#### **Get Ready for Chapter 2** Diagnose Readiness You have two options for checking Prerequisite Skills. **Text Option** Take the Quick Check below. Refer to the Quick Review for help. **Quick**Check **Quick**Review **EXAMPLE 1** Write an algebraic expression for each verbal expression. (Lesson 1-1) Write an algebraic expression for the phrase the product of eight and w increased by nine. **1.** four less than three times a number *n* the product of eight and *w* increased by nine **2.** a number *d* cubed less seven 9 8 W + **3.** the difference between two times *b* and eleven The expression is 8w + 9. EXAMPLE 2 Evaluate each expression. (Lesson 1-2) **4.** $(9-4)^2 + 3$ **5.** $\frac{3 \cdot 8 - 12 \div 2}{3^2}$ Evaluate 9 - $\left[\frac{8+2^2}{2} - 2(5 \times 2 - 8)\right]$ . **6.** $5(8-2) \div 3$ **7.** $\frac{1}{3}(21) + \frac{1}{8}(32)$ $9 - \left[\frac{8+2^2}{2} - 2(5 \times 2 - 8)\right]$ **Original expression 8.** $72 \div 9 + 3 \cdot 2^3$ **9.** $\frac{11-3}{2} + 7$ $=9 - \left[\frac{8+2^2}{2} - 2(2)\right]$ **Evaluate inside 10.** $2[(5-3)^2+8] + (3-1) \div 2$ the parentheses. $=9-\left(\frac{8+2^2}{2}-4\right)$ 11. BAKERY Sue buys 1 carrot cake for \$14, **Multiply.** 6 large chocolate chip cookies for \$1.50 $=9-\left(\frac{8+4}{2}-4\right)$ **Evaluate the** each, and a dozen doughnuts for \$0.45 power. each. How much money did Sue spend = 9 - (6 - 4)Add and then divide. at the bakery? = 7Simplify. **EXAMPLE 3** Find each percent. (Lesson 0-6) 32 is what percent of 40? **12.** What percent of 400 is 260? $\frac{a}{b} = \frac{p}{100}$ Use the percent proportion. **13.** Twelve is what percent of 60? $\frac{32}{40} = \frac{p}{100}$ 14. What percent of 25 is 75? Replace *a* with 32 and *b* with 40. 15. ICE CREAM What Favorite Number of 32(100) = 40pFind the cross products. percent of the Flavor Responses 3200 = 40p**Multiply.** people surveyed vanilla 82 prefer strawberry 80 = pDivide each side by 40. chocolate 76 ice cream? 32 is 80% of 40. strawberry 42

**Online Option** 

**Math Online** Take a self-check Chapter Readiness Quiz at glencoe.com.

# **Get Started on Chapter 2**

You will learn several new concepts, skills, and vocabulary terms as you study Chapter 2. To get ready, identify important terms and organize your resources. You may wish to refer to **Chapter 0** to review prerequisite skills.



X

# Then

You evaluated and simplified algebraic expressions. (Lesson 1-2)

#### Now/

- Translate sentences into equations.
- Translate equations into sentences.

#### New/ Vocabulary formula

# Math Online

- Extra Examples
- Personal Tutor
- Self-Check Quiz
- Homework Help

# **Writing Equations**

# Why?

The Daytona 500 is widely considered to be the most important event of the NASCAR circuit. The distance around the track is 2.5 miles, and the race is a total of 500 miles. We can write an equation to determine how many laps it takes to finish the race.



**Write Verbal Expressions** To write an equation, identify the unknown for which you are looking and assign a variable to it. Then, write the sentence as an equation. Look for key words such as *is, is as much as, is the same as,* or *is identical to* that indicate where you should place the equals sign.

Consider the Daytona 500 example above.



# EXAMPLE 1 Translate Sentences into Equations

Translate each sentence into an equation.

