Get Ready for Chapter 12

Diagnose Readiness You have two options for checking Prerequisite Skills.

QuickCheck

Determine the probability of each event if you randomly select a cube from a bag containing 6 red cubes, 4 yellow cubes, 3 blue cubes, and 1 green cube. (Lesson 0-11)

1. $P(\text{red})$
2. $P(\text{blue})$
3. $P(\text{not red})$
4. $P(\text{white})$

5. Jim rolls a die with 6 sides. What is the probability of rolling a 5? (Lesson 0-11)

6. Malika spins a spinner that is divided into 8 equal sections. Each section is a different color, including blue. What is the probability the spinner lands on the blue section?

QuickReview

EXAMPLE 1

Determine the probability of selecting a green cube if you randomly select a cube from a bag containing 6 red cubes, 4 yellow cubes, and 1 green cube.

There is 1 green cube and a total of 11 cubes in the bag.

$$\frac{1}{11} = \frac{\text{number of green cubes}}{\text{total number of cubes}}$$

The probability of selecting a green cube is $\frac{1}{11}$.

Find each product. (Lesson 0-5)

7. $\frac{5}{4} \cdot \frac{2}{3}$
8. $\frac{4}{19} \cdot \frac{7}{20}$
9. $\frac{4}{32} \cdot \frac{7}{32}$
10. $\frac{5}{12} \cdot \frac{6}{11}$
11. $\frac{56}{100} \cdot \frac{24}{100}$
12. $\frac{9}{34} \cdot \frac{17}{27}$

EXAMPLE 2

Find $\frac{4}{5} \cdot \frac{3}{4}$.

$\frac{4}{5} \cdot \frac{3}{4} = \frac{4 \cdot 3}{5 \cdot 4}$ **Multiply the numerators and the denominators.**

$= \frac{12}{20}$ **Simplify.**

$= \frac{3}{5}$ **Rename in simplest form.**

Write each fraction as a percent. Round to the nearest tenth. (Lesson 0-6)

13. $\frac{14}{17}$
14. $\frac{7}{8}$
15. $\frac{107}{125}$
16. $\frac{625}{1024}$

17. **SHOPPERS** At the mall, 700 of the 2000 people shopping were under the age of 21. What percent of the shoppers were under 21?

EXAMPLE 3

Write the fraction $\frac{33}{80}$ as a percent. Round to the nearest tenth.

$\frac{33}{80} \approx 0.413$ **Simplify and round.**

$0.413 \cdot 100 = 41.3$ **Multiply the decimal by 100.**

$\frac{33}{80}$ written as a percent is about 41.3%.

Get Started on Chapter 12

You will learn several new concepts, skills, and vocabulary terms as you study Chapter 12. To get ready, identify important terms and organize your resources. You may wish to refer to **Chapter 0** to review prerequisite skills.
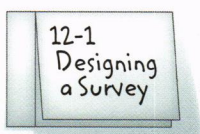
FOLDABLES® Study Organizer

Probability and Statistics Make this Foldable to help you organize your Chapter 12 notes about Probability and Statistics. Begin with 8 sheets of $8\frac{1}{2}$ by 11 paper.
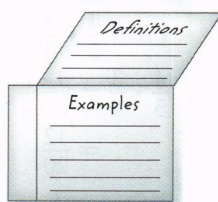
1. **Fold** each sheet of paper in half. Cut 1 inch from the end to the fold. Then cut 1 inch along the fold.
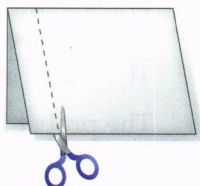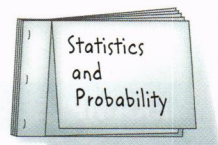
2. **Label** 7 of the 8 sheets with the lesson number and title.

   12-1
   Designing
   a Survey

3. **Label** the inside of each sheet with Definitions and Examples.

   Definitions

   Examples

4. **Stack** the sheets. Staple along the left side. Write the title of the chapter on the first page.

   Statistics
   and
   Probability

Math Online ▶ glencoe.com

- Study the chapter online
- Explore **Math in Motion**
- Get extra help from your own **Personal Tutor**
- Use **Extra Examples** for additional help
- Take a **Self-Check Quiz**
- **Review Vocabulary** in fun ways

New Vocabulary

English		Español
population	• p. 740 •	población
sample	• p. 740 •	muestra
biased sample	• p. 741 •	muestra sesgada
qualitative data	• p. 746 •	datos cualitativos
quantitative data	• p. 746 •	datos de cuantitativos
parameter	• p. 756 •	parámetro
statistic	• p. 756 •	estadística
standard deviation	• p. 757 •	desviación estándar
univariate data	• p. 757 •	data univariados
linear transformation	• p. 760 •	transfomación lineal
factorial	• p. 764 •	factorial
permutation	• p. 764 •	permutación
combination	• p. 765 •	combinación
compound event	• p. 771 •	evento compuesto
independent events	• p. 771 •	eventos independientes
complement	• p. 772 •	complemento
dependent events	• p. 772 •	eventos dependientes
mutually exclusive	• p. 773 •	mutuamente exclusivos
random variable	• p. 779 •	variable aleatoria
probability distribution	• p. 780 •	distribución de probabilidad
simulation	• p. 788 •	simulación

Review Vocabulary

probability • p. P33 • probilidad the ratio of favorable outcomes to the total possible outcomes

sample space • p. P33 • espacio muestral the list of all possible outcome

▶ Multilingual eGlossary glencoe.com

For a survey to be valid, it should contain no bias or favoritism. Even though you may question people who are chosen randomly, questions may be worded to influence responses. These two different surveys on Internet sales tax had different results.

ACTIVITY

Analyze the difference between the two survey questions.

Question 1

Should there be sales tax on purchases made on the Internet?

No
52%

Don't know
or refused
to answer
10%

Yes
38%
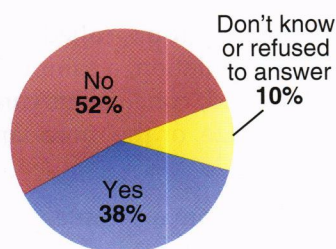
Question 2

Do you think people should or should not be required to pay the same sales tax for purchases made over the Internet as those made at a local store?

No
28%

Don't know
or refused
to answer
7%

Yes
65%
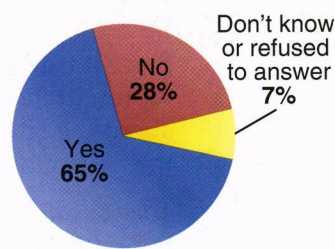
Notice that Question 2 includes more information. Pointing out that customers pay sales tax for items bought at a local store may give the people answering the survey a reason to say "yes."

Because they are random samples, the results of both of these surveys are accurate. However, the results could be used in a misleading way by someone with an interest in the issue. For example, an Internet retailer would prefer to state the results of Question 1.

Exercises

For Exercises 1 and 2, tell whether each question is likely to bias the results. Write *yes* or *no*. Explain your reasoning.

1. On a survey on environmental issues:
   a. "Due to diminishing resources, should a law be made to require recycling?"
   b. "Should the government require citizens to participate in recycling efforts?"

2. On a survey on education:
   a. "Should schools fund extracurricular sports programs?"
   b. "The budget of the River Valley School District is short of funds. Should taxes be raised in order for the district to fund extracurricular sports programs?"

3. You want to determine whether to serve hamburgers or pizza at a class party.
   a. Write a survey question that would likely produce biased results.
   b. Write a survey question that would likely produce unbiased results.

Designing a Survey

Why?

When manufacturing T-shirts, many steps and items must be checked for quality. These include fabrication, care labels, tags, trims, print artwork, and embroidery. It would be costly for a company to have each T-shirt inspected. Instead, they inspect a certain number of T-shirts.

All of the T-shirts that are made are a population, and the T-shirts that are inspected are a sample of the population. The inspectors draw conclusions about the sample and apply those conclusions to the entire population.

Design a Survey A **sample** is some portion of a larger group, called the **population**. Since it is impractical to examine every item in a population, a sample is selected to represent the population. After the sample is analyzed, conclusions can be drawn about the entire population. The larger the sample size, or the more samples taken, the more closely it approximates the population.

To accurately draw a conclusion from data received from a sample, you will need to first decide on the best method of collecting the data.

Key Concept — Data Collection Techniques — *For Your* FOLDABLE

Type	Definition/Use	Example
survey	• Data are from responses given by a sample of the population. • To make a general conclusion about the population.	To determine whether the student body is happy with the spring dance theme, the dance committee asks a random sample of 50 students for their opinion.
observational study	• Data are recorded after just observing the sample. • To compare reactions and draw a conclusion about responses of the population.	A toy company watches some children play and notes the toys they play with the most. They conclude that the population of two-year-olds prefers toys that sing to toys that do not make noise.
experiment	• Data are recorded after changing the sample. • To make general conclusions about what will happen during an event.	A quality control manager runs the assembly machines 10 times at a certain rate. Each time the product is defective. She concludes that it would happen every time the machine runs at that pace.

EXAMPLE 1 | **Classify Data Collection Techniques**

CHARITY A local charity is interested in finding out whether people are likely to give money to charity. They distributed 1000 questionnaires to people living in the neighborhood.

a. **Identify the sample, and determine the population from which it was selected.**

The sample is the 1000 people who received the questionnaires. The population is all the people in the neighborhood.

b. **Classify the type of data collection used by this charity.**

This is a survey. The data are from responses given by people in the sample.

✔ **Check Your Progress**

Identify each sample, and suggest a population from which it was selected. Then classify the type of data collection used.

1A. RESEARCH A research facility analyzed two groups of rats to determine their reaction to sugar.
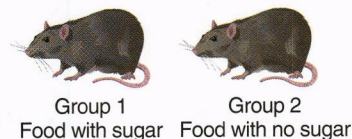
1B. RECYCLING The city council wants to start a recycling program. They send out a questionnaire to 1000 random citizens asking what they would recycle.

Group 1
Food with sugar

Group 2
Food with no sugar

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There are factors that affect the collection of data and the conclusions drawn. If a sample favors one group over another, then the data are invalid because it is a **biased sample**. A sample is *unbiased* if it is random. Members of a **random sample** have an equal probability of being chosen.

EXAMPLE 2 | **Identify if the Sample is Valid**

Identify each sample as *biased* or *unbiased*. Explain your reasoning.

a. **MUSIC** Every fifth person coming into a grocery store is asked to name a favorite radio station.

Unbiased; the sample is a random selection of people.

b. **MUSIC** Every fifth person at the Country Music Showcase is asked to name their favorite radio station.

Biased; because they are at a country music show, people may be more likely to select a country music station.

✔ **Check Your Progress**

2A. POLITICS A journalist visits a senior center and chooses 10 individuals randomly to poll about various political topics.

2B. SHOES A shoe company conducts an observation study that involves 10 girls and 2 boys to see which shoes are the most popular.

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Sampling Techniques Sample data are often used to estimate a characteristic within an entire population, such as voting preferences. A random sample of a population is selected so that it is representative of the entire population without any preference. Three common types of random samples are listed below.

	Key Concept	Random Samples	For Your FOLDABLE
Type	**Definition**	**Example**	
simple random sample	A sample that is equally likely to be chosen as any other sample from the population.	One hundred student ID numbers are randomly drawn from a hat, and those students are given a survey.	
stratified random sample	The population is first divided in similar, nonoverlapping groups. A random sample is then selected from each group.	To reflect the diversity of the country, a candidate surveys citizens of various groups, based on their percent of the population.	
systematic random sample	A sample in which the items in the sample are selected according to a specified time or item interval.	Every 10 minutes a toy is inspected. Or every 50th toy is inspected.	
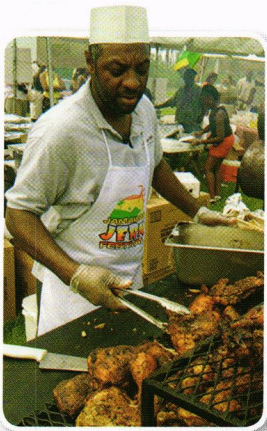
EXAMPLE 3   Classify a Random Sample

ZOOS Animals in a zoo are divided by continents. Then two animals were selected at random from each group to have their blood tested.

a. Identify the sample, and suggest a population from which it was selected.

The sample is the two animals selected from each continent. The population is the animals in the zoo.

b. Classify the sample as *simple*, *stratified*, or *systematic*. Explain your reasoning.

This is a stratified random sample. The animals are divided up into categories before there is a random selection.

✓ Check Your Progress

Identify each sample, and suggest a population from which it was selected. Then classify the sample as *simple*, *stratified*, or *systematic*. Explain your reasoning.

3A. CONTESTS Refer to the information at the left. The cooks lined up randomly within their category, and every tenth cook in each category was selected to explain the rules.

3B. FOOD At a popular hamburger restaurant, the manager checks the quality of the burgers every 20 minutes, starting at a randomly selected time.

3C. SHOWER At a bridal shower, a sticker was placed on the bottom of three random plates. The guests who receive the starred plates will win a prize.

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Example 1
p. 741

Identify each sample, and determine a population from which it was selected. Then classify the type of data collection used.

1. **MUSIC** A record company wants to test five designs for an album cover. They randomly invite ten teens from a local high school to view the album covers.

2. **PARTIES** Federico is trying to decide on a theme and a color scheme for his party. He sends a survey in each invitation, asking guests for their opinions.

Example 2
p. 741

Identify each sample as *biased* or *unbiased*. Explain your reasoning.

3. **POLITICS** A group of students stands at the door of the school and asks every tenth student who they would vote for in the upcoming election and why.

4. **SHOPPING** Every fifteenth shopper at a clothing store is asked what they would want most for their birthday.

Example 3
p. 742

Identify the sample, and suggest a population from which it was selected. Then classify the sample as *simple*, *stratified*, or *systematic*. Explain your reasoning.

5. **SPORTS CARDS** Greg divides his rookie baseball cards by teams. Then he randomly selects cards and records the players' RBIs.

6. **TELEVISION** A nostalgia television network wants to conduct a cartoon marathon. To choose the episodes, they mail a questionnaire to people selected at random throughout the country.

Practice and Problem Solving

● = **Step-by-Step Solutions** begin on page R12.
Extra Practice begins on page 815.

Example 1
p. 741

Identify each sample, and determine a population from which it was selected. Then classify the type of data collection used.

7 **FOOD** A frozen food company is considering creating frozen meals with tofu instead of meat. At a testing, they randomly give half of a group of 100 people the meals with meat and the other half the same meals with tofu and ask the people how they like the meals.

8. **PETS** The owners of dog care center want to know how many of each size crate they should order. They send flyers into the neighborhood to ask what size or breed of dog each person has.

9. **TRAVEL** A travel agency asks each of its customers for the past two years about their favorite and least favorite destinations.

Example 2
p. 741

Identify each sample as *biased* or *unbiased*. Explain your reasoning.

10. **MAGAZINES** A magazine publisher asks every tenth person at a fitness expo what magazines they have in their household.

11. **LIBRARY** The local library asks everyone who checks out a book if they also used the computers at the library.

12. **JEANS** A clothing chain gives its customers a card they can mail back that asks them questions about the customer's favorite brand of jeans.

13. **AMUSEMENT PARKS** An amusement park is deciding which rides to replace next year. As they leave the park, they ask teens what their least favorite ride is.

Example 3
p. 742

Identify the sample, and suggest a population from which it was selected. Then classify the sample as *simple*, *stratified*, or *systematic*. Explain your reasoning.

14. **TANNING** A tanning salon sorts its responses to a survey by the home states of the respondents. Then they are sorted to select teenagers.

15. **ART** Mitsu uses her blog about art to ask readers about their favorite medium and supply center. She then tabulates and publishes the results.

16. **CARS** The service manger at a car dealership inspects every fifth car to make sure cars are detailed after being serviced.

17. **MUSIC** A music store gives every fiftieth customer a free CD by a local artist.

18. **ELECTIONS** To estimate who the leading candidate is, the candidate's committee surveys a large group of people selected at random. The returns indicate that their candidate is leading 58% to 42%.

   a. Identify the sample. Suggest a population from which the sample was selected.

   b. State the method of data collection.

   c. Is the sample *biased* or *unbiased*? Explain.

   d. If unbiased, classify the random sample as *simple*, *stratified*, or *systematic*.
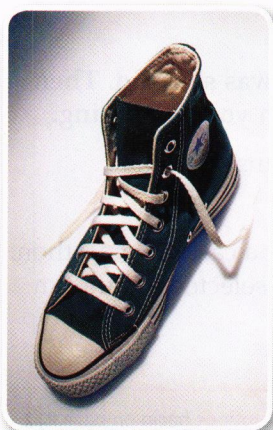
19. **SHOES** A shoe company's website keeps a count of styles and colors chosen by customers when ordering online.

   a. Identify the sample. From what population was the sample selected?

   b. State the method of data collection.

   c. Is the sample *biased* or *unbiased*? Explain.

   d. If unbiased, classify the sample as *simple*, *stratified*, or *systematic*.

20. **MULTIPLE REPRESENTATIONS** Design and conduct your own survey.

   a. **WRITING** Write a question you would like to answer through a survey. The question should be meaningful to you. Describe the method you will use to gather the data, and explain why you chose that method.

   b. **ANALYTICAL** Devise a method to conduct your survey using an unbiased sample. Explain why you chose your sample.

   c. **CONCRETE** Conduct your survey.

   d. **TABULAR** Record your results in a table.

   e. **GRAPHICAL** Use a graph (line, circle, histogram, etc…) or other visual or graphic method to present your results to the class.

H.O.T. Problems / Use **H**igher-**O**rder **T**hinking Skills

21. **CHALLENGE** Consider the following survey proposal.
   Question: How do students feel about the new dress code?
   Method: Divide the student body by their four grade levels. Then, take a simple random sample from each of the four grades. Conduct the survey using this sample.
   Discuss the strengths and weaknesses of this survey.

22. **REASONING** Compare and contrast the three data collection techniques described in this lesson.

23. **OPEN ENDED** Describe a real-world example of an observational study.

24. **WRITING IN MATH** Explain why accurate surveys are important to companies, and how the companies use them.

🌐 Real-World Link

The world's largest collection of Converse shoes is held by Joshua Mueller of Lakewood, Washington, consisting of 403 unique pairs.

Source: *The Guinness Book of World Records*

25. GRIDDED RESPONSE The first stage of a rocket burns 28 seconds longer than the second stage. If the total burning time is 152 seconds, how many seconds is the first stage?

26. Ms. Brinkman invested $30,000; part at 5%, and part at 8%. The total interest on the investment was $2100 after one year. How much did she invest at 8%?

   A $10,000   **C** $20,000

   B $15,000   **D** $25,000

27. A pair of $25 jeans is on sale for 15% off. What is the sale price?

   F $21.25   **H** $23.25

   G $24.25   **J** $22.25

28. GEOMETRY A piece of wire 42 centimeters long is bent into the shape of a rectangle with a width that is twice its length. Find the dimensions of the rectangle.

   A 5 cm, 12 cm   **C** 9 cm, 16 cm

   B 7 cm, 14 cm   **D** 11 cm, 18 cm

Spiral Review

Solve each equation. State any extraneous solutions. (Lesson 11-8)

29. $\dfrac{3}{c} = \dfrac{2}{c+2}$

30. $\dfrac{4}{f} = \dfrac{2}{f-3}$

31. $\dfrac{j}{j+2} = \dfrac{j-6}{j-2}$

32. $\dfrac{h-2}{h} = \dfrac{h-2}{h-5}$

33. $\dfrac{3m}{4} + \dfrac{1}{3} = \dfrac{3m+4}{6}$

34. $\dfrac{6}{5} + \dfrac{4p}{3} = \dfrac{8p}{5}$

35. $\dfrac{r-2}{r+2} - \dfrac{3r}{r-2} = -2$

36. $\dfrac{t-3}{t+3} - \dfrac{2t}{t-3} = -1$

37. $\dfrac{4v}{2v+3} - \dfrac{2v}{2v-3} = 1$

38. SPORTS When air is pumped into a ball, the pressure required can be computed by using the formula $P = \dfrac{3412.94}{\frac{4\pi r^3}{3}}$, where P represents the pressure in pound per square inch (psi), and r is the radius of the ball in inches.

  a. Simplify the complex fraction.

  b. Suppose the air pressure inside the ball is 8 psi. Approximate the radius of the ball to the nearest hundredth.

39. ROLLER COASTERS Suppose a roller coaster climbs 208 feet higher than its starting point, moving horizontally 360 feet. When it comes down, it moves horizontally 44 feet. (Lesson 10-5)

  a. How far will it travel to get to the top of the ride?

  b. How far will it travel on the downhill track?

