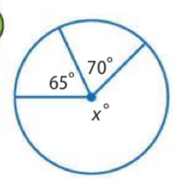
Extra Practice is on page R10.

Example 1 **Find the value of x.**

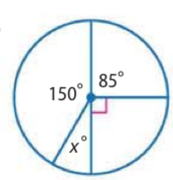
12.



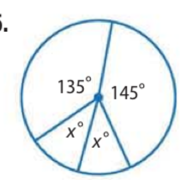
13.



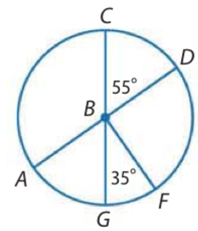
14.



15.



Example 2 $\overline{AD}$ and $\overline{CG}$ are diameters of ⊙B. Identify each arc as a *major arc, minor arc,* or *semicircle*. Then find its measure.



16. $m\overset{\frown}{CD}$ **17.** $m\overset{\frown}{AC}$ **18.** $m\overset{\frown}{CFG}$

19. $m\overset{\frown}{CGD}$ **20.** $m\overset{\frown}{GCF}$ **21.** $m\overset{\frown}{ACD}$

22. $m\overset{\frown}{AG}$ **23.** $m\overset{\frown}{ACF}$

Example 3

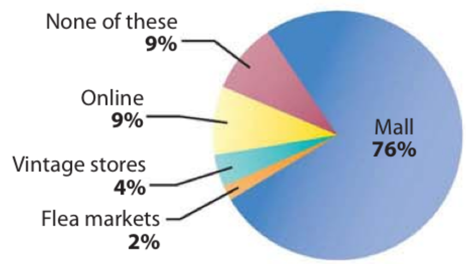
24. SHOPPING The graph shows the results of a survey in which teens were asked where the best place was to shop for clothes.

a. What would be the arc measures associated with the mall and vintage stores categories?

b. Describe the kinds of arcs associated with the category "Mall" and the category "None of these."

c. Are there any congruent arcs in this graph? Explain.

Best Places to Clothes Shop



25. CCSS **MODELING** The table shows the results of a survey in which Americans were asked how long food could be on the floor and still be safe to eat.

a. If you were to construct a circle graph of this information, what would be the arc measures associated with the first two categories?

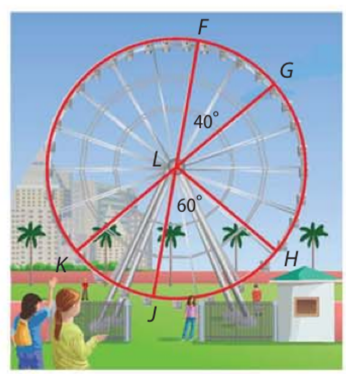
b. Describe the kind of arcs associated with the first category and the last category.

c. Are there any congruent arcs in this graph? Explain.

Dropped Food Do you eat food dropped on the floor?	
Not safe to eat	78%
Three-second rule*	10%
Five-second rule*	8%
Ten-second rule*	4%

Source: American Diabetic Association
* The length of time the food is on the floor.

Examples 2, 4 **ENTERTAINMENT** Use the Ferris wheel shown to find each measure.



26. $m\overset{\frown}{FG}$ **27.** $m\overset{\frown}{JH}$

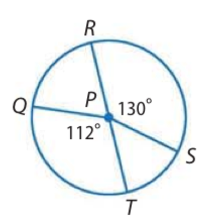
28. $m\overset{\frown}{JKF}$ **29.** $m\overset{\frown}{JFH}$

30. $m\overset{\frown}{GHF}$ **31.** $m\overset{\frown}{GHK}$

32. $m\overset{\frown}{HK}$ **33.** $m\overset{\frown}{JKG}$

34. $m\overset{\frown}{KFH}$ **35.** $m\overset{\frown}{HGF}$

Example 5 Use ⊙P to find the length of each arc. Round to the nearest hundredth.



36. $\overset{\frown}{RS}$, if the radius is 2 inches

37. $\overset{\frown}{QT}$, if the diameter is 9 centimeters

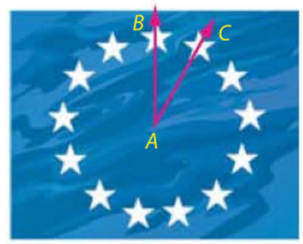
38. $\overset{\frown}{QR}$, if $PS = 4$ millimeters

39. $\overset{\frown}{RS}$, if $RT = 15$ inches

40. $\overset{\frown}{QRS}$, if $RT = 11$ feet

41. $\overset{\frown}{RTS}$, if $PQ = 3$ meters

HISTORY The figure shows the stars in the Betsy Ross flag referenced at the beginning of the lesson.



42. What is the measure of central angle A? Explain how you determined your answer.

43. If the diameter of the circle were doubled, what would be the effect on the arc length from the center of one star B to the next star C?

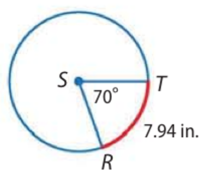
44. FARMS The *Pizza Farm* in Madera, California, is a circle divided into eight equal slices, as shown at the right. Each "slice" is used for growing or grazing pizza ingredients.

a. What is the total arc measure of the slices containing olives, tomatoes, and peppers?

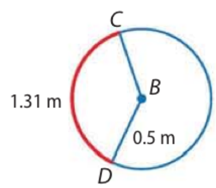
b. The circle is 125 feet in diameter. What is the arc length of one slice? Round to the nearest hundredth.

CCSS REASONING **Find each measure. Round each linear measure to the nearest hundredth and each arc measure to the nearest degree.**

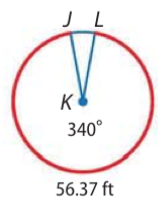
45. circumference of $\odot S$



46. $m\widehat{CD}$



47. radius of $\odot K$

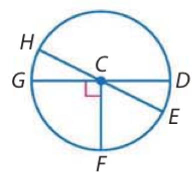


ALGEBRA **In $\odot C$, $m\angle HCG = 2x$ and $m\angle HCD = 6x + 28$. Find each measure.**

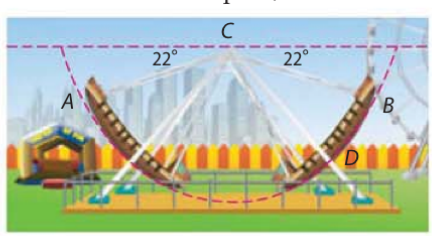
48. $m\widehat{EF}$

49. $m\widehat{HD}$

50. $m\widehat{HGF}$



51 RIDES A pirate ship ride follows a semicircular path, as shown in the diagram.



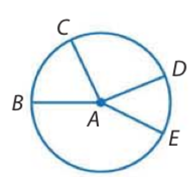
a. What is $m\widehat{AB}$?

b. If $CD = 62$ feet, what is the length of $\widehat{AB}$? Round to the nearest hundredth.

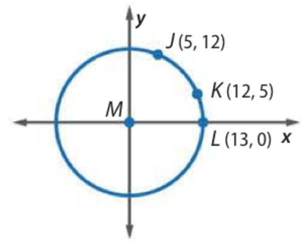
52. PROOF Write a two-column proof of Theorem 10.1.

Given: $\angle BAC \cong \angle DAE$

Prove: $\widehat{BC} \cong \widehat{DE}$



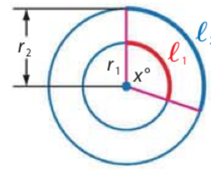
53. **COORDINATE GEOMETRY** In the graph, point M is located at the origin. Find each measure in $\odot M$. Round each linear measure to the nearest hundredth and each arc measure to the nearest tenth degree.



a. $m\widehat{JL}$ **b.** $m\widehat{KL}$ **c.** $m\widehat{JK}$

d. length of $\widehat{JL}$ **e.** length of $\widehat{JK}$

54. **ARC LENGTH AND RADIAN MEASURE** In this problem, you will use concentric circles to show that the length of the arc intercepted by a central angle of a circle is dependent on the circle's radius.



a. Compare the measures of arc ℓ_1 and arc ℓ_2. Then compare the lengths of arc ℓ_1 and arc ℓ_2. What do these two comparisons suggest?

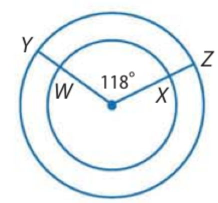
b. Use similarity transformations (dilations) to explain why the length of an arc ℓ intercepted by a central angle of a circle is proportional to the circle's radius r. That is, explain why we can say that for this diagram, $\frac{\ell_1}{r_1} = \frac{\ell_2}{r_2}$.

c. Write expressions for the lengths of arcs ℓ_1 and ℓ_2. Use these expressions to identify the constant of proportionality k in $\ell = kr$.

d. The expression that you wrote for k in part **c** gives the *radian measure* of an angle. Use it to find the radian measure of an angle measuring 90°.

H.O.T. Problems Use Higher-Order Thinking Skills

55. **ERROR ANALYSIS** Brody says that $\widehat{WX}$ and $\widehat{YZ}$ are congruent since their central angles have the same measure. Selena says they are not congruent. Is either of them correct? Explain your reasoning.



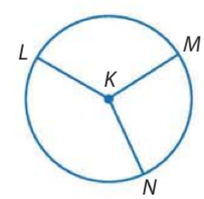
CCSS ARGUMENTS **Determine whether each statement is *sometimes, always,* or *never* true. Explain your reasoning.**

56. The measure of a minor arc is less than 180.

57. If a central angle is obtuse, its corresponding arc is a major arc.

58. The sum of the measures of adjacent arcs of a circle depends on the measure of the radius.

59. **CHALLENGE** The measures of $\widehat{LM}$, $\widehat{MN}$, and $\widehat{NL}$ are in the ratio 5:3:4. Find the measure of each arc.



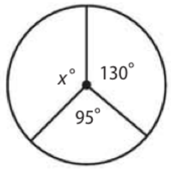
60. **OPEN ENDED** Draw a circle and locate three points on the circle. Estimate the measures of the three nonoverlapping arcs that are formed. Then use a protractor to find the measure of each arc. Label your circle with the arc measures.

61. **CHALLENGE** The time shown on an analog clock is 8:10. What is the measure of the angle formed by the hands of the clock?

62. **WRITING IN MATH** Describe the three different types of arcs in a circle and the method for finding the measure of each one.

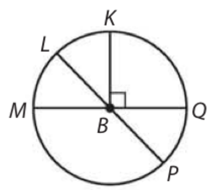
Standardized Test Practice

63. What is the value of x?



A 120
B 135
C 145
D 160

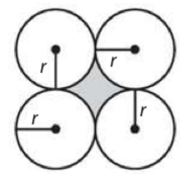
64. GRIDDED RESPONSE In $\odot B$, $m\angle LBM = 3x$ and $m\angle LBQ = 4x + 61$. What is the measure of $\angle PBQ$?



65. ALGEBRA A rectangle's width is represented by x and its length by y. Which expression best represents the area of the rectangle if the length and width are tripled?

F $3xy$
G $3(xy)^2$
H $9xy$
J $(xy)^3$

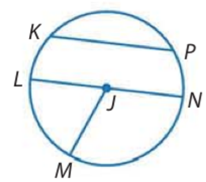
66. SAT/ACT What is the area of the shaded region if $r = 4$?



A $64 - 16\pi$
B $16 - 16\pi$
C $16 - 8\pi$
D $64 - 8\pi$
E $64\pi - 16$

Spiral Review

Refer to $\odot J$. (Lesson 10-1)



67. Name the center of the circle.

68. Identify a chord that is also a diameter.

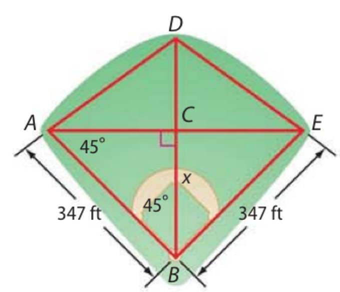
69. If $LN = 12.4$, what is JM?

Graph the image of each polygon with the given vertices after a dilation centered at the origin with the given scale factor. (Lesson 9-6)

70. $X(-1, 2)$, $Y(2, 1)$, $Z(-1, -2)$; $r = 3$

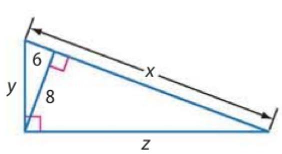
71. $A(-4, 4)$, $B(4, 4)$, $C(4, -4)$, $D(-4, -4)$; $r = 0.25$

72. BASEBALL The diagram shows some dimensions of Comiskey Park in Chicago, Illinois. $\overline{BD}$ is a segment from home plate to dead center field, and $\overline{AE}$ is a segment from the left field foul pole to the right field foul pole. If the center fielder is standing at C, how far is he from home plate? (Lesson 8-3)

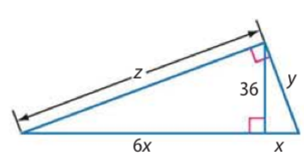


Find x, y, and z. (Lesson 8-1)

73.



74.



Skills Review

Find x.

75. $24^2 + x^2 = 26^2$

76. $x^2 + 5^2 = 13^2$

77. $30^2 + 35^2 = x^2$

LESSON

10-3 Arcs and Chords

Then	Now	Why?
You used the relationships between arcs and angles to find measures.	**1** Recognize and use relationships between arcs and chords. **2** Recognize and use relationships between arcs, chords, and diameters.	Embroidery hoops are used in sewing, quilting, and cross-stitching, as well as for embroidering. The endpoints of the snowflake shown are both the endpoints of a chord and the endpoints of an arc.

CCSS Common Core State Standards

Content Standards

G.C.2 Identify and describe relationships among inscribed angles, radii, and chords.

G.MG.3 Apply geometric methods to solve problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios). ★

Mathematical Practices

4 Model with mathematics.

3 Construct viable arguments and critique the reasoning of others.

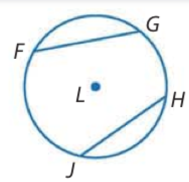
1 Arcs and Chords

A *chord* is a segment with endpoints on a circle. If a chord is not a diameter, then its endpoints divide the circle into a major and a minor arc.

Theorem 10.2

Words In the same circle or in congruent circles, two minor arcs are congruent if and only if their corresponding chords are congruent.

Example $\widehat{FG} \cong \widehat{HJ}$ if and only if $\overline{FG} \cong \overline{HJ}$.

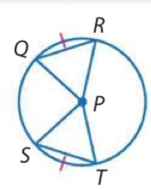


Proof Theorem 10.2 (part 1)

Given: $\odot P$; $\widehat{QR} \cong \widehat{ST}$

Prove: $\overline{QR} \cong \overline{ST}$

Proof:



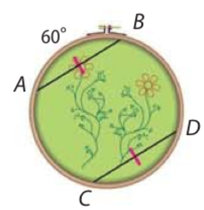
Statements	Reasons
1. $\odot P$, $\widehat{QR} \cong \widehat{ST}$	**1.** Given
2. $\angle QPR \cong \angle SPT$	**2.** If arcs are $\cong$, their corresponding central $\angle$s are $\cong$.
3. $\overline{QP} \cong \overline{PR} \cong \overline{SP} \cong \overline{PT}$	**3.** All radii of a circle are $\cong$.
4. $\triangle PQR \cong \triangle PST$	**4.** SAS
5. $\overline{QR} \cong \overline{ST}$	**5.** CPCTC

You will prove part 2 of Theorem 10.2 in Exercise 25.

Real-World Example 1 Use Congruent Chords to Find Arc Measure

CRAFTS In the embroidery hoop, $\overline{AB} \cong \overline{CD}$ and $m\widehat{AB} = 60$. Find $m\widehat{CD}$.

$\overline{AB}$ and $\overline{CD}$ are congruent chords, so the corresponding arcs $\widehat{AB}$ and $\widehat{CD}$ are congruent. $m\widehat{AB} = m\widehat{CD} = 60$

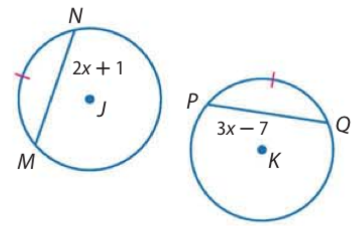


Guided Practice

1. If $m\widehat{AB} = 78$ in the embroidery hoop, find $m\widehat{CD}$.

Example 2 Use Congruent Arcs to Find Chord Lengths

ALGEBRA In the figures, $\odot J \cong \odot K$ and $\widehat{MN} \cong \widehat{PQ}$. Find PQ.



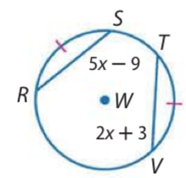
$\widehat{MN}$ and $\widehat{PQ}$ are congruent arcs in congruent circles, so the corresponding chords $\overline{MN}$ and $\overline{PQ}$ are congruent.

$MN = PQ$	Definition of congruent segments
$2x + 1 = 3x - 7$	Substitution
$8 = x$	Simplify.

So, $PQ = 3(8) - 7$ or 17.

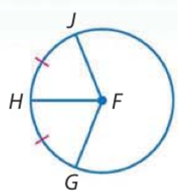
GuidedPractice

2. In $\odot W$, $\widehat{RS} \cong \widehat{TV}$. Find RS.



StudyTip

Arc Bisectors In the figure below, $\overline{FH}$ is an arc bisector of $\widehat{JG}$.



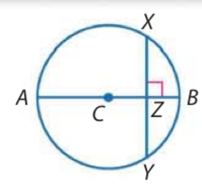
2 Bisecting Arcs and Chords

If a line, segment, or ray divides an arc into two congruent arcs, then it *bisects* the arc.

Theorems

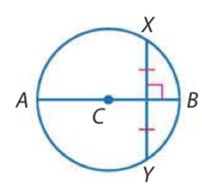
10.3 If a diameter (or radius) of a circle is perpendicular to a chord, then it bisects the chord and its arc.

Example If diameter $\overline{AB}$ is perpendicular to chord $\overline{XY}$, then $\overline{XZ} \cong \overline{ZY}$ and $\widehat{XB} \cong \widehat{BY}$.



10.4 The perpendicular bisector of a chord is a diameter (or radius) of the circle.

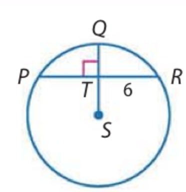
Example If $\overline{AB}$ is a perpendicular bisector of chord $\overline{XY}$, then $\overline{AB}$ is a diameter of $\odot C$.



You will prove Theorems 10.3 and 10.4 in Exercises 26 and 28, respectively.

Example 3 Use a Radius Perpendicular to a Chord

In $\odot S$, $m\widehat{PQR} = 98$. Find $m\widehat{PQ}$.



Radius $\overline{SQ}$ is perpendicular to chord $\overline{PR}$. So by Theorem 10.3, $\overline{SQ}$ bisects $\widehat{PQR}$. Therefore, $m\widehat{PQ} = m\widehat{QR}$. By substitution, $m\widehat{PQ} = \frac{98}{2}$ or 49.

GuidedPractice

3. In $\odot S$, find PR.

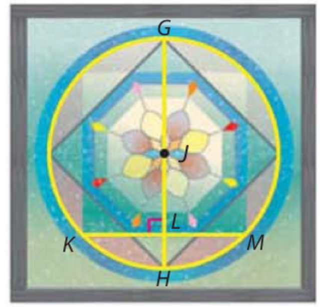
Real-WorldLink

To make stained glass windows, glass is heated to a temperature of 2000 degrees, until it is the consistency of taffy. The colors are caused by the addition of metallic oxides.

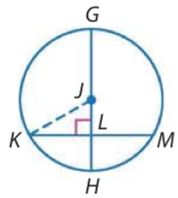
Source: *Artistic Stained Glass by Regg*

Real-World Example 4 Use a Diameter Perpendicular to a Chord

STAINED GLASS In the stained glass window, diameter $\overline{GH}$ is 30 inches long and chord $\overline{KM}$ is 22 inches long. Find JL.



Step 1 Draw radius $\overline{JK}$.



This forms right $\triangle JKL$.

Step 2 Find JK and KL.

Since $GH = 30$ inches, $JH = 15$ inches. All radii of a circle are congruent, so $JK = 15$ inches.

Since diameter $\overline{GH}$ is perpendicular to $\overline{KM}$, $\overline{GH}$ bisects chord $\overline{KM}$ by Theorem 10.3. So, $KL = \frac{1}{2}(22)$ or 11 inches.

Step 3 Use the Pythagorean Theorem to find JL.

$KL^2 + JL^2 = JK^2$	Pythagorean Theorem
$11^2 + JL^2 = 15^2$	$KL = 11$ and $JK = 15$
$121 + JL^2 = 225$	Simplify.
$JL^2 = 104$	Subtract 121 from each side.
$JL = \sqrt{104}$	Take the positive square root of each side.

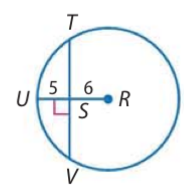
So, JL is $\sqrt{104}$ or about 10.20 inches long.

StudyTip

Drawing Segments You can add any known information to a figure to help you solve the problem. In Example 4, radius $\overline{JK}$ was drawn.

GuidedPractice

4. In $\odot R$, find TV. Round to the nearest hundredth.

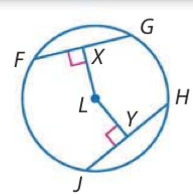


In addition to Theorem 10.2, you can use the following theorem to determine whether two chords in a circle are congruent.

Theorem 10.5

Words In the same circle or in congruent circles, two chords are congruent if and only if they are equidistant from the center.

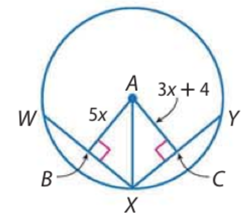
Example $\overline{FG} \cong \overline{JH}$ if and only if $LX = LY$.



You will prove Theorem 10.5 in Exercises 29 and 30.

Example 5 Chords Equidistant from Center

ALGEBRA In ⊙A, $WX = XY = 22$. Find AB.



Since chords $\overline{WX}$ and $\overline{XY}$ are congruent, they are equidistant from A. So, $AB = AC$.

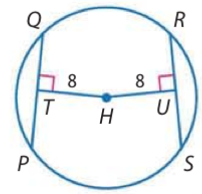
$AB = AC$

$5x = 3x + 4$ — Substitution

$x = 2$ — Simplify.

So, $AB = 5(2)$ or 10.

GuidedPractice

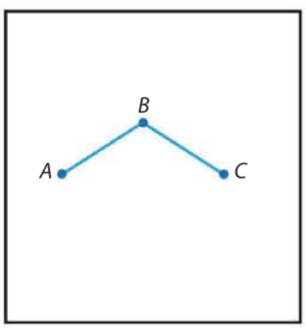


5. In ⊙H, $PQ = 3x - 4$ and $RS = 14$. Find x.

You can use Theorem 10.5 to find the point equidistant from three noncollinear points.

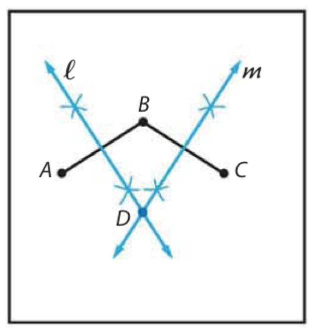
Construction Circle Through Three Noncollinear Points

Step 1



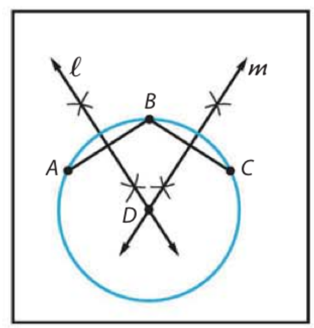
Draw three noncollinear points A, B, and C. Then draw segments $\overline{AB}$ and $\overline{BC}$.

Step 2



Construct the perpendicular bisectors ℓ and m of $\overline{AB}$ and $\overline{BC}$. Label the point of intersection D.

Step 3



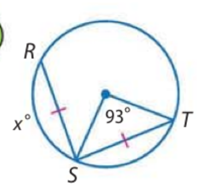
By Theorem 10.4, lines ℓ and m contain diameters of ⊙D. With the compass at point D, draw a circle through points A, B, and C.