**a**. Seven times a number squared is five times the difference of *k* and *m*.

Seven	times	n squared	is	five	times	the difference of k and m.
7	•	$n^2$	=	5	•	(k-m)
The ed	uation	is $7n^2 = 50$	k - n	n).		

#### **b**. Fifteen times a number subtracted from 80 is 25.

You can rewrite the verbal sentence so it is easier to translate. *Fifteen times a number subtracted from 80* is the same as *80 minus 15 times a number is 25*. Let *n* represent the number.

**80** minus 15 times a number is 25.  $80 - 15 \cdot n = 25$ The equation is 80 - 15n - 25

The equation is 80 - 15n = 25.

# Check Your Progress

**1A.** Two plus the quotient of a number and 8 is the same as 16.

**1B.** Twenty-seven times *k* is *h* squared decreased by 9.

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#### Real-World Link

In 1919, Britain and France offered a flight that carried two passengers at a time. Now there are approximately 45,000 flights each day in the U.S., carrying hundreds of passengers on each flight.

Source: Flightaware

Translating sentences to algebraic expressions and equations is a valuable skill in solving real-world problems.

## Real-World EXAMPLE 2 Use the Four-Step Problem-Solving Plan

# **AIR TRAVEL** Refer to the information at the left. In how many days will 180,000 flights have occurred in the United States?

- **Understand** The information given in the problem is that there are approximately 45,000 flights per day in the United States. We are asked to find how many days it will take for 180,000 flights to have occurred.
  - **Plan** Write an equation. Let *d* represent the number of days needed.

	45,000	times	the number of days	equals	180,000.	
	45,000	•	d	=	180,000	
е	45,000 đ	= 180,0	00 Find <i>d</i> by askin	ng, "What n	umber times 45	5,000 is 180,000?"

**Check** Check your answer by substituting 4 for *d* in the equation.

 $45,000(4) \stackrel{?}{=} 180,000$ 

d=4

 $180,000 = 180,000 \checkmark$  Multiply.

The answer makes sense and works for the original problem.

Substitute 4 for d.

## Check Your Progress

Solv

**2. GOVERNMENT** There are 50 members in the North Carolina Senate. This is 70 fewer than the number in the North Carolina House of Representatives. How many members are in the North Carolina House of Representatives?

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A rule for the relationship between certain quantities is called a **formula**. These equations use variables to represent numbers and form general rules.

## EXAMPLE 3 Write a Formula

**GEOMETRY** Translate the sentence into a formula.

The area of a triangle equals the product of  $\frac{1}{2}$  the length of the base and the height.

Words	The area of equals	the product of $\frac{1}{2}$ the length of the base and the height.
Variables	Let $A = $ area, $b = $ base,	and $h =$ height.
Equation	A =	$\frac{1}{2}bh$

The formula for the area of a triangle is  $A = \frac{1}{2}bh$ .

## Check Your Progress

**3. GEOMETRY** Translate the sentence into a formula. In a right triangle, the square of the measure of the hypotenuse *c* is equal to the sum of the squares of the measures of the legs, *a* and *b*.

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#### Math History Link

#### Ahmes

(about 1680–1620 в.с.) Ahmes was the Egyptian mathematician and scribe who copied the Rhind Mathematical Papyrus. The papyrus contains 87 algebra problems of the same type. The first set of problems asks how to divide *n* loaves of bread among 10 people. **Write Sentences from Equations** If you are given an equation, you can write a sentence or create your own word problem.



When given a set of information, you can create a problem that relates a story.

#### EXAMPLE 5 Write a Problem

#### Write a problem based on the given information.

t = the time that Maxine drove; t + 4 = the time that Tia drove; 2t + (t + 4) = 28

#### Sample problem:

Maxine and Tia went on a trip, and they took turns driving. During her turn, Tia drove 4 hours more than Maxine. Maxine took 2 turns, and Tia took 1 turn. Together they drove for 28 hours. How many hours did Maxine drive?