208 ft

360 ft   44 ft

Skills Review

40. PHYSICAL SCIENCE Mr. Blackwell's students recorded the height of an object above the ground after it was dropped from a height of 5 meters. (Lesson 1-7)

Time (s)	0	0.2	0.4	0.6	0.8	1
Height (cm)	500	480	422	324	186	10

Draw a graph showing the relationship between the height of the object and time.

Analyzing Survey Results

Why?

Companies like to use surveys to get feedback on how they are doing in areas ranging from sales to their Web site.

A company recently received these results from a survey about their Web site.

What do these values mean? How were the data collected? Is the sample an accurate representation of their customers?

Please rate the site on the following attributes.

1 = Poor, 4 = Outstanding   Overall Mean 2.2

Ease of Use
Visual appeal
Product ordering
Customer service
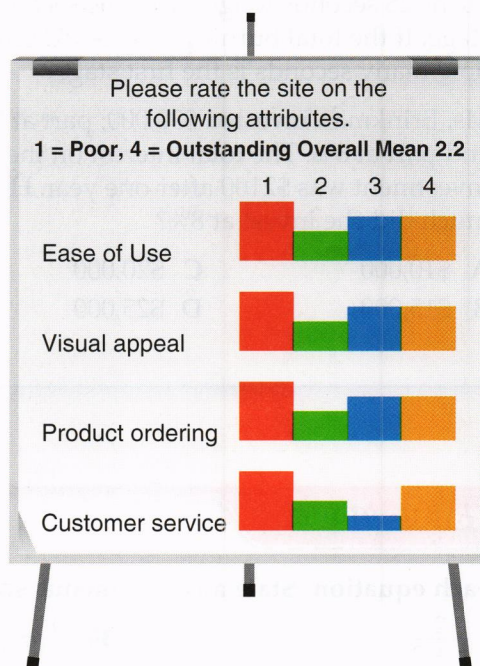
Summarize Survey Results Once data from a survey have been collected, they need to be summarized to be meaningful. We can summarize the data of a survey according to **measures of central tendency**.

Concept Summary   Measures of Central Tendency   For Your FOLDABLE

Type	Description	When Best Used
mean	the sum of the data divided by the number of items in the data set	The data sets have no outliers.
median	the middle number of the ordered data, or the mean of the middle two numbers	The data set has outliers, but there are no big gaps in the middle of the data.
mode	the number or numbers that occur most often	The data set has many repeated numbers.

Some data cannot be analyzed using statistical methods. **Quantitative data** can be given and analyzed as numerical values. Some examples of these are test scores, hours that you have studied, or the weight of objects. **Qualitative data** cannot be given a numerical value. Some examples of these are gender, nationality, or television show preference.

It is also possible to have quantitative data and still not be able to find a measure of central tendency. This occurs when the data do not represent the same thing.

EXAMPLE 1 **Select a Method to Summarize**

Which measure of central tendency best represents the data, if any? Justify your answer. Then find the measure.

Vegetable	Calories	Vegetable	Calories
asparagus	14	cauliflower	10
beans	30	celery	17
bell pepper	20	corn	66
broccoli	25	lettuce	9
cabbage	17	spinach	9
carrots	28	zucchini	17

a. **NUTRITION** The table shows the number of Calories per serving of each vegetable.

List the values from least to greatest: 9, 9, 10, 14, 17, 17, 17, 20, 25, 28, 30, 66.

There is one value that is much greater than the rest of the data, 66. Also, there does not appear to be a big gap in the middle of the data. There are only two sets of identical numbers. So, the median would best represent the data.

{9, 9, 10, 14, 17, 17, 17, 20, 25, 28, 30, 66}

The median is 17 Calories.

b. **CONCERTS** An amphitheater conducted a survey in which they asked 1000 adults the last time they attended a concert.

Results	
Response	**Percentage**
at least 3 years ago	8
1–3 years ago	15
6 months to 1 year ago	45
less than 6 months ago	32

A measure of central tendency cannot be calculated for this set of data. Each percentage in the table represents something different.

For example, 15% of the respondents attended a concert 1 to 3 years ago, while 32% saw a concert less than 6 months ago. So, the median value of 23.5% has no meaning in this situation.

☑ **Check Your Progress**

1A. **FINANCIAL LITERACY** An electronics store records the number of customers it has during each hour of the day.

Number of Customers			
86	71	79	86
79	32	88	86
82	69	71	70
86	81	85	86

1B. **BOOKS** In a survey, students between the ages of thirteen and eighteen reported where they get their books. The responses were: teachers, 420; school library, 1320; public library, 1020; parents, 720; bookstore, 1020; Internet, 540; friends, 540; as a gift, 1020.

Watch Out!

Percents Always make sure that a survey that gives data in percents tells the size of the sample.

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Evaluate Survey Results Once a survey has been conducted, data are summarized, a report of the findings and conclusions is made. However, bias can sometimes cause errors in the data, as well as how they are interpreted and reported.

You need to be able to judge the reliability of these survey reports. You can do this by making sure that the sample is random, large enough to be an accurate representation of the population, and that the source of the data is a reliable one.

Often newspaper, magazine, and television reports include the results of a survey. These surveys need to be judged for their validity before you make a decision based on them. Some questions that you may want to ask yourself are:

- What are the population and samples? Can I identify them easily? Are they biased?

- What is the source? Is the data source a reputable group? Could they be biased?

- Do the data actually support the conclusion?

EXAMPLE 2 Evaluate a Survey

YEARBOOKS Given the following portion of a survey report, evaluate the validity of the information and conclusion.

Question: Should the school have an electronic yearbook this year?

Sample: Ballots were placed in random students' lockers.

Conclusion: The school should only offer an electronic yearbook this year.

Results	
Choice	**Response**
electronic only	67%
traditional paper	22%
offer both	9%
no preference	2%

While the report states that students were chosen randomly, it does not say how many students were chosen. The results were given in percents. The 67% could mean about 16 out of 24. This may not be a large enough sample to represent a large school.

✓ Check Your Progress

2. **CONCERTS** At a sold-out concert in a 5000-seat concert hall, every 10th attendee completed a survey.

   Question: Did you feel that the price of the tickets was reasonable?

   **Conclusion:** The prices for the tickets are reasonable and should remain the same.

Results	
Choice	**Response**
very reasonable	56
reasonable	185
somewhat reasonable	132
unreasonable	69
very unreasonable	58

The way in which results are displayed can influence how you interpret those results. Here are some factors.

- If the scale of a line graph, bar graph, or histogram is large, any changes may appear to be small, when they actually could be quite significant. If the scale of a graph is small, small changes or differences can be made to appear quite large.

Notice the scale in the graph at the right. This graph is misleading because it appears that gas prices are not increasing too much, when they are actually quite significant.
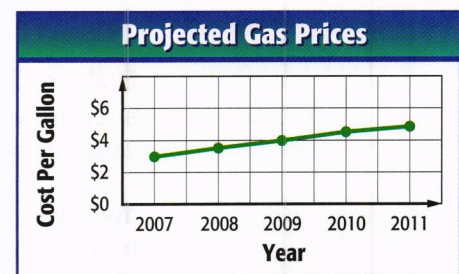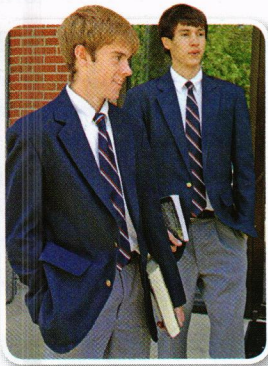
Projected Gas Prices

The following are features of a display that can influence the conclusion.

- The scales of graphs should be constant.

- Using percents rather than the actual numbers from a set of data can give a misleading result. However, if the numbers in the sample are large, percents best represent values.

- In a bar graph or histogram, all of the bars should have the same width. The heights of the bars represent the data values. Changing the width of a bar can exaggerate differences.

- If the colors on a circle graph, bar graph, or histogram are different shades of the same color, groups may visually blend together and influence how you interpret the results.

EXAMPLE 3   Misleading Results

UNIFORMS A high school principal is considering whether to institute a school uniform policy. She sends out a survey to the students at her high school to get their opinions.

Question: How would you feel about having a school uniform policy?

Conclusion: It would not bother students if a uniform policy were instituted.

Opinions Regarding School Uniforms
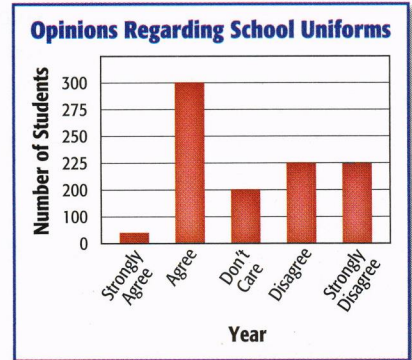
Determine whether the display gives an accurate picture of the survey results.

Upon first inspection, most of the students appear to agree with the uniforms. However, the scale in the graph is not constant. With a closer look, it appears that about 400 students either disagree or strongly disagree while a little more than 300 either agree or strongly agree.

In this case, the table is misleading, and the conclusion is invalid.

✓ Check Your Progress

3. The city council wanted to see how local companies were donating resources to charities. The bar graph shows the results.

   Question: How does your company contribute to local charities?

   Conclusion: Donating money is the least popular contribution by local companies.

   Determine whether the display gives an accurate picture of the survey results. Explain.
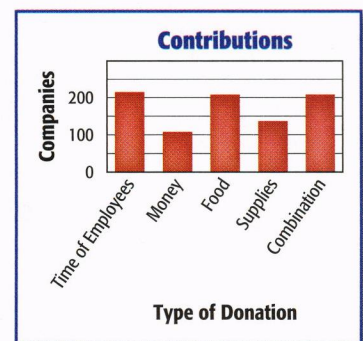
Contributions

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Example 1
p. 747

Which measure of central tendency best represents the data? Justify your answer. Then find the measure.

1. **RECYCLING** Archmont High School is recycling newspapers as a fundraiser to buy some benches for the courtyard. The newspapers are gathered into 5-inch bundles. Ms. Sato counted the bundles received each Friday for the first quarter of the school year: 15, 12, 14, 15, 18, 15, 13, 14, 13, 12.

2. **TRAVEL** An online travel agency wants to design tours for families with teens. So, they surveyed students about their favorite vacation destinations. The top five responses were: beach, 25%; theme parks, 22%; lakes, 21%; historical sites, 17%; mountains, 15%.

Example 2
p. 748

Given each survey report, evaluate the validity of the information and conclusion.

3. **RADIO** A radio station is considering changing its format. It mails 1000 surveys to randomly selected houses within their listening area. They receive 750 responses.

Question: What type of radio station do you like?

Conclusion: The radio station should have a format of rap/hip-hop.

Results	
Choice	**Response**
talk/sports	26%
classic rock	32%
rap/hip-hop	39%
country	3%

4. **SHOPPING** A shopping mall owner wants to know during what hours the most people prefer to shop. One day every 10th person who entered the mall was asked what times he or she preferred to shop.

Conclusion: The mall should remain open from 9:00 A.M. until 9:00 P.M.

Results	
Choice	**Response**
before 9:00 A.M.	26%
9:00 A.M.–12:00 noon	12%
12:00 noon–3:00 P.M.	26%
3:00 P.M.–6:00 P.M.	27%
6:00 P.M.–9:00 P.M.	9%

5. **VOLUNTEERING** Sample: 21,700 6th- to 12th-grade students surveyed by *USA Weekend*, Youthnoise.com, and a volunteer organization called Key Club.

Question: Are youth interested in volunteering?

Conclusion: Youth are interested in volunteering.

| How many hours a year do you volunteer?	
Number of Hours	**Percent (%)**
fewer than 20 hours	30
20 to 39 hours	35
40 to 59 hours	13
60 to 80 hours	7
more than 80 hours	15

6. **SPORTS** Sample: *Scholastic Magazine* asked about 3585 kids with online subscriptions about their favorite sports.

Question: What are kids' favorite sports to compete in?

Results: baseball/softball, 271; football, 436; basketball, 570; soccer, 279; hockey, 197; track, 209; swimming, 319; gymnastics, 197; skating, 289; bowling, 202; other, 616

Conclusion: Basketball is kids' favorite sport in which to compete.
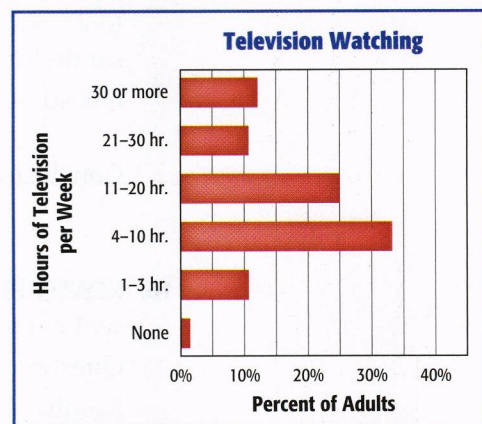
Example 3
p. 749

Determine whether each display gives an accurate picture of the survey results. Explain.

7. **CONVERSATION** A nationwide survey was conducted on the time students in grades six through twelve spend talking one-on-one with a family member. The responses were divided by regions and are displayed in the graph.

Conclusion: Students in the southeast talk with family members the most.

Conversation
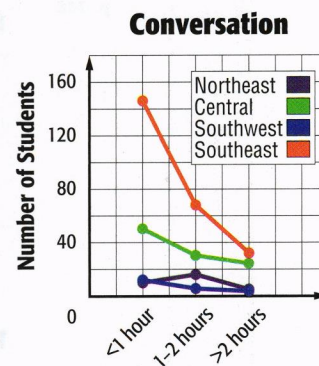
8. **TELEVISION** A survey conducted by a media network asked adults how many hours of television they watched each week. The results are displayed in the graph.

Television Watching

● = **Step-by-Step Solutions** begin on page R12.
Extra Practice begins on page 815.

Practice and Problem Solving

Example 1
p. 747

Which measure of central tendency best represents the data? Justify your answer. Then find the measure.

9. **FOOD** A sub shop adds a new bread to its menu. To see if they should keep it, the manager counts how many orders of that bread type are taken each day: 10, 16, 14, 13, 17, 15, 18, 16, 19.

10. **MOVIES** A video store wants to order additional movies. They conducted a survey to find their members' favorite type of movie. The responses were: comedy, 21%; drama, 19%; horror, 12%; science fiction, 17%; action and adventure, 18%; mystery and suspense, 13%.

11. **MOTORCYCLES** A motorcycle dealership conducted a telephone survey of its customers from the last five years about customer satisfaction. The results are listed in the table.

Response	Weight	Total
very satisfied	5	182
satisfied	4	252
okay	3	365
dissatisfied	2	169
very dissatisfied	1	54

12. **CONTESTS** A beverage company introduced a contest in which winning codes were printed on bottle caps. One code awarded the winner a $1 million prize, two codes awarded each winner a new car worth $20,000, and 50,000 codes awarded each winner a free beverage worth $1.

13. **SUMMER ACTIVITIES** In a survey, students were asked about their favorite summer activity. The responses are listed in the table.

Summer Activities			
swimming	650	camping	432
travel	885	reading	281
sports	1123	other	514

Example 2
p. 748

Given each survey report, evaluate the validity of the information and conclusion.

14. **SERVICES** A salon wants to know which of its services gets used the most. It surveyed 1090 customers between January and March.

Question: For which service did you come in today?

Conclusion: The salon mostly does coloring and highlighting.

Results	
Choice	**Response**
haircut	294
styling	185
coloring/highlights	349
perm	153
combination of services	109

15. **FOOD** The freshman class decided to have a picnic on National Sandwich Day. They took a survey of all freshmen to decide which sandwich to serve.

Question: Which sandwich would you eat on National Sandwich Day?

Conclusion: They should serve peanut butter and jelly sandwiches.

Results	
Choice	**Response**
grilled cheese	10.6%
reuben	17.4%
hamburger	18.2%
hot dog	16.3%
peanut butter and jelly	37.5%

16. **NEWSPAPERS** To determine the popularity of the horoscope section, a newspaper sent at random a survey to 1000 of its subscribers.

Question: How often do you read your horoscope?

Results: every day, 9.9%; most days, 9.7%; not very often, 39.1%; never, 41.3%

Conclusion: The paper should eliminate the horoscope.

17. **MUSIC** A music store wanted to know where people hear about new music.

Question: How do you hear about new music being released?

Results: radio, 27%; TV, 24%; magazines, 19%; friends, 25%; the Internet, 5%

Conclusion: People hear about new music from a variety of sources.

18. **DRIVING** *The Canton Repository* polled 750 people.

Question: Have you ever talked on a cell phone while driving a car?

Results: never, 20.7%; a few times, 48.7%; not anymore, 5.1%; always, 25.5%

Conclusion: The people of Canton are careless drivers.

19. **ENTERTAINMENT** A national survey of students in first through twelfth grades was published in *Scholastic Magazine*. They received 5564 total votes.

Question: What type of TV shows and movies do you watch most?

Results: action, 1329; cartoons, 1115; comedy, 1423; drama, 358; horror, 332; music video, 1007

Conclusion: Students prefer comedies to other types of shows.

20. **BUSINESS** A poll was conducted by Junior Achievement.

Question: Is starting a business challenging?

Results: very easy, 2.5%; easy, 8.9%; somewhat challenging, 43.4%; difficult but possible, 44.1%; almost impossible, 1.1%

Conclusion: Students are aware of the challenges in starting a business.

21. **READING** A survey conducted by Smart Girl asked students why they read.

Results: just for fun, 25%; to learn new things, 24%; because they have to for school, 18%; because they get bored and have nothing else to do, 17%; their friends like to read and talk about books, 16%

Conclusion: Students read for a variety of reasons.

Example 3
p. 749

Determine whether each display gives an accurate picture of the survey results.

22. MAGAZINES The school library is ordering magazines. To make sure that they get ones that students will read, the librarian sends out a survey to 100 randomly chosen students from each grade.

Question: What type of magazine do you enjoy reading?

Conclusion: The library should order fashion, sports, music, and game magazines.
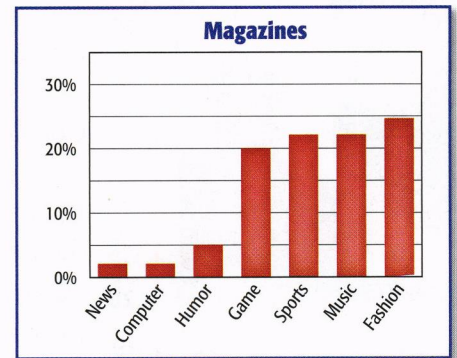
Magazines

23 ENVIRONMENT A survey of 10,000 randomly chosen subscribers to an environmental magazine was conducted.

Question: What will be the biggest environmental challenge in the 21st century?
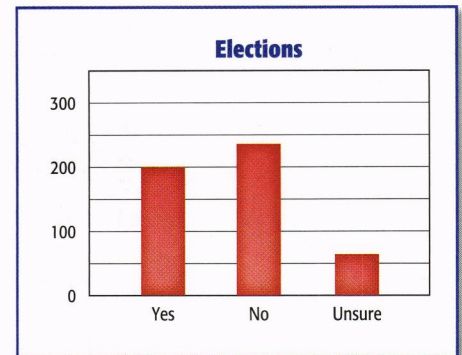
Conclusion: Finding places to put garbage is unimportant.

Environmental Challenge
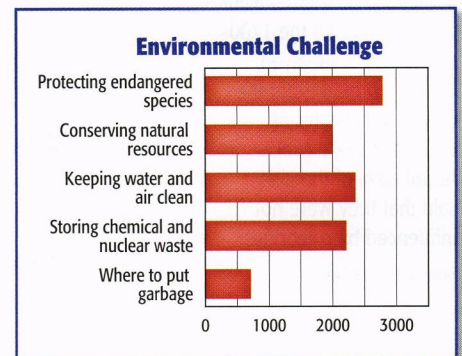
24. ELECTIONS A local elections board polled 498 high school seniors.

Question: Will elections ever be held online?

Conclusion: More seniors feel that elections will never be held online.

Elections

🌐 Real-World Link

First-time voters have become a focal point of political parties and interest groups. The number of new voters aged 18 to 24 has increased in recent years.

25. SAFETY TRAINING A chapter of the Red Cross offers classes designed for kids and teens in safety techniques. Of the participants, 74% take water safety class, 10% take babysitting classes, and 16% take first aid.
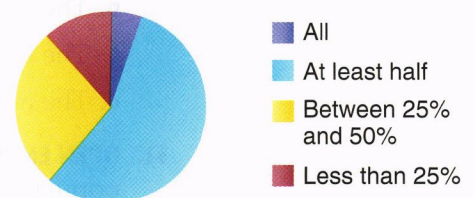
Question: Should the Red Cross continue babysitting classes? Write a valid conclusion using data to support your answer.