Check Your Understanding

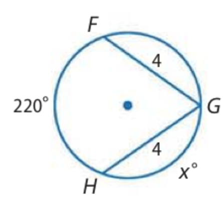
● = Step-by-Step Solutions begin on page R14.

Examples 1–2 ALGEBRA Find the value of x.

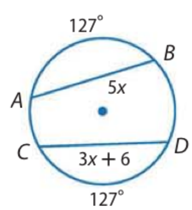
1.



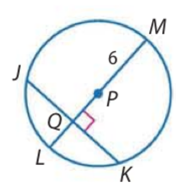
2.



3.



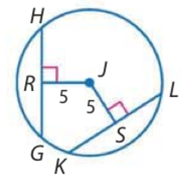
Examples 3–4 In ⊙P, $JK = 10$ and $m\widehat{JLK} = 134$. Find each measure. Round to the nearest hundredth.



4. $m\widehat{JL}$

5. PQ

Example 5 **6.** In $\odot J$, $GH = 9$, $KL = 4x + 1$. Find x.

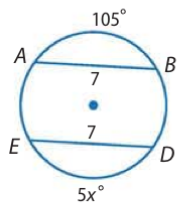


Practice and Problem Solving

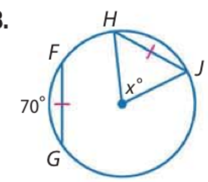
Extra Practice is on page R10.

Examples 1–2 **ALGEBRA** **Find the value of x.**

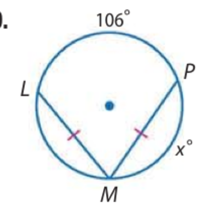
7.



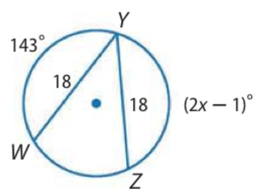
8.



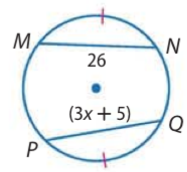
9.



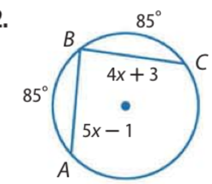
10.



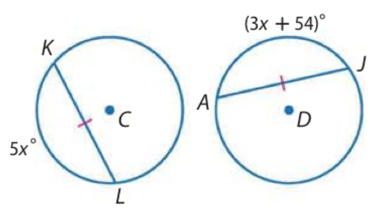
11.



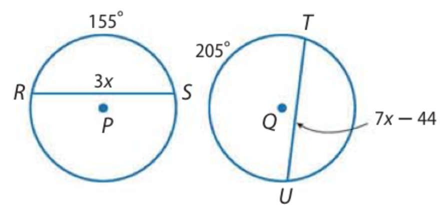
12.



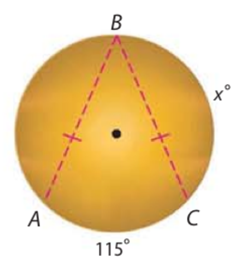
13 $\odot C \cong \odot D$



14. $\odot P \cong \odot Q$



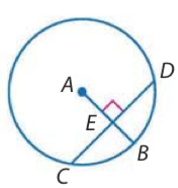
15. **CCSS MODELING** Angie is in a jewelry making class at her local arts center. She wants to make a pair of triangular earrings from a metal circle. She knows that $\widehat{AC}$ is 115°. If she wants to cut two equal parts off so that $\widehat{AB} = \widehat{BC}$, what is x?



Examples 3–4 **In $\odot A$, the radius is 14 and $CD = 22$. Find each measure. Round to the nearest hundredth, if necessary.**

16. CE

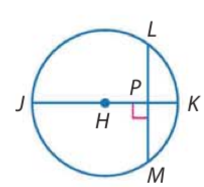
17. EB



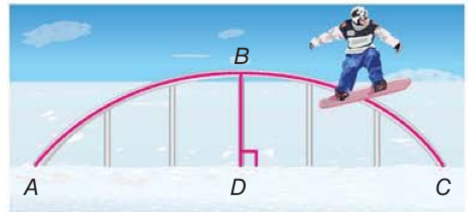
In $\odot H$, the diameter is 18, $LM = 12$, and $m\widehat{LM} = 84$. Find each measure. Round to the nearest hundredth, if necessary.

18. $m\widehat{LK}$

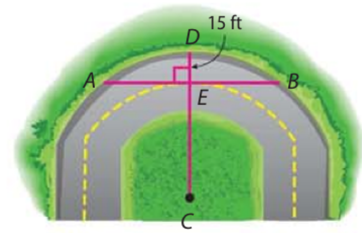
19. HP



20. **SNOWBOARDING** The snowboarding rail shown is an arc of a circle in which $\overline{BD}$ is part of the diameter. If $\widehat{ABC}$ is about 32% of a complete circle, what is $m\widehat{AB}$?

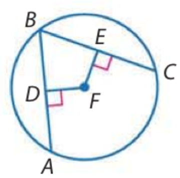


21. **ROADS** The curved road at the right is part of $\odot C$, which has a radius of 88 feet. What is AB? Round to the nearest tenth.

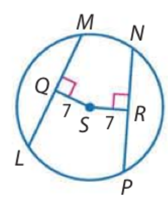


Example 5

22. **ALGEBRA** In $\odot F$, $\overline{AB} \cong \overline{BC}$, $DF = 3x - 7$, and $FE = x + 9$. What is x?



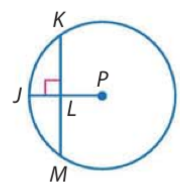
23. **ALGEBRA** In $\odot S$, $LM = 16$ and $PN = 4x$. What is x?



PROOF **Write a two-column proof.**

24. **Given:** $\odot P$, $\overline{KM} \perp \overline{JP}$

Prove: $\overline{JP}$ bisects $\overline{KM}$ and $\widehat{KM}$.

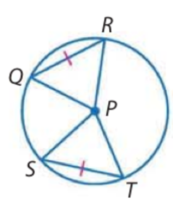


PROOF **Write the specified type of proof.**

25. paragraph proof of Theorem 10.2, part 2

Given: $\odot P$, $\overline{QR} \cong \overline{ST}$

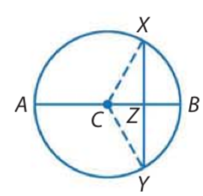
Prove: $\widehat{QR} \cong \widehat{ST}$



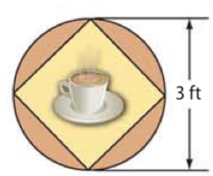
26. two-column proof of Theorem 10.3

Given: $\odot C$, $\overline{AB} \perp \overline{XY}$

Prove: $\overline{XZ} \cong \overline{YZ}$, $\widehat{XB} \cong \widehat{YB}$



27. **DESIGN** Roberto is designing a logo for a friend's coffee shop according to the design at the right, where each chord is equal in length. What is the measure of each arc and the length of each chord?



28. **CCSS ARGUMENTS** Write a two-column proof of Theorem 10.4.

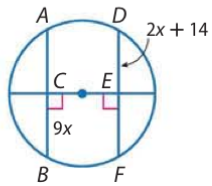
CCSS ARGUMENTS **Write a two-column proof of the indicated part of Theorem 10.5.**

29. In a circle, if two chords are equidistant from the center, then they are congruent.

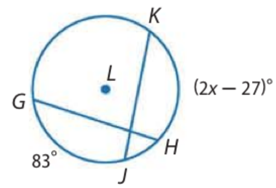
30. In a circle, if two chords are congruent, then they are equidistant from the center.

ALGEBRA **Find the value of *x*.**

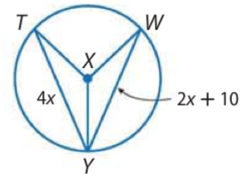
31. $\overline{AB} \cong \overline{DF}$



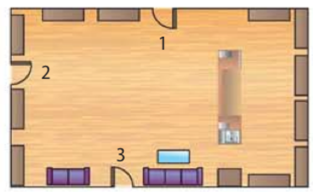
32. $\overline{GH} \cong \overline{KJ}$



33. $\widehat{WTY} \cong \widehat{TWY}$

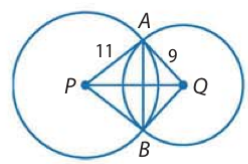


34. **ADVERTISING** A bookstore clerk wants to set up a display of new books. If there are three entrances into the store as shown in the figure at the right, where should the display be to get maximum exposure?



H.O.T. Problems Use Higher-Order Thinking Skills

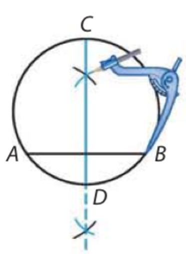
35. **CHALLENGE** The common chord $\overline{AB}$ between $\odot P$ and $\odot Q$ is perpendicular to the segment connecting the centers of the circles. If $AB = 10$, what is the length of $\overline{PQ}$? Explain your reasoning.



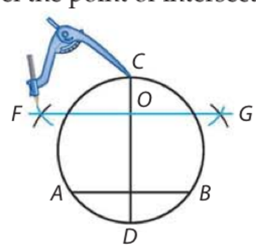
36. **REASONING** In a circle, $\overline{AB}$ is a diameter and $\overline{HG}$ is a chord that intersects $\overline{AB}$ at point X. Is it *sometimes*, *always*, or *never* true that $HX = GX$? Explain.

37. **CHALLENGE** Use a compass to draw a circle with chord $\overline{AB}$. Refer to this construction for the following problem.

Step 1 Construct $\overline{CD}$, the perpendicular bisector of $\overline{AB}$.



Step 2 Construct $\overline{FG}$, the perpendicular bisector of $\overline{CD}$. Label the point of intersection O.



a. Use an indirect proof to show that $\overline{CD}$ passes through the center of the circle by assuming that the center of the circle is *not* on $\overline{CD}$.

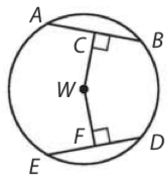
b. Prove that O is the center of the circle.

38. **OPEN ENDED** Construct a circle and draw a chord. Measure the chord and the distance that the chord is from the center. Find the length of the radius.

39. **WRITING IN MATH** If the measure of an arc in a circle is tripled, will the chord of the new arc be three times as long as the chord of the original arc? Explain your reasoning.

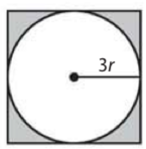
Standardized Test Practice

40. If $CW = WF$ and $ED = 30$, what is DF?



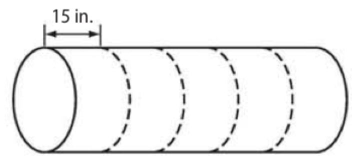
A 60
B 45
C 30
D 15

41. ALGEBRA Write the ratio of the area of the circle to the area of the square in simplest form.

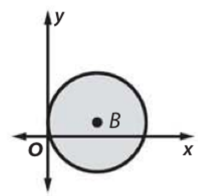


F $\frac{\pi}{4}$ **H** $\frac{3\pi}{4}$
G $\frac{\pi}{2}$ **J** π

42. SHORT RESPONSE The pipe shown is divided into five equal sections. How long is the pipe in feet (ft) and inches (in.)?



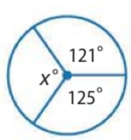
43. SAT/ACT Point B is the center of a circle, tangent to the y-axis, and the coordinates of Point B are (3, 1). What is the area of the circle?



A π units^2 **D** 6π units^2
B 3π units^2 **E** 9π units^2
C 4π units^2

Spiral Review

Find x. (Lesson 10-2)

44.


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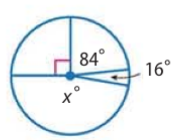

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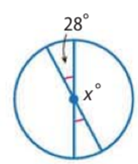


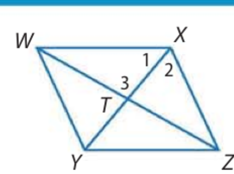
47. CRAFTS Ruby created a pattern to sew flowers onto a quilt by first drawing a regular pentagon that was 3.5 inches long on each side. Then she added a semicircle onto each side of the pentagon to create the appearance of five petals. How many inches of gold trim does she need to edge 10 flowers? Round to the nearest inch. (Lesson 10-1)

Determine whether each set of numbers can be the measures of the sides of a triangle. If so, classify the triangle as *acute*, *obtuse*, or *right*. Justify your answer. (Lesson 8-2)

48. 8, 15, 17 **49.** 20, 21, 31 **50.** 10, 16, 18

Skills Review

ALGEBRA Quadrilateral $WXZY$ is a rhombus. Find each value or measure.



51. If $m\angle 3 = y^2 - 31$, find y.

52. If $m\angle XZY = 56$, find $m\angle YWZ$.

LESSON 10-4 Inscribed Angles

:: Then

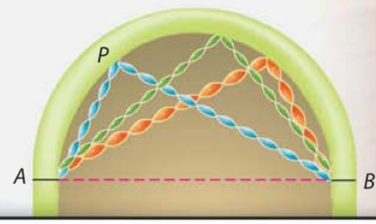
- You found measures of interior angles of polygons.

:: Now

1. Find measures of inscribed angles.
2. Find measures of angles of inscribed polygons.

:: Why?

- The entrance to a school prom has a semicircular arch. Streamers are attached with one end at point *A* and the other end at point *B*. The middle of each streamer can then be attached to a different point *P* along the arch.



NewVocabulary
inscribed angle
intercepted arc

Common Core State Standards

Content Standards

G.C.2 Identify and describe relationships among inscribed angles, radii, and chords.

G.C.3 Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.

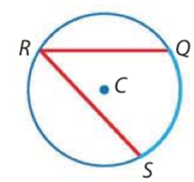
Mathematical Practices

7 Look for and make use of structure.

3 Construct viable arguments and critique the reasoning of others.

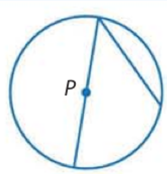
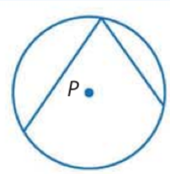
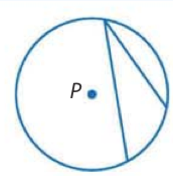
1 Inscribed Angles

Notice that the angle formed by each streamer appears to be congruent, no matter where point *P* is placed along the arch. An **inscribed angle** has a vertex on a circle and sides that contain chords of the circle. In ⊙*C*, $\angle QRS$ is an inscribed angle.



An **intercepted arc** has endpoints on the sides of an inscribed angle and lies in the interior of the inscribed angle. In ⊙*C*, minor arc $\overset{\frown}{QS}$ is intercepted by $\angle QRS$.

There are three ways that an angle can be inscribed in a circle.

Case 1	Case 2	Case 3
		
Center *P* is on a side of the inscribed angle.	Center *P* is inside the inscribed angle.	The center *P* is in the exterior of the inscribed angle.

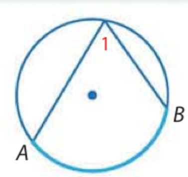
In Case 1, the side of the angle is a diameter of the circle.

For each of these cases, the following theorem holds true.

Theorem 10.6 Inscribed Angle Theorem

Words If an angle is inscribed in a circle, then the measure of the angle equals one half the measure of its intercepted arc.

Example $m\angle 1 = \frac{1}{2}m\overset{\frown}{AB}$ and $m\overset{\frown}{AB} = 2m\angle 1$



You will prove Cases 2 and 3 of the Inscribed Angle Theorem in Exercises 37 and 38.

VocabularyLink

Inscribed

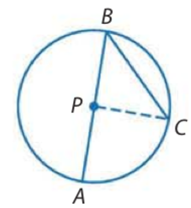
Everyday Use: written on or in a surface, such as inscribing the inside of a ring with an inscription

Math Use: touching only the sides (or interior) of another figure

Proof Inscribed Angle Theorem (Case 1)

Given: $\angle B$ is inscribed in $\odot P$.

Prove: $m\angle B = \frac{1}{2}m\widehat{AC}$



Proof:

Statements	Reasons
1. Draw an auxiliary radius $\overline{PC}$.	**1.** Two points determine a line.
2. $\overline{PB} \cong \overline{PC}$	**2.** All radii of a circle are $\cong$.
3. $\triangle PBC$ is isosceles.	**3.** Definition of isosceles triangle
4. $m\angle B = m\angle C$	**4.** Isosceles Triangle Theorem
5. $m\angle APC = m\angle B + m\angle C$	**5.** Exterior Angle Theorem
6. $m\angle APC = 2m\angle B$	**6.** Substitution (Steps 4, 5)
7. $m\widehat{AC} = m\angle APC$	**7.** Definition of arc measure
8. $m\widehat{AC} = 2m\angle B$	**8.** Substitution (Steps 6, 7)
9. $2m\angle B = m\widehat{AC}$	**9.** Symmetric Property of Equality
10. $m\angle B = \frac{1}{2}m\widehat{AC}$	**10.** Division Property of Equality

Example 1 Use Inscribed Angles to Find Measures

Find each measure.

a. $m\angle P$

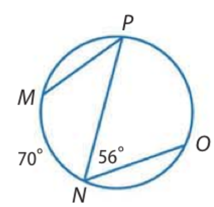
$m\angle P = \frac{1}{2}m\widehat{MN}$

$= \frac{1}{2}(70)$ or 35

b. $m\widehat{PO}$

$m\widehat{PO} = 2m\angle N$

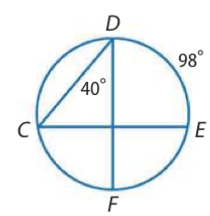
$= 2(56)$ or 112



GuidedPractice

1A. $m\widehat{CF}$

1B. $m\angle C$

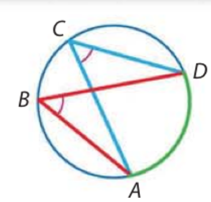


Two inscribed angles that intercept the same arc of a circle are related.

Theorem 10.7

Words If two inscribed angles of a circle intercept the same arc or congruent arcs, then the angles are congruent.

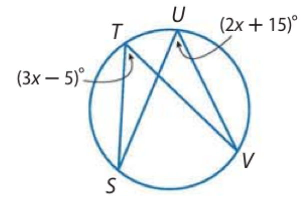
Example $\angle B$ and $\angle C$ both intercept $\widehat{AD}$. So, $\angle B \cong \angle C$.



You will prove Theorem 10.7 in Exercise 39.

Example 2 Use Inscribed Angles to Find Measures

ALGEBRA **Find $m\angle T$.**



$\angle T \cong \angle U$	$\angle T$ and $\angle U$ both intercept $\widehat{SV}$.
$m\angle T = m\angle U$	Definition of congruent angles
$3x - 5 = 2x + 15$	Substitution
$x = 20$	Simplify.

So, $m\angle T = 3(20) - 5$ or 55.

GuidedPractice

2. If $m\angle S = 3x$ and $m\angle V = (x + 16)$, find $m\angle S$.

StudyTip

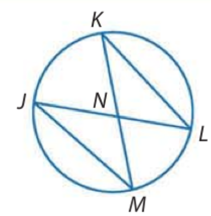
Inscribed Polygons Remember that for a polygon to be an inscribed polygon, *all* of its vertices must lie on the circle.

Example 3 Use Inscribed Angles in Proofs

Write a two-column proof.

Given: $\widehat{JM} \cong \widehat{KL}$

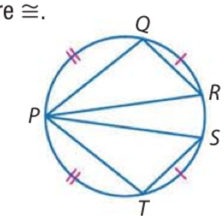
Prove: $\triangle JMN \cong \triangle KLN$



Proof:

Statements	Reasons
1. $\widehat{JM} \cong \widehat{KL}$	**1.** Given
2. $\overline{JM} \cong \overline{KL}$	**2.** If minor arcs are ≅, their corresponding chords are ≅.
3. $\angle M$ intercepts $\widehat{JK}$. $\angle L$ intercepts $\widehat{JK}$.	**3.** Definition of intercepted arc
4. $\angle M \cong \angle L$	**4.** Inscribed ∡ of same arc are ≅.
5. $\angle JNM \cong \angle KNL$	**5.** Vertical ∡ are ≅.
6. $\triangle JMN \cong \triangle KLN$	**6.** AAS

GuidedPractice



3. Given: $\widehat{QR} \cong \widehat{ST}$, $\widehat{PQ} \cong \widehat{PT}$

Prove: $\triangle PQR \cong \triangle PTS$

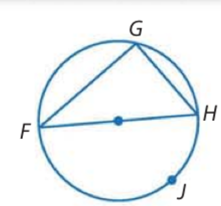
2 Angles of Inscribed Polygons

Triangles and quadrilaterals that are inscribed in circles have special properties.

Theorem 10.8

Words An inscribed angle of a triangle intercepts a diameter or semicircle if and only if the angle is a right angle.

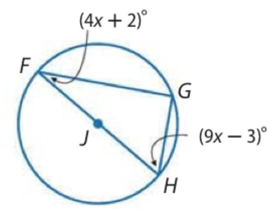
Example If $\widehat{FJH}$ is a semicircle, then $m\angle G = 90$. If $m\angle G = 90$, then $\widehat{FJH}$ is a semicircle and $\overline{FH}$ is a diameter.



You will prove Theorem 10.8 in Exercise 40.

Example 4 Find Angle Measures in Inscribed Triangles

ALGEBRA Find $m\angle F$.



$\triangle FGH$ is a right triangle because $\angle G$ inscribes a semicircle.

$m\angle F + m\angle G + m\angle H = 180$	Angle Sum Theorem
$(4x + 2) + 90 + (9x - 3) = 180$	Substitution
$13x + 89 = 180$	Simplify.
$13x = 91$	Subtract 89 from each side.
$x = 7$	Divide each side by 13.

So, $m\angle F = 4(7) + 2$ or 30.

GuidedPractice

4. If $m\angle F = 7x + 2$ and $m\angle H = 17x - 8$, find x.

While many different types of triangles, including right triangles, can be inscribed in a circle, only certain quadrilaterals can be inscribed in a circle.

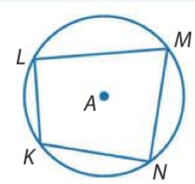
StudyTip

CCSS Arguments Theorem 10.9 can be verified by considering that the arcs intercepted by opposite angles of an inscribed quadrilateral form a circle.

Theorem 10.9

Words If a quadrilateral is inscribed in a circle, then its opposite angles are supplementary.

Example If quadrilateral *KLMN* is inscribed in $\odot A$, then $\angle L$ and $\angle N$ are supplementary and $\angle K$ and $\angle M$ are supplementary.



You will prove Theorem 10.9 in Exercise 31.

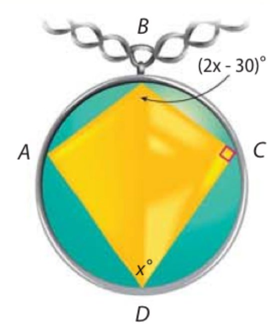
Real-WorldLink

Charms for jewelry first became popular during the age of the Egyptian Pharaohs. They were repopularized by Queen Victoria in the early 20th century and by Louis Vuitton in 2001.

Source: *My Mother's Charms*

Real-World Example 5 Find Angle Measures

JEWELRY The necklace charm shown uses a quadrilateral inscribed in a circle. Find $m\angle A$ and $m\angle B$.



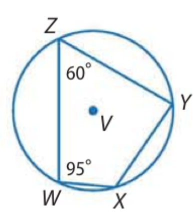
Since *ABCD* is inscribed in a circle, opposite angles are supplementary.

$m\angle A + m\angle C = 180$	$m\angle B + m\angle D = 180$
$m\angle A + 90 = 180$	$(2x - 30) + x = 180$
$m\angle A = 90$	$3x - 30 = 180$
	$3x = 210$
	$x = 70$

So, $m\angle A = 90$ and $m\angle B = 2(70) - 30$ or 110.

GuidedPractice

5. Quadrilateral *WXYZ* is inscribed in $\odot V$. Find $m\angle X$ and $m\angle Y$.



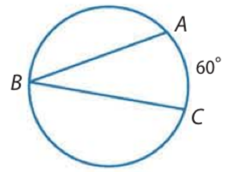
Steve Gorton/Dorling Kindersley/Getty Images

Check Your Understanding

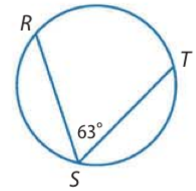
= Step-by-Step Solutions begin on page R14.