#### Check Your Progress

**5.** p = Beth's salary; 0.1p = bonus; p + 0.1p = 525

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# 🗹 Check Your Understanding

- Example 1 p. 75
- Translate each sentence into an equation.
  - **1.** Three times *r* less than 15 equals 6.
  - **2.** The sum of *q* and four times *t* is equal to 29.
  - A number n squared plus 12 is the same as the quotient of p and 4.
  - **4.** Half of *j* minus 5 is the sum of *k* and 13.
  - **5.** The sum of 8 and three times *k* equals the difference of 5 times *k* and 3.
  - 6. Three fourths of *w* plus 5 is one half of *w* increased by nine.
  - **7.** The quotient of 25 and *t* plus 6 is the same as twice *t* plus 1.
  - 8. Thirty-two divided by *y* is equal to the product of three and *y* minus four.

<b>Example 2</b> p. 76	<b>9. FINANCIAL LITERACY</b> Samuel has \$1900 in the bank. He wishes to increase his account to a total of \$2500 by depositing \$30 per week from his paycheck. Write and solve an equation to find how many weeks he needs to reach his goal.					
	<b>10. PAINTING</b> Miguel is earning extra money by painting houses. He charges a \$200 fee plus \$12 per can of paint needed to complete the job. Write and use an equation to find how many cans of paint he needs for a \$260 job.					
Example 3	Translate each sentence into a formula.					
р. 76	<b>11.</b> The perimeter of a regular pentagon is 5 times the length of each side.					
	<b>12.</b> The area of a circle is the product of $\pi$ and the radius <i>r</i> squared.					
	<b>13.</b> Four times $\pi$ times the radius squared is the surface area of a sphere.					
	<b>14.</b> One third the product of the length of the side squared and the height is the volume of a pyramid with a square base.					
Example 4	Translate each equation into a sentence.					
p. 77	<b>15.</b> $7m - q = 23$ <b>16.</b> $6 + 9k + 5j = 54$					
	<b>17.</b> $3(g+8) = 4h - 10$ <b>18.</b> $6d^2 - 7f = 8d + f^2$					
Example 5	Write a problem based on the given information.					
p. 77	<b>19.</b> $g = \text{gymnasts on a team}; 3g = 45$					
	<b>20.</b> $c = \text{cost of a notebook}; 0.25c = \text{markup}; c + 0.25c = 3.75$					
	= Step-by-Step Solutions begin on page R12.					

# **Practice and Problem Solving**

Example 1 p. 75

#### Translate each sentence into an equation.

- **21.** The difference of *f* and five times *g* is the same as 25 minus *f*.
- **22.** Three times *b* less than 100 is equal to the product of 6 and *b*.
- **23.** Four times the sum of 14 and *c* is *a* squared.
- Example 2 p. 76
- **24. MUSIC** A piano has 52 white keys. Write and use an equation to find the number of octaves on a piano keyboard.
- **25. GARDENING** A flat of plants contains 12 plants. Yoshi wants a garden that has three rows with 10 plants per row. Write and solve an equation for the number of flats Yoshi should buy.

Translate each sentence into a formula.



Extra Practice begins on page 815.

**Example 3** 

p. 76

p. 77

- **26.** The perimeter of a rectangle is equal to 2 times the length plus twice the width.
  - 27 Celsius temperature *C* is five ninths times the difference of the Fahrenheit temperature *F* and 32.
- **28.** The density of an object is the quotient of its mass and its volume.
- **29.** Simple interest is computed by finding the product of the principal amount *p*, the interest rate *r*, and the time *t*.