26. BRACES A journal for dentists conducted a nationwide survey of 5000 dentists. Does this graph accurately represent the data and conclusion? Justify your answer.

Question: What percent of patients do you refer to an orthodontist?

Percentage of Patients Referred

- All
- At least half
- Between 25% and 50%
- Less than 25%

Conclusion: Many dentists refer more than half of their patients to orthodontists.

27. CELEBRITIES A celebrity gossip magazine conducted a survey of their subscribers.

Question: Are you influenced by celebrities?

Conclusion: Their subscribers are not influenced by celebrities.

Does this graph accurately represent the data and conclusion? Justify your answer.

Influenced by Celebrities

- very influenced
- influenced
- slightly influenced
- not influenced

5%
12%
27%
56%
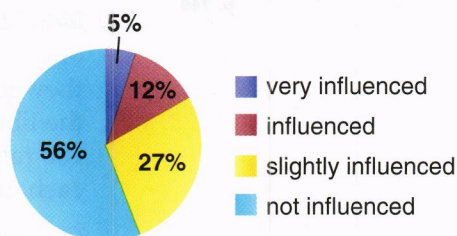
28. ⬧ **MULTIPLE REPRESENTATIONS** In this problem, you will explore another way to analyze data.

a. **CONCRETE** Below is a distribution of coins in unequal stacks. Duplicate these stacks with coins.

2   3   3   4   5   6   6   8   8

b. **GRAPHICAL** Make a line plot of the stacks. Above each of these columns, record how much each stack differs from the mean (the number of coins per stack). Find the absolute value of each of these values.

c. **ANALYTICAL** Move the coins one at a time to make the stacks equal. Avoid unnecessary moves. Count the moves. The number of moves tells us how much the original set of stacks differs from the set of equal stacks.

d. **ANALYTICAL** Find the mean of the absolute values. Describe what this value is and what it means in these circumstances.

H.O.T. Problems   Use **H**igher-**O**rder **T**hinking Skills

29. FIND THE ERROR Pepita and Ben are asked to decide which measure of central tendency to use given the data of test scores 84, 82, 80, 32, 87, 83, 85. Is either of them correct? Explain your reasoning.

Pepita

The mean is the best measure of central tendency because the data are clumped together, and there are no repeated values.

Ben

The median is the best measure of central tendency because the data are clumped together, but there is an outlier, and there are no repeated values.

30. CHALLENGE Find a set of numbers that satisfies each list of conditions.

a. The mean, median, and mode are all the same number.

b. The mean is greater than the median.

c. The mode is 10 and the median is greater then the mean.

d. The mean is 6, the median is 5.5, and the mode is 9.

31. OPEN ENDED Describe a survey you would like to conduct. Include the sample, population, method of questioning, and how you would display the results.

32. WRITING IN MATH Explain why a company may display survey results inaccurately. Give one example of how they might accomplish this.

33. At the county fair, 1000 tickets were sold. Adult tickets cost $8.50, children's tickets cost $4.50, and a total of $7300 was collected. How many children's tickets were sold?

  A 700   **C** 400

  B 600   **D** 300

34. Edward has 20 dimes and nickels, which together total $1.40. How many nickels does he have?

  F 12   **H** 8

  G 10   **J** 6

35. If 4.5 kilometers is about 2.8 miles, about how many miles is 6.1 kilometers?

  A 3.2 miles   **C** 3.8 miles

  B 3.6 miles   **D** 4.0 miles

36. EXTENDED RESPONSE Three times the width of a certain rectangle exceeds twice its length by three inches, and four times its length is twelve more than its perimeter.

  a. Translate the sentences into equations.

  b. Find the dimensions of the rectangle.

  c. What is the area of the rectangle?

Spiral Review

Identify each sample and suggest a population from which it is selected. State whether the sample is *biased* or *unbiased*. If unbiased, classify the sample as *simple, stratified,* or *systematic.* (Lesson 12-1)

37. SCHOOL Twenty names were drawn from a container containing identical pieces of paper with the names of every member of the senior class. These seniors were then asked who they would choose for senior class president.

38. BOOKS To check the quality of the books being manufactured, an inspector checks every 50th book that comes off the line.

39. COMPUTERS Mayfield High School participated in a survey to find out how teens feel about certain issues involving social networks. The responses are divided into group by each age and then tallied for each question.
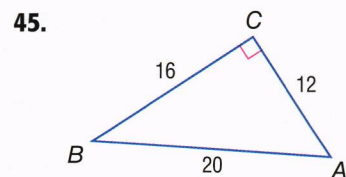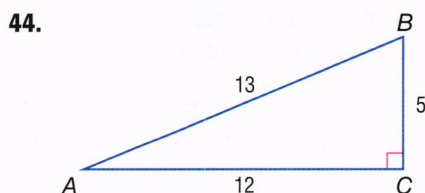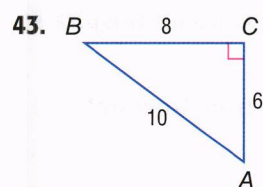
Find the zeros of each function. (Lesson 11-8)

40. $f(x) = \dfrac{x^2 - 8x + 15}{x^2 + 5x - 6}$

41. $f(x) = \dfrac{x^2 - x - 12}{x^2 - 6x + 8}$

42. $f(x) = \dfrac{x^2 - x - 30}{x^2 - 3x - 18}$

Find the values of the three trigonometric ratios for angle *A*. (Lesson 10-8)

43.

44.

45.

Skills Review

Find the mean, median, and mode to the nearest tenth for each set of data. (Lesson 0-12)

46. 100, 105, 100, 105, 100, 110

47. 12, 25, 14, 35, 42, 27, 31, 48

48. 90, 85, 92, 99, 78, 82, 92, 90

49. 1, 5, 3, 7, 6, 2, 9, 2, 5, 1, 9, 1

50. 55, 65, 45, 35, 65, 25, 85

51. 25, 28, 21, 26, 25, 27, 29, 30

Statistics and Parameters

Why?

At the start of every class period for one week, each of Mr. Day's algebra students randomly draws 9 pennies from a jar of 1000 pennies. Each student calculates the mean age of the random sample of pennies drawn and then returns the pennies to the jar.

How does the mean age for 9 pennies compare to the mean age of all 1000 pennies?
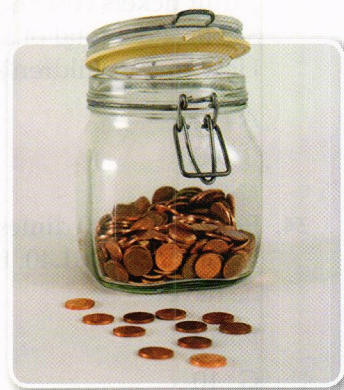
Statistics and Parameters In this situation, the statistics of a sample are used to draw conclusions about the entire population. This is called **statistical inference**.

In the scenario above, each student takes a random sample of pennies from the jar. The jar of 1000 pennies represents the population. A **statistic** is a measure that describes a characteristic of a sample. A **parameter** is a measure that describes a characteristic of a population. Parameters are usually estimated values based on the statistics of a carefully chosen random sample. A statistic can and usually will vary from sample to sample. A parameter will not change, for it represents the entire population.

EXAMPLE 1 Identify Statistics and Parameters

Identify the sample and the population for each situation. Then describe the sample statistic and the population parameter.

a. At a local university, a random sample of 40 scholarship applicants is selected. The mean grade-point average of the 40 applicants is calculated.

Sample: the group of 40 scholarship applicants
Population: all applicants
Sample statistic: mean grade-point average of the sample
Population parameter: mean grade-point average of all applicants

b. A stratified random sample of registered nurses is selected from all hospitals in a three-county area, and the median salary is calculated.

Sample: randomly selected registered nurses from hospitals in three-county area
Population: all nurses at the hospitals in the same region
Sample statistic: median salary of nurses in the sample
Population parameter: median salary of all nurses in sampled hospitals

✓ **Check Your Progress**

1. CEREAL Starting with a randomly selected box of Co-Co-Chunks from the manufacturing line, every 50th box of cereal is removed and weighed. The mode weight of a day's sample is calculated.

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Statistical Analysis Data that involve only one variable are called **univariate data**. This kind of data can be represented by measures of central tendency, such as the mean, median, and mode. Univariate data can also be represented by **measures of variation**, such as range, quartiles, and interquartile range.

|	**Concept Summary**	**Measures of Variation**	**For Your FOLDABLE**
Type	**Description**	**When Best Used**	
range	the difference between the greatest and least values	to describe which numbers are included in the data set	
quartile	the values that divide the data set into four equal parts	to determine values in the upper or lower portions of a data set	
interquartile range	the range of the middle half of a data set; the difference between the upper and lower quartiles	to determine what values lie in the middle half of the data set	

The **mean absolute deviation** is the average of the absolute values of the differences between the mean and each value in the data set. Recall that absolute value is the distance from a number to zero on a number line.

Key Concept	**Mean Absolute Deviation**	**For Your FOLDABLE**

Step 1 Find the mean.

Step 2 Find the sum of the absolute values of the differences between each value in the set of data and the mean.

Step 3 Divide the sum by the number of values in the set of data.
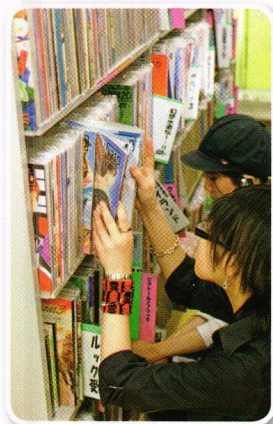
EXAMPLE 2 **Use the Mean Absolute Deviation**

MARKETING Each person that visited the Comic Book Shoppe's web site was asked to enter the number of times each month they buy a comic book. They received the following responses in one day: 2, 2, 3, 4, 14. Find the mean absolute deviation to the nearest tenth.

Step 1 The mean of this set of data is 5.

Step 2 Find the sum of the absolute values of the differences between each value and the mean.
$|2 - 5| + |2 - 5| + |3 - 5| + |4 - 5| + |14 - 5| = 3 + 3 + 2 + 1 + 9$ or 18

Step 3 Divide the sum by the number of values: $18 \div 5 = 3.6$.

☑ **Check Your Progress**

2. **DANCES** The prom committee kept count of how many tickets it sold each day during lunch: 12, 32, 36, 41, 22, 47, 51, 33, 37, 49. Find the mean absolute deviation of these data.

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The **standard deviation** is a calculated value that shows how the data deviates from the mean of the data. The standard deviation is represented by the lower-case Greek symbol sigma, σ. The **variance** of the data is the square of the standard deviation. Use the method below to calculate the variance and standard deviation.

Key Concept   Variance and Standard Deviation   For Your FOLDABLE

Step 1 Find the mean, $\bar{x}$.

Step 2 Find the square of the difference between each value in the set of data and the mean. Then sum the squares and divide by the number of values in the set of data. The result is the variance.

Step 3 Take the square root of the variance to find the standard deviation.

EXAMPLE 3   **Find the Variance and Standard Deviation**

Find the mean, variance, and standard deviation of 3, 6, 11, 12, and 13 to the nearest tenth.

Step 1 To find the mean, add the numbers and then divide by the number of values in the data set.

$$\bar{x} = \frac{3 + 6 + 11 + 12 + 13}{5} = \frac{45}{5} \text{ or } 9$$

Step 2 To find the variance, square the difference between each number and the mean. Then add the squares, and divide by the number of values.

$$\sigma^2 = \frac{(3-9)^2 + (6-9)^2 + (11-9)^2 + (12-9)^2 + (13-9)^2}{5}$$

$$= \frac{(-6)^2 + (-3)^2 + 2^2 + 3^2 + 4^2}{5}$$

$$= \frac{36 + 9 + 4 + 9 + 16}{5} \text{ or } \frac{74}{5}$$

Step 3 The standard deviation is the square root of the variance.

$$\sigma^2 = \frac{74}{5} \qquad \text{Variance}$$

$$\sqrt{\sigma^2} = \sqrt{\frac{74}{5}} \qquad \text{Take the square root of the variance.}$$

$$\sigma \approx 3.8 \qquad \text{Use a calculator.}$$

The mean is 9, the variance is $\frac{74}{5}$ or 14.8, and the standard deviation is about 3.8.

✔ **Check Your Progress**

Find the mean, variance, and standard deviation of each set of data to the nearest tenth.

3A. 6, 10, 15, 11, 8   **3B.** 92, 84, 71, 83, 100

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The standard deviation illustrates the spread of a set of data. For example, when the mean is 75 and the standard deviation is 3, we know that almost all of the data values are very close to the mean. When the mean is 75 and the standard deviation is 15, then the data are more spread out and there may be an outlier.

🌐 **Real-World EXAMPLE 4** **Statistical Analysis**

NUTRITION Caleb kept track of the Calories he ate each day. Find the standard deviation of the data set.

Day	Sun	Mon	Tues	Wed	Thurs	Fri	Sat
Calories	1800	2000	2100	2250	1900	2500	2000

Use a graphing calculator to find the standard deviation. Clear all lists. Press [STAT] [ENTER], and enter each data value into L1, pressing [ENTER] after each value. To view the statistics, press [STAT] [▶] 1 [ENTER]. So, the standard deviation is about 216.9.

```
1-Var Stats
 x̄=2078.571429
 Σx=14550
 Σx²=30572500
 Sx=234.2668401
 σx=216.8889493
↓n=7
```
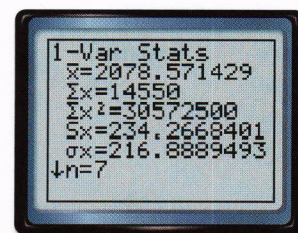
✅ **Check Your Progress**

4. Caleb tracked his Calorie intake for another week: 1950, 2000, 2100, 2000, 1900, 2100, 2000. Find the standard deviation of his Calorie intake for this week.

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✅ Check Your Understanding

Example 1
p. 756

Identify the sample and the population for each situation. Then describe the sample statistic and the population parameter.

1. **POLITICS** A random sample of 1003 Mercy County voters is asked if they would vote for the incumbent for governor. The percent responding *yes* is calculated.

2. **BOOKS** A random sample of 1000 U.S. college students is surveyed about how much they spend on books per year.

Example 2
p. 757

Find the mean absolute deviation to the nearest tenth.

3. **FINANCIAL LITERACY** Iye is waiting tables at the Pizza Pan Restaurant. He is keeping track of the tips that he receives each hour: $20, $31, $24, $22, $35, $12.

4. **PARTIES** Dalila kept an account of what each cousin gave toward their grandmother's birthday party: $25, $24, $36, $28, $34, $25, $17.

Example 3
p. 758

Find the mean, variance, and standard deviation of each set of data to the nearest tenth.

5. 3, 4, 18, 21, 17

6. 12, 15, 18, 21

Example 4
p. 759

7. **ELECTRONICS** Ed surveyed his classmates to find out how many electronic gadgets each person has in their home. Find the standard deviation of the data set to the nearest tenth: 3, 10, 11, 10, 9, 11, 12, 8, 11, 8, 7, 12, 11, 11, 5.

Practice and Problem Solving

● = **Step-by-Step Solutions** begin on page R12.
Extra Practice begins on page 815.

Example 1
p. 756

Identify the sample and the population for each situation. Then describe the sample statistic and the population parameter.

8. A stratified random sample of high school students from each school in the county was polled about the time spent each week on extracurricular activities.

9. A stratified random sample of 2500 high school students across the country was asked how much money they spent each month.

Example 2
p. 757

Find the mean absolute deviation to the nearest tenth.

10. **DVDS** Mr. Robinson asked his students to count the number of DVDs they owned.

Number of DVDs					
26	39	5	82	12	14
0	3	15	19	41	6
2	0	11	1	19	29

11. **SALES** An amusement park manager wanted to keep track of how many bags of cotton candy were sold each hour: 16, 24, 15, 17, 22, 16, 18, 24, 17, 13, 25, 21.

Example 3
p. 758

Find the mean, variance, and standard deviation of each set of data to the nearest tenth.

12. 3, 8, 7, 12

13. 76, 78, 83, 74, 75

14. 0.01, 0.03, 0.1, 0.5

15. 0.8, 0.01, 0.06, 0.02, 0.4, 0.8, 0.5

Example 4
p. 759

16. **ONLINE AUCTIONS** Scott makes keychains and sells them on an online auction site. He tracks the selling price of each keychain: $3.25, $4.50, $5.00, $5.75, $2.25, $8.50, $6.00, $3.50, $4.50, $5.00. Find the standard deviation to the nearest tenth.

17. **PART-TIME JOBS** Ms. Johnson asks all of the girls on the tennis team how many hours each week they work at part-time jobs: 10, 12, 0, 6, 9, 15, 12, 10, 11, 20. Find the standard deviation to the nearest tenth.

18. **BASKETBALL** The heights of players on an NBA team are shown.

a. Find the mean and standard deviation.

Heights of Professional Basketball Players (in.)				
80	77	83	74	78
80	83	74	83	69
78	85	81	81	79

b. Suppose the 5'9" player was traded for Earl Boykins. Find the mean and the standard deviation. Describe the effect this trade has on the calculations.

c. Performing one or more operations on a data set transforms the data. If the operation can be written as a linear function, the transformation is a ==linear transformation==. Convert each measure to centimeters, and then find the mean. Next, convert the mean to inches. How does this calculation compare to the mean found in part **a**?

19. **PENNIES** Mr. Day has another jar of pennies on his desk. There are 30 pennies in this jar. Theo looks at 5 pennies from the jar and replaces them. Lydia looks at 10 pennies and replaces them, and Peter looks at 20 pennies and replaces them.

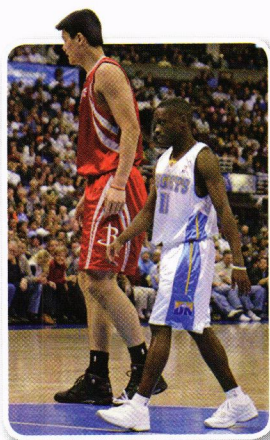a. Identify the sample and the population for each situation. Then describe a statistic and a parameter.

Years of Pennies in Jar					
2001	1990	2000	1982	1991	1975
2007	1981	2005	2007	2003	2005
1997	1974	1992	1994	1991	1992
2000	1995	1999	2005	2006	2005
2004	2004	1998	2001	2002	2006

b. The years of Theo's pennies are 1974, 1975, 1981, 1999, 1992. Find the mean and mean absolute deviation.

c. The years of Lydia's pennies are 2004, 1999, 2004, 2005, 1991, 2003, 2005, 2000, 2001, 1998. Find the mean and mean absolute deviation.

d. The years of Peter's pennies are 2007, 2005, 1975, 2003, 2005, 1997, 1992, 1994, 1991, 1992, 2000, 1999, 2005, 1982, 2005, 2004, 1998, 2001, 2002, 2006. Find the mean and mean absolute deviation.

e. Find the mean and mean absolute deviation for all of the pennies in the jar. Which sample was more similar to the full population? Explain.
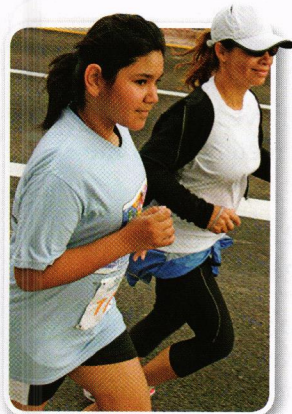
20. BABYSITTING Samantha wants to see if she is getting a fair wage for babysitting at $8.50 per hour. She takes a survey of her friends to see what they charge per hour. The results are $8, $8.50, $9, $7.50, $10, $8.25, $8.75. Find the mean absolute deviation of the data to the nearest tenth.

21. RUNNING The results of a 5K race are published in a local paper. Over a thousand people participated, but only the times of the top 15 finishers are listed.

15th Annual 5K Road Race					
Place	**Time (min:s)**	**Place**	**Time (min:s)**	**Place**	**Time (min:s)**
1	15:56	6	16:34	11	17:14
2	16:06	7	16:41	12	17:46
3	16:11	8	16:54	13	17:56
4	16:21	9	17:00	14	17:57
5	16:26	10	17:03	15	18:03

a. Find the mean and mean absolute deviation of the top 15 running times to the nearest tenth. (*Hint:* Convert each time to seconds.)

b. Identify the sample and population.

c. Analyze the sample. Classify the data as *qualitative* or *quantitative*. Can a statistical analysis of the sample be applied to the population? Explain.

H.O.T. Problems   Use Higher-Order Thinking Skills

22. FIND THE ERROR Amy and Esteban are describing one way to increase the accuracy of a survey. Is either of them correct? Explain your reasoning.