Example 1 **Find each measure.**

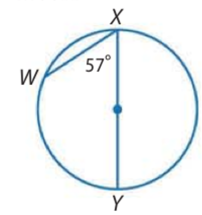
1. $m\angle B$



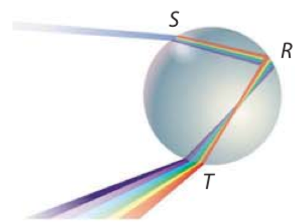
2. $m\widehat{RT}$



3. $m\widehat{WX}$

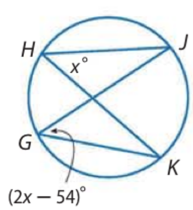


4. SCIENCE The diagram shows how light bends in a raindrop to make the colors of the rainbow. If $m\widehat{ST} = 144$, what is $m\angle R$?

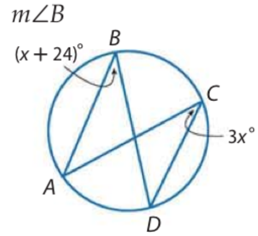


Example 2 **ALGEBRA** **Find each measure.**

5. $m\angle H$



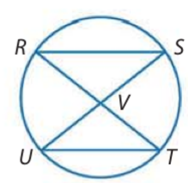
6. $m\angle B$



Example 3 **7. PROOF** Write a two-column proof.

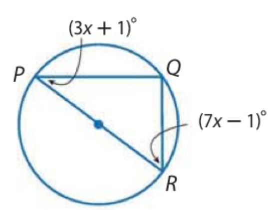
Given: $\overline{RT}$ bisects $\overline{SU}$.

Prove: $\triangle RVS \cong \triangle UVT$

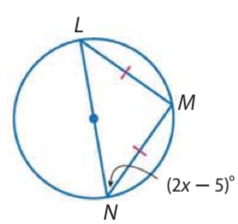


Examples 4–5 CCSS **STRUCTURE** **Find each value.**

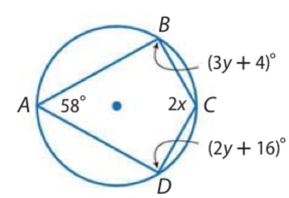
8. $m\angle R$



9. x



10. $m\angle C$ and $m\angle D$

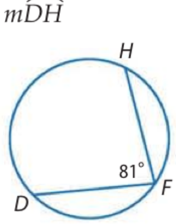


Practice and Problem Solving

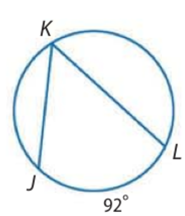
Extra Practice is on page R10.

Example 1 **Find each measure.**

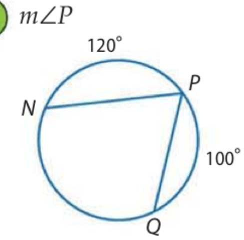
11. $m\widehat{DH}$



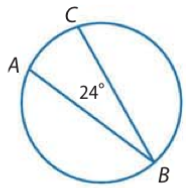
12. $m\angle K$



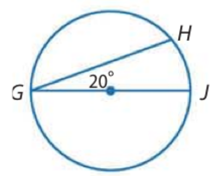
13. $m\angle P$



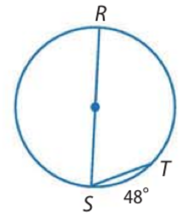
14. $m\widehat{AC}$



15. $m\widehat{GH}$



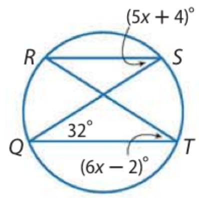
16. $m\angle S$



Example 2 **ALGEBRA** **Find each measure.**

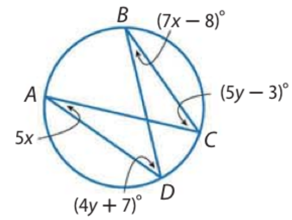
17. $m\angle R$

18. $m\angle S$



19. $m\angle A$

20. $m\angle C$

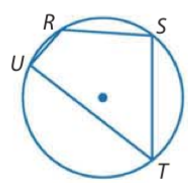


Example 3 **PROOF** **Write the specified type of proof.**

21. paragraph proof

Given: $m\angle T = \frac{1}{2}m\angle S$

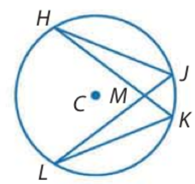
Prove: $m\widehat{TUR} = 2m\widehat{URS}$



22. two-column proof

Given: $\odot C$

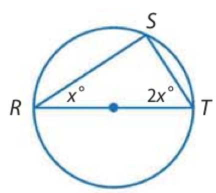
Prove: $\triangle KML \sim \triangle JMH$



Example 4 **ALGEBRA** **Find each value.**

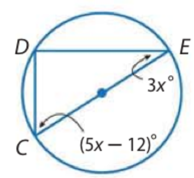
23. x

24. $m\angle T$



25. x

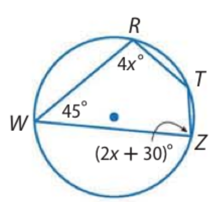
26. $m\angle C$



Example 5 **CCSS STRUCTURE** **Find each measure.**

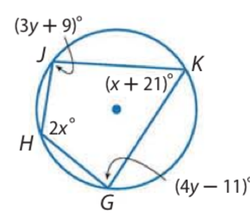
27. $m\angle T$

28. $m\angle Z$



29. $m\angle H$

30. $m\angle G$



31. **PROOF** Write a paragraph proof for Theorem 10.9.

SIGNS **A stop sign in the shape of a regular octagon is inscribed in a circle. Find each measure.**

32. $m\widehat{NQ}$

33. $m\angle RLQ$

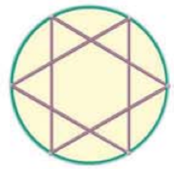
34. $m\angle LRQ$

35. $m\angle LSR$

36. **ART** Four different string art star patterns are shown. If all of the inscribed angles of each star shown are congruent, find the measure of each inscribed angle.

a.

b.



c.

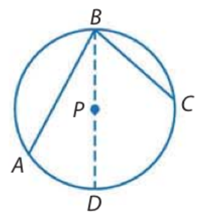
d.

PROOF **Write a two-column proof for each case of Theorem 10.6.**

37. **Case 2**

Given: P lies inside $\angle ABC$.
$\overline{BD}$ is a diameter.

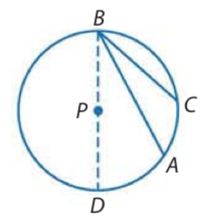
Prove: $m\angle ABC = \frac{1}{2}m\widehat{AC}$



38. **Case 3**

Given: P lies outside $\angle ABC$.
$\overline{BD}$ is a diameter.

Prove: $m\angle ABC = \frac{1}{2}m\widehat{AC}$



PROOF **Write the specified proof for each theorem.**

39. Theorem 10.7, two-column proof

40. Theorem 10.8, paragraph proof

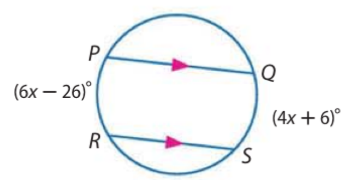
41. **MULTIPLE REPRESENTATIONS** In this problem, you will investigate the relationship between the arcs of a circle that are cut by two parallel chords.

a. Geometric Use a compass to draw a circle with parallel chords $\overline{AB}$ and $\overline{CD}$. Connect points A and D by drawing segment $\overline{AD}$.

b. Numerical Use a protractor to find $m\angle A$ and $m\angle D$. Then determine $m\widehat{AC}$ and $m\widehat{BD}$. What is true about these arcs? Explain.

c. Verbal Draw another circle and repeat parts **a** and **b**. Make a conjecture about arcs of a circle that are cut by two parallel chords.

d. Analytical Use your conjecture to find $m\widehat{PR}$ and $m\widehat{QS}$ in the figure at the right. Verify by using inscribed angles to find the measures of the arcs.



H.O.T. Problems Use Higher-Order Thinking Skills

CCSS ARGUMENTS **Determine whether the quadrilateral can *always*, *sometimes*, or *never* be inscribed in a circle. Explain your reasoning.**

42. square **43.** rectangle **44.** parallelogram **45.** rhombus **46.** kite

47. **CHALLENGE** A square is inscribed in a circle. What is the ratio of the area of the circle to the area of the square?

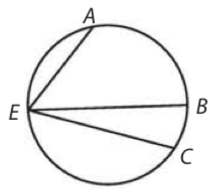
48. **WRITING IN MATH** A 45°-45°-90° right triangle is inscribed in a circle. If the radius of the circle is given, explain how to find the lengths of the right triangle's legs.

49. **OPEN ENDED** Find and sketch a real-world logo with an inscribed polygon.

50. **WRITING IN MATH** Compare and contrast inscribed angles and central angles of a circle. If they intercept the same arc, how are they related?

Standardized Test Practice

51. In the circle below, $m\widehat{AC} = 160$ and $m\angle BEC = 38$. What is $m\angle AEB$?

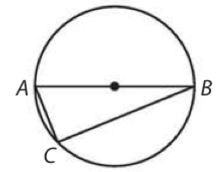


A 42 **C** 80
B 61 **D** 84

52. ALGEBRA Simplify $4(3x - 2)(2x + 4) + 3x^2 + 5x - 6$.

F $9x^2 + 3x - 14$ **H** $27x^2 + 37x - 38$
G $9x^2 + 13x - 14$ **J** $27x^2 + 27x - 26$

53. SHORT RESPONSE In the circle below, $\overline{AB}$ is a diameter, $AC = 8$ inches, and $BC = 15$ inches. Find the diameter, the radius, and the circumference of the circle.

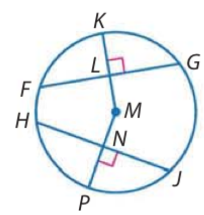


54. SAT/ACT The sum of three consecutive integers is −48. What is the least of the three integers?

A −15 **D** −18
B −16 **E** −19
C −17

Spiral Review

In ⊙M, $FL = 24$, $HJ = 48$, and $m\widehat{HP} = 65$. Find each measure. (Lesson 10-3)

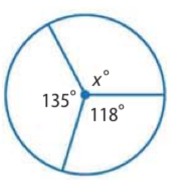


55. FG **56.** $m\widehat{PJ}$

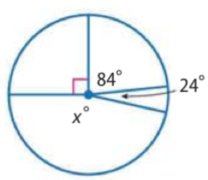
57. NJ **58.** $m\widehat{HJ}$

Find x. (Lesson 10-2)

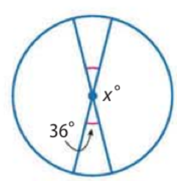
59.



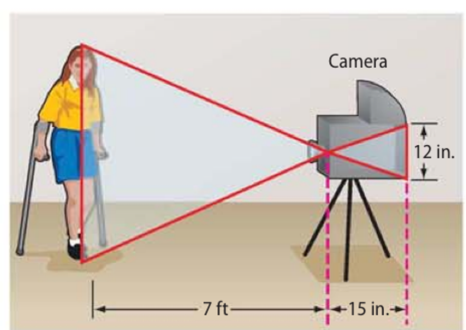
60.



61.



62. PHOTOGRAPHY In one of the first cameras invented, light entered an opening in the front. An image was reflected in the back of the camera, upside down, forming similar triangles. Suppose the image of the person on the back of the camera is 12 inches, the distance from the opening to the person is 7 feet, and the camera itself is 15 inches long. How tall is the person being photographed? (Lesson 7-3)



Skills Review

ALGEBRA Suppose B is the midpoint of $\overline{AC}$. Use the given information to find the missing measure.

63. $AB = 4x - 5$, $BC = 11 + 2x$, $AC = ?$

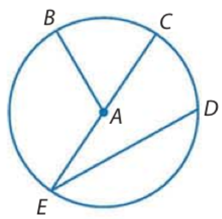
64. $AB = 6y - 14$, $BC = 10 - 2y$, $AC = ?$

65. $BC = 6 - 4m$, $AC = 8$, $m = ?$

66. $AB = 10s + 2$, $AC = 40$, $s = ?$

Chapter 10 Mid-Chapter Quiz
Lessons 10-1 through 10-4

For Exercises 1–3, refer to ⊙A. (Lesson 10-1)



1. Name the circle.

2. Name a diameter.

3. Name a chord that is not a diameter.

4. **BICYCLES** A bicycle has tires that are 24 inches in diameter. (Lesson 10-1)

a. Find the circumference of one tire.

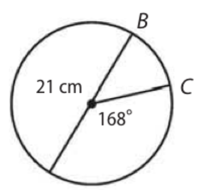
b. How many inches does the tire travel after 100 rotations?

Find the diameter and radius of a circle with the given circumference. Round to the nearest hundredth. (Lesson 10-1)

5. $C = 23$ cm

6. $C = 78$ ft

7. **MULTIPLE CHOICE** Find the length of $\widehat{BC}$. (Lesson 10-2)



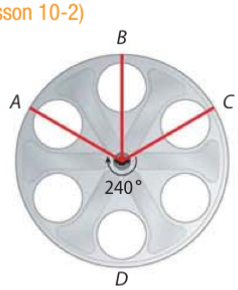
A 18°

B 2.20 cm

C 168°

D 30.79 cm

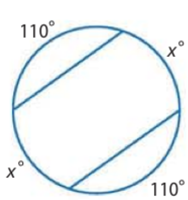
8. **MOVIES** The movie reel shown below has a diameter of 14.5 inches. (Lesson 10-2)



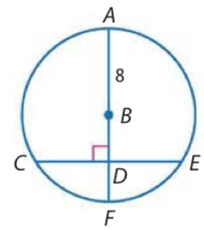
a. Find $m\widehat{ADC}$.

b. Find the length of $\widehat{ADC}$.

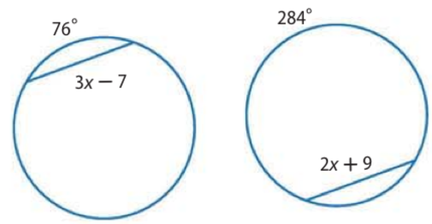
9. Find the value of x. (Lesson 10-3)



10. In ⊙B, $CE = 13.5$. Find BD. Round to the nearest hundredth. (Lesson 10-3)

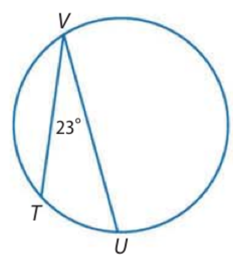


11. The two circles shown are congruent. Find x and the length of the chord. (Lesson 10-3)

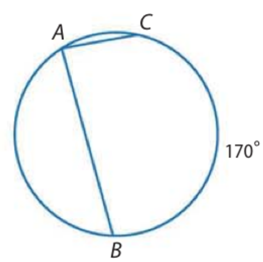


Find each measure. (Lesson 10-4)

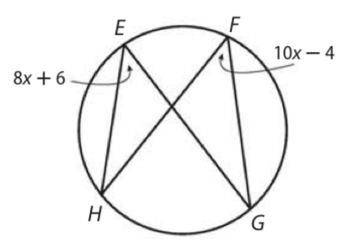
12. $m\widehat{TU}$



13. $m\angle A$



14. **MULTIPLE CHOICE** Find x. (Lesson 10-4)



F 1.8

G 5

H 46

J 90

15. If a square with sides of 14 inches is inscribed in a circle, what is the diameter of the circle? (Lesson 10-4)

LESSON

10-5 Tangents

Then

- You used the Pythagorean Theorem to find side lengths of right triangles.

Now

- 1 Use properties of tangents.
- 2 Solve problems involving circumscribed polygons.

Why?

- The first bicycles were moved by pushing your feet on the ground. Modern bicycles use pedals, a chain, and gears. The chain loops around circular gears. The length of the chain between these gears is measured from the points of tangency.

New Vocabulary

tangent
point of tangency
common tangent

Common Core State Standards

Content Standards

G.CO.12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.).

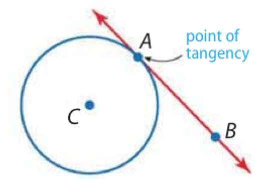
G.C.4 Construct a tangent line from a point outside a given circle to the circle.

Mathematical Practices

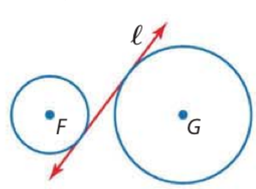
1 Make sense of problems and persevere in solving them.

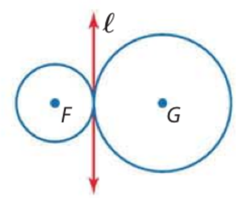
2 Reason abstractly and quantitatively.

1 Tangents A **tangent** is a line in the same plane as a circle that intersects the circle in exactly one point, called the **point of tangency**. $\overleftrightarrow{AB}$ is tangent to $\odot C$ at point A. $\overline{AB}$ and $\overrightarrow{AB}$ are also called tangents.



A **common tangent** is a line, ray, or segment that is tangent to two circles in the same plane. In each figure below, line ℓ is a common tangent of circles F and G.

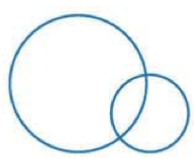




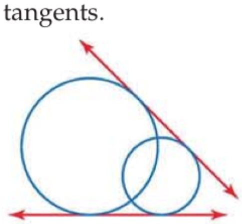
Example 1 Identify Common Tangents

Copy each figure and draw the common tangents. If no common tangent exists, state *no common tangent*.

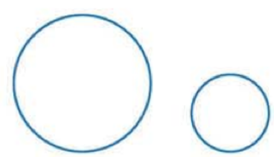
a.



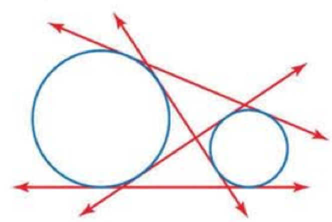
These circles have two common tangents.



b.

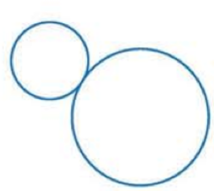


These circles have 4 common tangents.



Guided Practice

1A.



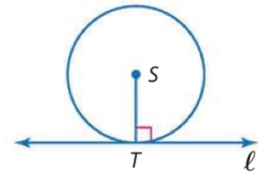
1B.

The shortest distance from a tangent to the center of a circle is the radius drawn to the point of tangency.

Theorem 10.10

Words In a plane, a line is tangent to a circle if and only if it is perpendicular to a radius drawn to the point of tangency.

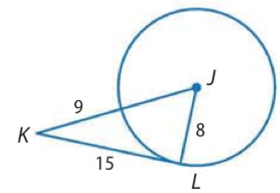
Example Line ℓ is tangent to $\odot S$ if and only if $\ell \perp \overline{ST}$.



You will prove both parts of Theorem 10.10 in Exercises 32 and 33.

Example 2 Identify a Tangent

$\overline{JL}$ is a radius of $\odot J$. Determine whether $\overline{KL}$ is tangent to $\odot J$. Justify your answer.



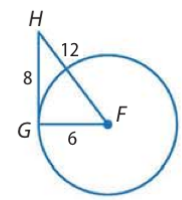
Test to see if $\triangle JKL$ is a right triangle.

$8^2 + 15^2 \stackrel{?}{=} (8 + 9)^2$ Pythagorean Theorem

$289 = 289$ ✓ Simplify.

$\triangle JKL$ is a right triangle with right angle JLK. So $\overline{KL}$ is perpendicular to radius $\overline{JL}$ at point L. Therefore, by Theorem 10.10, $\overline{KL}$ is tangent to $\odot J$.

GuidedPractice

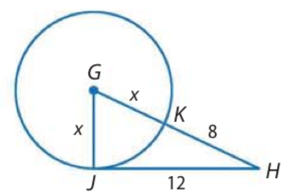


2. Determine whether $\overline{GH}$ is tangent to $\odot F$. Justify your answer.

You can also use Theorem 10.10 to identify missing values.

Example 3 Use a Tangent to Find Missing Values

$\overline{JH}$ is tangent to $\odot G$ at J. Find the value of x.



By Theorem 10.10, $\overline{JH} \perp \overline{GJ}$. So, $\triangle GHJ$ is a right triangle.

$GJ^2 + JH^2 = GH^2$ Pythagorean Theorem

$x^2 + 12^2 = (x + 8)^2$ $GJ = x$, $JH = 12$, and $GH = x + 8$

$x^2 + 144 = x^2 + 16x + 64$ Multiply.

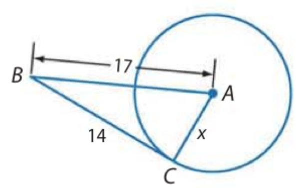
$80 = 16x$ Simplify.

$5 = x$ Divide each side by 16.

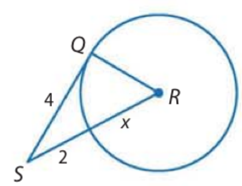
GuidedPractice

Find the value of x. Assume that segments that appear to be tangent are tangent.

3A.

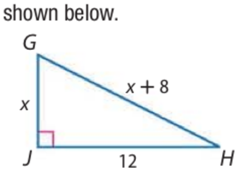


3B.



Problem-SolvingTip

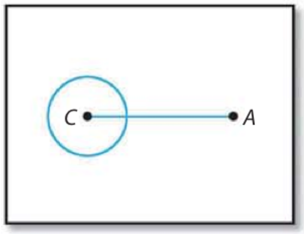
CCSS Sense-Making You can use the *solve a simpler problem* strategy by sketching and labeling the right triangles without the circles. A drawing of the triangle in Example 3 is shown below.



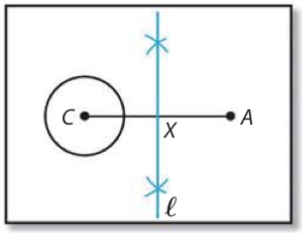
You can use Theorems 10.8 and 10.10 to construct a line tangent to a circle.

Construction Line Tangent to a Circle Through an External Point

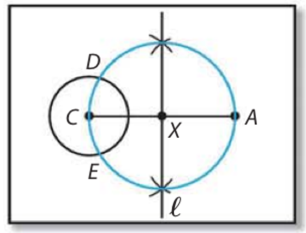
Step 1 Use a compass to draw circle C and a point A outside of circle C. Then draw $\overline{CA}$.



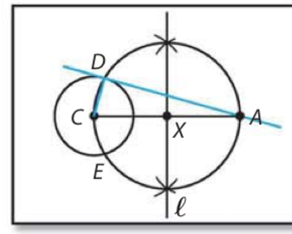
Step 2 Construct line ℓ, the perpendicular bisector of $\overline{CA}$. Label the point of intersection X.



Step 3 Construct circle X with radius $\overline{XC}$. Label the points of intersection of the two circles D and E.



Step 4 Draw $\overleftrightarrow{AD}$ and $\overline{DC}$. $\triangle ADC$ is inscribed in a semicircle. So, $\angle ADC$ is a right angle and $\overleftrightarrow{AD}$ is tangent to $\odot C$.



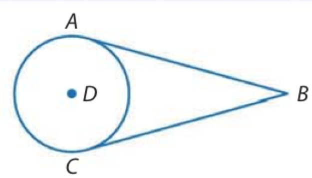
You will justify this construction in Exercise 36 and construct a line tangent to a circle through a point on the circle in Exercise 34.

More than one line can be tangent to the same circle.

Theorem 10.11

Words If two segments from the same exterior point are tangent to a circle, then they are congruent.

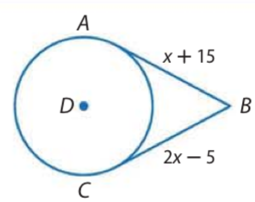
Example If $\overline{AB}$ and $\overline{CB}$ are tangent to $\odot D$, then $\overline{AB} \cong \overline{CB}$.



You will prove Theorem 10.11 in Exercise 28.

Example 4 Use Congruent Tangents to Find Measures

ALGEBRA $\overline{AB}$ **and** $\overline{CB}$ **are tangent to** $\odot D$**. Find the value of** x**.**

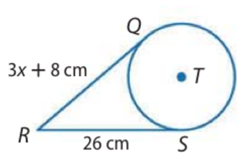


$AB = CB$	Tangents from the same exterior point are congruent.
$x + 15 = 2x - 5$	Substitution
$15 = x - 5$	Subtract x from each side.
$20 = x$	Add 5 to each side.