#### **Example 4** Translate each equation into a sentence.

<b>30.</b> $j + 16 = 35$	<b>31.</b> 4 <i>m</i> = 52	<b>32.</b> $7(p + 23) = 102$
<b>33.</b> $r^2 - 15 = t + 19$	<b>34.</b> $\frac{2}{5}v + \frac{3}{4} = \frac{2}{3}x^2$	<b>35.</b> $\frac{1}{3} - \frac{4}{5}z = \frac{4}{3}y^3$

Example 5

p. 77

#### Write a problem based on the given information.

- **36.** q =quarts of strawberries; 2.50q = 10
- **37.** p = the principal amount; 0.12p = the interest charged; p + 0.12p = 224
- **38.** m = number of movies rented; 10 + 1.50m = 14.50
- **39.** p = the number of players in the game; 5p + 7 = number of cards in a deck

#### For Exercises 40–43, match each sentence with an equation.

<b>A.</b> $g^2 = 2(g - 10)$	<b>C.</b> $g^3 = 24g + 4$
<b>B.</b> $\frac{1}{2}g + 32 = 15 + 6g$	<b>D.</b> $3g^2 = 30 + 9g$

- **40.** One half of *g* plus thirty-two is as much as the sum of fifteen and six times *g*.
- **41.** A number *g* to the third power is the same as the product of 24 and *g* plus 4.
- **42.** The square of *g* is the same as two times the difference of *g* and 10.
- **43.** The product of 3 and the square of *g* equals the sum of thirty and the product of nine and *g*.
- **44. FINANCIAL LITERACY** Tim's bank contains quarters, dimes, and nickels. He has three more dimes than quarters and 6 fewer nickels than quarters. If he has 63 coins, write and solve an equation to find how many quarters Tim has.
- **SHOPPING** Pilar bought 17 items for her camping trip, including tent stakes, packets of drink mix, and bottles of water. She bought 3 times as many packets of drink mix as tent stakes. She also bought 2 more bottles of water than tent stakes. Write and solve an equation to discover how many tent stakes she bought.
- **46. WULTIPLE REPRESENTATIONS** In this problem, you will explore how to translate relations with powers.

x	2	3	4	5	6
у	5	10	17	26	37

- **a. VERBAL** Write a sentence to describe the relationship between *x* and *y* in the table.
- **b. ALGEBRAIC** Write an equation that represents the data in the table.
- **c. GRAPHICAL** Graph each ordered pair and draw the function. Describe the graph as discrete or continuous.

#### H.O.T. Problems

Use Higher-Order Thinking Skills

- **47. OPEN ENDED** Write a problem about your favorite television show that uses the equation x + 8 = 30.
- **48. REASONING** The surface area of a three-dimensional object is the sum of the areas of the faces. If  $\ell$  represents the length of the side of a cube, write a formula for the surface area of the cube.
- **49. CHALLENGE** Given the perimeter *P* and width *w* of a rectangle, write a formula to find the length  $\ell$ .
- **50.** WRITING IN MATH Explain how to translate a verbal sentence into an algebraic equation. Include any tips that you may have for your fellow students.



Real-World Link

There are more than 16,000 commercial and public campgrounds nationwide. Camping is the number one outdoor vacation activity in America.

Source: Travel Industry Association of America

# **Standardized Test Practice**

**51.** Which equation *best* represents the relationship between the number of hours an electrician works *h* and the total charges *c*?

Cost of Electrician						
Emergency House Call	\$30 one time fee					
Rate	\$55/hour					

- **A** c = 30 + 55
- **B** c = 30h + 55
- **C** c = 30 + 55h
- **D** c = 30h + 55h
- **52.** A car traveled at 55 miles per hour for 2.5 hours and then at 65 miles per hour for 3 hours. How far did the car travel in all?
  - F
     300.5 mi
     H
     330 mi

     G
     305 mi
     J
     332.5 mi

**53. SHORT RESPONSE** Suppose each dimension of rectangle *ABCD* is doubled. What is the perimeter of the new *ABCD*?



- **54. STATISTICS** Stacy's first five science test scores were 95, 86, 83, 95, and 99. Which of the following is a true statement?
  - A The mode is the same as the median.
  - **B** The median is the same as the mean.
  - **C** The range is the same as the mode.
  - **D** The mode is the same as the mean.