> **Amy**
> The survey should include as many people in the population as possible.

> **Esteban**
> The sample for the survey should be chosen randomly. Several random samples should be taken.

23. CHALLENGE Find the mean and standard deviation of the population of data represented by the histogram.

24. REASONING Determine whether the following statement is *sometimes, always,* or *never* true. Explain. *Two random samples taken from the same population will have the same mean and standard deviation.*
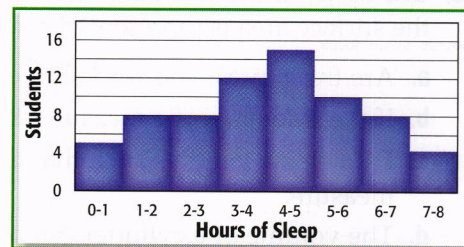
25. WRITING IN MATH Compare and contrast statistics and parameters. Include an example of each.

26. OPEN ENDED Describe a real-world situation in which it would be useful to use a sample mean to help estimate a population mean. Describe how you could collect a random sample from this population.

27. WRITING IN MATH Compare and contrast standard deviation and mean absolute deviation.

28. Melina bought a shirt that was marked 20% off for $15.75. What was the original price?

   A $16.69   **C** $18.69

   B $17.69   **D** $19.69

29. SHORT RESPONSE A group of students visited the Capitol building. Twenty students met with the local representative. This was 16% of the students. How many student ambassadors were there altogether?

30. The tallest 7 trees in a park have heights of 19, 24, 17, 26, 24, 20, and 18 meters. Find the median of their heights.

   F 17   **H** 21

   G 20   **J** 24

31. It takes 3 hours for a boat to travel 27 miles upstream. The same boat travels 30 miles downstream in 2 hours. Find the speed of the boat.

   A 12 mph   **C** 3 mph

   B 14 mph   **D** 5 mph

Spiral Review

Tell which measure of central tendency best represents the data. Justify your answer. Then find the measure. (Lesson 12-2)

32. FOOD DRIVE A high school is offering an incentive to classes that bring in the canned goods. The pounds of food brought by the classes are represented by the data set 8, 12, 18, 25, 21, 5, 10, and 14.

33. TEST SCORES The results of the math test are 78, 81, 85, 86, 88, 85, 90, 91, 85, 95, and 98.

Identify each sample as *biased* or *unbiased*. Explain your reasoning. (Lesson 12-1)

34. SHOPPING Every tenth person walking into the mall is asked to name their favorite store.

35. MUSIC Every fifth person at a rock concert is asked to name their favorite radio station.

36. GEOMETRY If the side length of a cube is s, the volume is represented by s^3, and the surface area is represented by $6s^2$. (Lesson 7–1)

   a. Are the expressions for volume and surface area monomials? Explain.

   b. If the side of a cube measures 3 feet, find the volume and surface area.

   c. Find a side length s such that the volume and surface area have the same measure.

   d. The volume of a cylinder can be found by $V = \pi r^2 h$. Suppose you have two cylinders. Each dimension of the second is twice the measure of the first, so $V = \pi(2r)^2(2h)$. What is the ratio of the volume of the first cylinder to the second?
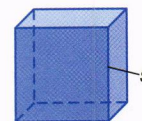
Skills Review

A bowl contains 3 red chips, 6 green chips, 5 yellow chips, and 8 orange chips. A chip is drawn randomly. Find each probability. (Lesson 0–11)

37. red

38. orange

39. yellow or green

40. not orange

41. not green

42. red or orange

Identify each sample, and suggest a population from which it was selected. Then classify the type of data collection used. (Lesson 12-1)

1. **CEREAL** A cereal company invites 100 children and parents to test a new cereal.

2. **SCHOOL LUNCH** A school is creating a new lunch menu. They send out a questionnaire to all students with odd homeroom numbers to see what items should be on the new menu.

3. **MEDICINE** A research facility gave a new medicine to hamsters and determined that 1 out of every 50 hamsters that took the medicine lost its hair. They conclude that the same thing will happen to every 50 people who take the medicine.

4. **MASCOTS** The cheerleaders send out a flyer with pictures of options for the new mascot to all the girls in the school. The new mascot is chosen from the favorite from the survey.

Identify each sample as *biased* or *unbiased*. Explain your reasoning. (Lesson 12-1)

5. **ART** Every fifth person leaving the art museum is asked to name their favorite piece.

6. **SHOPPING** Each person leaving the Earring Pagoda is asked to name their favorite store in the mall.

7. **FOOTBALL** Every 10th student leaving the student union at Ohio State is asked to name their favorite college football team.

8. **CLASSES** Every 5th person leaving the school is asked to name their favorite class.

9. **MULTIPLE CHOICE** Every 10 minutes, Kaleigh writes down whether the TV is showing a commercial or a program. Which of the following best describes the sample? (Lesson 12-1)

   A simple
   B stratified
   C systematic
   D none of the above

Which measure of central tendency best represents the data? Justify your answer. Then find the measure. (Lesson 12-2)

10. **PLAY AREA** Ian listed the ages of the children playing at the play area at the mall.
    2, 3, 2, 2, 4, 2, 3, 2, 8, 3, 4, 2

11. **RECYCLING** Marielle is in charge of recycling cans at her school. She counts the number of cans recycled each week.
    22, 10, 23, 25, 24, 23, 25, 19

12. Does the following give an accurate picture of the survey results? (Lesson 12-2)

    A survey of 500 students was conducted.
    Question: What is the most important aspect of school?
    Conclusion: Preparing for the future is not important at all.
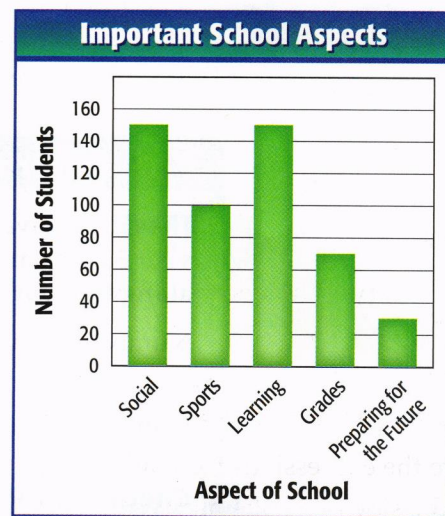
Important School Aspects

Find the mean, variance, and standard deviation to the nearest tenth for each set of data. (Lesson 12-3)

13. 2, 4, 5, 7, 7, 9   14. 13, 14, 18, 21

15. **MULTIPLE CHOICE** Several friends are chipping in to buy a gift for their teacher. Ignati is keeping track of how much each friend gives for the gift. Find the mean absolute deviation. (Lesson 12-3)

    $10, $5, $3, $6, $7, $8

    F 2.22   H 1.833
    G 6.5   J 2.4

Permutations and Combinations

Why?

Angie's coach told her that she would bat sixth in the softball game. When a coach decides on the team's lineup, the order in which she fills in the names determines the order in which the players will bat.

Permutations The list of all of the people or objects in a group is called the **sample space**. When the objects are arranged so that order is important and every possible order of the objects is provided, the arrangement is called a **permutation**.

Suppose Angie's coach has 4 players in mind for the first 4 spots in the lineup. The Fundamental Counting Principle can be used to determine the number of permutations. A batter cannot bat first and second, so once that player is chosen, she is not available for the next choice.

number of permutations		choices for 1st batter		choices for 2nd batter		choices for 3rd batter		choices for 4th batter
P	$=$	4	$\cdot$	3	$\cdot$	2	$\cdot$	1
	$= 24$							

There are 24 different ways to arrange the first four batters.

🌐 Real-World EXAMPLE 1 Permutation

TRAVEL A travel agency is planning a vacation package in which travelers will visit 5 cities around Europe. How many ways can the agency arrange the 5 cities along the tour?

Number of ways to arrange the cities $= 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$
$ = 120$

There are 120 ways to arrange the cities.

✔ Check Your Progress

1. **MOVIES** Lloyd and five friends go to a movie. In how many different ways can they sit together in a row of 6 empty seats?

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The expression used in Example 1 to calculate the number of permutations of the five cities, $5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$, can be written as 5!, which is read *5 factorial*.

🔷 Key Concept Factorial
For Your FOLDABLE

Words	The **factorial** of a positive integer n is the product of the positive integers less than or equal to n.
Symbols	$n! = n \cdot (n-1) \cdot (n-2) \cdot \ldots \cdot 1$, where $n \geq 1$. Also, $0! = 1$.

Suppose Angie's coach has 5 players in mind for the top 3 spots in the lineup. The Fundamental Counting Principle can be used to determine the number of permutations.

choices for 1st batter		choices for 2nd batter		choices for 3rd batter		
5	·	4	·	3	=	60 permutations

Notice that $5 \cdot 4 \cdot 3$ is the same as $\dfrac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1}$. This relationship is expressed in the following formula.

Key Concept — Permutation Formula

For Your **FOLDABLE**

Words   The number of permutations of n objects taken r at a time is the quotient of $n!$ and $(n - r)!$.

Symbols $P(n, r) = \dfrac{n!}{(n - r)!}$

Real-World EXAMPLE 2   Use the Permutation Formula

LIBRARY The librarian is placing 6 of 10 magazines on a shelf in a showcase. How many ways can she arrange the magazines in the case?

$P(n, r) = \dfrac{n!}{(n - r)!}$ Permutation Formula

$P(10, 6) = \dfrac{10!}{(10 - 6)!}$ $n = 10$ and $r = 6$

$= \dfrac{10!}{4!}$ Simplify.

$= \dfrac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}{\cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}$ Divide by common factors.

$= 151{,}200$ Simplify.

There are 151,200 ways for the librarian to arrange the magazines.

✓ Check Your Progress

2. FASHION A designer has created 15 outfits and needs to select 10 for a fashion show. How many ways can the designer arrange the outfits for the show?

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Combinations A selection of objects in which order is not important is called a **combination**. To find all two-letter combinations of *A*, *B*, and *C*, you would list all of the arrangements of two letters, which are listed below.

AB	*BA*	*AC*	*CA*	*BC*	*CB*

Because order does not matter, *AB* and *BA* are the same, so there are 2! ways to write the same letters. Divide $P(n, r)$ by 2! to remove the groups with identical objects. So there are 3 two-letter combinations.
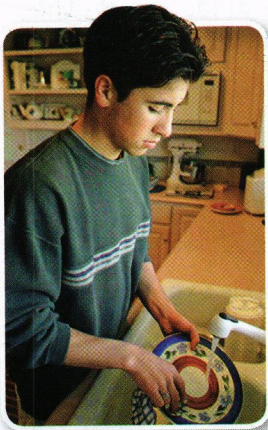
Real-World EXAMPLE 3 — Combination

FAMILY Horacio has 2 brothers and 2 sisters. Their parents assign chores to them each week. How many ways can two children be chosen to wash the dishes?

Since the order in which the children are chosen does not matter, we need to find the number of combinations of 5 children taken 2 at a time.

$$C(n, r) = \frac{\text{number of permutations}}{\text{number of permutations with identical objects}}$$

First find the number of permutations.

$$P(5, 2) = \frac{5!}{3!} \text{ or } 20 \qquad n = 5 \text{ and } r = 2$$

Because we are choosing 2, there are $2! = 2$ permutations with identical objects.

$$C(n, r) = \frac{20}{2} \text{ or } 10$$

There are 10 possible ways to choose 2 children.

✓ Check Your Progress

3. TEST Louis is given the option of answering any 10 out of the 12 questions on his history test. How many ways can he complete the test?

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We can now state a formula for combinations.

Key Concept — Combination Formula

For Your FOLDABLE

Words   The number of combinations of n objects taken r at a time is the quotient of $n!$ and $(n - r)!r!$.

Symbols $C(n, r) = \dfrac{n!}{(n - r)!r!}$

Real-World EXAMPLE 4 — Use the Combination Formula

RETAIL Marques works part-time at a local department store. His manager asked him to choose for display 5 different styles of shirts from the wall of the store that has 8 shirts on it to put in a display. How many ways can Marques choose the shirts?

$$C(n, r) = \frac{n!}{(n - r)!r!} \qquad \text{Combination Formula}$$

$$= \frac{8!}{(8 - 5)!5!} \qquad n = 8 \text{ and } r = 5$$

$$= \frac{8!}{3!5!} \qquad \text{Simplify.}$$

$$= \frac{8 \cdot 7 \cdot 6 \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}{3 \cdot 2 \cdot 1 \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}} \qquad \text{Divide by common factors.}$$

$$= \frac{336}{6} \text{ or } 56 \qquad \text{There are 56 ways for Marques to select 5 shirts.}$$

✓ Check Your Progress

4. SPRING DANCE A group of four students is selecting corsages and boutonnières to wear to the spring dance. They can choose from 18 different flowers which consist of 4 roses, 6 carnations, and 8 tulips. In how many ways can 4 flowers be chosen to wear?

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We can use permutations or combinations to find the probability of an event.

🌐 Real-World EXAMPLE 5   Probability Using a Permutation

BICYCLES A combination lock requires a three-digit code made up of the digits 0 through 9. No number can be used more than once.

a. How many different arrangements are possible?

Since the order of the numbers in the code is important, this is a permutation of 10 digits taken 3 at a time.

$$P(n, r) = \frac{n!}{(n - r)!}$$ **Permutation Formula**

$$= \frac{10!}{(10 - 3)!}$$ **$n = 10$ and $r = 3$**

$$= \frac{10!}{7!}$$ **Simplify.**

$$= \frac{10 \cdot 9 \cdot 8 \cdot \cancel{7} \cdot \cancel{6} \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}{\cancel{7} \cdot \cancel{6} \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}$$ **Divide by common factors.**

$$= 720$$ **Simplify.**

There are 720 possible codes.

b. What is the probability that all of the digits are odd?

Use the Fundamental Counting Principle to determine the number of ways for the three digits to be odd.

- There are five odd digits: 1, 3, 5, 7, and 9.
- The number of choices for the three digits, if they are odd, is $5 \cdot 4 \cdot 3$. So, the number of favorable outcomes is 60.

$$P(\text{all digits odd}) = \frac{60}{720} \quad \longleftarrow \text{ number of favorable outcomes}$$
$$\longleftarrow \text{ number of possible outcomes}$$

$$= \frac{1}{12} \quad \textbf{Simplify.}$$

The probability that all of the digits are odd numbers is $\frac{1}{12}$ or about 8%.

✓ Check Your Progress

SPANISH CLUB The Spanish club is electing a president, vice president, secretary, and treasurer. Rebekah and Lydia are among the nine students who are running.

5A. How many ways can the Spanish club choose their officers?

5B. Assuming that the positions are chosen at random, what is the probability that either Rebekah or Lydia will be chosen as president or vice president?

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✓ Check Your Understanding

Example 1
p. 764

1. CHARITY A youth charity group is holding a raffle and wants to display a picture of the 6 prizes on a flyer. How many ways can they arrange the prizes in a row?

Examples 2–4
pp. 765–766

Evaluate each expression.

2. $P(7, 2)$ **③** $P(9, 3)$ **4.** $C(6, 4)$ **5.** $C(5, 2)$

6. RECYCLING Juana is setting recycling bins out for pick-up. She has one bin each for glass, plastic, paper and aluminum. How many ways can she arrange the bins in a row?

7. **FOOD** Linda is preparing to bake a cake. She gets the ingredients out and sets them on the counter top. Six of the 14 ingredients are spices. How many ways can she arrange the spices in a row on the countertop?

8. **ICE CREAM** The Dairy Barn offers 5 varieties of chocolate ice cream, 4 varieties of candy-flavored ice cream, and 6 varieties of berry-flavored ice cream.

    a. In how many ways can a customer choose 3 different flavors of ice cream?

    b. Does the selection involve a *permutation* or a *combination*?

    c. If the ice cream flavors are chosen randomly, what is the probability that a customer will select all chocolate?

Practice and Problem Solving

9 **PHOTOGRAPHY** The four captains of the football team are being arranged in a row for a newspaper photograph. How many ways can the photographer arrange the players for the photograph?

10. **SCIENCE FAIR** There are 8 finalists in a science fair competition. How many ways can they stand in a row on the stage?

11. **AMUSEMENT PARKS** Tino is entering an amusement park with 5 of his friends. At the gate they must go through a turnstile one by one. How many ways can Tino and his friends go in?

12. **JOBS** At a fast food restaurant there are 4 employees that are capable of running the cash registers. How many ways can the manager arrange the employees at the four front counter registers?

Evaluate each expression.

13. $P(6, 6)$

14. $P(5, 1)$

15. $P(4, 1)$

16. $P(7, 3)$

17. $C(7, 6)$

18. $C(5, 3)$

19. $C(5, 5)$

20. $C(3, 0)$

21. **DANCE** At the spring dance, Christy and 7 of her friends sit on one side of a table. How many ways can they fill the 10 empty seats?

22. **JEWELRY** Jewel works at the jewelry store in the mall. Her manager asks her to place 3 of the 12 birthstone necklaces in the front display case. How many ways can she arrange the necklaces in the display case?

23. **MARBLES** Fifteen marbles out of 20 must be randomly selected. There are 7 red marbles, 8 purple marbles, and 5 green marbles from which to choose. What is the probability that 5 of each color are selected?

24. **SCHOOL PLAY** Westerville High school is seeking volunteers to help decorate for the winter dance. In all, 4 freshmen, 5 sophomores, 6 juniors, and 8 seniors tried out for the 12 open spots.

    a. How many ways can the 12 spots be chosen?

    b. If the students are chosen randomly, what is the probability that at least one senior will be chosen?

Determine whether each situation involves a *permutation* or a *combination*.

25. choosing 3 different pizza toppings from a list of 12

26. selecting 4 different ingredients out of 8 for a salad

27. choosing team captains for a football team

28. choosing the first-, second-, and third-place winner of an art competition

29. selecting 5 books to read from a list of 8

30. an arrangement of the letters in the word *probability*

31. **GAMES** Tonisha is playing a game in which you make words to score points. There are 12 letters in the box, and she must choose 4. She cannot see the letters.

   a. Suppose the 12 letters are all different. In how many ways can she choose 4?

   b. She chooses *A*, *T*, *R*, and *E*. How many different arrangements of three letters can she make?

   c. How many of the three-letter arrangements are words? List them.

32. **PAGEANTS** The Teen Miss USA pageant has 51 contestants. The judges choose Teen Miss USA and four runners-up.

   a. Does the selection involve a *permutation* or a *combination*? Explain.

   b. In how many ways can the judges choose Teen Miss USA and four runners-up?

33. **BASKETBALL** The coach had to select 5 out of 12 players on the team to start the game. How many different groups of players could be selected as starters?

34. **LOCKER** Christopher cannot remember the order of his locker combination. He only remembers that it contains the numbers 5, 16, and 31. What is the maximum number of attempts Christopher could make?
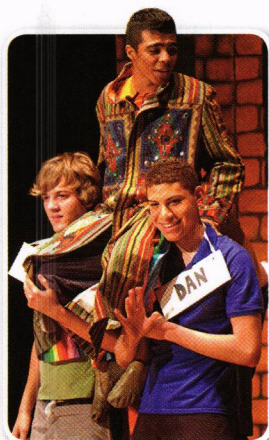
H.O.T. Problems   Use **H**igher-**O**rder **T**hinking Skills

35. **FIND THE ERROR** Sydney and Ming want to form a 4-person committee to be in charge of decorations for the dance. They are determining how many committees are possible if 10 people are available. Is either of them correct?

   Sydney
   $$P(10, 4) = \frac{10!}{(10 - 4)!}$$
   $$= 5040$$

   Ming
   $$C(10, 4) = \frac{10!}{(10 - 4)!4!}$$
   $$= 210$$

36. **CHALLENGE** Seven identical mathematics books and 4 identical science books are to be stored on one shelf. In how many different ways can the books be arranged?

37. **WHICH ONE DOESN'T BELONG?** Determine which situation does not belong. Explain.

choosing 5 players on a quiz team	choosing 10 colored marbles from a bag
choosing 4 horses from 6 to run race	ranking students in a senior class

38. **REASONING** Determine whether the statement $P(n, r) = C(n, r)$ is *sometimes*, *always*, or *never* true. Explain your reasoning.

39. **WRITING IN MATH** Write a situation in which order is not important when 3 of 8 objects are being selected.

40. A gardener wants to plant 3 different types of flowers along a path. If she is choosing from 8 different types of flowers, how many ways can the 3 flowers be planted?