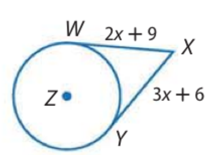
GuidedPractice

ALGEBRA Find the value of x**. Assume that segments that appear to be tangent are tangent.**

4A.



4B.



2 Circumscribed Polygons

A polygon is circumscribed about a circle if every side of the polygon is tangent to the circle.

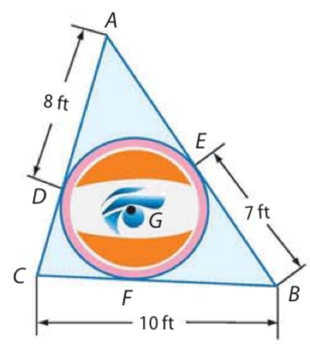
> **WatchOut!**
> **Identifying Circumscribed Polygons** Just because the circle is tangent to one or more of the sides of a polygon does not mean that the polygon is circumscribed about the circle, as shown in the second set of figures.

Circumscribed Polygons	Polygons Not Circumscribed
	

You can use Theorem 10.11 to find missing measures in circumscribed polygons.

Real-World Example 5 Find Measures in Circumscribed Polygons

GRAPHIC DESIGN A graphic designer is giving directions to create a larger version of the triangular logo shown. If $\triangle ABC$ is circumscribed about $\odot G$, find the perimeter of $\triangle ABC$.



Step 1 Find the missing measures.

Since $\triangle ABC$ is circumscribed about $\odot G$, $\overline{AE}$ and $\overline{AD}$ are tangent to $\odot G$, as are $\overline{BE}$, $\overline{BF}$, $\overline{CF}$, and $\overline{CD}$. Therefore, $\overline{AE} \cong \overline{AD}$, $\overline{BF} \cong \overline{BE}$, and $\overline{CF} \cong \overline{CD}$.

So, $AE = AD = 8$ feet, $BF = BE = 7$ feet.

By Segment Addition, $CF + FB = CB$, so $CF = CB - FB = 10 - 7$ or 3 feet. So, $CD = CF = 3$ feet.

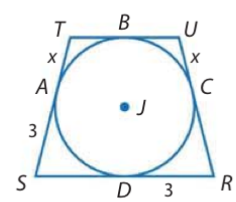
Step 2 Find the perimeter of $\triangle ABC$.

$$\text{perimeter} = AE + EB + BC + CD + DA$$
$$= 8 + 7 + 10 + 3 + 8 \text{ or } 36$$

So, the perimeter of $\triangle ABC$ is 36 feet.

GuidedPractice

5. Quadrilateral $RSTU$ is circumscribed about $\odot J$. If the perimeter is 18 units, find x.



Check Your Understanding

= Step-by-Step Solutions begin on page R14.

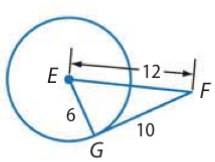
Example 1

1. Copy the figure shown, and draw the common tangents. If no common tangent exists, state *no common tangent*.

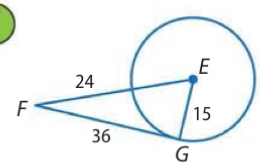
Example 2

Determine whether $\overline{FG}$ is tangent to $\odot E$. Justify your answer.

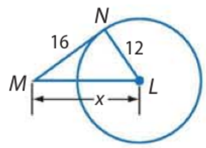
2.

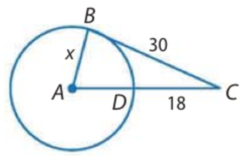


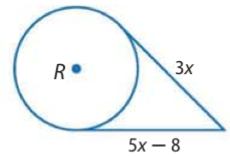
3



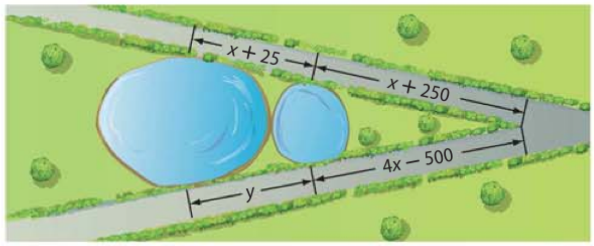
Examples 3–4 **Find x. Assume that segments that appear to be tangent are tangent.**

4.


5.


6.


7. **LANDSCAPE ARCHITECT** A landscape architect is paving the two walking paths that are tangent to two approximately circular ponds as shown. The lengths given are in feet. Find the values of x and y.

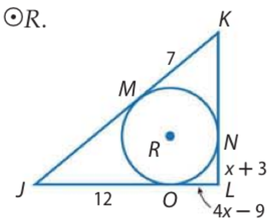


Example 5

8. CCSS **SENSE-MAKING** Triangle JKL is circumscribed about $\odot R$.

a. Find x.

b. Find the perimeter of $\triangle JKL$.



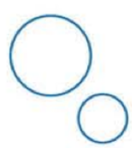
Practice and Problem Solving

Extra Practice is on page R10.

Example 1 **Copy each figure and draw the common tangents. If no common tangent exists, state *no common tangent*.**

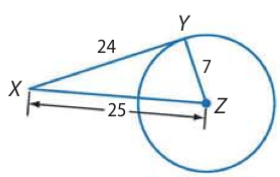
9.

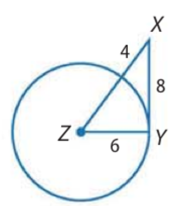

10.

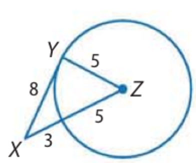

11.


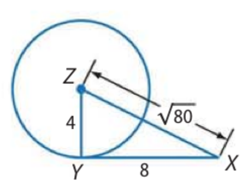
12.


Example 2 **Determine whether each $\overline{XY}$ is tangent to the given circle. Justify your answer.**

13.


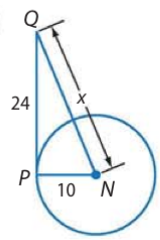
14.


15.


16.


Examples 3–4 **Find x. Assume that segments that appear to be tangent are tangent. Round to the nearest tenth if necessary.**

17.



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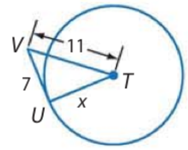

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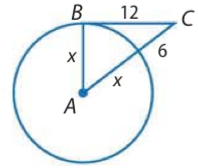

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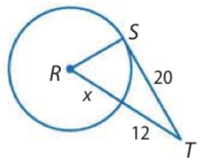

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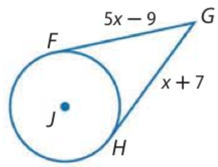

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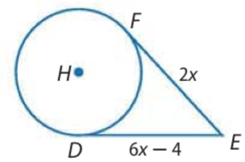
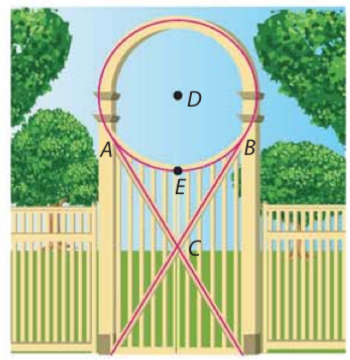


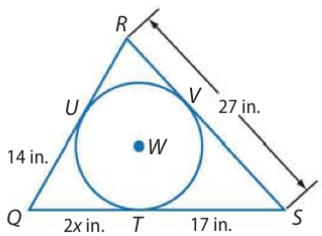
23. ARBORS In the arbor shown, $\overline{AC}$ and $\overline{BC}$ are tangents to $\odot D$. The radius of the circle is 26 inches and $EC = 20$ inches. Find each measure to the nearest hundredth.

a. AC **b.** BC

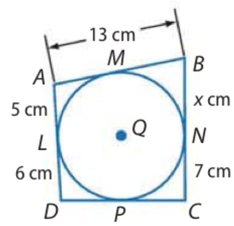


Example 5 **CCSS SENSE-MAKING** **Find the value of x. Then find the perimeter.**

24.

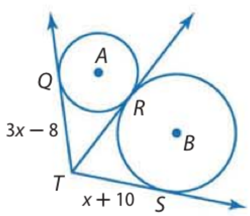


25.

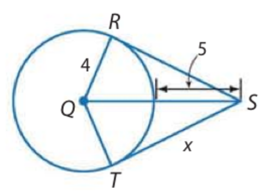


Find x to the nearest hundredth. Assume that segments that appear to be tangent are tangent.

26.



27.

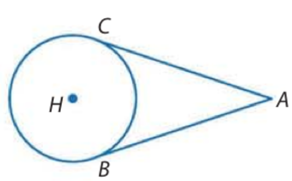


Write the specified type of proof.

28. two-column proof of Theorem 10.11

Given: $\overline{AC}$ is tangent to $\odot H$ at C.
$\overline{AB}$ is tangent to $\odot H$ at B.

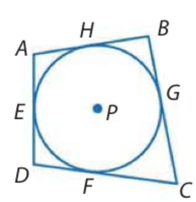
Prove: $\overline{AC} \cong \overline{AB}$



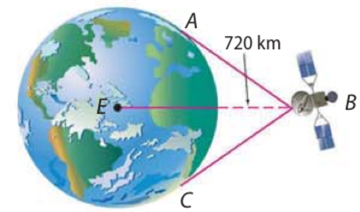
29. two-column proof

Given: Quadrilateral $ABCD$ is circumscribed about $\odot P$.

Prove: $AB + CD = AD + BC$



30. **SATELLITES** A satellite is 720 kilometers above Earth, which has a radius of 6360 kilometers. The region of Earth that is visible from the satellite is between the tangent lines $\overline{BA}$ and $\overline{BC}$. What is BA? Round to the nearest hundredth.



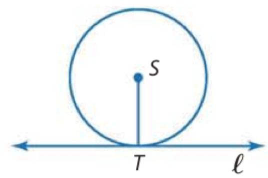
31 **SPACE TRASH** *Orbital debris* refers to materials from space missions that still orbit Earth. In 2007, a 1400-pound ammonia tank was discarded from a space mission. Suppose the tank has an altitude of 435 miles. What is the distance from the tank to the farthest point on Earth's surface from which the tank is visible? Assume that the radius of Earth is 4000 miles. Round to the nearest mile, and include a diagram of this situation with your answer.

32. **PROOF** Write an indirect proof to show that if a line is tangent to a circle, then it is perpendicular to a radius of the circle. (Part 1 of Theorem 10.10)

Given: ℓ is tangent to $\odot S$ at T; $\overline{ST}$ is a radius of $\odot S$.

Prove: $\ell \perp \overline{ST}$

(*Hint:* Assume ℓ is *not* $\perp$ to $\overline{ST}$.)

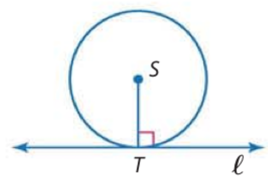


33. **PROOF** Write an indirect proof to show that if a line is perpendicular to the radius of a circle at its endpoint, then the line is a tangent of the circle. (Part 2 of Theorem 10.10)

Given: $\ell \perp \overline{ST}$; $\overline{ST}$ is a radius of $\odot S$.

Prove: ℓ is tangent to $\odot S$.

(*Hint:* Assume ℓ is *not* tangent to $\odot S$.)

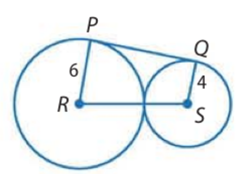


34. CCSS **TOOLS** Construct a line tangent to a circle through a point on the circle.

Use a compass to draw $\odot A$. Choose a point P on the circle and draw $\overleftrightarrow{AP}$. Then construct a segment through point P perpendicular to $\overleftrightarrow{AP}$. Label the tangent line t. Explain and justify each step.

H.O.T. Problems Use Higher-Order Thinking Skills

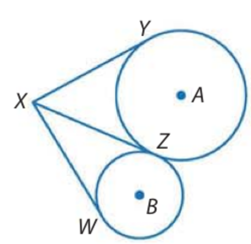
35. **CHALLENGE** $\overline{PQ}$ is tangent to circles R and S. Find PQ. Explain your reasoning.



36. **WRITING IN MATH** Explain and justify each step in the construction on page 734.

37. **OPEN ENDED** Draw a circumscribed triangle and an inscribed triangle.

38. **REASONING** In the figure, $\overline{XY}$ and $\overline{XZ}$ are tangent to $\odot A$. $\overline{XZ}$ and $\overline{XW}$ are tangent to $\odot B$. Explain how segments $\overline{XY}$, $\overline{XZ}$, and $\overline{XW}$ can all be congruent if the circles have different radii.



39. **WRITING IN MATH** Is it possible to draw a tangent from a point that is located anywhere outside, on, or inside a circle? Explain.

Standardized Test Practice

40. $\odot P$ has a radius of 10 centimeters, and $\overline{ED}$ is tangent to the circle at point D. F lies both on $\odot P$ and on segment $\overline{EP}$. If $ED = 24$ centimeters, what is the length of $\overline{EF}$?

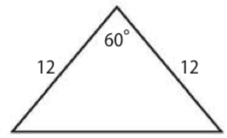
A 10 cm **C** 21.8 cm
B 16 cm **D** 26 cm

41. SHORT RESPONSE A square is inscribed in a circle having a radius of 6 inches. Find the length of each side of the square.

42. ALGEBRA Which of the following shows $25x^2 - 5x$ factored completely?

F $5x(x)$ **H** $x(x - 5)$
G $5x(5x - 1)$ **J** $x(5x - 1)$

43. SAT/ACT What is the perimeter of the triangle shown below?

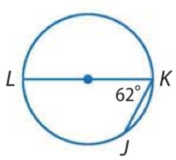


A 12 units **D** 36 units
B 24 units **E** 104 units
C 34.4 units

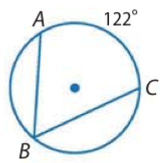
Spiral Review

Find each measure. (Lesson 10-4)

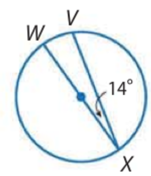
44. $m\widehat{JK}$



45. $m\angle B$

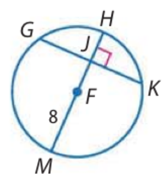


46. $m\widehat{VX}$

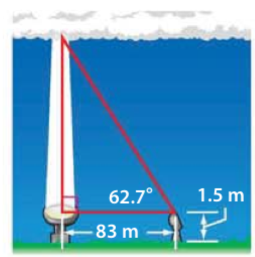


In $\odot F$, $GK = 14$ and $m\widehat{GHK} = 142$. Find each measure. Round to the nearest hundredth. (Lesson 10-3)

47. $m\widehat{GH}$ **48.** JK **49.** $m\widehat{KM}$

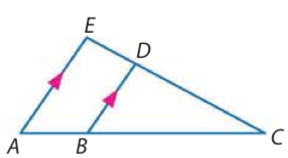


50. METEOROLOGY The altitude of the base of a cloud formation is called the *ceiling*. To find the ceiling one night, a meteorologist directed a spotlight vertically at the clouds. Using a theodolite, an optical instrument with a rotatable telescope, placed 83 meters from the spotlight and 1.5 meters above the ground, he found the angle of elevation to be 62.7°. How high was the ceiling? (Lesson 8-5)

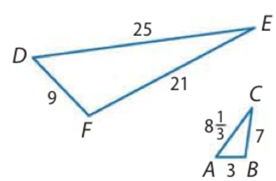


Determine whether the triangles are similar. If so, write a similarity statement. Explain your reasoning. (Lesson 7-3)

51.



52.



Skills Review

Solve each equation.

53. $15 = \frac{1}{2}[(360 - x) - 2x]$

54. $x + 12 = \frac{1}{2}[(180 - 120)]$

55. $x = \frac{1}{2}[(180 - 64)]$

EXTEND 10-5

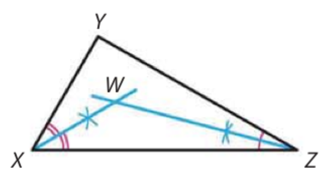
Geometry Lab
Inscribed and Circumscribed Circles

In this lab, you will perform constructions that involve inscribing or circumscribing a circle.

CCSS Common Core State Standards
Content Standards
G.CO.13 Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.
G.C.3 Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.
Mathematical Practices 5

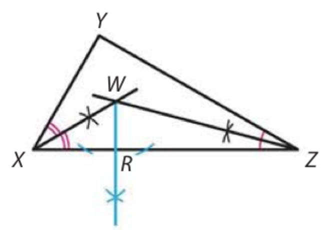
Activity 1 Construct a Circle Inscribed in a Triangle

Step 1



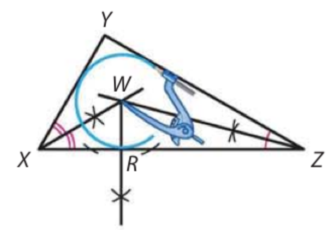
Draw a triangle XYZ and construct two angle bisectors of the triangle to locate the incenter W.

Step 2



Construct a segment perpendicular to a side through the incenter. Label the intersection R.

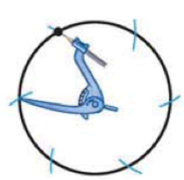
Step 3



Set a compass of the length of $\overline{WR}$. Put the point of the compass on W and draw a circle with that radius.

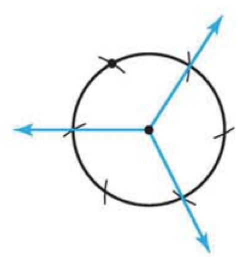
Activity 2 Construct a Triangle Circumscribed About a Circle

Step 1



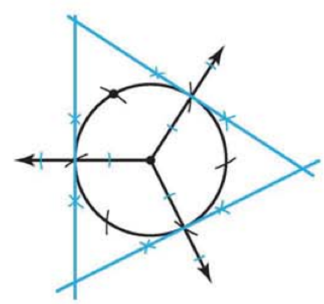
Construct a circle and draw a point. Use the same compass setting you used to construct the circle to construct an arc on the circle from the point. Continue as shown.

Step 2



Draw rays from the center through every other arc.

Step 3



Construct a line perpendicular to each of the rays.

Model

1. Draw a right triangle and inscribe a circle in it.
2. Inscribe a regular hexagon in a circle. Then inscribe an equilateral triangle in a circle. (*Hint:* The first step of each construction is identical to Step 1 in Activity 2.)
3. Inscribe a square in a circle. Then circumscribe a square about a circle.
4. **CHALLENGE** Circumscribe a regular hexagon about a circle.

LESSON 10-6

Secants, Tangents, and Angle Measures

Then

- You found measures of segments formed by tangents to a circle.

Now

1. Find measures of angles formed by lines intersecting on or inside a circle.
2. Find measures of angles formed by lines intersecting outside the circle.

Why?

- An average person's field of vision is about 180°. Most cameras have a much narrower viewing angle of between 20° and 50°. This viewing angle determines how much of a curved object a camera can capture on film.

NewVocabulary
secant

Common Core State Standards

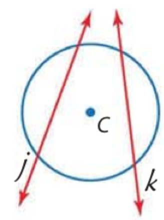
Content Standards
Reinforcement of G.C.4 Construct a tangent line from a point outside a given circle to the circle.

Mathematical Practices
3 Construct viable arguments and critique the reasoning of others.
1 Make sense of problems and persevere in solving them.

Image Source/Getty Images

1 Intersections On or Inside a Circle

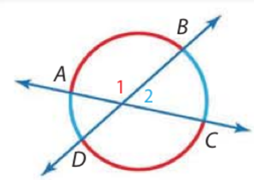
A **secant** is a line that intersects a circle in exactly two points. Lines j and k are secants of $\odot C$.



When two secants intersect inside a circle, the angles formed are related to the arcs they intercept.

Theorem 10.12

Words If two secants or chords intersect in the interior of a circle, then the measure of an angle formed is one half the *sum* of the measure of the arcs intercepted by the angle and its vertical angle.



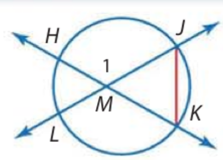
Example $m\angle 1 = \frac{1}{2}(m\overset{\frown}{AB} + m\overset{\frown}{CD})$ and $m\angle 2 = \frac{1}{2}(m\overset{\frown}{DA} + m\overset{\frown}{BC})$

Proof

Given: $\overleftrightarrow{HK}$ and $\overleftrightarrow{JL}$ intersect at M.

Prove: $m\angle 1 = \frac{1}{2}(m\overset{\frown}{JH} + m\overset{\frown}{LK})$

Proof:

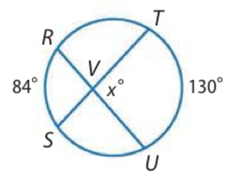


Statements	Reasons
1. $\overleftrightarrow{HK}$ and $\overleftrightarrow{JL}$ intersect at M.	1. Given
2. $m\angle 1 = m\angle MJK + m\angle MKJ$	2. Exterior Angle Theorem
3. $m\angle MJK = \frac{1}{2}m\angle\overset{\frown}{LK}$, $m\angle MKJ = \frac{1}{2}m\angle\overset{\frown}{JH}$	3. The measure of an inscribed ∠ equals half the measure of the intercepted arc.
4. $m\angle 1 = \frac{1}{2}m\angle\overset{\frown}{LK} + \frac{1}{2}m\angle\overset{\frown}{JH}$	4. Substitution
5. $m\angle 1 = \frac{1}{2}(m\overset{\frown}{JH} + m\overset{\frown}{LK})$	5. Distributive Property

Example 1 Use Intersecting Chords or Secants

Find x.

a.

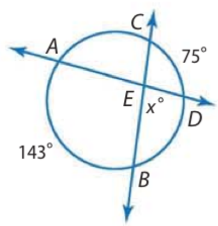


$m\angle TVU = \frac{1}{2}(m\overset{\frown}{RS} + m\overset{\frown}{TU})$ Theorem 10.12

$x = \frac{1}{2}(84 + 130)$ Substitution

$= \frac{1}{2}(214)$ or 107 Simplify.

b.



Step 1 Find $m\angle AEB$.

$m\angle AEB = \frac{1}{2}(m\overset{\frown}{AB} + m\overset{\frown}{CD})$ Theorem 10.12

$= \frac{1}{2}(143 + 75)$ Substitution

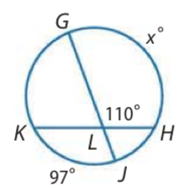
$= \frac{1}{2}(218)$ or 109 Simplify.

Step 2 Find x, the measure of $\angle DEB$.

$\angle AEB$ and $\angle DEB$ are supplementary angles.

So, $x = 180 - 109$ or 71.

c.



$m\angle GLH = \frac{1}{2}(m\overset{\frown}{GH} + m\overset{\frown}{KJ})$ Theorem 10.12

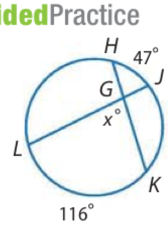
$110 = \frac{1}{2}(x + 97)$ Substitution

$220 = (x + 97)$ Multiply each side by 2.

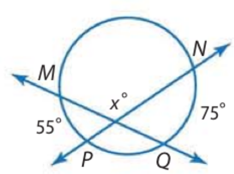
$123 = x$ Subtract 97 from each side.

Guided Practice

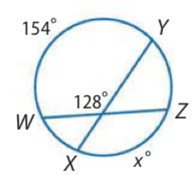
1A.



1B.



1C.



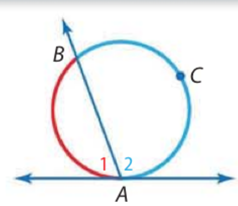
> **Study Tip**
>
> **Alternative Method**
> In Example 1b, $m\angle DEB$ can also be found by first finding the sum of the measures of $\overset{\frown}{AC}$ and $\overset{\frown}{BD}$.
>
> $m\overset{\frown}{AC} + m\overset{\frown}{BD}$
> $= 360 - (m\overset{\frown}{AC} + m\overset{\frown}{CD})$
> $= 360 - (143 + 75)$
> $= 142$
>
> $m\angle DEB$
> $= \frac{1}{2}(m\overset{\frown}{AC} + m\overset{\frown}{BD})$
> $= \frac{1}{2}(142)$ or 71

Recall that Theorem 10.6 states that the measure of an inscribed angle is half the measure of its intercepted arc. If one of the sides of this angle is tangent to the circle, this relationship still holds true.

Theorem 10.13

Words If a secant and a tangent intersect at the point of tangency, then the measure of each angle formed is one half the measure of its intercepted arc.

Example $m\angle 1 = \frac{1}{2}m\overset{\frown}{AB}$ and $m\angle 2 = \frac{1}{2}m\overset{\frown}{ACB}$

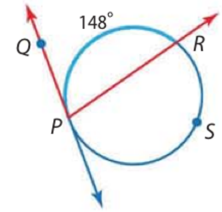


You will prove Theorem 10.13 in Exercise 33.

Example 2 Use Intersecting Secants and Tangents

Find each measure.

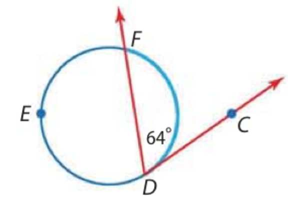
a. $m\angle QPR$



$m\angle QPR = \frac{1}{2}m\widehat{PR}$ — Theorem 10.13

$= \frac{1}{2}(148)$ or 74 — Substitute and simplify.

b. $m\widehat{DEF}$



$m\angle CDF = \frac{1}{2}m\widehat{FD}$ — Theorem 10.13

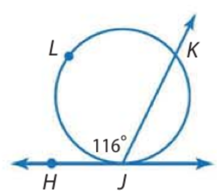
$64 = \frac{1}{2}m\widehat{FD}$ — Substitution

$128 = m\widehat{FD}$ — Multiply each side by 2.

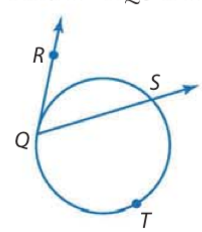
$m\widehat{DEF} = 360 - m\widehat{FD} = 360 - 128$ or 232

GuidedPractice

2A. Find $m\widehat{JLK}$.



2B. Find $m\angle RQS$ if $m\widehat{QTS} = 238$.



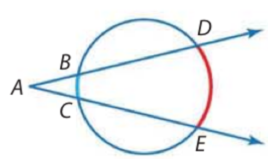
2 Intersections Outside a Circle

Secants and tangents can also meet outside a circle. The measure of the angle formed also involves half of the measures of the arcs they intercept.

Theorem 10.14

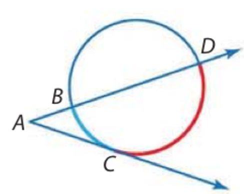
Words If two secants, a secant and a tangent, or two tangents intersect in the exterior of a circle, then the measure of the angle formed is one half the *difference* of the measures of the intercepted arcs.

Examples



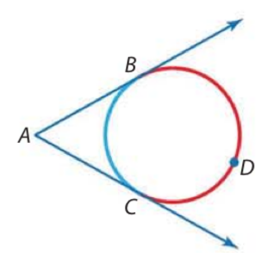
Two Secants

$m\angle A = \frac{1}{2}(m\widehat{DE} - m\widehat{BC})$



Secant-Tangent

$m\angle A = \frac{1}{2}(m\widehat{DC} - m\widehat{BC})$



Two Tangents

$m\angle A = \frac{1}{2}(m\widehat{BDC} - m\widehat{BC})$

You will prove Theorem 10.14 in Exercises 30–32.

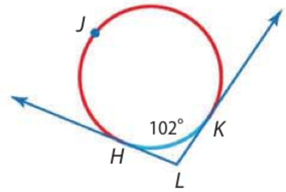
StudyTip

Absolute Value The measure of each $\angle A$ can also be expressed as half the absolute value of the difference of the arc measure. In this way, the order of the arc measures does not affect the outcome of the calculation.

Example 3 Use Tangents and Secants that Intersect Outside a Circle

Find each measure.

a. $m\angle L$

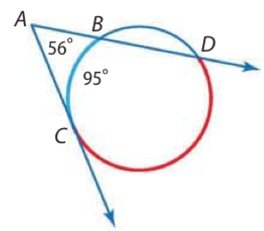


$m\angle L = \frac{1}{2}(m\widehat{HJK} - m\widehat{HK})$ Theorem 10.14

$= \frac{1}{2}(360 - 102) - 102$ Substitution

$= \frac{1}{2}(258 - 102)$ or 78 Simplify.

b. $m\widehat{CD}$



$m\angle A = \frac{1}{2}(m\widehat{CD} - m\widehat{BC})$ Theorem 10.14

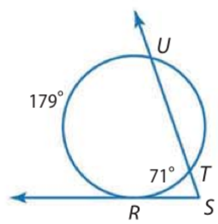
$56 = \frac{1}{2}(m\widehat{CD} - 95)$ Substitution

$112 = m\widehat{CD} - 95$ Multiply each side by 2.

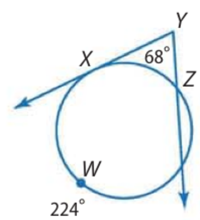
$207 = m\widehat{CD}$ Add 95 to each side.

GuidedPractice

3A. $m\angle S$



3B. $m\widehat{XZ}$



You can apply the properties of intersecting secants to solve real-world problems.

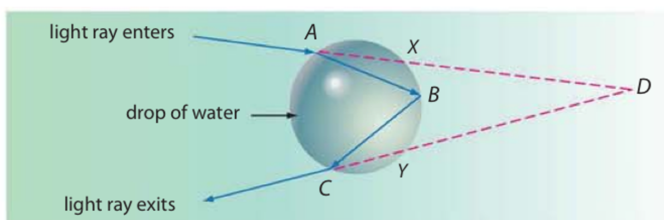
Real-WorldLink

There is a difference in the *index of refraction* between the two mediums such as air and glass. The index of refraction N is given by the equation $N = \frac{c}{V}$, where c is the speed of light and V is the velocity of light in that material.

Source: Microscopy Resource Center

Real-World Example 4 Apply Properties of Intersecting Secants

SCIENCE The diagram shows the path of a light ray as it hits a drop of water. The ray is bent, or *refracted*, at points A, B, and C. If $m\widehat{AC} = 128$ and $m\widehat{XBY} = 84$, what is $m\angle D$?



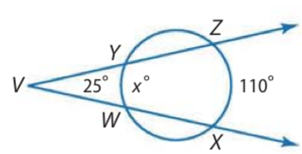
$m\angle D = \frac{1}{2}(m\widehat{AC} - m\widehat{XBY})$ Theorem 10.14

$= \frac{1}{2}(128 - 84)$ Substitution

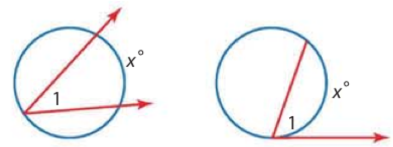
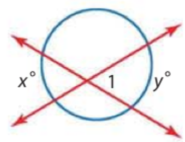
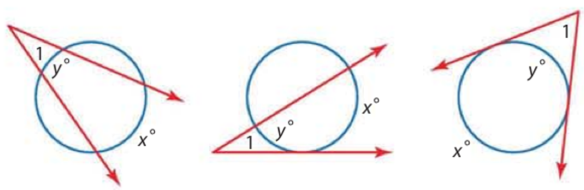
$= \frac{1}{2}(44)$ or 22 Simplify.

GuidedPractice

4. Find the value of x.



KeyConcept Circle and Angle Relationships

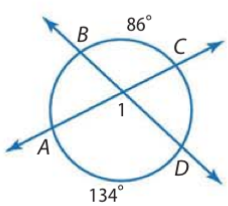
Vertex of Angle	Model(s)	Angle Measure
on the circle		one half the measure of the intercepted arc $m\angle 1 = \frac{1}{2}x$
inside the circle		one half the measure of the sum of the intercepted arc $m\angle 1 = \frac{1}{2}(x + y)$
outside the circle		one half the measure of the difference of the intercepted arcs $m\angle 1 = \frac{1}{2}(x - y)$

Check Your Understanding

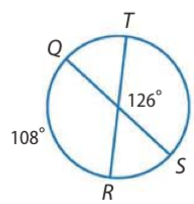
● = Step-by-Step Solutions begin on page R14.

Examples 1–2 **Find each measure. Assume that segments that appear to be tangent are tangent.**

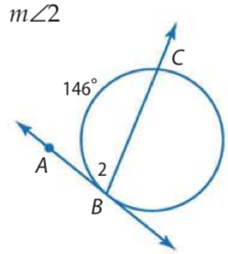
1. $m\angle 1$



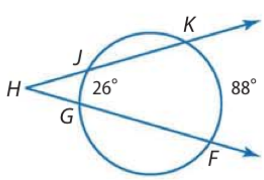
2. $m\widehat{TS}$



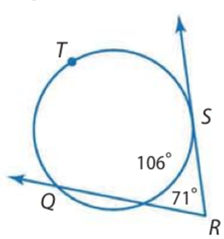
3. $m\angle 2$



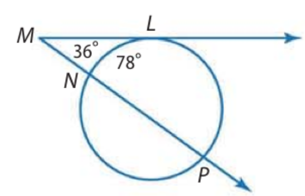
Examples 3–4 **4.** $m\angle H$



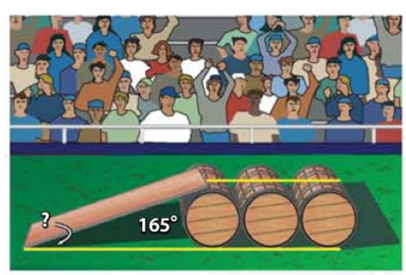
5. $m\widehat{QTS}$



6. $m\widehat{LP}$



7 **STUNTS** A ramp is attached to the first of several barrels that have been strapped together for a circus motorcycle stunt as shown. What is the measure of the angle the ramp makes with the ground?

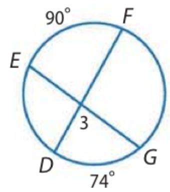


Practice and Problem Solving

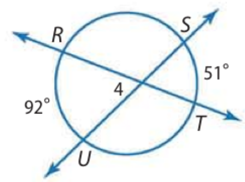
Extra Practice is on page R10.

Examples 1–2 **Find each measure. Assume that segments that appear to be tangent are tangent.**

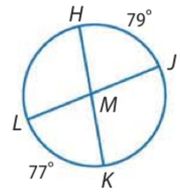
8. $m\angle 3$



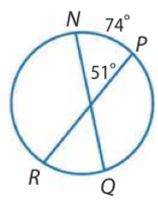
9. $m\angle 4$



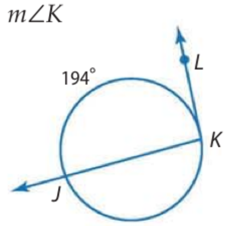
10. $m\angle JMK$



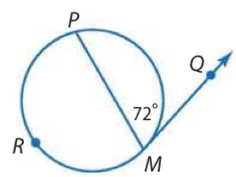
11. $m\widehat{RQ}$



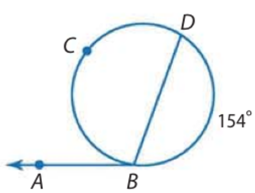
12. $m\angle K$



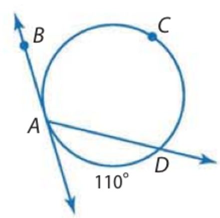
13. $m\widehat{PM}$



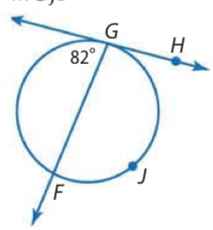
14. $m\angle ABD$



15. $m\angle DAB$



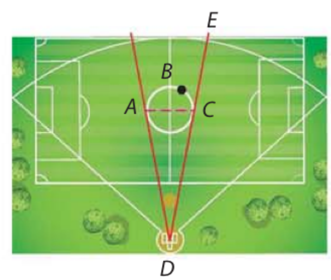
16. $m\widehat{GJF}$



17. **SPORTS** The multi-sport field shown includes a softball field and a soccer field. If $m\widehat{ABC} = 200$, find each measure.

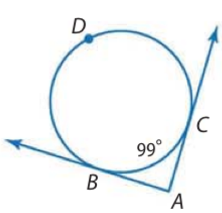
a. $m\angle ACE$

b. $m\angle ADC$

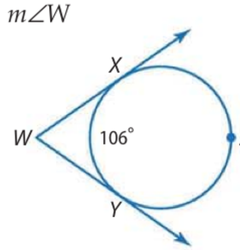


Examples 3–4 **CCSS STRUCTURE** **Find each measure.**

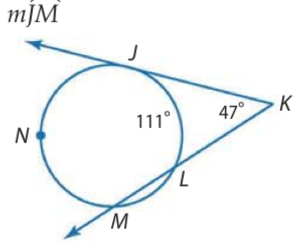
18. $m\angle A$



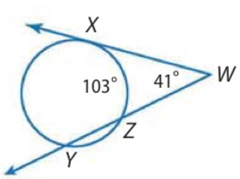
19. $m\angle W$



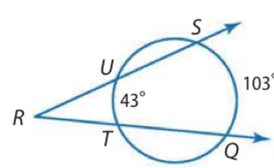
20. $m\widehat{JM}$



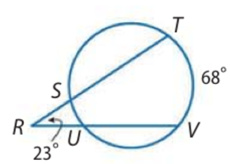
21. $m\widehat{XY}$



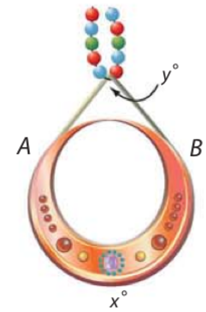
22. $m\angle R$



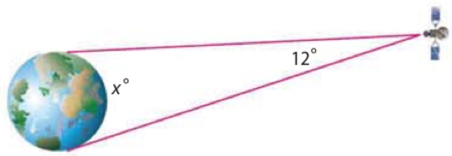
23. $m\widehat{SU}$



24. **JEWELRY** In the circular necklace shown, A and B are tangent points. If $x = 260$, what is y?

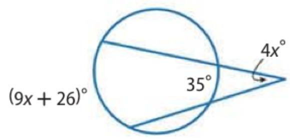


25. **SPACE** A satellite orbits above Earth's equator. Find x, the measure of the planet's arc, that is visible to the satellite.

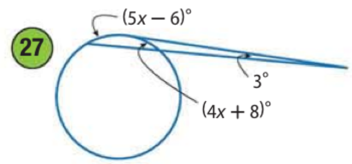


ALGEBRA **Find the value of x.**

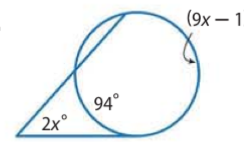
26.



27.



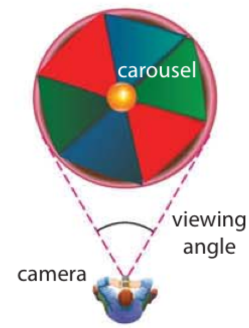
28.



29. **PHOTOGRAPHY** A photographer frames a carousel in his camera shot as shown so that the lines of sight form tangents to the carousel.

a. If the camera's viewing angle is 35°, what is the arc measure of the carousel that appears in the shot?

b. If you want to capture an arc measure of 150° in the photograph, what viewing angle should be used?

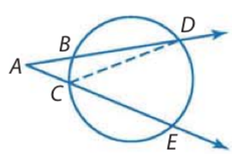


CCSS ARGUMENTS **For each case of Theorem 10.14, write a two-column proof.**

30. **Case 1**

Given: secants $\overrightarrow{AD}$ and $\overrightarrow{AE}$

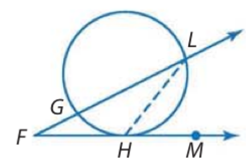
Prove: $m\angle A = \frac{1}{2}(m\widehat{DE} - m\widehat{BC})$



31. **Case 2**

Given: tangent $\overrightarrow{FM}$ and secant $\overrightarrow{FL}$

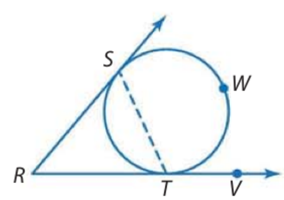
Prove: $m\angle F = \frac{1}{2}(m\widehat{LH} - m\widehat{GH})$



32. **Case 3**

Given: tangents $\overrightarrow{RS}$ and $\overrightarrow{RV}$

Prove: $m\angle R = \frac{1}{2}(m\widehat{SWT} - m\widehat{ST})$

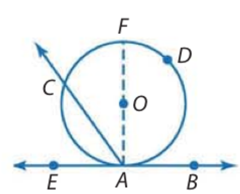


33. **PROOF** Write a paragraph proof of Theorem 10.13.

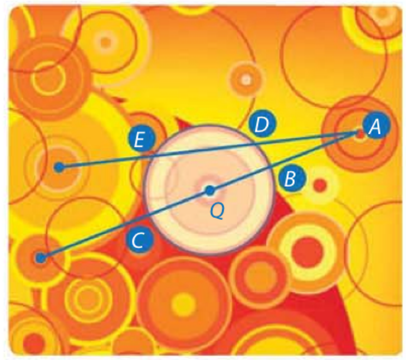
a. **Given:** $\overleftrightarrow{AB}$ is a tangent of $\odot O$.
$\overrightarrow{AC}$ is a secant of $\odot O$.
$\angle CAE$ is acute.

Prove: $m\angle CAE = \frac{1}{2}m\widehat{CA}$

b. Prove that if $\angle CAB$ is obtuse, $m\angle CAB = \frac{1}{2}m\widehat{CDA}$.



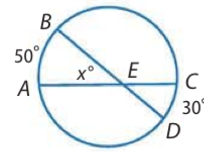
34. **WALLPAPER** In the wallpaper design shown, $\overline{BC}$ is a diameter of $\odot Q$. If $m\angle A = 26$ and $m\widehat{CE} = 67$, what is $m\widehat{DE}$?



35. **MULTIPLE REPRESENTATIONS** In this problem, you will explore the relationship between Theorems 10.12 and 10.6.

a. Geometric Copy the figure shown. Then draw three successive figures in which the position of point *D* moves closer to point *C*, but points *A*, *B*, and *C* remain fixed.

b. Tabular Estimate the measure of $\widehat{CD}$ for each successive circle, recording the measures of $\widehat{AB}$ and $\widehat{CD}$ in a table. Then calculate and record the value of *x* for each circle.



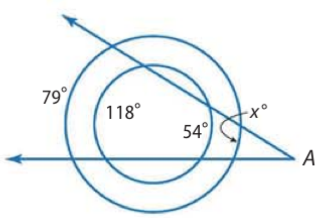
c. Verbal Describe the relationship between $m\widehat{AB}$ and the value of *x* as $m\widehat{CD}$ approaches zero. What type of angle does $\angle AEB$ become when $m\widehat{CD} = 0$?

d. Analytical Write an algebraic proof to show the relationship between Theorems 10.12 and 10.6 described in part **c**.

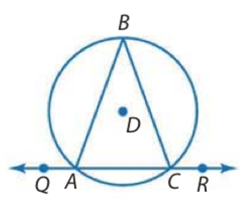
H.O.T. Problems Use Higher-Order Thinking Skills

36. **WRITING IN MATH** Explain how to find the measure of an angle formed by a secant and a tangent that intersect outside a circle.

37. **CHALLENGE** The circles below are concentric. What is *x*?



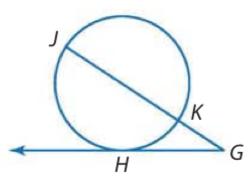
38. **REASONING** Isosceles $\triangle ABC$ is inscribed in $\odot D$. What can you conclude about $m\widehat{AB}$ and $m\widehat{BC}$? Explain.



39. **CCSS ARGUMENTS** In the figure, $\overline{JK}$ is a diameter and $\overrightarrow{GH}$ is a tangent.

a. Describe the range of possible values for $m\angle G$. Explain.

b. If $m\angle G = 34$, find the measures of minor arcs *HJ* and *KH*. Explain.

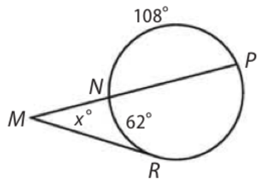


40. **OPEN ENDED** Draw a circle and two tangents that intersect outside the circle. Use a protractor to measure the angle that is formed. Find the measures of the minor and major arcs formed. Explain your reasoning.

41. **WRITING IN MATH** A circle is inscribed within $\triangle PQR$. If $m\angle P = 50$ and $m\angle Q = 60$, describe how to find the measures of the three minor arcs formed by the points of tangency.

Standardized Test Practice

42. What is the value of x if $m\widehat{NR} = 62$ and $m\widehat{NP} = 108$?

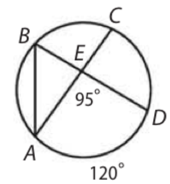


A 23° **C** 64°
B 31° **D** 128°

43. ALGEBRA Points $A(-4, 8)$ and $B(6, 2)$ are both on circle C, and $\overline{AB}$ is a diameter. What are the coordinates of C?

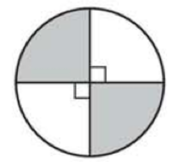
F (2, 10) **H** (5, −3)
G (10, −6) **J** (1, 5)

44. GRIDDED RESPONSE If $m\angle AED = 95$ and $m\widehat{AD} = 120$, what is $m\angle BAC$?



45. SAT/ACT If the circumference of the circle below is 16π units, what is the total area of the shaded regions?

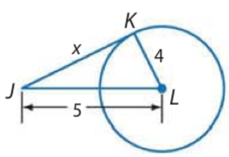
A 64π units^2 **D** 8π units^2
B 32π units^2 **E** 2π units^2
C 12π units^2



Spiral Review

Find x. Assume that segments that appear to be tangent are tangent. (Lesson 10-5)

46.



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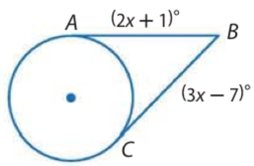
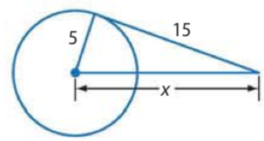


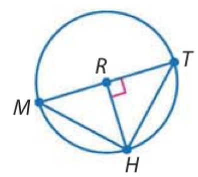
48.



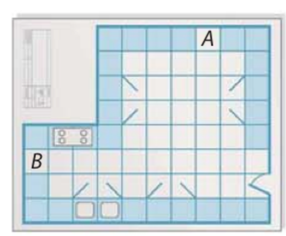
49. PROOF Write a two-column proof. (Lesson 10-4)

Given: $\widehat{MHT}$ is a semicircle; $\overline{RH} \perp \overline{TM}$.

Prove: $\frac{TR}{RH} = \frac{TH}{HM}$



50. REMODELING The diagram at the right shows the floor plan of Trent's kitchen. Each square on the diagram represents a 3-foot by 3-foot area. While remodeling his kitchen, Trent moved his refrigerator from square A to square B. Describe one possible combination of transformations that could be used to make this move. (Lesson 9-4)



COORDINATE GEOMETRY Find the measure of each angle to the nearest tenth of a degree by using the Distance Formula and an inverse trigonometric ratio. (Lesson 8-4)

51. $\angle C$ in triangle BCD with vertices $B(-1, -5)$, $C(-6, -5)$, and $D(-1, 2)$

52. $\angle X$ in right triangle XYZ with vertices $X(2, 2)$, $Y(2, -2)$, and $Z(7, -2)$

Skills Review

Solve each equation.

53. $x^2 + 13x = -36$

54. $x^2 - 6x = -9$

55. $3x^2 + 15x = 0$

56. $28 = x^2 + 3x$

57. $x^2 + 12x + 36 = 0$

58. $x^2 + 5x = -\frac{25}{4}$

LESSON 10-7 Special Segments in a Circle

Then

- You found measures of diagonals that intersect in the interior of a parallelogram.

Now

1. Find measures of segments that intersect in the interior of a circle.
2. Find measures of segments that intersect in the exterior of a circle.

Why?

- A large circular cake is cut lengthwise instead of into wedges to serve more people for a party. Only a small portion of the original cake remains. Using the geometry of circles, you can determine the diameter of the original cake.

NewVocabulary
chord segment
secant segment
external secant segment
tangent segment

Common Core State Standards

Content Standards
Reinforcement of G.C.4
Construct a tangent line from a point outside a given circle to the circle.