   A 342   **B** 338   **C** 336   **D** 328

41. If Jack can tie 21 knots in 15 minutes, how many can he tie in 25 minutes if he continues at the same pace?

   F 18   **G** 35   **H** 36   **J** 37

42. Shante has 30 coins, quarters and dimes, that total $5.70. How many quarters does she have?

   A 12   **C** 18
   B 15   **D** 20

43. SHORT RESPONSE There are 3 red candies in a bag of 20 candies. If you draw one without looking, what is the probability of drawing a red candy?

Spiral Review

Find the mean, variance, and standard deviation of each set of data to the nearest tenth. (Lesson 12-3)

44. 76, 47, 59, 47, 72, 89

45. 20, 30, 10, 40, 20, 12, 50

46. 44, 34, 64, 74, 94, 104, 55

47. 1, 5, 9, 4, 2, 4, 8, 4, 2, 1

48. SURVEY A soda manufacturing company surveyed customers to find the number of cans of soda they drank in a week. They received the following responses: 14, 7, 3, 0, 10, 12, and 10. Which measure of central tendency best represents the data? Justify your answer. Then find the measure. (Lesson 12-2)

49. PET CARE Kendra takes care of pets while their owners are away. One week she has three dogs that all eat the same dog food at the rates shown. How many bags of food should Kendra buy for that week? (Lesson 11-6)

Max   Miles   Stormy
12 days/bag 15 days/bag 16 days/bag

Find each product. (Lesson 11-4)

50. $\dfrac{8}{x^2} \cdot \dfrac{x^4}{4x}$

51. $\dfrac{10r^3}{6n^3} \cdot \dfrac{42n^2}{35r^3}$

52. $\dfrac{10y^3z^2}{6wx^3} \cdot \dfrac{12w^2x^2}{25y^2z^4}$

53. $\dfrac{(n-1)(n+1)}{(n+1)} \cdot \dfrac{(n-4)}{(n-1)(n+4)}$

54. $\dfrac{(x-8)}{(x+8)(x-3)} \cdot \dfrac{(x+4)(x-3)}{(x-8)}$

55. $\dfrac{3a^2b}{2gh} \cdot \dfrac{24g^2h}{15ab^2}$

56. COOKING The formula $t = \dfrac{40(25 + 1.85a)}{50 - 1.85a}$ relates the time t in minutes that it takes to cook an average-size potato in an oven to the altitude a in thousands of feet. (Lesson 11-3)

  a. What is the value of t for an altitude of 4500 feet?

  b. Calculate the time it takes to cook a potato at 3500 feet and at 7000 feet. How do your cooking times compare?

Skills Review

Ten red tiles, 12 blue tiles, 8 green tiles, 4 yellow tiles, and 12 black tiles are placed in a bag and selected at random. Find each probability. (Lesson 0-11)

57. P(blue)

58. P(red)

59. P(black or yellow)

60. P(green or red)

61. P(not blue)

62. P(not green)

Probability of Compound Events

Why?

Then
You calculated simple probability.
(Lesson 0-11)

Now
- Find probabilities of independent and dependent events.
- Find probabilities of mutually exclusive events.

New Vocabulary
compound event
independent events
dependent events
mutually exclusive events

Math Online

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- Extra Examples
- Personal Tutor
- Self-Check Quiz
- Homework Help
- Math in Motion

Evita is flying from Cleveland to Honolulu. The airline reports that the flight from Cleveland to Honolulu has a 40% on-time record. The airline also reported that they lose luggage 5% of the time. What is the probability that both the flight will be on time and Evita's luggage will arrive?

Independent and Dependent Events Recall that one event, like flying from Cleveland to Honolulu, is called a *simple event*. A **compound event** is made up of two or more simple events. So, the probability that the flight will be on time and the luggage arrives is an example of a compound event. The plane being on time may not affect whether luggage is lost. These two events are called **independent events** because the outcome of one event does not affect the outcome of the other.

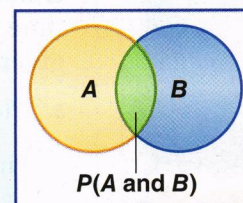> **Key Concept**   **Probability of Independent Events**   For Your **FOLDABLE**
>
> **Words**   If two events, A and B, are independent, then the probability of both events occurring is the product of the probability of A and the probability of B.
>
> **Model**
>
> $P(A \text{ and } B)$
>
> **Symbols** $P(A \text{ and } B) = P(A) \cdot P(B)$
>
> ▶ Math *in Motion*, BrainPOP® glencoe.com

> 🌎 **Real-World EXAMPLE 1**   **Independent Events**
>
> **MARBLES** A bag contains 6 black marbles, 9 blue marbles, 4 yellow marbles, and 2 green marbles. A marble is selected, replaced, and a second marble is selected. Find the probability of selecting a black marble, then a yellow marble.
>
> First marble: $P(\text{black}) = \dfrac{6}{21}$ ⟵ number of black marbles ⟵ total number of marbles
>
> Second marble: $P(\text{yellow}) = \dfrac{4}{21}$ ⟵ number of yellow marbles ⟵ total number of marbles
>
> $P(\text{black, yellow}) = P(\text{black}) \cdot P(\text{yellow})$ **Probability of independent events**
>
> $= \dfrac{6}{21} \cdot \dfrac{4}{21} \text{ or } \dfrac{24}{441}$ **Substitution**
>
> The probability is $\dfrac{24}{441}$ or about 5.4%.

✔ **Check Your Progress**

Find each probability.

1A. $P(\text{blue, green})$ **1B.** $P(\text{not black, blue})$

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When the outcome of one event affects the outcome of another event, they are **dependent events**. In Example 1, if the marble was not placed back in the bag, then drawing the two marbles would have been dependent events. The probability of drawing the second marble depends on what marble was drawn first.

Recall that the complement of a set is the set of all objects that do *not* belong to the given set. In a standard deck of cards, the complement of drawing a heart is drawing a diamond, club, or spade. So, the probability of drawing a heart is $\frac{13}{52}$, and the probability of not drawing a heart is $\frac{52-13}{52}$ or $\frac{39}{52}$.

The sum of the probabilities for any two complementary events is 1.

Real-World EXAMPLE 2 Dependent Events

CARDS Cynthia randomly draws three cards from a standard deck one at a time without replacement. Find the probability that the cards are drawn in the given order.

a. $P(\text{diamond, spade, diamond})$

First card: $P(\text{diamond}) = \frac{13}{52}$ or $\frac{1}{4}$ ← number of diamonds / total number of cards

Second card: $P(\text{spade}) = \frac{13}{51}$ ← number of spades / number of cards remaining

Third card: $P(\text{diamond}) = \frac{12}{50}$ or $\frac{6}{25}$ ← number of diamonds remaining / number of cards remaining

$P(\text{diamond, spade, diamond}) = P(\text{diamond}) \cdot P(\text{spade}) \cdot P(\text{diamond})$

$= \frac{1}{4} \cdot \frac{13}{51} \cdot \frac{6}{25}$ or $\frac{13}{850}$ **Substitution**

The probability is $\frac{13}{850}$ or about 1.5%.

b. $P(\text{four, four, not a jack})$

After Cynthia draws the first two fours from the deck of 52 cards, there are 50 cards left. Since neither of these cards are jacks, there are still four jacks left in the deck. So, there are $52 - 2 - 4$ or 46 cards that are not jacks.

$P(\text{four, four, not a jack}) = P(\text{four}) \cdot P(\text{four}) \cdot P(\text{not a jack})$

$= \frac{4}{52} \cdot \frac{3}{51} \cdot \frac{46}{50}$

$= \frac{552}{132,600}$ or $\frac{23}{5525}$

The probability is $\frac{23}{5525}$ or about 0.4%.

✓ Check Your Progress Find each probability.

2A. $P(\text{two, five, not a five})$ **2B.** $P(\text{heart, not a heart, heart})$

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Real-World Link

A standard deck of cards consists of 52 cards. There are 4 suits: hearts, diamonds, clubs, and spades. There are 13 cards of each suit. An ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, jack, queen, and king. The hearts and diamonds are red, and the clubs and spades are black.

Problem-Solving Tip

▶ **Act It Out** Acting out the situation can help you understand what the question is asking. Use a deck of cards to represent the situation described in the problem.
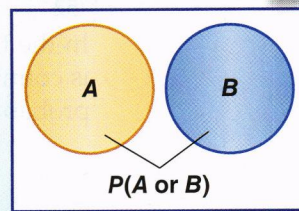
Mutually Exclusive Events Events that cannot occur at the same time are called ==mutually exclusive events==. Suppose you wanted to find the probability of drawing a heart or a diamond. Since a card cannot be both a heart and a diamond, the events are mutually exclusive.

Key Concept   **Mutually Exclusive Events**   For Your **FOLDABLE**

Words If two events, A and B, are mutually exclusive, then the probability that either A or B occurs is the sum of their probabilities.

Model

$P(A \text{ or } B)$

Symbols $P(A \text{ or } B) = P(A) + P(B)$

🌐 **Real-World EXAMPLE 3**   **Mutually Exclusive Events**

A die is being rolled. Find each probability.

a. $P(3 \text{ or } 5)$

Since a die cannot show both a 3 and a 5 at the same time, these events are mutually exclusive.

$P(\text{rolling a 3}) = \dfrac{1}{6}$ ⟵ number of sides with a 3
⟵ total number of sides

$P(\text{rolling a 5}) = \dfrac{1}{6}$ ⟵ number of sides with a 5
⟵ total number of sides

$P(3 \text{ or } 5) = P(\text{rolling a 3}) + P(\text{rolling a 5})$ **Probability of mutually exclusive events**

$= \dfrac{1}{6} + \dfrac{1}{6}$ **Substitution**

$= \dfrac{2}{6}$ or $\dfrac{1}{3}$ **Add.**

The probability of rolling a 3 or a 5 is $\dfrac{1}{3}$ or about 33%.

b. $P(\text{at least } 4)$

Rolling at least a 4 means you can roll either a 4, 5, or a 6. So, you need to find the probability of rolling a 4, 5, or a 6.

$P(\text{rolling a 4}) = \dfrac{1}{6}$ ⟵ number of sides with a 4
⟵ total number of sides

$P(\text{rolling a 5}) = \dfrac{1}{6}$ ⟵ number of sides with a 5
⟵ total number of sides

$P(\text{rolling a 6}) = \dfrac{1}{6}$ ⟵ number of sides with a 6
⟵ total number of sides

$P(\text{at least } 4) = P(\text{rolling a 4}) + P(\text{rolling a 5}) + P(\text{rolling a 6})$ **Mutually exclusive events**

$= \dfrac{1}{6} + \dfrac{1}{6} + \dfrac{1}{6}$ **Substitution**

$= \dfrac{3}{6}$ or $\dfrac{1}{2}$ **Add.**

The probability of rolling at least a 4 is $\dfrac{1}{2}$ or about 50%.

✔ **Check Your Progress**

3A. $P(\text{less than } 3)$ **3B.** $P(\text{even})$

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ReadingMath

A or B Unlike everyday language, the expression *A or B* allows the possibility of both *A* and *B* occurring.

Suppose you want to find the probability of randomly drawing a 2 or a diamond from a standard deck of cards. Since it is possible to draw a card that is both a 2 and a diamond, these events are not mutually exclusive.

P(2)	P(diamond)	P(2, diamond)
$\frac{4}{52}$	$\frac{13}{52}$	$\frac{1}{52}$
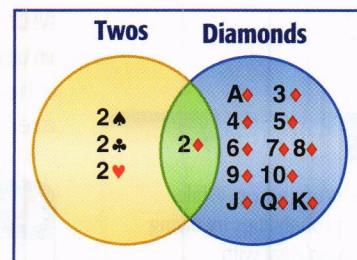
In the first two fractions above, the probability of drawing the two of diamonds is counted twice, once for a two and once for a diamond. To find the correct probability, subtract P(2 of diamonds) from the sum of the first two probabilities.

$$P(2 \text{ or a diamond}) = P(2) + P(\text{diamond}) - P(2 \text{ of diamonds})$$
$$= \frac{4}{52} + \frac{13}{52} - \frac{1}{52}$$
$$= \frac{16}{52} \text{ or } \frac{4}{13} \qquad \text{The probability is } \frac{4}{13} \text{ or about 31\%.}$$

Watch Out!

Intersection of Events When determining the probability of events that are not mutually exclusive, you may count the intersection of the events twice since it occurs in both events. It only actually occurs once.

Key Concept   Events that are Not Mutually Exclusive   For Your FOLDABLE

Words   If two events, *A* and *B*, are not mutually exclusive, then the probability that either *A* or *B* occurs is the sum of their probabilities decreased by the probability of both occurring.

Model
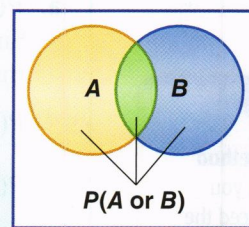
P(A or B)

Symbols $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

Real-World EXAMPLE 4   Events that are Not Mutually Exclusive

STUDENT ATHLETES Of 240 girls, 176 are on the Honor Roll, 48 play sports, and 36 are on the Honor Roll and play sports. What is the probability that a randomly selected student plays sports or is on the Honor Roll?

Since some students play sports and are on the Honor Roll, the events are not mutually exclusive.

$$P(\text{sports}) = \frac{48}{240} \qquad P(\text{Honor Roll}) = \frac{176}{240} \qquad P(\text{sports and Honor Roll}) = \frac{36}{240}$$

$$P(\text{sports or Honor Roll}) = P(\text{sports}) + P(\text{HR}) - P(\text{sports and HR})$$
$$= \frac{48}{240} + \frac{176}{240} - \frac{36}{240} \qquad \textbf{Substitution}$$
$$= \frac{188}{240} \text{ or } \frac{47}{60} \qquad \textbf{Simplify.}$$

The probability is $\frac{47}{60}$ or about 78%.

✓ Check Your Progress

4. PETS Out of 5200 households surveyed, 2107 had a dog, 807 had a cat, and 303 had both a dog and a cat. What is the probability that a randomly selected household has a dog or a cat?

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Examples 1 and 2
pp. 771–772

Determine whether the events are *independent* or *dependent*. Then find the probability.

1. **BABYSITTING** A toy bin contains 12 toys, 8 stuffed animals, and 3 board games. Marsha randomly chooses 2 items for the child she is babysitting. What is the probability that she chose 2 stuffed animals as the first two choices?

2. **FRUIT** A basket contains 6 apples, 5 bananas, 4 oranges, and 5 peaches. Drew randomly chooses one piece of fruit, eats it, and chooses another. What is the probability that he chose a banana and then an apple?

3. **MONEY** Nakos has 4 quarters, 3 dimes, and 2 nickels in his pocket. Nakos randomly picks two coins out of his pocket. What is the probability that he did not choose a dime either time, if he replaced the first coin before choosing a second coin?

4. **BOOKS** Joanna needs a book to prop up a table leg. She randomly selects a book, puts it back on the shelf, and selects another book. What is the probability that Joanna selected two math books?
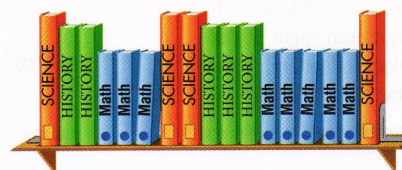
Examples 3 and 4
pp. 773–774

A card is drawn from a standard deck of playing cards. Determine whether the events are *mutually exclusive* or *not* mutually exclusive. Then find the probability.

5. P(two or queen)

6. P(diamond or heart)

7. P(seven or club)

8. P(spade or ace)

Practice and Problem Solving

● = **Step-by-Step Solutions** begin on page R12.
Extra Practice begins on page 815.

Examples 1 and 2
pp. 771–772

Determine whether the events are *independent* or *dependent*. Then find the probability.

9. **COINS** If a coin is tossed 4 times, what is the probability of getting tails all 4 times?

10. **DICE** A die is rolled twice. What is the probability of rolling two different numbers?

11. **CANDY** A box of chocolates contains 10 milk chocolates, 8 dark chocolates, and 6 white chocolates. Sung randomly chooses a chocolate, eats it, and then randomly chooses another. What is the probability that Sung chose a milk chocolate and then a white chocolate?

12. **DICE** A die is rolled twice. What is the probability of rolling the same numbers?

13. **PETS** Chuck and Rashid went to a pet store to buy dog food. They chose from 10 brands of dry food, 6 brands of canned food, and 3 brands of pet snacks. What is the probability that both chose dry food, if Chuck randomly chose first and liked the first brand he picked up?

14. **CARS** A rental agency has 12 white sedans, 8 gray sedans, 6 red sedans, and 3 green sedans for rent. Mr. Escobar rents a sedan, returns it because the radio is broken, and gets another sedan. Assuming the returned sedan remains in circulation, what is the probability that Mr. Escobar was given a green sedan and then a gray sedan?
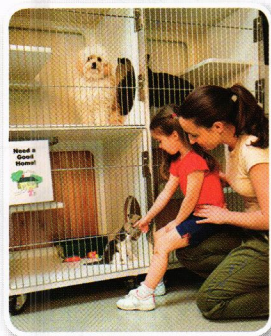
● Real-World Link

About 65% of pet owners acquire their pets free or at low cost.

Source: National Council or Pet Population Study and Policy

Determine whether the events are *mutually exclusive* or *not* mutually exclusive. Then find the probability.

15. BOWLING Cindy's bowling records indicate that for any frame, the probability that she will bowl a strike is 30%, a spare 45%, and neither 25%. What is the probability that she will bowl either a spare or a strike for any given frame?

16. SPORTS CARDS Dario owns 145 baseball cards, 102 football cards, and 48 basketball cards. What is the probability that he randomly selects a baseball or a football card?

17. SCHOLARSHIPS 3000 essays were received for a $5000 college scholarship. 2865 essays were the required length, 2577 of the applicants had the minimum required grade-point average, and 2486 had the required length and minimum grade-point average. What is the probability that an essay selected at random will have the required length or the required grade-point average?

18. KITTENS Ruby's cat had 8 kittens. The litter included 2 orange females, 3 mixed-color females, 1 orange male, and 2 mixed-color males. Ruby wants to keep one kitten. What is the probability that she randomly chooses a kitten that is female or orange?

Real-World Link

A bowling museum and hall of fame is located in St. Louis, Missouri. The museum spans 50,000 square feet and is 3 stories tall.

Source: International Bowling Museum and Hall of Fame

CHIPS A restaurant serves red, blue, and yellow tortilla chips. The bowl of chips Gabriel receives has 10 red chips, 8 blue chips, and 12 yellow chips. After Gabriel chooses a chip, he eats it. Find each probability.

19. P(red, blue)

20. P(blue, yellow)

21. P(yellow, not blue)

22. P(red, not yellow)

23 SOCKS Damon has 14 white socks, 6 black socks, and 4 blue socks in his drawer. If he chooses two socks at random, what is the probability that the first two socks are white?

Cards are being randomly drawn from a standard deck of cards. Once a card is drawn, it is not replaced. Find each probability.

24. P(heart or spade)

25. P(spade or club)

26. P(queen, then heart)

27. P(jack, then spade)

28. P(five, then red)

29. P(ace or black)

30. CANDY A bag contains 10 red, 6 green, 7 yellow, and 5 orange jelly beans. What is the probability of randomly choosing a red jelly bean, replacing, randomly choosing another red jelly bean, replacing, and then randomly choosing an orange jelly bean?

31. SPORTS The extracurricular activities in which the senior class at Valley View High School participate are shown in the Venn diagram.

  a. How many students are in the senior class?

  b. How many students participate in athletics?

  c. If a student is randomly chosen, what is the probability that the student participates in athletics or drama?

  d. If a student is randomly chosen, what is the probability that the student participates in only drama and band?
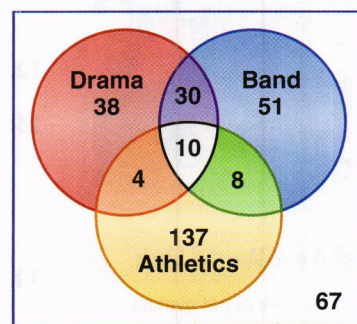
32. TILES Kirsten and José are playing a game. Kirsten places tiles numbered 1 to 50 in a bag. José selects a tile at random. If he selects a prime number or a number greater than 40, then he wins the game. What is the probability that José will win on his first turn?

33 **MULTIPLE REPRESENTATIONS** In this problem, you will explore conditional probability. **Conditional probability** is the probability that event B occurs given that event A has already occurred. It is calculated by dividing the probability of the occurrence of both events by the probability of the occurrence of the first event. The notation for conditional probability is $P(B|A)$.

a. GRAPHICAL Draw a Venn diagram to illustrate $P(A \text{ and } B)$.

b. VERBAL Write the formula for $P(B|A)$ given the Venn diagram.

c. ANALYTICAL A jar contains 12 marbles, of which 8 marbles are red and 4 marbles are green. If marbles are chosen without replacement, find $P(\text{red})$ and $P(\text{red, green})$.

d. ANALYTICAL Using the probabilities from part **c** and the Venn diagram in part **a**, determine the probability of choosing a green marble on the second selection, given that the first marble selected was red.

e. ANALYTICAL Write a formula for finding a conditional probability.

f. ANALYTICAL Use the definition from part **e** to answer the following: At a basketball game, 80% of the fans cheered for the home team. In the same crowd, 20% of the fans were waving banners and cheering for the home team. What is the probability that a fan waved a banner given that the fan cheered for the home team?

H.O.T. Problems   Use **H**igher-**O**rder **T**hinking Skills

34. FIND THE ERROR George and Aliyah are determining the probability of randomly choosing a blue or red marble from a bag of 8 blue marbles, 6 red marbles, 8 yellow marbles, and 4 white marbles. Is either of them correct? Explain.

George

$P(\text{blue or red}) = P(\text{blue}) \cdot P(\text{red})$

$= \dfrac{8}{26} \cdot \dfrac{6}{26}$

$= \dfrac{48}{676}$

about 7%

Aliyah

$P(\text{blue or red}) = P(\text{blue}) + P(\text{red})$

$= \dfrac{8}{26} + \dfrac{6}{26}$

$= \dfrac{14}{26}$

about 54%

35. CHALLENGE In some cases, if one bulb in a string of holiday lights fails to work, the whole string will not light. If each bulb in a set has a 99.5% chance of working, what is the maximum number of lights that can be strung together with at least a 90% chance that the whole string will light?

36. REASONING Suppose there are three events A, B, and C that are not mutually exclusive. List all of the probabilities you would need to consider in order to calculate $P(A \text{ or } B \text{ or } C)$. Then write the formula you would use to calculate it.

37. OPEN ENDED Describe a situation in your life that involves dependent and independent events. Explain why the events are dependent or independent.

38. WRITING IN MATH Explain why the subtraction occurs when finding the probability of two events that are not mutually exclusive.

39. In how many ways can a committee of 4 be selected from a group of 12 people?

    A 48

    B 483

    C 495

    D 11,880

40. A total of 925 tickets were sold for $5925. If adult tickets cost $7.50 and children's tickets cost $3.00, how many adult tickets were sold?

    F 700   **H** 325

    G 600   **J** 225

41. **SHORT RESPONSE** A circular swimming pool with a diameter of 28 feet has a deck of uniform width built around it. If the area of the deck is 60π square feet, find its width.

42. The probability of heads landing up when you flip a coin is $\frac{1}{2}$. What is the probability of getting tails if you flip it again?

    A $\frac{1}{4}$ **C** $\frac{1}{2}$

    B $\frac{1}{3}$ **D** $\frac{3}{4}$

Spiral Review

43. **SHOPPING** The Millers have twelve grandchildren, 5 boys and 7 girls. For their anniversary, the grandchildren decided to pool their money and have three of them shop for the entire group. (Lesson 12-4)

    a. Does this situation represent a *combination* or *permutation*?

    b. How many ways are there to choose the three?

    c. What is the probability that all three will be girls?

44. **ECOLOGY** A group of 1000 randomly selected teens were asked if they believed there was global warming. The results are shown in the table. Find the mean absolute deviation to the nearest tenth. (Lesson 12-3)

Teen Ecology Survey Results	
Response	**Number**
Yes, strongly agree	312
Yes, mildly agree	340
No, I don't think so	109
No, absolutely not	116
Not sure	123

Solve each equation. State any extraneous solutions. (Lesson 11-8)

45. $\frac{4}{a} = \frac{3}{a-2}$

46. $\frac{3}{x} = \frac{1}{x-2}$

47. $\frac{x}{x+1} = \frac{x-6}{x-1}$

48. $\frac{2n}{3} + \frac{1}{2} = \frac{2n-3}{6}$

49. **COOKING** Hannah was making candy using a two-quart pan. As she stirred the mixture, she noticed that the pan was about $\frac{2}{3}$ full. If each piece of candy has a volume of about $\frac{3}{4}$ ounce, approximately how many pieces of candy will Hannah make? (*Hint:* There are 32 ounces in a quart.) (Lesson 11-3)

50. **GEOMETRY** A rectangle has a width of $3\sqrt{5}$ centimeters and a length of $4\sqrt{10}$ centimeters. Find the area of the rectangle. Write as a simplified radical expression. (Lesson 10-2)

Skills Review

Solve each equation. Check your solution. (Lesson 10-4)

51. $\sqrt{-3a} = 6$

52. $\sqrt{a} = 100$

53. $\sqrt{-k} = 4$

54. $5\sqrt{2} = \sqrt{x}$

55. $3\sqrt{7} = \sqrt{-y}$

56. $3\sqrt{4a} - 2 = 10$

Probability Distributions

Why?

A gaming software company with five online games on the market is interested in how many games their customers play. They surveyed 1000 randomly chosen customers. The results of the survey are shown.

Number of Computer Games	Number of Customers
1	130
2	110
3	150
4	500
5	110

Random Variables and Probability A variable with a value that is the numerical outcome of a random event is called a **random variable**. A random variable with a finite number of possibilities is a **discrete random variable**. We can let the random variable G represent the number of different games. So, G can equal 1, 2, 3, 4, or 5.

EXAMPLE 1   Random Variables

A graduation supply company offers 5 items that can be purchased for graduation: a diploma frame, graduation picture, cap and gown, senior key ring, and class pin. The school takes a poll of the seniors to see how many of these items each senior is buying. The results are shown.

Number of Items Being Purchased	Number of Seniors
0	12
1	122
2	134
3	115
4	145
5	97

a. Find the probability that a randomly chosen senior is buying exactly 3 items.

Let X represent the number of items being purchased. There is only one outcome in which 3 items are being purchased, and there are 625 seniors.

$$P(X = 3) = \frac{\text{3 items being purchased}}{\text{seniors surveyed}}$$

$P(X = n)$ is the probability of X occurring n times.

$$= \frac{115}{625} \text{ or } \frac{23}{125}$$

The probability is $\frac{23}{125}$ or 18.4%.

b. Find the probability that a randomly chosen senior buys at least 4 items.

There $145 + 97$ or 242 seniors who are purchasing at least 4 items.

$$P(X \geq 4) = \frac{242}{625}$$

The probability is $\frac{242}{625}$ or about 38.7%.

✓ Check Your Progress

GRADES After an algebra test, there are 7 students with As, 9 with Bs, 11 with Cs, 3 with Ds, and 2 with Fs.

1A. Find the probability that a randomly chosen student has a C.

1B. Find the probability that a randomly chosen student has at least a B.

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Probability Distributions A **probability distribution** is the probability of every possible value of the random variable. A **probability graph** is a bar graph that displays a probability distribution.

Key Concept **Properties of Probability Distributions** **For Your FOLDABLE**

• The probability of each value of X is greater than or equal to 0 and is less than or equal to 1.

• The sum of the probabilities of all values of X is 1.

EXAMPLE 2 **Probability Distribution**

PIZZA The table shows the probability distribution of the number of times a customer orders pizza each month.

Pizzas Ordered Per Month	
X = Number of Pizzas	Probability
0	0.10
1	0.12
2	0.44
3	0.24
4+	0.10

a. Show that the distribution is valid.

• For each value of X, the probability is greater than or equal to 0 and less than or equal to 1.

• The sum of the probabilities, $0.10 + 0.12 + 0.44 + 0.24 + 0.10$, is 1.

b. What is the probability that a customer orders pizza fewer than three times per month?

The probability of a compound event is the sum of the probabilities of each individual event. The probability of a customer ordering fewer than 3 times per month is the sum of the probability of ordering 2 times per month plus the probability of ordering one time per month.

$$P(X < 3) = P(X = 2) + P(X = 1) + P(X = 0)$$ Sum of individual probabilities
$$= 0.44 + 0.12 + 0.10$$ $P(X = 2) = 0.44$, $P(X = 1) = 0.12$, and $P(X = 0) = 0.10$
$$= 0.66$$ Add.

c. Make a probability graph of the data.

Use the data from the probability distribution table to draw a bar graph. Remember to label each axis and give the graph a title.

Pizzas Ordered Per Month
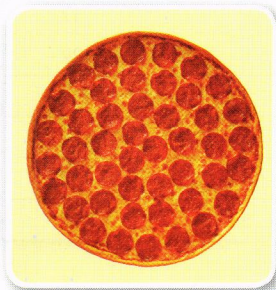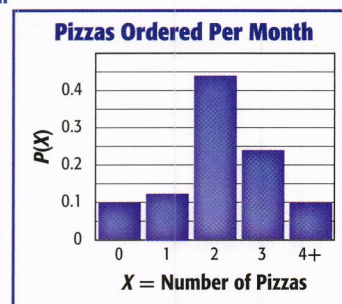
🌐 **Real-World Link**

In October of 2007, Joseph Jones, then a high school senior, ate 83 slices of pepperoni pizza within 10 minutes at an eating competition.

Source: About Pizza

✔ **Check Your Progress**

The table shows the probability distribution of adults who play golf by age range.

2A. Show that the distribution is valid.

2B. What is the probability that an adult golfer is 35 years old or older?

2C. Make a probability graph of the data.

Golfers By Age	
A = Ages	Probability
18–24	0.13
25–34	0.18
35–44	0.21
45–54	0.19
55–64	0.12
65+	0.17

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Example 1
p. 779

1. **GPS** A car dealership surveys 10,000 of its customers who have a GPS system to ask how often they have used the system within the past year. The results are shown.

   a. Find the probability that a randomly chosen customer will have used the GPS system more than 20 times.

   b. Find the probability that a randomly chosen customer will have used the GPS system no more than 10 times.

Customers Using the GPS System	
Uses	**Customers**
0	1382
1–5	2350
6–10	2010
11–15	1863
16–20	1925
21+	470

Example 2
p. 780

2. **JEANS** A fashion boutique ordered jeans with different numbers of stripes down the outside seams. The table shows the probability distribution of the number of each type of jean sold in a particular week.

   a. Show that the distribution is valid.

   b. What is the probability that a randomly chosen pair of jeans has fewer than 3 stripes?

   c. Make a probability graph of the data.

Types of Jeans Sold	
X = Number of Stripes	Probability
0	0.15
1	0.19
2	0.26
3	0.22
4	0.18

Example 1
p. 779

3. **HOME THEATER** An electronics store sells the components and speakers for home theaters. The store surveyed its customers to see how many of the 10 components they bought. The results are shown.

   a. Find the probability that a randomly chosen customer bought 5 or 6 components.

   b. Find the probability that a randomly chosen customer bought fewer than 5 components.

Home Theater Components Purchased	
Components	**Customers**
0–2	26
3–4	42
5–6	33
7–8	24
9–10	40

Practice and Problem Solving

● = **Step-by-Step Solutions** begin on page R12.
Extra Practice begins on page 815.

Example 1
p. 779

4. **FOOD DRIVE** Ms. Valdez's biology class held a food drive. The class kept track of the types of food donated.

   a. Find the probability that a randomly chosen product will be soup.

   b. Find the probability that a randomly chosen product will be a boxed dinner or pasta.

Food Drive Donations Count	
Product	**Packages**
boxed dinner	36
pasta	22
juice	12
soup	45

5. **SCHOOL SPIRIT** The student council wants to organize a spirit club to cheer at school sporting events. They surveyed the student body and asked students how many sporting events they typically attend each year.

Number of Sporting Events	0–5	6–10	11–15	16–20	21+
Number of Students	96	112	204	108	80

   a. Find the probability a randomly chosen student attended at most 10 events.

   b. Find the probability a randomly chosen student attended at least 16 events.
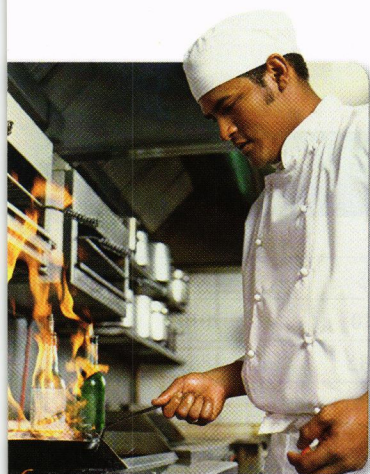
6. RESTAURANTS Kwan Chinese Restaurant has a delivery service. Mr. Kwan is keeping track of how many deliveries they have each week for a year. The results are shown.

   a. Find the probability that there will be more than 20 deliveries in a randomly chosen week.

   b. Find the probability that there will be fewer than 21 deliveries in a randomly chosen week.

Kwan Chinese Restaurant Deliveries	
Deliveries Per Week	Weeks
0–5	0
6–10	9
11–20	18
21–25	12
26+	13

7. PARTY Chrystal owns a company that plans parties for children. Throughout the year she has kept a count of each party theme she has used. The table shows the results of her tally.

Theme of Party	animal	circus	superhero	sports	music	other
Number of Parties	42	15	9	45	32	35

   a. Find the probability that a randomly chosen theme will be animal or sports.

   b. Find the probability that a randomly chosen theme will not be animal or sports.

Example 2
p. 780

8. MUSIC A Web site conducted a survey on the format of music teens listened to. The table shows the probability distribution of the results.

   a. Show that the distribution is valid.

   b. What is the probability that the type of format randomly chosen will be an MP3 or online?

   c. Make a probability graph of the data.

Formats for Music	
Format	Probability
CDs	0.35
radio	0.31
mini-disc	0.02
MP3	0.11
online	0.19
other	0.02

9 GRADES Mr. Rockwell's Algebra class took a chapter test last week. The table shows the probability distribution of the results.

   a. Show that the distribution is valid.

   b. What is the probability that a student chosen at random will have no higher than a B?

   c. Make a probability graph of the data.

Algebra Test Grades	
Grade	Probability
A	0.29
B	0.43
C	0.17
D	0.11
F	0

10. SKATE PARKS The park department asked the counties that had skate parks what equipment was allowed to be used in their park. The table shows the probability distribution of the results.
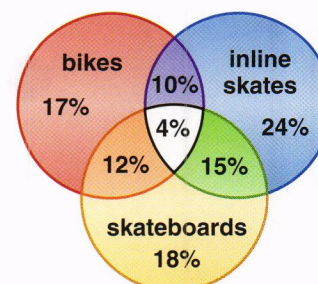
   a. Show that the distribution is valid.

   b. What is the probability that a park chosen at random allows bikes or skateboards?

   c. Make a probability graph of the data.

bikes 17%
inline skates 24%
10%
4%
12%
15%
skateboards 18%
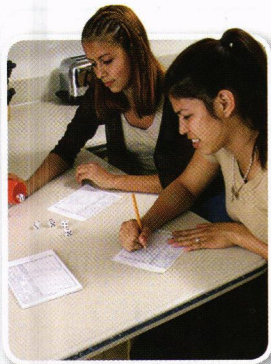
11 **MARKETING** A retail marketing group conducted a survey on teen shopping habits and asked where the teens did most of their holiday shopping. The table shows the probability distribution of the results

Types of Stores	malls	individual stores	online	catalogs	other
Probability	0.35	0.32	0.17	0.11	0.05

a. Show that the distribution is valid.

b. What is the probability that a shopper chosen at random will shop online or in a catalog?

c. Make a probability graph of the data.

12. **SPORTS CARDS** Joshua mixed up all of his sports cards and placed them in a bag. Then he told his sister Drea that she could keep whatever card she randomly drew out of the bag.

a. What is the probability that a randomly chosen card is hockey or football?

b. Make a probability distribution table for the data. Round to the nearest hundredth.

c. Is the distribution valid? Why or why not?
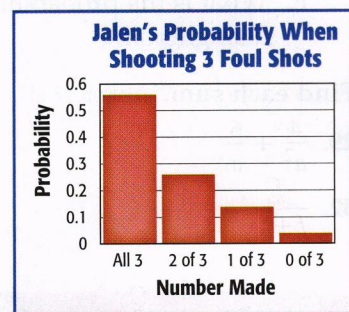
d. Make a probability graph of the data.

Joshua's Sports Cards	
Sport	**Number Sold**
baseball	53
football	27
basketball	39
hockey	21

13. **MULTIPLE REPRESENTATIONS** In this problem, you will explore the differences between a prediction and what actually happens.

a. **VERBAL** What is the probability of rolling a 2 on a die? What is the probability of rolling a 1 or a 6? What is the probability of rolling an odd number?

b. **ANALYTICAL** Roll the die 20 times. Record the value of the die after each roll.

c. **ANALYTICAL** Determine the probability distribution for X = value of the die.

d. **VERBAL** From your probability distribution, what is the probability of rolling a 2? What is the probability of rolling a 1 or a 6? What is the probability of rolling an odd number? Explain why the numbers may not be the same.

H.O.T. Problems   Use **H**igher-**O**rder **T**hinking Skills

14. **CHALLENGE** What is wrong with the probability distribution shown? Explain your reasoning.

15. **REASONING** Suppose two dice are rolled twelve times. Which sum is most likely to occur? Make a table to show the probability distribution. Then make a probability graph to confirm your answer.

Jalen's Probability When Shooting 3 Foul Shots

16. **REASONING** Explain why the sum of the probabilities in a probability distribution should always be 1. Include an example.

17. **OPEN ENDED** Write a real-world problem in which you could find a probability distribution. Create a probability graph for your data.

18. **WRITING IN MATH** Write a real-world story in which you are the owner of a business. Explain how you could use a probability distribution to help you make a business decision.

19. A bucket contains 10 balls numbered 1, 1, 2, 3, 4, 4, 4, 5, 6, and 6. A ball is randomly chosen from the bucket. What is the probability of drawing a ball with a number greater than 6?

   A $\frac{1}{5}$ **C** 1

   B $\frac{3}{10}$ **D** 0

20. SHORT RESPONSE Mr. Bahn has $20,000 to invest. He invests part at 6% and the rest at 7%. He earns $1280 in interest within a year. How much did he invest at 7%?

21. Suppose there are 10 tickets in a box for a drawing numbered as follows: 1, 2, 2, 3, 4, 4, 6, 6, 9, and 9. A single ticket is randomly chosen from the box. What is the probability of drawing a ticket with a number less than 10?

   F 0   **G** $\frac{1}{5}$ **H** $\frac{3}{10}$ **J** 1

22. GEOMETRY The height of a triangle is 5 inches less than the length of its base. If the area of the triangle is 52 square inches, find the base and the height.

   A 15 in., 9 in.   **C** 13 in., 8 in.

   B 11 in., 7 in.   **D** 17 in., 11 in.

Spiral Review

23. PET TOYS A pet store has a bin of clearance items that contains 6 balls, 5 tug toys, 8 rawhide chews, and 4 chew toys, all in equal-sized boxes. If Johnda reaches in the box and pulls out two items, what is the probability that she will pull out a tug toy each time? (Lesson 12-5)

A die is rolled and a spinner is spun like the one shown. Find the probability. (Lesson 12-4)

24. $P(3 \text{ and } Y)$ **25.** $P(\text{even and } G)$

26. $P(\text{prime number and } R \text{ or } B)$ **27.** $P(4 \text{ and not } Y)$
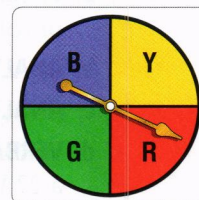
28. GAMES For a certain game, each player rolls four dice at the same time. (Lesson 12-3)

  a. Do the outcomes of rolling the four dice represent permutations or combinations? Explain.

  b. How many outcomes are possible?

  c. What is the probability that four dice show the same number on a single roll?

Find each sum. (Lesson 11-6)

29. $\dfrac{4}{a^2} + \dfrac{6}{a}$ **30.** $\dfrac{3}{b^3} + \dfrac{7}{b^2}$ **31.** $\dfrac{4}{d+6} + \dfrac{5}{d-5}$

32. $\dfrac{f}{f+5} + \dfrac{4}{f-4}$ **33.** $\dfrac{8h}{h+6} + \dfrac{h}{h-3}$ **34.** $\dfrac{7k}{k-3} + \dfrac{k}{k+2}$

Skills Review

35. Write an expression to represent the probability of tossing a coin n times and getting n heads. Express as a power of 2. (Lesson 7-2)

36. Write an expression to represent the probability of rolling a die n times and getting 3 n times. Express as a power of 6. (Lesson 7-2)

37. Write an expression to represent the probability of rolling a die n times and getting a prime number n times. Express as a power. (Lesson 7-2)

EXTEND
12-6

Graphing Technology Lab
The Normal Curve

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• Other Calculator Keystrokes
• Graphing Technology Personal Tutor

When there are a large number of values in a data set, the frequency distribution tends to cluster around the mean of the set in a distribution (or shape) called a **normal distribution**. The graph of a normal distribution is called a **normal curve**. Since the shape of the graph resembles a bell, the graph is also called a *bell curve*.