Mathematical Practices
1 Make sense of problems and persevere in solving them.
7 Look for and make use of structure.

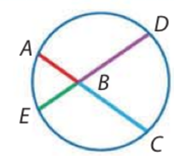
1 Segments Intersecting Inside a Circle

When two chords intersect inside a circle, each chord is divided into two segments, called **chord segments**.

Theorem 10.15 Segments of Chords Theorem

Words If two chords intersect in a circle, then the products of the lengths of the chord segments are equal.

Example $AB \cdot BC = DB \cdot BE$

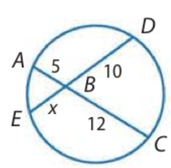


You will prove Theorem 10.15 in Exercise 23.

Example 1 Use the Intersection of Two Chords

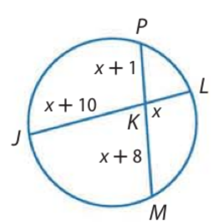
Find x.

a.



$AB \cdot BC = EB \cdot BD$	Theorem 10.15
$5 \cdot 12 = x \cdot 10$	Substitution
$60 = 10x$	Multiply.
$6 = x$	Divide each side by 10.

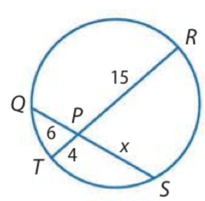
b.



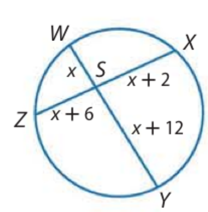
$JK \cdot KL = PK \cdot KM$	Theorem 10.15
$(x + 10) \cdot x = (x + 1)(x + 8)$	Substitution
$x^2 + 10x = x^2 + 9x + 8$	Multiply.
$10x = 9x + 8$	Subtract x^2 from each side.
$x = 8$	Subtract $9x$ from each side.

GuidedPractice

1A.



1B.



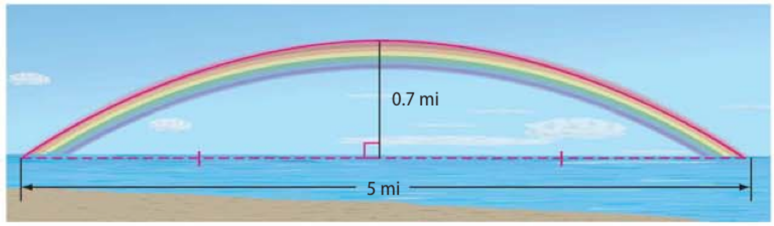
Real-World Link

The lower the Sun is to the horizon, the more of a rainbow you can see. At sunset, you could see a full semicircle of a rainbow with the top of the arch 42 degrees above the horizon.

Source: The National Center for Atmospheric Research

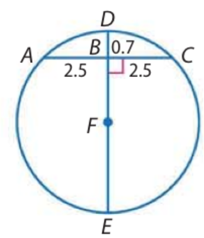
Real-World Example 2 Find Measures of Segments in Circles

SCIENCE **The true shape of a rainbow is a complete circle. However, we see only the arc of the circle that appears above Earth's horizon. What is the radius of the circle containing the arc of the rainbow shown?**



Understand You know that the rainbow's arc is part of a whole circle. $\overline{AC}$ is a chord of this circle, and $\overline{DB}$ is a perpendicular bisector of $\overline{AC}$.

Plan Draw a model. Since it bisects chord $\overline{AC}$, $\overline{DE}$ is a diameter of the circle. Use the products of the lengths of the intersecting chords to find the length of the diameter.

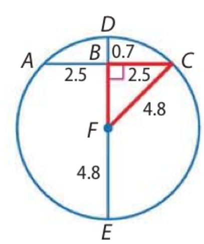


Solve

$AB \cdot BC = DB \cdot BE$	Theorem 10.15
$2.5 \cdot 2.5 = 0.7 \cdot BE$	Substitution
$6.25 = 0.7BE$	Multiply.
$8.9 \approx BE$	Divide each side by 0.7.
$DE = DB + BE$	Segment Addition Postulate
$= 0.7 + 8.9$	Substitution
$= 9.6$	Add.

Since the diameter of the circle is about 9.6 miles, the radius is about 9.6 ÷ 2 or 4.8 miles.

Check Use the Pythagorean Theorem to check the triangle in the circle formed by the radius, the chord, and part of the diameter.

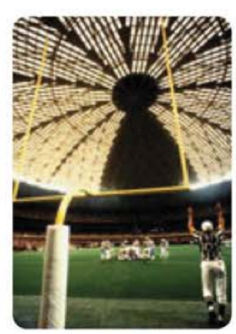


$DB + BF = DF$	Segment Addition Postulate
$0.7 + BF = 4.8$	Substitution
$BF = 4.1$	Subtract 0.7 from each side.
$BF^2 + BC^2 = CF^2$	Pythagorean Theorem
$4.1^2 + 2.5^2 \stackrel{?}{=} 4.8^2$	Substitution
$23.06 \approx 23.04$ ✓	Simplify.

Problem-Solving Tip

Make a Drawing When solving word problems involving circles, it is helpful to make a drawing and label all parts of the circle that are known. Use a variable to label the unknown measure.

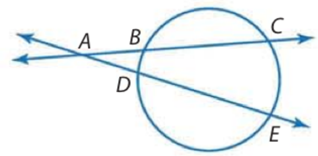
Guided Practice



2. ASTRODOME The highest point, or apex, of the Astrodome is 208 feet high, and the diameter of the circle containing the arc is 710 feet. How long is the stadium from one side to the other?

2 Segments Intersecting Outside a Circle

A **secant segment** is a segment of a secant line that has exactly one endpoint on the circle. In the figure, $\overline{AC}$, $\overline{AB}$, $\overline{AE}$ and $\overline{AD}$ are secant segments.



A secant segment that lies in the exterior of the circle is called an **external secant segment**. In the figure, $\overline{AB}$ and $\overline{AD}$ are external secant segments.

A special relationship exists among secants and external secant segments.

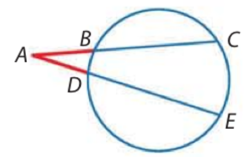
StudyTip

Simplify the Theorem Each side of the equation in Theorem 10.16 is the product of the lengths of the exterior part and the whole segment.

Theorem 10.16 Secant Segments Theorem

Words If two secants intersect in the exterior of a circle, then the product of the measures of one secant segment and its external secant segment is equal to the product of the measures of the other secant and its external secant segment.

Example $AC \cdot AB = AE \cdot AD$



You will prove Theorem 10.16 in Exercise 24.

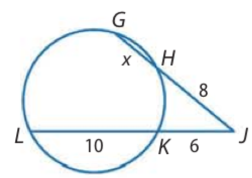
WatchOut!

Use the Correct Equation Be sure to multiply the length of the secant segment by the length of the external secant segment. Do not multiply the length of the internal secant segment, or chord, by the length of the external secant segment.

Example 3 Use the Intersection of Two Chords

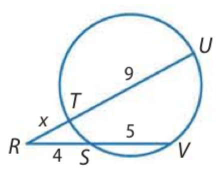
Find x.

$JG \cdot JH = JL \cdot JK$	Theorem 10.16
$(x + 8)8 = (10 + 6)6$	Substitution
$8x + 64 = 96$	Multiply.
$8x = 32$	Subtract 64 from each side.
$x = 4$	Divide each side by 8.

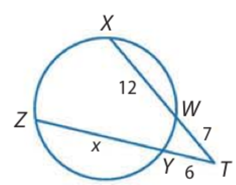


GuidedPractice

3A.



3B.

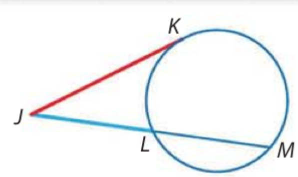


An equation similar to the one in Theorem 10.16 can be used when a secant and a tangent intersect outside a circle. In this case, the **tangent segment**, or segment of a tangent with one endpoint on the circle, is both the exterior and whole segment.

Theorem 10.17

Words If a tangent and a secant intersect in the exterior of a circle, then the square of the measure of the tangent is equal to the product of the measures of the secant and its external secant segment.

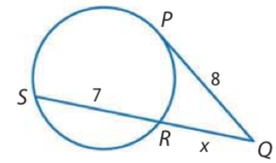
Example $JK^2 = JL \cdot JM$



You will prove Theorem 10.17 in Exercise 25.

Example 4 Use the Intersection of a Secant and a Tangent

$\overline{PQ}$ is tangent to the circle. Find x. Round to the nearest tenth.



$PQ^2 = QR \cdot QS$	Theorem 10.17
$8^2 = x(x + 7)$	Substitution
$64 = x^2 + 7x$	Multiply.
$0 = x^2 + 7x - 64$	Subtract 64 from each side.

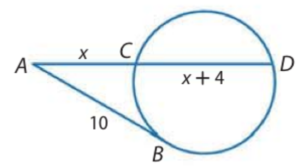
Since the expression is not factorable, use the Quadratic Formula.

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	Quadratic Formula
$= \frac{-7 \pm \sqrt{7^2 - 4(1)(-64)}}{2(1)}$	$a = 1$, $b = 7$, and $c = -64$
$= \frac{-7 \pm \sqrt{305}}{2}$	Simplify.
≈ 5.2 or -12.2	Use a calculator.

Since lengths cannot be negative, the value of x is about 5.2.

GuidedPractice

4. $\overline{AB}$ is tangent to the circle. Find x. Round to the nearest tenth.

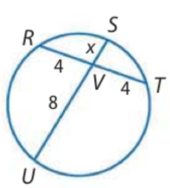


Check Your Understanding

● = Step-by-Step Solutions begin on page R14.

Examples 1, 3 and 4 **Find x. Assume that segments that appear to be tangent are tangent.**

1.



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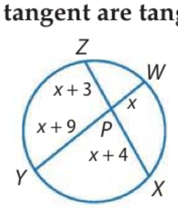

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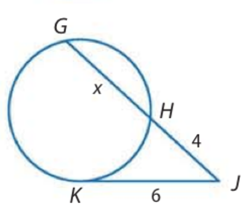

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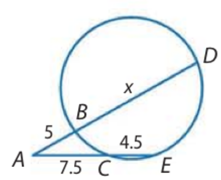
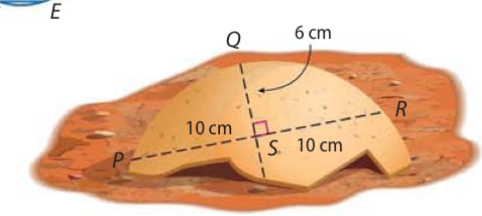


Example 2

5 **SCIENCE** A piece of broken pottery found at an archaeological site is shown. $\overline{QS}$ lies on a diameter of the circle. What was the circumference of the original pottery? Round to the nearest hundredth.

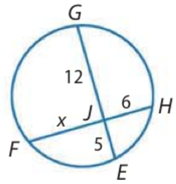


Practice and Problem Solving

Extra Practice is on page R10.

Examples 1, 3 and 4 **Find x to the nearest tenth. Assume that segments that appear to be tangent are tangent.**

6.



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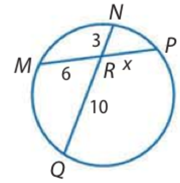

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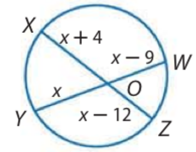

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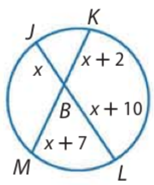

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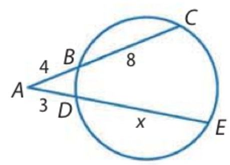

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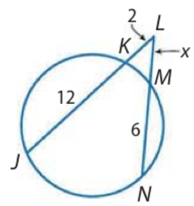

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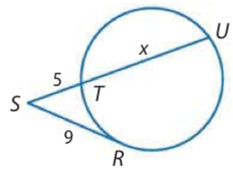

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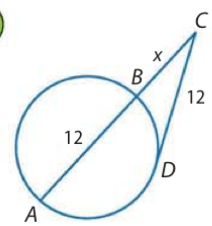
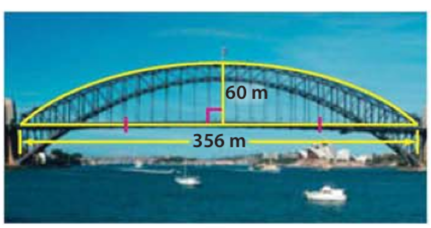
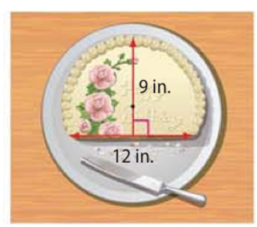


Example 2

14. BRIDGES What is the diameter of the circle containing the arc of the Sydney Harbour Bridge shown? Round to the nearest tenth.

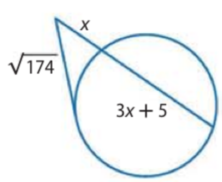


15. CAKES Sierra is serving cake at a party. If the dimensions of the remaining cake are shown below, what was the original diameter of the cake?



CCSS STRUCTURE **Find each variable to the nearest tenth. Assume that segments that appear to be tangent are tangent.**

16.



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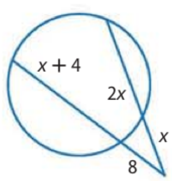
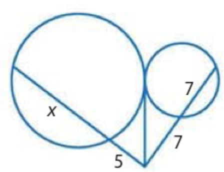


18.



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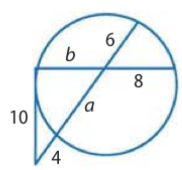

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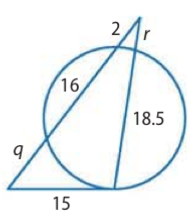

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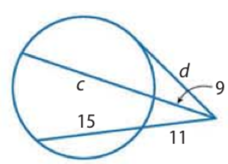


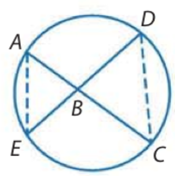
22. **INDIRECT MEASUREMENT** Gwendolyn is standing 16 feet from a giant sequoia tree and Chet is standing next to the tree, as shown. The distance between Gwendolyn and Chet is 27 feet. Draw a diagram of this situation, and then find the diameter of the tree.

PROOF **Prove each theorem.**

23. two-column proof of Theorem 10.15

Given: $\overline{AC}$ and $\overline{DE}$ intersect at B.

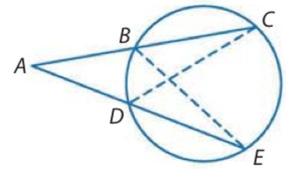
Prove: $AB \cdot BC = EB \cdot BD$



24. paragraph proof of Theorem 10.16

Given: Secants $\overline{AC}$ and $\overline{AE}$

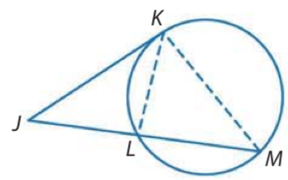
Prove: $AB \cdot AC = AD \cdot AE$



25. two-column proof of Theorem 10.17

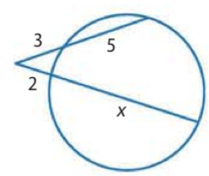
Given: tangent $\overline{JK}$, secant $\overline{JM}$

Prove: $JK^2 = JL \cdot JM$



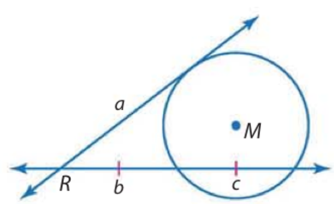
H.O.T. Problems Use Higher-Order Thinking Skills

26. **CCSS CRITIQUE** Tiffany and Jun are finding the value of x in the figure at the right. Tiffany wrote $3(5) = 2x$, and Jun wrote $3(8) = 2(2 + x)$. Is either of them correct? Explain your reasoning.



27. **WRITING IN MATH** Compare and contrast the methods for finding measures of segments when two secants intersect in the exterior of a circle and when a secant and a tangent intersect in the exterior of a circle.

28. **CHALLENGE** In the figure, a line tangent to circle M and a secant line intersect at R. Find a. Show the steps that you used.



29. **REASONING** When two chords intersect at the center of a circle, are the measures of the intercepting arcs *sometimes*, *always*, or *never* equal to each other?

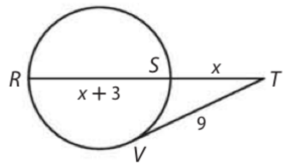
30. **OPEN ENDED** Investigate Theorem 10.17 by drawing and labeling a circle that has a secant and a tangent intersecting outside the circle. Measure and label the two parts of the secant segment to the nearest tenth of a centimeter. Use an equation to find the measure of the tangent segment. Verify your answer by measuring the segment.

31. **WRITING IN MATH** Describe the relationship among segments in a circle when two secants intersect inside a circle.

Standardized Test Practice

32. $\overline{TV}$ is tangent to the circle, and R and S are points on the circle. What is the value of x to the nearest tenth?

A 7.6 **C** 5.7
B 6.4 **D** 4.8



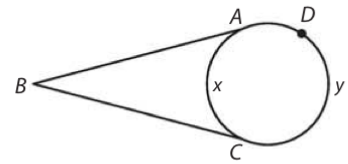
33. ALGEBRA A department store has all of its jewelry discounted 40%. It is having a sale that says you receive an additional 20% off the already discounted price. How much will you pay for a ring with an original price of $200?

F $80 **H** $120
G $96 **J** $140

34. EXTENDED RESPONSE The degree measures of minor arc $\widehat{AC}$ and major arc $\widehat{ADC}$ are x and y, respectively.

a. If $m\angle ABC = 70°$, write two equations relating x and y.

b. Find x and y.

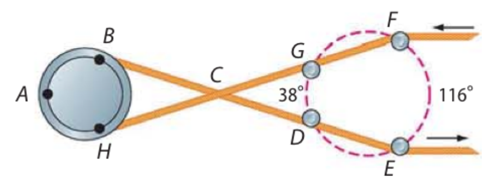


35. SAT/ACT During the first two weeks of summer vacation, Antonia earned $100 per week. During the next six weeks, she earned $150 per week. What was her average weekly pay?

A $50 **D** $135
B $112.50 **E** $137.50
C $125

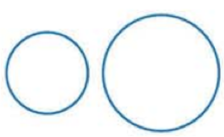
Spiral Review

36. WEAVING Once yarn is woven from wool fibers, it is often dyed and then threaded along a path of pulleys to dry. One set of pulleys is shown. Note that the yarn appears to intersect itself at C, but in reality it does not. Use the information from the diagram to find $m\widehat{BH}$. (Lesson 10-6)



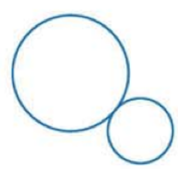
Copy the figure shown and draw the common tangents. If no common tangent exists, state *no common tangent*. (Lesson 10-5)

37.



38.

39.



40.

COORDINATE GEOMETRY **Graph each figure and its image along the given vector.** (Lesson 9-2)

41. $\triangle KLM$ with vertices $K(5, -2)$, $L(-3, -1)$, and $M(0, 5)$; $\langle -3, -4 \rangle$

42. quadrilateral $PQRS$ with vertices $P(1, 4)$, $Q(-1, 4)$, $R(-2, -4)$, and $S(2, -4)$; $\langle -5, 3 \rangle$

43. $\triangle EFG$ with vertices $E(0, -4)$, $F(-4, -4)$, and $G(0, 2)$; $\langle 2, -1 \rangle$

Skills Review

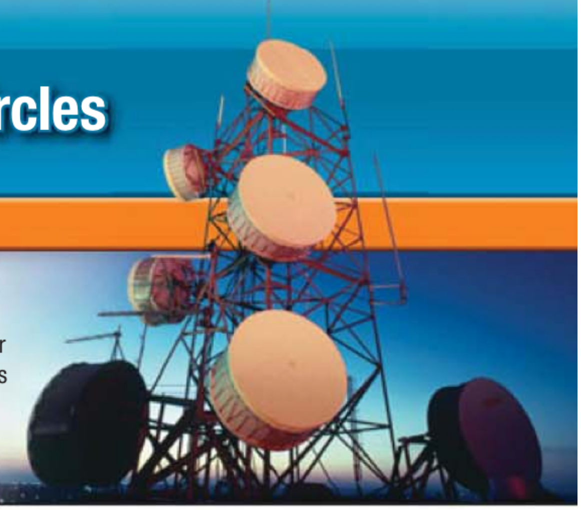
Write an equation in slope-intercept form of the line having the given slope and y-intercept.

44. m: 3, y-intercept: -4 **45.** m: 2, $(0, 8)$ **46.** m: $\frac{5}{8}$, $(0, -6)$

47. m: $\frac{2}{9}$, y-intercept: $\frac{1}{3}$ **48.** m: -1, b: -3 **49.** m: $-\frac{1}{12}$, b: 1

LESSON

10-8 Equations of Circles



Then

- You wrote equations of lines using information about their graphs.

Now

1. Write the equation of a circle.
2. Graph a circle on the coordinate plane.

Why?

- Telecommunications towers emit radio signals that are used to transmit cellular calls. Each tower covers a circular area, and towers are arranged so that a signal is available at any location in the coverage area.

NewVocabulary
compound locus

Common Core State Standards

Content Standards

G.GPE.1 Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.

G.GPE.6 Find the point on a directed line segment between two given points that partitions the segment in a given ratio.

Mathematical Practices

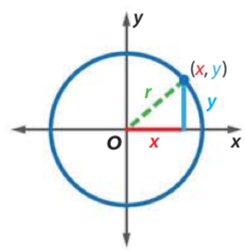
2 Reason abstractly and quantitatively.

7 Look for and make use of structure.

1 Equation of a Circle

Since all points on a circle are equidistant from the center, you can find an equation of a circle by using the Distance Formula.

Let (x, y) represent a point on a circle centered at the origin. Using the Pythagorean Theorem, $x^2 + y^2 = r^2$.



Now suppose that the center is not at the origin, but at the point (h, k). You can use the Distance Formula to develop an equation for the circle.

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ Distance Formula

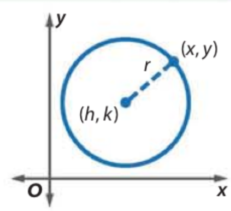
$r = \sqrt{(x - h)^2 + (y - k)^2}$ $d = r$, $(x_1, y_1) = (h, k)$, $(x_2, y_2) = (x, y)$

$r^2 = (x - h)^2 + (y - k)^2$ Square each side.

KeyConcept Equation of a Circle in Standard Form

The standard form of the equation of a circle with center at (h, k) and radius r is $(x - h)^2 + (y - k)^2 = r^2$.

The standard form of the equation of a circle is also called the *center-radius* form.



Example 1 Write an Equation Using the Center and Radius

Write the equation of each circle.

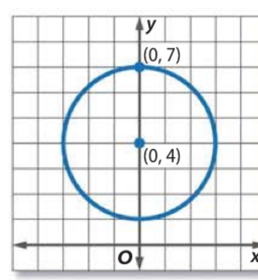
a. center at (1, −8), radius 7

$(x - h)^2 + (y - k)^2 = r^2$ Equation of a circle

$(x - 1)^2 + [y - (-8)]^2 = 7^2$ $(h, k) = (1, -8)$, $r = 7$

$(x - 1)^2 + (y + 8)^2 = 49$ Simplify.

b. the circle graphed at the right



The center is at (0, 4) and the radius is 3.

$(x - h)^2 + (y - k)^2 = r^2$ Equation of a circle

$(x - 0)^2 + (y - 4)^2 = 3^2$ $(h, k) = (0, 4)$, $r = 3$

$x^2 + (y - 4)^2 = 9$ Simplify.

GuidedPractice

1A. center at origin, radius $\sqrt{10}$

1B. center at (4, −1), diameter 8

Example 2 Write an Equation Using the Center and a Point

Write the equation of the circle with center at (−2, 4), that passes through (−6, 7).

Step 1 Find the distance between the points to determine the radius.

$r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ — Distance Formula

$= \sqrt{[-6 - (-2)]^2 + (7 - 4)^2}$ — $(x_1, y_1) = (-2, 4)$ and $(x_2, y_2) = (-6, 7)$

$= \sqrt{25}$ or 5 — Simplify.