Data sets that have a normal distribution include reaction times of drivers that are the same age, achievement test scores, and the heights of people that are the same age.
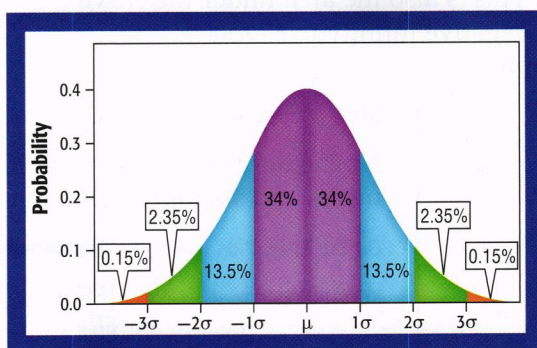
You can use a graphing calculator to graph and analyze a normal distribution if the mean and standard deviation of the data are known.

ACTIVITY 1   Graph a Normal Distribution

HEIGHT The mean height of 15-year-old boys in the city where Isaac lives is 67 inches, with a standard deviation of 2.8 inches. Use a normal distribution to represent these data.

Step 1 Set the viewing window.

- Xmin   67 ⎼ 3 ⊠ 2.8 ENTER **58.6**
- Xmax   67 ⊞ 3 ⊠ 2.8 ENTER **75.4**
- Xscl   2.8 ENTER
- Ymin   0 ENTER
- Ymax   1 ÷ ⎛ 2 ⊠ 2.8 ⎞ ENTER
  .17857142…
- Yscale   1
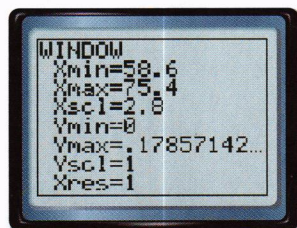
Step 2 By entering the mean and standard deviation into the calculator, we can graph the corresponding normal curve. Enter the values using the following keystrokes.

KEYSTROKES: Y= 2nd [DISTR] ENTER
X,T,θ,n , 67 , 2.8
) Graph
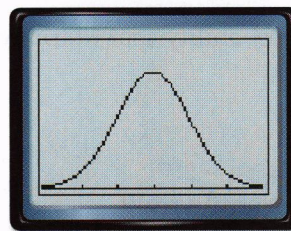
[58.6, 75.4] scl: 2.8 by [0, 0.17857142] scl: 1

The probability of a range of values is the area under the curve.

Analyze a Normal Distribution

Use the graph to answer questions about the data. What is the probability that Isaac will be at most 67 inches tall when he is 15?

The sum of all the y-values up to $x = 67$ would give us the probability that Isaac's height will be less than or equal to 67 inches. This is also the area under the curve. We will shade the area under the curve from negative infinity to 67 inches and find the area of the shaded portion of the graph.

Step 1 ShadeNorm Function

KEYSTROKES: [2nd] [DISTR] [▶] [ENTER]

Step 2 Shade the graph.

Next enter the lowest value, highest value, mean, and standard deviation.

On the TI-84 Plus, -1×10^{99} represents negative infinity.

KEYSTROKES: [(−)] 1 [2nd] [EE] 99 [,] 67 [,] 67 [,] 2.8 [ )] [ENTER]

```
DISTR DRAW
1:ShadeNorm(
2:Shade_t(
3:ShadeX²(
4:ShadeF(
```
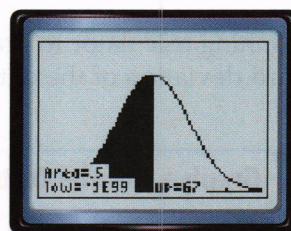
```
Area=.5
low=-1E99    up=67
```
[58.6, 75.4] scl: 2.8 by [0, 0.17857142] scl: 1

The area is given as 0.5. The probability that Isaac will be 67 inches tall is 0.5 or 50%. Since the mean value is 67, we expect the probability to be 50%.

Exercises

1. What is the probability that Isaac will be at least 6 feet tall when he is 15?

2. What is the probability that Isaac will be between 65 and 68 inches?

3. If the mean height of 15-year-old girls in the same city is 64 inches with a standard deviation of 2.1, what is the probability that Isaac's sister, Maria, will be at least 64 inches tall when she is 15?

4. What is the probability that Maria will be no taller than 5 feet when she is 15 years old?

Extension

Refer to the curve at the right.

5. Compare this curve to the normal curve in Activity 1.

6. Describe where an outlier of the data set would be graphed on this curve.

Probability Simulations

Why?

Alex has been practicing his penalty kicks. He expects to be able to make at least 66% of his penalty kicks. To test this, he takes 50 penalty kicks, of which he makes 33.
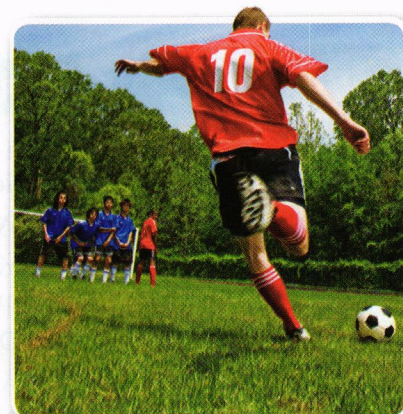
Experimental Probability and Expected Value

Experimental probability is determined using data from tests or experiments. Alex's experimental probability is 66%. Therefore, he *expects* to make 66% of his future kicks.

<div style="float:left; width:25%;">

Then
You used probability distributions.
(Lesson 12-6)

Now
- Design simulations to estimate probabilities.
- Summarize data from simulations.

New Vocabulary
experimental probability
theoretical probability
relative frequency
empirical study
expected value
simulation

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</div>

Experimental probability should not be confused with theoretical probability. **Theoretical probability** is the likeliness of an event happening. For example, when tossing a coin, the theoretical probability of it landing on heads is always 0.5, while an experiment tossing many coins may produce a different *experimental* probability.

Experimental probability is the ratio of the number of times an outcome occurs to the total number of events or trials. The ratio is also known as the **relative frequency.**

$$\text{experiment probability} = \frac{\text{frequency of an outcome}}{\text{total number of trials}}$$

It is often useful to perform an **empirical study**. In this study, an experiment is performed repeatedly, data are collected and combined, the results are analyzed, and an expected value can be calculated. The **expected value** is the average value that is expected for the outcome of one trial.

⊕ Real-World EXAMPLE 1   Experimental Probability

a. **SOCCER** What is the experimental probability that Alex successfully makes his goal kicks?

$$\text{experimental probability} = \frac{33}{50} \leftarrow \begin{array}{l}\text{frequency of successes}\\ \text{total number of goal kicks}\end{array}$$

The experimental probability of the test is $\frac{33}{50}$ or 66%.

b. **SOCCER** Alex takes 50 kicks two more times. He makes 29 of the first 50 kicks and 34 of the second 50. What is the experimental probability of all three tests?

$$\text{experimental probability} = \frac{96}{150} \text{ or } \frac{16}{25}$$

The experimental probability of the three tests was $\frac{16}{25}$ or 64%.

✓ Check Your Progress

1. **GAMES** Hakeem rolls a die 20 times. A 4 appears 5 times. What is the experimental probability of rolling a 4?

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Performing Simulations A **simulation** allows you to find an experimental probability by using objects to act out an event that would be difficult or impractical to perform. You can conduct simulations using one or more objects such as dice, coins, marbles, or spinners. The theoretical probability of objects you choose should be identical to the experimental probability.

🌐 Real-World EXAMPLE 2   Simulation

In a trivia game, a player answers one of three questions on a card, two of which are multiple choice.

a. What could be used to simulate getting a multiple-choice question? Explain.

You could use a die, where rolling 1 through 4 represents a multiple-choice question and rolling 5 or 6 does not.

b. Describe a way to conduct the simulation if 3 out of 5 are multiple-choice.

A spinner with five equally sized spaces, three of which are colored red, could be used. If the spinner lands on red, then the question is multiple choice.

✔️ Check Your Progress

2. Of the 48 games remaining on Bobbie's favorite basketball team's schedule, 24 will not be televised.

   A. What could be used to simulate whether the next game will be televised?

   B. Describe a simulation if 36 of the 48 games will not be televised.

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🌐 Real-World EXAMPLE 3   Experimental Probability and Expected Value

QUALITY CONTROL Eloy inspects automobile frames as they come through the assembly line. From previous observations, he expects to find a weld defect in one out of ten of the frames each day.

a. What objects can be used to model the possible outcomes of the automobile inspection? Explain.

Use a simulation that has 10 objects, where 1 out of the 10 objects represents a defect. One possible simulation would be to place 10 marbles in a bag. Let 1 red marble represent the defects and 9 yellow marbles represent the good automobiles. One marble at a time can be drawn out of the bag, the results recorded, and the marble replaced in the bag. Repeat this 9 more times representing 10 automobiles.

b. What is the expected value that there is one automobile frame found with defects in a certain day?

On average, 1 out of 10 automobiles has a defect. So, the expected value is $\frac{1}{10}$ or 10%.

c. Run the simulation using Eloy's expectation of defects. What is the experimental probability of having a defect for every ten frames?

After 50 simulations, or 500 drawings, 57 had a defect, so the experimental probability of getting a defect is $\frac{57}{500}$ or 11.4%.

Defects	Frequency	Total
0	12	0
1	24	24
2	10	20
3	3	9
4	1	4
		57

d. **How does the experimental probability compare to the expected value?**

The expected value is 10%, and the experimental probability is 11.4%. They are relatively close. As the number of trials increased it would get closer to the expected value.

✓ **Check Your Progress**

3. **TELEVISION** Anthony's favorite show is going to be replayed by a cable channel. They are going to play one episode each day in a random order. Anthony missed 3 of the 22 episodes.

  A. What objects can be used to model the possible outcomes of one of the shows that he missed being aired on the first day?

  B. What is the expected value that it will be a show he missed?

  C. The results of a simulation Anthony performed are shown. What is the experimental probability that it will be a show he missed?

Show	Frequency
missed	5
watched	45

  D. How does the experimental probability compare to the expected value?

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✓ **Check Your Understanding**

Example 1
p. 787

1. **GAMES** Games at the fair require the majority of players to lose in order for game owners to make a profit. New games are tested to make sure they have sufficient difficulty. The results of three test groups are listed in the table. The owners want a maximum of 33% of players to win. There were 50 participants in each test group.

  a. What is the experimental probability that the participant was a winner in the second group?

Result	Group 1	Group 2	Group 3
Winners	13	15	19
Losers	37	35	31

  b. What is the experimental probability of winning for all three groups?

2. **BATTING AVERAGE** In a computer baseball game, a baseball player has a batting average of 300. That is, he gets a hit 300 out of 1000, or 30%, of the times he is at bat. What could be used to simulate the player taking a turn at bat?

Example 2
p. 788

3 **TEST** On a true-false test, Marlene answered 16 out of the 20 questions correctly by guessing randomly.

  a. What could be used to simulate her correctly answering a question? Explain.

  b. Describe a way to simulate the next 20 questions.

Example 3
p. 788

4. **LOTTERIES** In a certain state, lottery numbers are five-digit numbers. Each digit can be 1, 2, 3, 4, 5, or 6. Once a week, a winning number is chosen randomly.

  a. How many five-digit numbers are possible? Explain how you calculated the number of possible outcomes.

  b. Perform a simulation for winning the lottery. Describe the objects you used.

  c. According to your experiment, if you buy one ticket, what is the experimental probability of winning?

  d. How does your experimental probability compare to the theoretical probability of winning?

Practice and Problem Solving

Example 1
p. 787

5. CARDS Javier is drawing a card from a standard deck of cards, recording the suit, and then replacing the card in the deck. The table below shows his results.

Suit	clubs	diamonds	hearts	spades
Frequency	7	4	5	9

a. Find the experimental probability of drawing a heart.

b. Find the experimental probability of drawing a black card.

c. Javier repeated his test. The results are shown below. Find the experimental probability of drawing a spade for both tests.

Suit	clubs	diamonds	hearts	spades
Frequency	5	8	6	6

Example 2
p. 788

6. CDs There are 6 country CDs, 8 pop CDs, 3 rap CDs, and 7 rock CDs in a storage case. What could be used for a simulation to determine the probability of randomly selecting any one type of CD?

7. TESTS What could be used to simulate guessing on a multiple-choice test with 4 possible answers for each question?

Example 3
p. 788
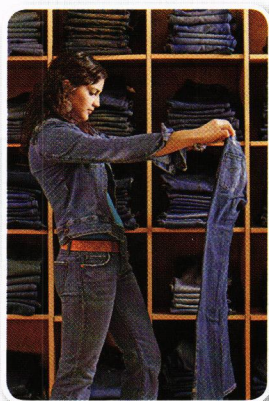
8. JEANS Julie examines the stitching on pairs of jeans that are produced at a manufacturing plant. She expects to find defects in 1 out of every 16 pairs.

a. What can be used to model the possible outcome of a pair of jeans having defects? Explain.

b. What is the expected value that a random pair of jeans has a defect?

c. The results of simulations using Julie's expectations are shown. What is the experimental probability that a random pair of jeans will have a defect?

d. How does the experimental probability compare to the expected value?

Defects	Frequency
0	71
1	9
2	11
3	6
4	3

9. DIE ROLL Roll a die 25 times and record your results. Find each probability based on your results.

a. What is the probability of rolling a 2?

b. What is the probability of rolling a prime number?

c. What is the probability of rolling an even number or 3?

d. Compare your results to the theoretical probabilities.

10. PRIZES For its tenth anniversary, a video store randomly gives each customer a prize from the following choices: a free movie rental, a free video game rental, a free bag of popcorn, or a free pre-viewed movie. The chance of winning each prize is equal.

a. What could be used to perform a simulation of this situation? Explain.

**b.** How could you use this simulation to model the next 50 free items?

🌐 Real-World Link

Women between the ages of 16 and 24 have an average of 8 pairs of jeans.

Source: Cotton Incorporated

11 **COIN TOSS** Toss 4 coins, one at a time, 20 times, and record the number of heads and tails. Find each probability based on your results.

 a. P(any three coins will show tails)

 b. P(any two coins will show heads)

 c. P(the first coin will show heads and the fourth coin will show tails)

12. PRIZES For a promotion, the concession stands at a football stadium are giving away free items. Every time a customer buys something, a wheel is spun to choose the customer's prize. Each prize is equally likely.
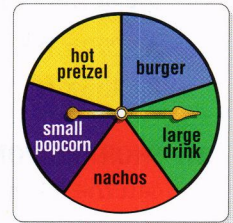
 a. Other than a spinner, what object could be used to simulate this situation? Explain.

 b. Perform the simulation until you have received at least one of each item.

 c. In your simulation, how many items must be bought to win every prize? Choose another representation to solve the problem. How do the solutions compare?

13. GRAPHING CALCULATOR With every purchase at a fast-food restaurant, you receive a scratchoff game card with two circles. You choose one circle to scratch. One reveals a prize, and the other reveals "Sorry. Try Again." The chance that the prize on a card is cash is 5%, a sandwich is 20%, a drink is 50%, and fries is 25%. Determine how many game cards you must scratch to win the cash.

 a. Generate a list of 100 random 0s and 1s in L1 of the calculator. 0 means you did not win a prize, and 1 means you won a prize.

 b. Generate a list of 100 random integers from 1 to 20 in L2. Let each 1 represent the cash prize.

 c. In L3, multiply the values in L1 and L2 together. What value represents that you won the cash prize? Explain.

 d. From your simulation, how many game cards had to be scratched to win the cash?

14. ROLLING A DIE Multiply each number on a die by the probability of rolling that number. Add these values.

 a. What is the expected value of one roll of a die?

 b. Find the expected value of the sum of the numbers on two dice.

H.O.T. Problems Use Higher-Order Thinking Skills

15. REASONING The experimental probability of heads when a coin is tossed 15 times is *sometimes*, *never*, or *always* equal to the theoretical probability. Explain.

16. CHALLENGE Lenora tested her tennis ball machine by running 5 simulations. The experimental probability of the machine being accurate is 7% higher than the results of the 5th simulation. Determine the results of the 5th simulation.

Simulation	Accuracy
1	95%
2	85%
3	90%
4	85%

17. REASONING Find a counterexample to the following statement. Explain.

It is possible for an experimental probability to be 0 if the theoretical probability is 1.

18. OPEN ENDED Describe a situation at your school that could be represented by a simulation. What could you use to simulate the situation?

19. WRITING IN MATH Compare and contrast experimental and theoretical probability.

20. GEOMETRY Suppose a covered water tank in the shape of a right circular cylinder is thirty feet long and eight feet in diameter. What is the surface area of the cylinder?

A 272π ft^2 **C** 286π ft^2

B 153π ft^2 **D** 248π ft^2

21. SHORT RESPONSE How many different ways can the letters P, Q, R, S be arranged?

22. In how many ways can the letters in the word STATISTICS be arranged?

F 50,400   **H** 15,400

G 20,800   **J** 3480

23. Two consecutive numbers have a sum of 91. What are the numbers?

A 41, 50   **C** 45, 46

B 44, 47   **D** 49, 42

Spiral Review

24. The table shows a class's grade distribution, where $A = 4.0$, $B = 3.0$, $C = 2.0$, $D = 1.0$, and $F = 0$. (Lesson 12-6)

$G =$ Grade	0	1.0	2.0	3.0	4.0
Probability	0.05	0.05	0.30	0.35	0.25

a. Is the probability distribution valid? Explain.

b. What is the probability that a student passes the course?

c. What is the probability that a student chosen at random from the class receives a grade of C or better?

Review problems are color coded by lesson. Each student draws and then returns a colored ball from a bucket to see which lesson to review. There are 5 red, 10 yellow, 5 blue, and 2 green balls. Find each probability described. (Lesson 12-4)

25. $P(\text{yellow})$ **26.** $P(\text{red})$ **27.** $P(\text{red, blue})$ **28.** $P(\text{yellow, green})$

29. PARTIES Student Council is planning a party for the school volunteers. There are five 66-ounce unopened bottles of soda left from a recent dance. When poured over ice, $5\frac{1}{2}$ ounces of soda fills a cup. How many servings of soda do they have? (Lesson 11-7)

Write an inverse variation equation that relates x and y. Assume that y varies inversely as x. Then solve. (Lesson 11-1)

30. If $y = 8.5$ when $x = -1$, find x when $y = -1$.

31. If $y = 8$ when $x = 1.55$, find x when $y = -0.62$.

32. If $y = 6.4$ when $x = 4.4$, find x when $y = 3.2$.

33. TOPOGRAPHY To determine the mileage between landmarks, the U.S. military superimposes a coordinate grid over a map of the region. The units on this grid are approximately equal to 50,000 feet. So, a distance of 3 units on the grid equals an actual distance of 3(50,000) or 150,000 feet. Suppose the locations of two landmarks are at (132, 428) and (254, 105). Find the actual distance between these landmarks to the nearest mile. (Lesson 10-6)

Write each fraction as a percent rounded to the nearest whole number. (Lesson 0-6)

34. $\frac{26}{58}$ **35.** $\frac{55}{125}$ **36.** $\frac{14}{128}$ **37.** $\frac{82}{110}$ **38.** $\frac{76}{124}$ **39.** $\frac{23}{86}$

Chapter Summary

Key Concepts

Designing a Survey and Analyzing Results
(Lessons 12-1 and 12-2)

• The three methods for collecting data are surveys, observational studies, and experiments.

• A sample is biased if one group is favored over another.

• Data can be organized by mean, median, mode, range, quartile and interquartile range.

Statistics (Lesson 12-3)

• A parameter is a characteristic of a whole population.

• The mean absolute value is the average of the absolute values of differences between the mean and each value and the data set.

Permutations and Combinations (Lesson 12-4)

• In a permutation, the order of objects is important,
$P(n,r) = \dfrac{n!}{(n-r)!}$

• In a combination, the order of objects is not important,
$C(n,r) = \dfrac{n!}{(n-r)!r!}$

Probability Distributions and Simulations
(Lessons 12-6 and 12-7)

• For each value of X, $0 \le P(X) \le 1$. The sum of the probabilities for all values of X is 1.

• Theoretical probability describes expected outcomes, while experimental probability describes tested outcomes.

• Simulations are used to perform experiments that would be difficult or impossible to perform in real life.