Step 2 Write the equation using $h = -2$, $k = 4$, and $r = 5$.

$(x - h)^2 + (y - k)^2 = r^2$ — Equation of a circle

$[x - (-2)]^2 + (y - 4)^2 = 5^2$ — $h = -2$, $k = 4$, and $r = 5$

$(x + 2)^2 + (y - 4)^2 = 25$ — Simplify.

GuidedPractice

2. Write the equation of the circle with center at (−3, −5) that passes through (0, 0).

2 Graph Circles

You can use the equation of a circle to graph it on a coordinate plane. To do so, you may need to write the equation in standard form first.

Example 3 Graph a Circle

The equation of a circle is $x^2 + y^2 - 8x + 2y = -8$. State the coordinates of the center and the measure of the radius. Then graph the equation.

Write the equation in standard form by completing the square.

$x^2 + y^2 - 8x + 2y = -8$ — Original equation

$x^2 - 8x + y^2 + 2y = -8$ — Isolate and group like terms.

$x^2 - 8x + 16 + y^2 + 2y + 1 = -8 + 16 + 1$ — Complete the squares.

$(x - 4)^2 + (y + 1)^2 = 9$ — Factor and simplify.

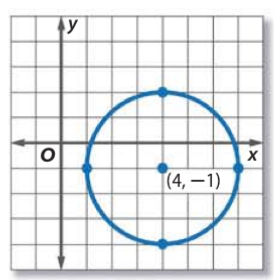
$(x - 4)^2 + [y - (-1)]^2 = 3^2$ — Write +1 as − (−1) and 9 as 3^2.

> **StudyTip**
> **Completing the Square** To complete the square for any quadratic expression of the form $x^2 + bx$, follow these steps.
> **Step 1** Find one half of b.
> **Step 2** Square the result in Step 1.
> **Step 3** Add the result of Step 2 to $x^2 + bx$.

With the equation now in standard form, you can identify h, k, and r.

$$(x - 4)^2 + [y - (-1)]^2 = 3^2$$
$$\uparrow \quad \uparrow \quad \uparrow$$
$$(x - h)^2 + (y - k)^2 = r^2$$

So, $h = 4$, $k = -1$, and $r = 3$. The center is at (4, −1), and the radius is 3. Plot the center and four points that are 3 units from this point. Sketch the circle through these four points.



GuidedPractice

For each circle with the given equation, state the coordinates of the center and the measure of the radius. Then graph the equation.

3A. $x^2 + y^2 - 4 = 0$

3B. $x^2 + y^2 + 8x - 14y + 40 = 0$

Warren Faidley/Corbis

Real-WorldLink

About 1000 tornadoes are reported across the United States each year. The most violent tornadoes have wind speeds of 250 mph or more. Damage paths can be a mile wide and 50 miles long.

Source: National Oceanic & Atmospheric Administration

Real-World Example 4 Use Three Points to Write an Equation

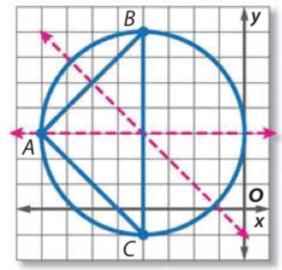
PT

TORNADOES Three tornado sirens are placed strategically on a circle around a town so they can be heard by all. Write the equation of the circle on which they are placed if the coordinates of the sirens are $A(-8, 3)$, $B(-4, 7)$, and $C(-4, -1)$.

Understand You are given three points that lie on a circle.

Plan Graph $\triangle ABC$. Construct the perpendicular bisectors of two sides to locate the center of the circle. Then find the radius.

Use the center and radius to write an equation.



Solve The center appears to be at $(-4, 3)$. The radius is 4. Write an equation.

$$(x - h)^2 + (y - k)^2 = r^2$$

$$[x - (-4)]^2 + (y - 3)^2 = 4^2$$

$$(x + 4)^2 + (y - 3)^2 = 16$$

Check Verify the center by finding the equations of the two bisectors and solving the system of equations. Verify the radius by finding the distance between the center and another point on the circle. ✔

GuidedPractice

4. Write an equation of a circle that contains $R(1, 2)$, $S(-3, 4)$, and $T(-5, 0)$.

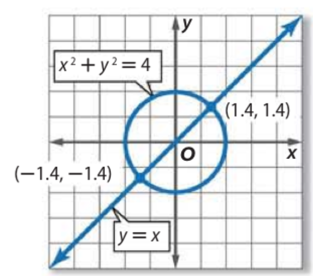
A line can intersect a circle in at most two points. You can find the point(s) of intersection between a circle and a line by applying techniques used to find the intersection between two lines and techniques used to solve quadratic equations.

Example 5 Intersections with Circles

PT

Find the point(s) of intersection between $x^2 + y^2 = 4$ and $y = x$.

Graph these equations on the same coordinate plane. The points of intersection are solutions of both equations. You can estimate these points on the graph to be at about $(-1.4, -1.4)$ and $(1.4, 1.4)$. Use substitution to find the coordinates of these points algebraically.



$x^2 + y^2 = 4$	Equation of circle
$x^2 + x^2 = 4$	Since $y = x$, substitute x for y.
$2x^2 = 4$	Simplify.
$x^2 = 2$	Divide each side by 2.
$x = \pm\sqrt{2}$	Take the square root of each side.

So $x = \sqrt{2}$ or $x = -\sqrt{2}$. Use the equation $y = x$ to find the corresponding y-values.

$y = x$	Equation of line	$y = x$
$y = \sqrt{2}$	$x = \sqrt{2}$ or $x = -\sqrt{2}$	$y = -\sqrt{2}$

The points of intersection are located at $(\sqrt{2}, \sqrt{2})$ and $(-\sqrt{2}, -\sqrt{2})$ or at about $(-1.4, -1.4)$ and $(1.4, 1.4)$. Check these solutions in both of the original equations.

StudyTip

Quadratic Techniques In addition to taking square roots, other quadratic techniques that you may need to apply in order to solve equations of the form $ax^2 + bx + c = 0$ include completing the square, factoring, and the Quadratic Formula, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

GuidedPractice

5. Find the point(s) of intersection between $x^2 + y^2 = 8$ and $y = -x$.

Check Your Understanding

= Step-by-Step Solutions begin on page R14.

Examples 1–2 **Write the equation of each circle.**

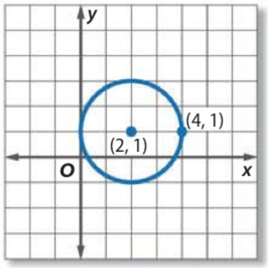
1. center at (9, 0), radius 5

2. center at (3, 1), diameter 14

3 center at origin, passes through (2, 2)

4. center at (−5, 3), passes through (1, −4)

5.



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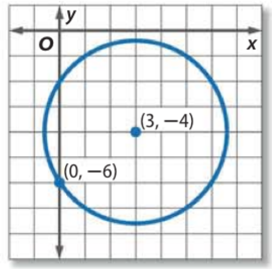


Example 3 **For each circle with the given equation, state the coordinates of the center and the measure of the radius. Then graph the equation.**

7. $x^2 - 6x + y^2 + 4y = 3$

8. $x^2 + (y + 1)^2 = 4$

Example 4 **9. RADIOS** Three radio towers are modeled by the points $R(4, 5)$, $S(8, 1)$, and $T(-4, 1)$. Determine the location of another tower equidistant from all three towers, and write an equation for the circle.

10. COMMUNICATION Three cell phone towers can be modeled by the points $X(6, 0)$, $Y(8, 4)$, and $Z(3, 9)$. Determine the location of another cell phone tower equidistant from the other three, and write an equation for the circle.

Example 5 **Find the point(s) of intersection, if any, between each circle and line with the equations given.**

11. $(x - 1)^2 + y^2 = 4$
$y = x + 1$

12. $(x - 2)^2 + (y + 3)^2 = 18$
$y = -2x - 2$

Practice and Problem Solving

Extra Practice is on page R10.

Examples 1–2 CCSS **STRUCTURE** **Write the equation of each circle.**

13. center at origin, radius 4

14. center at (6, 1), radius 7

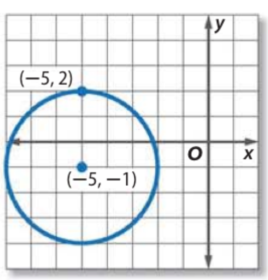
15. center at (−2, 0), diameter 16

16. center at (8, −9), radius $\sqrt{11}$

17. center at (−3, 6), passes through (0, 6)

18. center at (1, −2), passes through (3, –4)

19.



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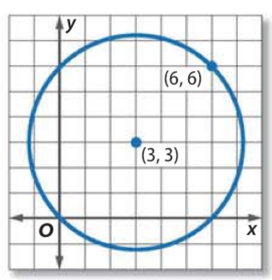


21 **WEATHER** A Doppler radar screen shows concentric rings around a storm. If the center of the radar screen is the origin and each ring is 15 miles farther from the center, what is the equation of the third ring?

22. GARDENING A sprinkler waters a circular area that has a diameter of 10 feet. The sprinkler is located 20 feet north of the house. If the house is located at the origin, what is the equation for the circle of area that is watered?

Example 3 **For each circle with the given equation, state the coordinates of the center and the measure of the radius. Then graph the equation.**

23. $x^2 + y^2 = 36$

24. $x^2 + y^2 - 4x - 2y = -1$

25. $x^2 + y^2 + 8x - 4y = -4$

26. $x^2 + y^2 - 16x = 0$

Example 4 **Write an equation of a circle that contains each set of points. Then graph the circle.**

27. $A(1, 6)$, $B(5, 6)$, $C(5, 0)$

28. $F(3, -3)$, $G(3, 1)$, $H(7, 1)$

Example 5 **Find the point(s) of intersection, if any, between each circle and line with the equations given.**

29. $x^2 + y^2 = 5$
$y = \frac{1}{2}x$

30. $x^2 + y^2 = 2$
$y = -x + 2$

31. $x^2 + (y + 2)^2 = 8$
$y = x - 2$

32. $(x + 3)^2 + y^2 = 25$
$y = -3x$

33. $x^2 + y^2 = 5$
$y = 3x$

34. $(x - 1)^2 + (y - 3)^2 = 4$
$y = -x$

Write the equation of each circle.

35. a circle with a diameter having endpoints at $(0, 4)$ and $(6, -4)$

36. a circle with $d = 22$ and a center translated 13 units left and 6 units up from the origin

37. CCSS **MODELING** Different-sized engines will launch model rockets to different altitudes. The higher a rocket goes, the larger the circle of possible landing sites becomes. Under normal wind conditions, the landing radius is three times the altitude of the rocket.

a. Write the equation of the landing circle for a rocket that travels 300 feet in the air.

b. What would be the radius of the landing circle for a rocket that travels 1000 feet in the air? Assume the center of the circle is at the origin.

38. **SKYDIVING** Three of the skydivers in the circular formation shown have approximate coordinates of $G(13, -2)$, $H(-1, -2)$, and $J(6, -9)$.

a. What are the approximate coordinates of the center skydiver?

b. If each unit represents 1 foot, what is the diameter of the skydiving formation?

39. **DELIVERY** Pizza and Subs offers free delivery within 6 miles of the restaurant. The restaurant is located 4 miles west and 5 miles north of Consuela's house.

a. Write and graph an equation to represent this situation if Consuela's house is at the origin of the coordinate system.

b. Can Consuela get free delivery if she orders pizza from Pizza and Subs? Explain.

40. **INTERSECTIONS OF CIRCLES** Graph $x^2 + y^2 = 4$ and $(x - 2)^2 + y^2 = 4$ on the same coordinate plane.

a. Estimate the point(s) of intersection between the two circles.

b. Solve $x^2 + y^2 = 4$ for y.

c. Substitute the value you found in part **b** into $(x - 2)^2 + y^2 = 4$ and solve for x.

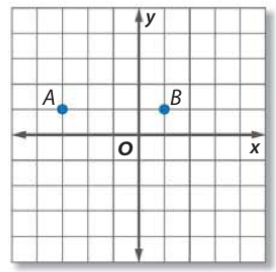
d. Substitute the value you found in part **c** into $x^2 + y^2 = 4$ and solve for y.

e. Use your answers to parts **c** and **d** to write the coordinates of the points of intersection. Compare these coordinates to your estimate from part **a**.

f. Verify that the point(s) you found in part **d** lie on both circles.

41. Prove or disprove that the point $(1, 2\sqrt{2})$ lies on a circle centered at the origin and containing the point $(0, -3)$.

42. **MULTIPLE REPRESENTATIONS** In this problem, you will investigate a compound locus for a pair of points. A **compound locus** satisfies more than one distinct set of conditions.



a. Tabular Choose two points A and B in the coordinate plane. Locate 5 coordinates from the locus of points equidistant from A and B.

b. Graphical Represent this same locus of points by using a graph.

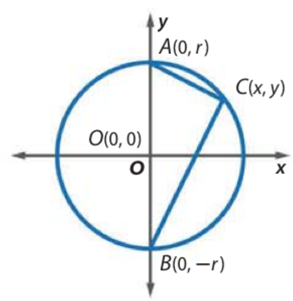
c. Verbal Describe the locus of all points equidistant from a pair of points.

d. Graphical Using your graph from part **b**, determine and graph the locus of all points in a plane that are a distance of AB from B.

e. Verbal Describe the locus of all points in a plane equidistant from a single point. Then describe the locus of all points that are both equidistant from A and B and are a distance of AB from B. Describe the graph of the compound locus.

43. A circle with a diameter of 12 has its center in the second quadrant. The lines $y = -4$ and $x = 1$ are tangent to the circle. Write an equation of the circle.

H.O.T. Problems Use Higher-Order Thinking Skills



44. CHALLENGE Write a coordinate proof to show that if an inscribed angle intercepts the diameter of a circle, as shown, the angle is a right angle.

45. CCSS **REASONING** A circle has the equation $(x - 5)^2 + (y + 7)^2 = 16$. If the center of the circle is shifted 3 units right and 9 units up, what would be the equation of the new circle? Explain your reasoning.

46. OPEN ENDED Graph three noncollinear points and connect them to form a triangle. Then construct the circle that circumscribes it.

47. WRITING IN MATH Seven new radio stations must be assigned broadcast frequencies. The stations are located at $A(9, 2)$, $B(8, 4)$, $C(8, 1)$, $D(6, 3)$, $E(4, 0)$, $F(3, 6)$, and $G(4, 5)$, where 1 unit = 50 miles.

a. If stations that are more than 200 miles apart can share the same frequency, what is the least number of frequencies that can be assigned to these stations?

b. Describe two different beginning approaches to solving this problem.

c. Choose an approach, solve the problem, and explain your reasoning.

CHALLENGE Find the coordinates of point P on $\overrightarrow{AB}$ that partitions the segment into the given ratio AP to PB.

48. $A(0, 0)$, $B(3, 4)$, 2 to 3

49. $A(0, 0)$, $B(-8, 6)$, 4 to 1

50. WRITING IN MATH Describe how the equation for a circle changes if the circle is translated a units to the right and b units down.

Standardized Test Practice

51. Which of the following is the equation of a circle with center (6, 5) that passes through (2, 8)?

A $(x - 6)^2 + (y - 5)^2 = 5^2$
B $(x - 5)^2 + (y - 6)^2 = 7^2$
C $(x + 6)^2 + (y + 5)^2 = 5^2$
D $(x - 2)^2 + (y - 8)^2 = 7^2$

52. ALGEBRA What are the solutions of $n^2 - 4n = 21$?

F 3, 7
G 3, −7
H −3, 7
J −3, −7

53. SHORT RESPONSE Solve: $5(x - 4) = 16$.

Step 1: $5x - 4 = 16$
Step 2: $5x = 20$
Step 3: $x = 4$

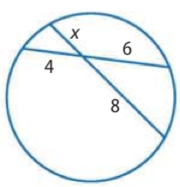
Which is the first incorrect step in the solution shown above?

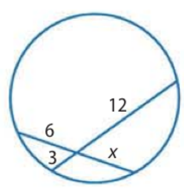
54. SAT/ACT The center of ⊙F is at (−4, 0) and has a radius of 4. Which point lies on ⊙F?

A (4, 0)
B (0, 4)
C (4, 3)
D (−4, 4)
E (0, 8)

Spiral Review

Find *x*. (Lesson 10-7)

55.


56.


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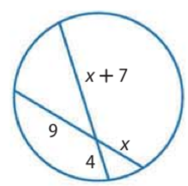
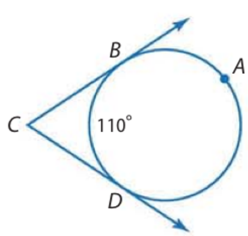
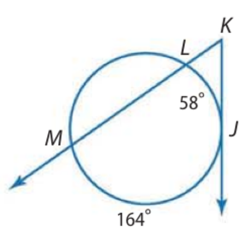
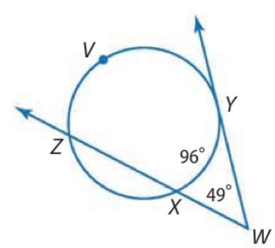


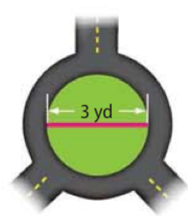
Find each measure. (Lesson 10-6)

58. $m\angle C$


59. $m\angle K$


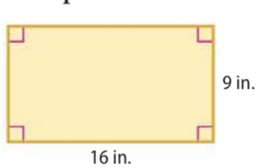
60. $m\widehat{YXZ}$


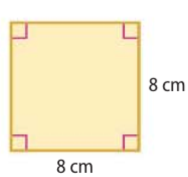
61. STREETS The neighborhood where Vincent lives has roundabouts where certain streets meet. If Vincent rides his bike once around the very edge of the grassy circle, how many feet will he have ridden? (Lesson 10-1)

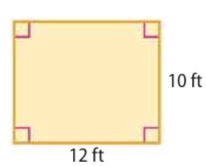


Skills Review

Find the perimeter and area of each figure.

62.


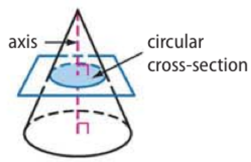
63.


64.


EXTEND 10-8

Geometry Lab
Parabolas

A circle is one type of cross-section of a right circular cone. Such cross-sections are called **conic sections** or **conics**. A circular cross-section is formed by the intersection of a cone with a plane that is perpendicular to the axis of the cone. You can find other conic sections using concrete models of cones.



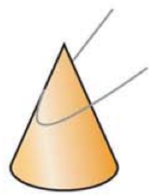
CCSS Common Core State Standards
Content Standards
G.GPE.2 Derive the equation of a parabola given a focus and directrix.
Mathematical Practices 5

Activity 1 Intersection of Cone and Plane

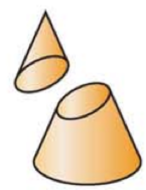
Sketch the intersection of a cone and a plane that lies at an angle to the axis of the cone but does not pass through its base.

Step 1 Fill a conical paper cup with modeling compound. Then peel away the cup.

Step 2 Draw dental floss through the cone model at an angle to the axis that does not pass through the base.

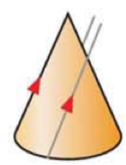


Step 3 Pull the pieces of the cone apart and trace the cross-section onto your paper.



Model and Analyze

1. The conic section in Activity 1 is called an ellipse. What shape is an ellipse?
2. Repeat Activity 1, drawing the dental floss through the model at an angle parallel to an imaginary line on the side of the cone through the cone's base. Describe the resulting shape.

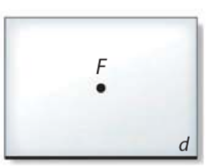


The conic section you found in Exercise 2 is called a **parabola**. In Algebra 1, a parabola was defined as the shape of the graph of a quadratic function, such as $y = x^2$. Like a circle and all conics, a parabola can also be defined as a locus of points. You can explore the loci definition of a parabola using paper folding.

Activity 2 Shape of Parabola

Use paper folding to approximate the shape of a parabola.

Step 1 Mark and label the bottom edge of a rectangular piece of wax paper *d*. Label a point *F* at the center.



Step 2 Fold *d* up so that it touches *F*. Make a sharp crease. Then open the paper and smooth it flat.

Step 3 Repeat Step 2 at least 20 times, folding the paper to a different point on *d* each time. Trace the curve formed.

Model and Analyze

3. Label a point P on the parabola and draw $\overline{PF}$. Then use a protractor to find a point D on line d such that $\overline{PD} \perp d$. Describe the relationship between $\overline{PF}$ and $\overline{PD}$.

Repeat Activity 2, making the indicated change on a new piece of wax paper. Describe the effect on the parabola formed.

4. Place line d along the edge above point F.

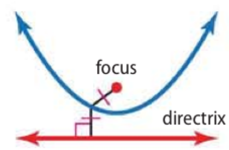
5. Place line d along the edge to the right of point F.

6. Place line d along the edge to left of point F.

7. Place point F closer to line d.

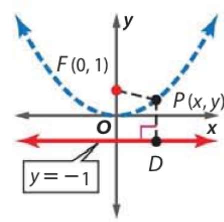
8. Place point F farther away from line d.

Geometrically, a parabola is the locus of all points in a plane equidistant from a fixed point, called the **focus**, and a fixed line, called the **directrix**. Recall that the distance between a fixed point and a line is the length of the segment perpendicular to the line through that point. You can find an equation of a parabola on the coordinate plane using its locus definition and the Distance Formula.



Activity 3 Equation of Parabola

Find an equation of the parabola with focus at (0, 1) and directrix $y = -1$.



Step 1 Graph $F(0, 1)$ and $y = -1$. Sketch a U-shaped curve for the parabola between the point and line as shown. Label a point $P(x, y)$ on the curve.

Step 2 Label a point D on $y = -1$ such that $\overline{PD}$ is perpendicular to the line $y = -1$. The coordinates of this point must therefore be $D(x, -1)$.

Step 3 Use the Distance Formula to find PD and PF.

$$PD = \sqrt{(x - x)^2 + [y - (-1)]^2} \quad D(x, -1), P(x, y), F(0, 1) \qquad PF = \sqrt{(x - 0)^2 + (y - 1)^2}$$

$$= \sqrt{(y + 1)^2} \quad \text{Simplify.} \qquad = \sqrt{x^2 + (y - 1)^2}$$

Step 4 Since $PD = PF$, set these expressions equal to each other.

$$\sqrt{(y + 1)^2} = \sqrt{x^2 + (y - 1)^2} \quad PD = PF$$

$$(y + 1)^2 = x^2 + (y - 1)^2 \quad \text{Square each side.}$$

$$y^2 + 2y + 1 = x^2 + y^2 - 2y + 1 \quad \text{Square each binomial.}$$

$$4y = x^2 \text{ or } y = \frac{1}{4}x^2 \quad \text{Subtract } y^2 - 2y + 1 \text{ from each side.}$$

An equation of the parabola with focus at (0, 1) and directrix $y = -1$ is $y = \frac{1}{4}x^2$.

Model and Analyze

Find an equation of the parabola with the focus and directrix given.

9. $(0, -2), y = 2$

10. $\left(0, \frac{1}{2}\right), y = -\frac{1}{2}$

11. $(1, 0), x = -1$

12. $(-3, 0), x = 3$

A line can intersect a parabola in 0, 1, or 2 points. Find the point(s) of intersection, if any, between each parabola and line with the given equations.

13. $y = x^2, y = x + 2$

14. $y = 2x^2, y = 4x - 2$

15. $y = -3x^2, y = 6x$

16. $y = -(x + 1)^2, y = -x$

CHAPTER 10

Study Guide and Review

Study Guide

KeyConcepts

Circles and Circumference (Lesson 10-1)

- The circumference of a circle is equal to πd or $2\pi r$.

Angles, Arcs, Chords, and Inscribed Angles
(Lessons 10-2 to 10-4)

- The sum of the measures of the central angles of a circle is 360°.
- The length of an arc is proportional to the length of the circumference.
- Diameters perpendicular to chords bisect chords and intercepted arcs.
- The measure of an inscribed angle is half the measure of its intercepted arc.

Tangents, Secants, and Angle Measures
(Lessons 10-5 and 10-6)

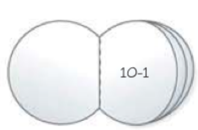
- A line that is tangent to a circle intersects the circle in exactly one point and is perpendicular to a radius.
- Two segments tangent to a circle from the same exterior point are congruent.
- The measure of an angle formed by two secant lines is half the positive difference of its intercepted arcs.
- The measure of an angle formed by a secant and tangent line is half its intercepted arc.

Special Segments and Equation of a Circle
(Lessons 10-7 and 10-8)

- The lengths of intersecting chords in a circle can be found by using the products of the measures of the segments.
- The equation of a circle with center (h, k) and radius r is $(x - h)^2 - (y - k)^2 = r^2$.

FOLDABLES StudyOrganizer

Be sure the Key Concepts are noted in your Foldable.



KeyVocabulary

adjacent arcs (p. 708)
arc (p. 706)
arc length (p. 709)
center (p. 697)
central angle (p. 706)
chord (p. 697)
chord segment (p. 750)
circle (p. 697)
circumference (p. 699)
circumscribed (p. 700)
common tangent (p. 732)
compound locus (p. 762)
concentric circles (p. 698)
congruent arcs (p. 707)
diameter (p. 697)
external secant segment (p. 752)
inscribed (p. 700)
inscribed angle (p. 723)
intercepted arc (p. 723)
major arc (p.707)
minor arc (p. 707)
pi (π) (p. 699)
point of tangency (p. 732)
radius (p. 697)
secant (p. 741)
secant segment (p. 752)
semicircle (p. 707)
tangent (p. 732)

VocabularyCheck

State whether each sentence is *true* or *false*. If *false*, replace the underlined word or phrase to make a true sentence.

1. Any segment with both endpoints on the circle is a radius of the circle.

2. A chord passing through the center of a circle is a diameter.

3. A central angle has the center as its vertex and its sides contain two radii of the circle.

4. An arc with a measure of less than 180° is a major arc.

5. An intercepted arc is an arc that has its endpoints on the sides of an inscribed angle and lies in the interior of the inscribed angle.

6. A common tangent is the point at which a line in the same plane as a circle intersects the circle.

7. A secant is a line that intersects a circle in exactly one point.

8. A secant segment is a segment of a diameter that has exactly one endpoint on the circle.

9. Two circles are concentric circles if and only if they have congruent radii.

Lesson-by-Lesson Review

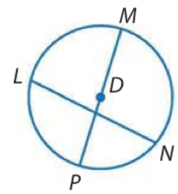
10-1 Circles and Circumference

For Exercises 10–12, refer to ⊙*D*.

10. Name the circle.

11. Name a radius.

12. Name a chord that is not a diameter.



Find the diameter and radius of a circle with the given circumference. Round to the nearest hundredth.

13. $C = 43$ cm

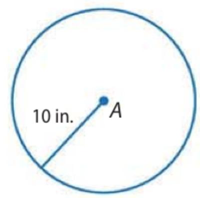
14. $C = 26.7$ yd

15. $C = 108.5$ ft

16. $C = 225.9$ mm

Example 1

Find the circumference of ⊙*A*.



$C = 2\pi r$ — Circumference formula

$= 2\pi(10)$ — Substitution

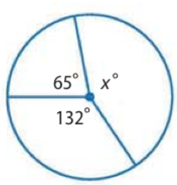
≈ 62.83 — Use a calculator.

The circumference of ⊙*A* is about 62.83 inches.

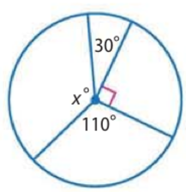
10-2 Measuring Angles and Arcs

Find the value of *x*.

17.

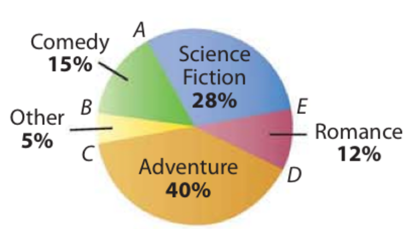


18.



19. **MOVIES** The pie chart below represents the results of a survey taken by Mrs. Jameson regarding her students' favorite types of movies. Find each measure.

Mrs. Jameson's Students' Favorite Types of Movies



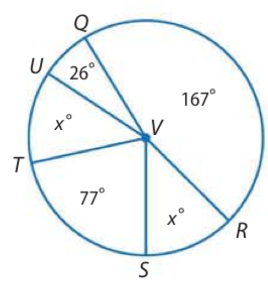
a. $m\widehat{AE}$

b. $m\widehat{BC}$

c. Describe the type of arc that the category Adventure represents.

Example 2

Find the value of *x*.



$m\angle QVR + m\angle RVS + m\angle SVT + m\angle TVU + m\angle UVQ = 360$ — Sum of Central Angles

$167 + x + 77 + x + 26 = 360$ — Substitution

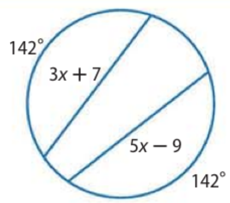
$270 + 2x = 360$ — Simplify.

$2x = 90$ — Subtract.

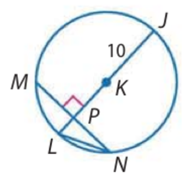
$x = 45$ — Divide.

10-3 Arcs and Chords

20. Find the value of x.



In ⊙K, $MN = 16$ and $m\overset{\frown}{MN} = 98$. Find each measure. Round to the nearest hundredth.

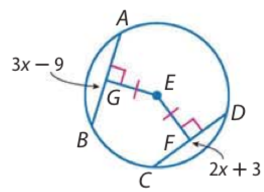


21. $m\overset{\frown}{NJ}$ **22.** LN

23. GARDENING The top of the trellis shown is an arc of a circle in which $\overline{CD}$ is part of the diameter and $\overline{CD} \perp \overline{AB}$. If $\overset{\frown}{ACB}$ is about 28% of a complete circle, what is $m\overset{\frown}{CB}$?

Example 3

ALGEBRA In ⊙E, $EG = EF$. Find AB.



Since chords $\overline{EG}$ and $\overline{EF}$ are congruent, they are equidistant from E. So, $AB = CD$.

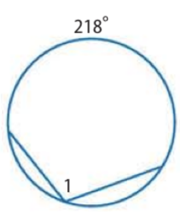
$AB = CD$	Theorem 10.5
$3x - 9 = 2x + 3$	Substitution
$3x = 2x + 12$	Add.
$x = 12$	Simplify.

So, $AB = 3(12) - 9$ or 27.

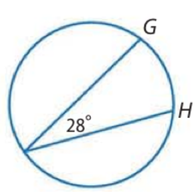
10-4 Inscribed Angles

Find each measure.

24. $m\angle 1$



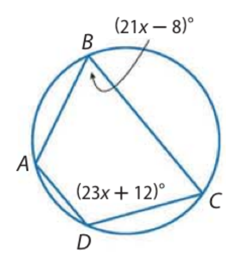
25. $m\overset{\frown}{GH}$



26. MARKETING In the logo at the right, $m\angle 1 = 42$. Find $m\angle 5$.

Example 4

Find $m\angle D$ and $m\angle B$.



Since $ABCD$ is inscribed in a circle, opposite angles are supplementary.

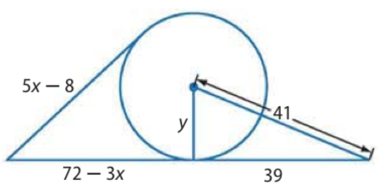
$m\angle D + m\angle B = 180$	Definition of supplementary
$23x + 12 + 21x - 8 = 180$	Substitution
$44x + 4 = 180$	Simplify.
$44x = 176$	Subtract.
$x = 4$	Divide.

So, $m\angle D = 23(4) + 12$ or 104 and $m\angle B = 21(4) - 8$ or 76.

10-5 Tangents

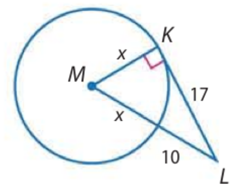
27. SCIENCE FICTION In a story Todd is writing, instantaneous travel between a two-dimensional planet and its moon is possible when the time-traveler follows a tangent. Copy the figures below and draw all possible travel paths.

28. Find x and y. Assume that segments that appear to be tangent are tangent. Round to the nearest tenth if necessary.



Example 5

In the figure, $\overline{KL}$ is tangent to $\odot M$ at K. Find the value of x.



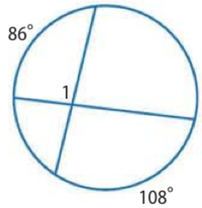
By Theorem 10.9, $\overline{MK} \perp \overline{KL}$. So, $\triangle MKL$ is a right triangle.

$KM^2 + KL^2 = ML^2$	Pythagorean Theorem
$x^2 + 17^2 = (x + 10)^2$	Substitution
$x^2 + 289 = x^2 + 20x + 100$	Multiply.
$289 = 20x + 100$	Simplify.
$189 = 20x$	Subtract.
$9.45 = x$	Divide.

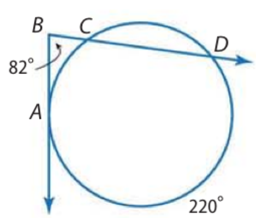
10-6 Secants, Tangents, and Angle Measures

Find each measure. Assume that segments that appear to be tangent are tangent.

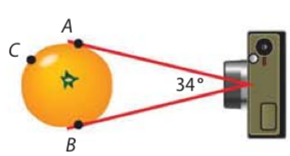
29. $m\angle 1$



30. $m\widehat{AC}$

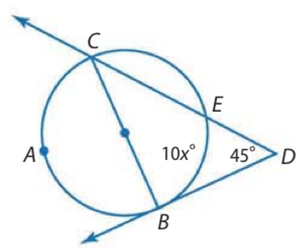


31. PHOTOGRAPHY Ahmed needs to take a close-up shot of an orange for his art class. He frames a shot of an orange as shown below, so that the lines of sight form tangents to the orange. If the measure of the camera's viewing angle is 34°, what is $m\widehat{ACB}$?



Example 6

Find the value of x.



$\widehat{CAB}$ is a semicircle because $\overline{CB}$ is a diameter.

So, $m\widehat{CAB} = 180$.

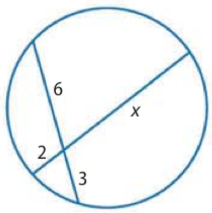
$m\angle D = \frac{1}{2}(m\widehat{CB} - m\widehat{EB})$	Theorem 10.14
$45 = \frac{1}{2}(180 - 10x)$	Substitution
$90 = 180 - 10x$	Multiply.
$-90 = -10x$	Subtract.
$9 = x$	Divide.

Study Guide and Review *Continued*

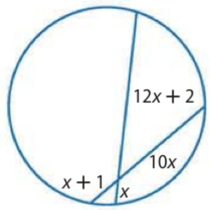
10-7 Special Segments in a Circle

Find x. Assume that segments that appear to be tangent are tangent.

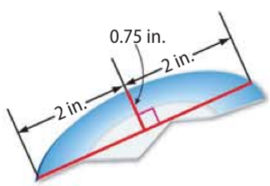
32.



33.

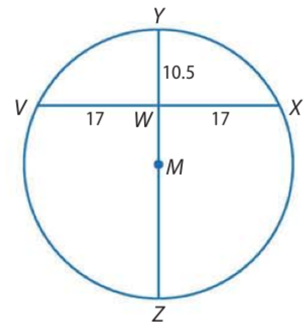


34. ARCHAEOLOGY While digging a hole to plant a tree, Henry found a piece of a broken saucer. What was the circumference of the original saucer? Round to the nearest hundredth.



Example 7

Find the diameter of circle M.



$VW \cdot WX = YW \cdot WZ$	Theorem 10.14
$17 \cdot 17 = 10.5 \cdot WZ$	Substitution
$289 = 10.5 \cdot WZ$	Simplify.
$27.5 \approx WZ$	Divide each side by 10.5.
$YZ = YW + WZ$	Segment Addition Postulate
$YZ = 10.5 + 27.5$	Substitution
$YZ = 38$	Simplify.

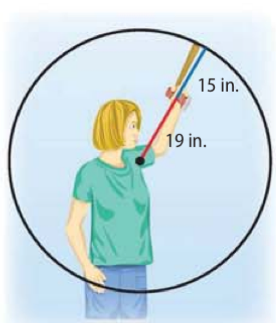
10-8 Equations of Circles

Write the equation of each circle.

35. center at $(-2, 4)$, radius 5

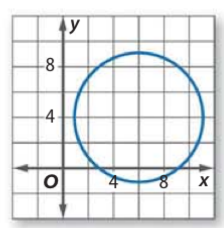
36. center at $(1, 2)$, diameter 14

37. FIREWOOD In an outdoor training course, Kat learns a wood-chopping safety check that involves making a circle with her arm extended, to ensure she will not hit anything overhead as she chops. If her reach is 19 inches, the hatchet handle is 15 inches, and her shoulder is located at the origin, what is the equation of Kat's safety circle?



Example 8

Write the equation of the circle graphed below.



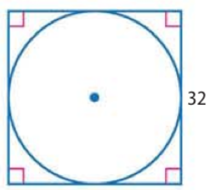
The center is at (6, 4) and the radius is 5.

$(x - h)^2 + (y - k)^2 = r^2$	Equation of a circle
$(x - 6)^2 + (y - 4)^2 = 5^2$	$(h, k) = (6, 4)$ and $r = 5$
$(x - 6)^2 + (y - 4)^2 = 25$	Simplify.

CHAPTER 10 Practice Test

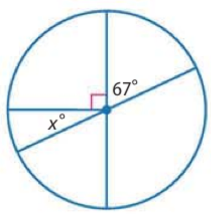
1. **POOLS** Amanda's family has a swimming pool that is 4 feet deep in their backyard. If the diameter of the pool is 25 feet, what is the circumference of the pool to the nearest foot?

2. Find the exact circumference of the circle below.

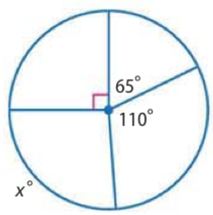


Find the value of x.

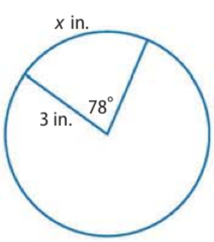
3.



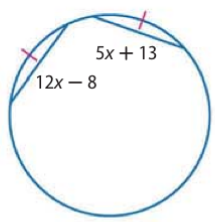
4.



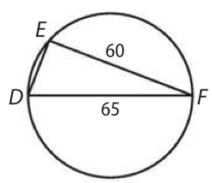
5.



6.

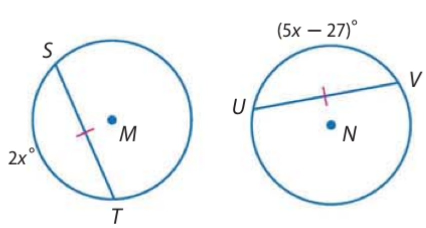


7. **MULTIPLE CHOICE** What is ED?



A 15
B 25
C 88.5
D not enough information

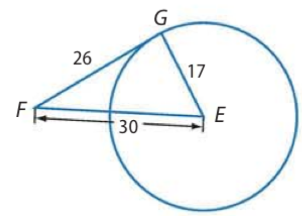
8. Find x if $\odot M \cong \odot N$.



9. **MULTIPLE CHOICE** How many points are shared by concentric circles?

F 0
G 1
H 2
J infinite points

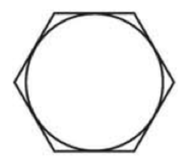
10. Determine whether $\overline{FG}$ is tangent to $\odot E$. Justify your answer.



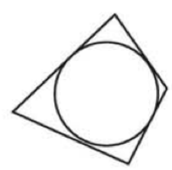
11. **MULTIPLE CHOICE** Which of the figures below shows a polygon circumscribed about a circle?

A

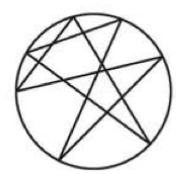
C



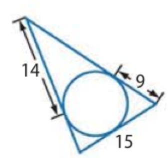
B



D

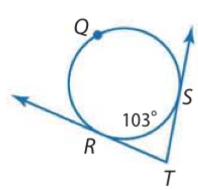


12. Find the perimeter of the triangle at the right. Assume that segments that appear to be tangent are tangent.

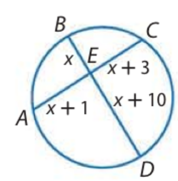


Find each measure.

13. $m\angle T$



14. x

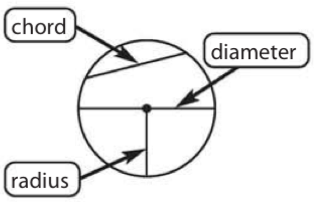


15. **FLOWERS** Hannah wants to encircle a tree trunk with a flower bed. If the center of the tree trunk is the origin and Hannah wants the flower bed to extend to 3 feet from the center of the tree, what is the equation that would represent the flower bed?

CHAPTER 10 Preparing for Standardized Tests

Properties of Circles

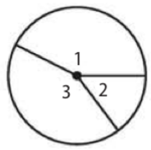
A circle is a unique shape in which the angles, arcs, and segments intersecting the circle have special properties and relationships. You should be able to identify the parts of a circle, write the equation of a circle, and solve for arc, angle, and segment measures in a circle.



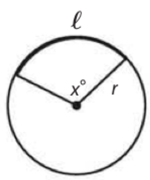
$r = \frac{1}{2}d$

$d = 2r$

$C = 2\pi r$ or πd



$m\angle 1 + m\angle 2 + m\angle 3 = 360$



$\ell = \frac{x}{360} \cdot 2\pi r$

Strategies for Applying the Properties of Circles

Step 1

Review the parts of a circle and their relationships.

- Some key parts include: **radius, diameter, arc, chord, tangent, secant**
- Study the key theorems and the properties of circles as well as the relationships between the parts of a circle.

Step 2

Read the problem statement and study any figure you are given carefully.

- Determine what you are being asked to find.
- Fill in any information in the figure that you can.
- Determine which theorems or properties apply to the problem situation.

Step 3

Solve the problem and check your answer.

- Apply the theorems or properties to solve the problem.
- Check your answer to be sure it makes sense.

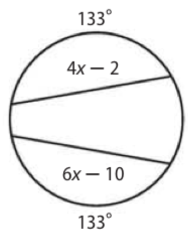
Standardized Test Example

Read the problem. Identify what you need to know. Then use the information in the problem to solve.

Solve for x in the figure.

A 2 **C** 4

B 3 **D** 6



Read the problem statement and study the figure carefully. You are given a circle with two chords that correspond to congruent minor arcs. One important property of circles is that two chords are congruent if and only if their corresponding minor arcs are congruent. You can use this property to set up and solve an equation for x.

$4x - 2 = 6x - 10$ Definition of Congruent Segments

$4x - 6x = -10 + 2$ Subtract.

$-2x = -8$ Simplify.

$\frac{-2x}{-2} = \frac{-8}{-2}$ Divide each side by −2.

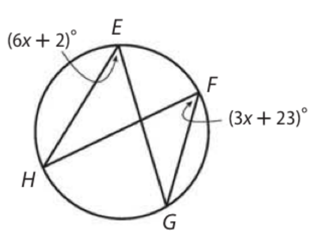
$x = 4$ Simplify.

So, the value of x is 4. The answer is C. You can check your answer by substituting 4 into each expression and making sure both chords have the same length.

Exercises

Read each problem. Identify what you need to know. Then use the information in the problem to solve.

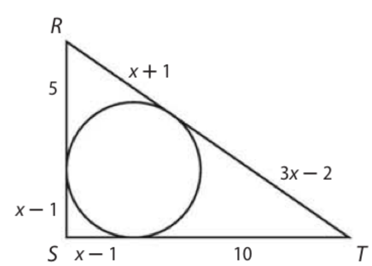
1. Solve for x in the figure below.



A 4 **C** 6

B 5 **D** 7

2. Triangle RST is circumscribed about the circle below. What is the perimeter of the triangle?



F 33 units **H** 37 units

G 36 units **J** 40 units

CHAPTER 10

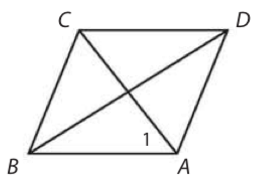
Standardized Test Practice

Cumulative, Chapters 1 through 10

Multiple Choice

Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.

1. If $ABCD$ is a rhombus, and $m\angle ABC = 70°$, what is $m\angle 1$?



A 45°
B 55°
C 70°
D 125°

2. Karen argues that if you live in Greensboro, North Carolina, then you live in Guilford County. Which assumption would you need to make to form an indirect proof of this claim?

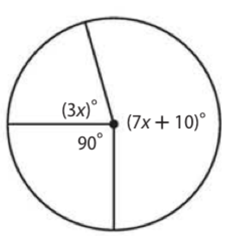
F Suppose someone lives in Guilford County, but not in Greensboro.

G Suppose someone lives in Greensboro, but not in Guilford County.

H Suppose someone lives in Greensboro and in Guilford County.

J Suppose someone lives in Guilford County and in Greensboro.

3. What is the value of x in the figure?

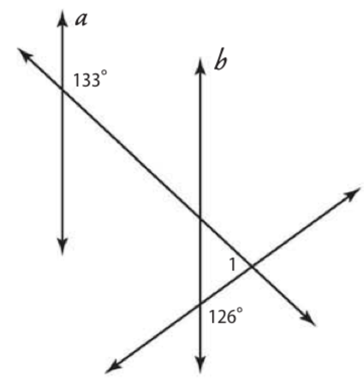


A 19
B 23
C 26
D 28

Test-Taking Tip

Question 3 Use the properties of circles to set up and solve an equation to find x.

4. Given $a \parallel b$, find $m\angle 1$.



F 47°
G 54°
H 79°
J 101°

5. Which of the following conditions would *not* guarantee that a quadrilateral is a parallelogram?

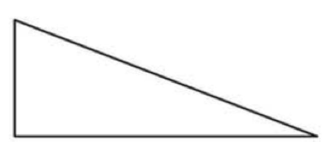
A both pairs of opposite sides congruent

B both pairs of opposite angles congruent

C diagonals bisect each other

D one pair of opposite sides parallel

6. The ratio of the measures of the angles of the triangle below is 3:2:1. Which of the following is *not* an angle measure of the triangle?

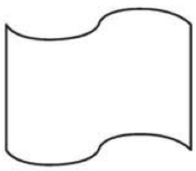


F 30°
G 45°
H 60°
J 90°

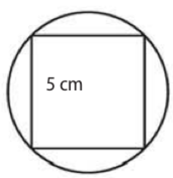
Short Response/Gridded Response

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

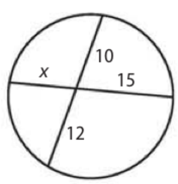
7. Does the figure shown have rotational symmetry? If so, give the order of symmetry.



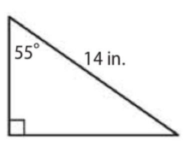
8. **GRIDDED RESPONSE** A square with 5-centimeter sides is inscribed in a circle. What is the circumference of the circle? Round your answer to the nearest tenth of a centimeter.



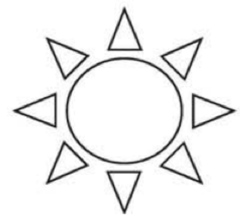
9. Solve for x in the figure. Show your work.



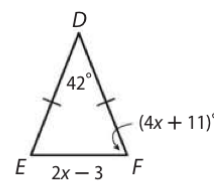
10. **GRIDDED RESPONSE** What is the perimeter of the right triangle below? Round your answer to the nearest tenth if necessary.



11. **GRIDDED RESPONSE** State the magnitude of rotational symmetry of the figure. Express your answer in degrees.



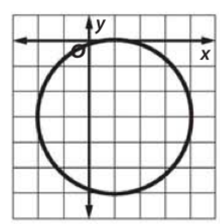
12. What is the length of $\overline{EF}$?



Extended Response

Record your answers on a sheet of paper. Show your work.

13. Use the circle shown to answer each question.



a. What is the center of the circle?

b. What is the radius of the circle?

c. Write an equation for the circle.

Need ExtraHelp?

If you missed Question...	1	2	3	4	5	6	7	8	9	10	11	12	13
Go to Lesson...	6-5	5-4	10-2	3-2	6-3	8-3	9-5	10-1	10-7	8-4	9-5	4-6	10-8