FOLDABLES Study Organizer

Be sure the Key Concepts are noted in your Foldable.
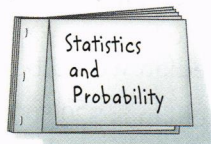
Key Vocabulary

biased sample (p. 741)
combination (p. 765)
complement (p. 772)
compound event (p. 771)
conditional probability (p. 777)
dependent events (p. 772)
discrete random variable (p. 779)
empirical study (p. 787)
experiment (p. 740)
experimental probability (p.787)
factorial (p. 764)
independent events (p. 771)
mean absolute deviation (p. 757)
mutually exclusive (p. 773)
parameter (p. 756)
permutation (p. 764)
population (p. 740)
probability distribution (p. 780)
qualitative data (p. 758)
quantitative data (p. 758)
random variable (p. 779)
relative frequency (p. 787)
sample (p. 740)
simple random sample (p. 742)
simulation (p. 788)
standard deviation (p. 758)
statistic (p. 756)
statistical inference (p. 756)
stratified random sample (p. 742)
survey (p. 740)
systemic random sample (p. 742)
theoretical probability (p. 787)
univariate data (p. 757)
variance (p. 758)

Vocabulary Check

Choose the word or term that best completes each sentence.

1. The arrangement in which order is important is called a (combination, permutation).

2. Rolling one die and then another are (dependent, independent) events.

3. The sum of probabilities of complements equals (0, 1).

4. Randomly drawing a marble from a jar and then drawing another marble are dependent events if the marbles (are, are not) replaced.

5. Events that cannot occur at the same time are (mutually exclusive, inclusive).

Lesson-by-Lesson Review

12-1 Designing a Survey (pp. 739–745)

6. **SCHOOL DANCE** The homecoming dance committee is trying to decide on a theme. They send out a questionnaire to all of the girls in the school. Identify the sample and suggest a population from which it was selected. Then, classify the type of data collection used.

7. **GOVERNMENT** To determine whether voters support a new trade agreement, 5 people from the list of registered voters in each state and in the District of Columbia are selected at random. Is the sample *biased* or *unbiased*?

8. **CANDY BARS** To ensure that all of the chocolate bars are the appropriate weight, every 50th bar on the conveyor belt in the candy factory is removed and weighed. Is the sample *simple*, *stratified*, or *systematic*?

EXAMPLE 1

For the situation, identify the sample and suggest a population from which it was selected. Then, classify the type of data collection used.

An artist is trying to choose a cover for a children's book. She sends out a flyer with the two covers to all of the students at one school. She asks them to check their favorite cover.

The sample is all of the students at the one school.
The population is all children who read books.
The type of data collection is a survey.

EXAMPLE 2

People listening to a country music radio station are asked to name their favorite type of music. Identify the sample as *biased* or *unbiased*.

The sample is biased because people listening to a country music station are more likely to vote for country music as their favorite.

12-2 Analyzing Survey Results (pp. 746–755)

Which measure of central tendency best represents the data? Justify your answer. Then find the measure.

9. **CLASSROOM** Sophia keeps track of the ages of the students in her class. She wants to best represent the ages of her classmates: 13, 14, 13, 13, 14, 13, 13, 15, 14, 13, 14, 14, 10.

10. **PETS** Jason conducts a survey about the number of pets his friends have. He wants to best represent the number of pets: 0, 2, 1, 2, 4, 0, 2, 3, 1, 2, 1, 2, 3, 2, 4, 3, 10.

11. **LUNCH PRICES** The cafeteria wants to show the best representation of how much students spend on lunch: 2, 3, 4, 3, 2, 4, 1, 3, 4, 2, 3, 3, 4, 3.

EXAMPLE 3

FINANCIAL LITERACY A company wants to show the best representation of an employee's salary. The salaries of the employees in the company are $25,000, $30,000, $28,000, $29,000, $30,000, and $65,000. Tell which measure of center best represents the data. Then find the measure of center.

List the values from least to greatest: $25,000, $28,000, $29,000, $30,000, $30,000, $65,000.

There is one value that is much greater than the rest of the data, $65,000. Also, there does not appear to be a big gap in the middle of the data. The median would best represent the data.

The median is $29,500.

12-3 **Statistics and Parameters** (pp. 756–762)

Find the mean absolute deviation to the nearest tenth.

12. SHOVELING SIDEWALKS Ben is shoveling sidewalks to raise money over break. He is keeping track of how many he shovels each day: 2, 4, 3, 5, 3.

13. CANDY BARS Luci is keeping track of the number of candy bars each member of the drill team sold.

20, 25, 30, 50, 40, 60, 20, 10, 42

Find the mean, variance, and standard deviation to the nearest tenth for each set of data.

14. 1, 1, 3, 4, 6

15. 10, 11, 11, 10, 12, 13, 14, 10

16. 3, 5, 6, 2, 1, 5

17. 10, 11, 10, 11, 12

18. 15, 16, 16, 15, 16, 17, 18

19. FOOD A fast food company polled a random sample of its customers to find how many times a month they eat out: 10, 3, 12, 15, 7, 8, 4, 12, 9, 14, 12. Find the mean absolute deviation of the data set to the nearest tenth.

EXAMPLE 4

GIFTS Joshua is collecting money from his family for his grandmother. He keeps track of how much was donated: 10, 5, 20, 15, 10. Find the mean absolute deviation.

First, find the mean of the data.

$$\frac{10 + 5 + 20 + 15 + 10}{5} = \frac{60}{5} = 12$$

Next, find the absolute value of the difference between the mean and each value.

$|10 - 12| = 2; |5 - 12| = 7; |20 - 12| = 8;$
$|15 - 12| = 3; |10 - 12| = 2$

Now, find the mean of the differences.

$$\frac{2 + 7 + 8 + 3 + 2}{5} = \frac{22}{5} = 4.4$$

The mean absolute deviation is 4.4.

EXAMPLE 5

Find the mean, variance, and standard deviation for 2, 4, 3, 5, and 6.

mean: $\frac{2 + 4 + 3 + 5 + 6}{5} = \frac{20}{5} = 4$

$\sigma^2 = \frac{(2 - 4)^2 + (4 - 4)^2 + (3 - 4)^2 + (5 - 4)^2 + (6 - 4)^2}{5}$

$\sigma^2 = \frac{4 + 0 + 1 + 1 + 4}{5}$ or 2

$\sigma = \sqrt{2}$ or about 1.4

12-4 **Probability with Permutations and Combinations** (pp. 764–770)

Evaluate each expression.

20. $C(10, 3)$ **21.** $C(9, 5)$

22. $P(6, 3)$ **23.** $P(5, 4)$

24. PHOTOS The Spanish teacher at South High School wants to arrange 7 students who traveled to Mexico for a yearbook photo.

a. Is this a permutation or combination?

b. How many ways can the students be arranged?

EXAMPLE 6

Find $C(8, 3)$.

$C(8, 3) = \frac{8!}{(8 - 3)!3!} = \frac{8!}{5!3!} = \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 3 \cdot 2 \cdot 1}$

$= \frac{8 \cdot 7 \cdot 6}{6} = 56$

EXAMPLE 7

Find $P(4, 2)$.

$P(4, 2) = \frac{4!}{(4 - 2)!} = \frac{4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1} = 4 \cdot 3 = 12$

12-5 Probability of Compound Events (pp. 771–778)

A box contains 8 red chips, 6 blue chips, and 12 white chips. Three chips are randomly drawn from the box and are not replaced.

25. P(red, white, blue) **26.** P(red, red, red)

27. P(red, white, white) **28.** P(blue, blue)

One card is randomly drawn from a standard deck of 52 cards. Find each probability.

29. P(heart or red)

30. P(10 or spade)

EXAMPLE 8

A bag of colored paper clips contains 30 red clips, 22 blue clips, and 22 green clips. Find each probability if three clips are drawn randomly from the bag and are not replaced. Find P(blue, red, green).

First clip: $P(\text{blue}) = \frac{22}{74}$ Second clip: $P(\text{red}) = \frac{30}{73}$

Third clip: $P(\text{green}) = \frac{22}{72}$

$P(\text{blue, red, green}) = \frac{22}{74} \cdot \frac{30}{73} \cdot \frac{22}{72} = \frac{605}{16{,}206}$

12-6 Probability Distributions (pp. 779–786)

A local cable provider asked its subscribers how many television sets they had in their homes. The results of their survey are shown in the probability distribution.

X = Number of Televisions	Probability
1	0.18
2	0.36
3	0.34
4	0.08
5+	0.04

31. Show that the distribution is valid.

32. If a household is selected at random, what is the probability that it has fewer than 4 televisions?

EXAMPLE 9

The table shows the probability distribution for the number of activities in which students at Midpark High School participate.

X = Number of Activities	Probability
0	0.04
1	0.12
2	0.37
3	0.30
4+	0.17

What is the probability that a randomly chosen student participates in 1 to 3 activities?

$P(1 \le X \le 3) = P(X = 1) + P(X = 2) + P(X = 3)$

$= 0.12 + 0.37 + 0.30$

$= 0.79 \text{ or } 79\%$

12-7 Probability Simulations (pp. 787–792)

The results of a simulation of coin flipping are shown.

Outcome	Frequency
heads	25
tails	75

33. What is the experimental probability of heads?

34. What is the experimental probability of tails?

35. What is the theoretical probability of heads?

36. How can guessing randomly on a true-false question be simulated?

EXAMPLE 10

On a multiple-choice test with four choices, Maya answered 18 out of 20 correctly by guessing randomly. Describe a way to simulate this method.

A spinner with 4 equal sections could be spun 20 times with the results of each spin being recorded.

Identify each sample, and suggest a population from which it was selected. Then classify the type of data collection used.

1. **TOYS** A toy company invites 50 children in to test a new toy and records the reactions.

2. **FLOWERS** A nursery is sending out questionnaires to determine which flowers people like best. They send the questionnaires out to all people over 50 on their mailing list.

3. **MULTIPLE CHOICE** On a multiple-choice test with four choices, Zack answered 12 out of 20 questions correctly. What could be used to simulate his correctly answering a question?

   A tossing a coin

   B rolling a six-sided number cube

   C spinning a spinner with four equal sections

   D rolling a three-sided number cube

Evaluate each expression.

4. $P(7, 5)$

5. $C(10, 4)$

6. $C(7, 2)$

7. $P(6, 3)$

Which measure of central tendency best represents the data? Justify your answer. Then find the measure.

8. **VOTING** The polling place kept a list of all the ages of the people who voted: 21, 25, 32, 41, 32, 20, 65, 33, 30, 72.

9. **SHOPPING** A department store kept track of the number of items shoppers purchased on a given day: 3, 5, 4, 3, 4, 5, 5, 3, 2, 3, 2, 10.

Find the mean, variance, and standard deviation to the nearest tenth for each set of data.

10. 4, 5, 5, 6, 6, 8, 9, 10

11. 22, 25, 27, 30

12. 10, 10, 12, 14

13. **SALES** Nate is keeping track of how much people spent at the school bookstore in one day. Find the mean absolute deviation for the data to the nearest tenth: 1, 1, 2, 3, 4, 5, 12.

Identify each sample as *biased* or *unbiased*. Explain your reasoning.

14. **NEWSPAPERS** A survey is sent to all people who subscribe to *The Dispatch* to determine what newspaper people prefer to read.

15. **SHOPPING** Each person leaving the Maxtowne Mall is asked to name their favorite clothing store in the mall.

16. **PIZZA** How many ways can 3 different toppings be chosen from a list of 10 toppings?

17. What is the theoretical probability of tossing heads when a coin is tossed?

18. A die is rolled twice. What is the probability of getting a 2 then a 3?

19. **EDUCATION** Kristin surveys 200 people in her school to determine how many nights a week students do homework. The results are shown.

Number of Nights	Number of Students
0	10
1	30
2	50
3	90
4	10
5 or more	10

   a. Find the probability that a randomly chosen student will have studied more than 4 nights.

   b. Find the probability that a randomly chosen student will have studied no more that 3 nights.

20. **MULTIPLE CHOICE** The second graders are divided into boys and girls. Then 2 girls and 2 boys are chosen at random to represent the class at the Pride Assembly. Which of the following best describes the sample?

   F simple

   G stratified

   H systematic

   J none of the above

Organize Data

Sometimes you may be given a set of data that you need to analyze in order to solve problems on a standardized test. Use this lesson to practice organizing data to help you solve problems.

Strategies for Organizing Data

Step 1

When you are given a problem statement containing data, consider:

- **making a list** of the data.

- **using a table** to organize the data.

- **using a data display** (such as a bar graph, Venn diagram, circle graph, line graph, or box-and-whisker plot) to organize the data.
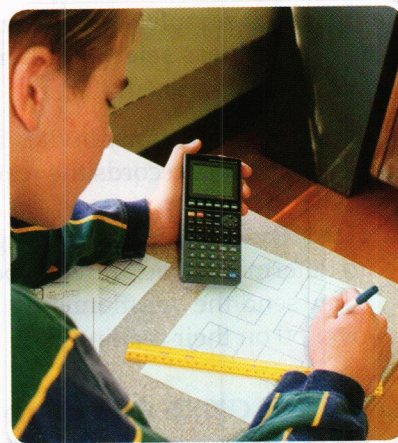
Step 2

Organize the data.

- Create your table, list, or data display.

- If possible, fill in any missing values that can be found by intermediate computations.

Step 3

Analyze the data to solve the problem.

- Reread the problem statement to determine what you are being asked to solve.

- Use the properties of algebra to work with the organized data and solve the problem.

- If time permits, go back and check your answer.

EXAMPLE

Read the problem. Identify what you need to know. Then use the information in the problem to solve. Show your work.

Of the 24 students in a music class, 10 play the flute, 14 play the piano, and 13 play the guitar. Two students play the flute only, 5 the piano only, and 7 the guitar only. One student plays the flute and the guitar but not the piano. Two students play the piano and guitar but not the flute. Three students play all the instruments. If a student is selected at random, what is the probability that he or she plays the piano and flute, but not the guitar?

| Scoring Rubric	
Criteria	**Score**
Full Credit: The answer is correct and a full explanation is provided that shows each step.	2
Partial Credit: • The answer is correct, but the explanation is incomplete. • The answer is incorrect, but the explanation is correct.	1
No Credit: Either an answer is not provided or the answer does not make sense.	0

Read the problem carefully. The data is difficult to analyze as it is presented. Use a Venn diagram to organize the data and solve the problem.

Example of a 2-point response:

Use a Venn diagram to organize the data. Fill in all of the information given in the problem statement. There are 14 students who play the piano, so $14 - 5 - 2 - 3$ or 4 students play the piano and the flute, but not the guitar. Find the probability.

$P(\text{piano and flute}) = \frac{4}{24}$ or $\frac{1}{6}$

So, the probability that a randomly selected student plays the piano and flute but not the guitar is $\frac{1}{6}$.
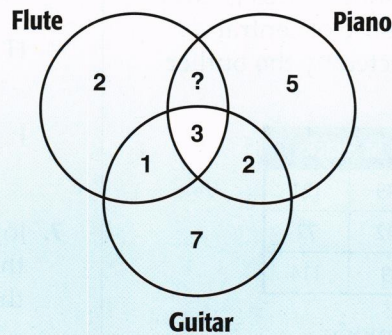
Exercises

Read the problem. Identify what you need to know. Then use the information in the problem to solve. Show your work.

1. There are 40 students, 9 camp counselors, and 5 teachers at Camp Kern. Each person is assigned to one activity this afternoon. There are 9 students going hiking and 17 students going horseback riding. Of the camp counselors, 2 will supervise the hike and 3 will help with the canoe trip. There are 2 teachers helping with the canoe trip and 2 going horseback riding. Suppose a person is selected at random during the afternoon activities. What is the probability that the one selected is a student on the canoe trip or a camp counselor on a horse? Express your answer as a fraction.

2. The table shows the number of coins in a piggy bank.

Coin	Number
Penny	16
Nickel	18
Dime	20
Quarter	10

   a. Find the probability that a randomly selected coin will be a dime.

   b. Find the probability that a randomly selected coin will be either a nickel or a quarter.

3. It takes Craig 40 minutes to mow his family's lawn. His brother Jacob can do the same job in 50 minutes. How long would it take them to mow the lawn together? Round your answer to the nearest tenth of a minute.

Multiple Choice

Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.

1. What are the excluded values of the variable in the expression $\dfrac{x^2 - x - 12}{x^2 - x - 2}$?

   A −1, 2

   B −2, 2

   C −2, 1

   D −3, 4

2. The table shows the number of Calories in twelve different snacks. Which measure of central tendency would be *most* affected by the outlier 342 Calories?

Number of Calories in Snacks			
122	87	149	121
64	138	342	72
179	105	99	114

   F mean

   G median

   H mode

   J range

3. Which of the following is *not* a factor of $x^4 - 6x^2 - 27$?

   A $x^2 + 3$

   B $x - 3$

   C $x + 3$

   D $x^2 - 3$

4. Eduardo has 20 CDs. He wants to choose 3 of them at random to take on a road trip. How many different ways can he do this if the order is *not* important?

   F 60

   G 84

   H 1,140

   J 6,840

5. Which of the following does *not* accurately describe the graph $y = -2x^2 + 4$?

   A The parabola is symmetric about the *y*-axis.

   B The parabola opens downward.

   C The parabola has the origin as its vertex.

   D The parabola crosses the *x*-axis in two different places.

6. The highest point in North Carolina is Mt. Mitchell at an elevation of 2,037 meters above sea level. Suppose the position of a hiker is given by the function $p(t) = -2.5t + 2,037$, where t is the number of minutes. Which of the following is the best interpretation of the slope of the function?

   F The hiker's initial position was 2,037 feet below sea level.

   G The hiker's initial position was 2,037 feet above sea level.

   H The hiker is descending at a rate of 2.5 meters per minute.

   J The hiker is ascending at a rate of 2.5 meters per minute

7. Jorge has made 39 out of 52 free throw attempts this season. What is the experimental probability that he makes a free throw?

   A 54%

   B 68%

   C 75%

   D 79%

8. Which equation passes through the points $(-1, -3)$ and $(-2, 3)$.

   F $y = -6x - 9$

   G $y = -\dfrac{1}{4}x + 3$

   H $y = 4x - 5$

   J $y = \dfrac{2}{3}x + 1$

9. At a museum, each child admission costs $5.75 and each adult costs $8.25. How much does it cost a family that consists of 2 adults and 4 children?

   A $34.50

   B $39.50

   C $44.50

   D $49.50

Test-TakingTip

Question 4 Since order is not important, you are looking for the number of combinations of CDs that can be chosen.
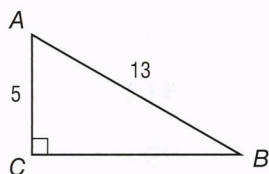
Short Response/Gridded Response

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

10. **GRIDDED RESPONSE** Suppose Colleen spins the spinner below 80 times and records the results in a frequency table. How many times should she expect to spin a vowel?
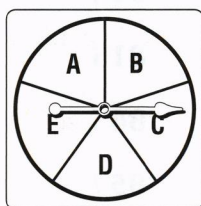
11. What is the value of sin B? Express your answer as a fraction.

12. Graph $f(x) \geq |x - 2|$ on a coordinate grid.

13. **GRIDDED RESPONSE** Find the standard deviation of the set of data below to the nearest tenth.

14	11	9	6
10	16	15	13
9	12	19	10

14. Larissa has 5 peanut butter cookies, 7 chocolate chip cookies, 4 sugar cookies, and 9 oatmeal raisin cookies in a jar. If she picks two cookies at random without replacing them, what is the probability that she will choose a peanut butter cookie then a sugar cookie? Express your answer as a fraction.

15. Write an expression that describes the area in square units of a triangle with a height of $4c^3d^2$ and a base of $3cd^4$.

16. Casey made 84 field goals during the basketball season for a total of 183 points. Each field goal was worth either 2 or 3 points. How many 2-point and 3-point field goals did Casey make during the season?

17. **GRIDDED RESPONSE** The booster club pays $180 to rent a concession stand at a football game. They purchase cans of soda for $0.25 and sell them at the game for $1.15. How many cans of soda must they sell to break even?

Extended Response

Record your answers on a sheet of paper. Show your work.

18. To predict whether or not an issue on a ballot will pass or fail, a committee randomly calls 250 houses with area codes that are inside the voting district and asks the opinions of registered voters. Based on these efforts, the committee determines that 71% ($\pm$2.5%) of the voting population supports the issue. The committee concludes that the issue will pass.

   a. Identify the sample.

   b. Describe the population.

   c. What method of data collection did the committee use: survey, experiment, or observational survey? Explain.

   d. Is the sample *biased* or *unbiased*. Explain.

   e. If unbiased, classify the sample as *simple*, *stratified*, or *systematic*. Explain.

Need Extra Help?

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Reference