# **Spiral Review**

#### Write a counterexample for each conditional statement. (Lesson 1-8)

- **55.** If you were born in Florida, then you live in Florida.
- 56. If the product of two numbers is an even number, then both factors must be even numbers.
- **57.** If a number is divisible by 2, then it is divisible by 4.
- 58. SHOPPING Cuties is having a sale on earrings (Lesson 1-7)
  - **a.** Make a table that shows the cost of buying 1 to 5 pairs of earrings.
  - **b.** Write the data as a set of ordered pairs.
  - **c.** Graph the data.
- 59. GEOMETRY Refer to the table below. (Lesson 1-6)



Polygon	triangle	quadrilateral	pentagon	hexagon	heptagon
Number of Sides	3	4	5	6	7
Interior Angle Sum	180	360	540	720	900

- a. Identify the independent and dependent variables.
- **b.** Identify the domain and range for this situation.
- c. State whether the function is *discrete* or *continuous*. Explain.

# **Skills Review**

**Evaluate each expression.** (Lesson 1-1) **60.** 9<sup>2</sup> **61.** 10<sup>6</sup>

**62.** 3<sup>5</sup>

**63.** 5<sup>3</sup>

You can use **algebra tiles** to model solving equations. To **solve an equation** means to find the value of the variable that makes the equation true. An  $\begin{bmatrix} x \\ -x \end{bmatrix}$  tile represents the variable *x*. The **1** tile represents a positive 1. The **1** tile represents a negative 1. And, the **1** tile represents the variable negative *x*. The goal is to get the *x*-tile by itself on one side of the mat by using the rules stated below.



# ACTIVITY 1 Addition Equation

EXPLORE

Use an equation model to solve x + 3 = -4.

- **Step 1** Model the equation. Place 1 *x*-tile and 3 positive 1-tiles on one side of the mat. Place 4 negative 1-tiles on the other side of the mat.
- **Step 2** Isolate the *x*-term. Add 3 negative 1-tiles to each side. The resulting equation is x = -7.



# ACTIVITY 2 Subtraction Equation

Use an equation model to solve x - 2 = 1.



Place 1 *x*-tile and 2 negative 1-tiles on one side of the mat. Place 1 positive 1-tile on the other side of the mat. Then add 2 positive 1-tiles to each side.



Group the tiles to form zero pairs. Then remove all the zero pairs. The resulting equation is x = 3.

(continued on the next page)

#### **Model and Analyze**

Use algebra tiles to solve each equation.

**1.** x + 4 = 9 **2.** x + (-3) = -4 **3.** x + 7 = -2 **4.** x + (-2) = 11

**5.** WRITING IN MATH If a = b, what can you say about a + c and b + c? about a - c and b - c?

When solving multiplication equations, the goal is still to get the *x*-tile by itself on one side of the mat by using the rules for dividing.



#### ACTIVITY 3 Multiplication Equation

#### Use an equation model to solve 3x = 12.

- **Step 1** Model the equation. Place 3 *x*-tiles on one side of the mat. Place 12 positive 1-tiles on the other side of the mat.
- **Step 2** Isolate the *x*-term. Separate the tiles into 3 equal groups to match the 3 *x*-tiles. Each *x*-tile is paired with 4 positive 1-tiles. The resulting equation is x = 4.



#### **Model and Analyze**

Use algebra tiles to solve each equation.

**6.** 5x = -15

**7.** -3x = -9

**8.** 4x = 8

**9.** -6x = 18

- **10. MAKE A CONJECTURE** How would you use algebra tiles to solve  $\frac{x}{4} = 5$ ? Discuss the steps you would take to solve this equation algebraically.
- 82 Chapter 2 Linear Equations

# **Solving One-Step Equations**

# Why?

A record for the most snow angels made at one time was set in Michigan when 3784 people participated. North Dakota had 8910 people register to break the record. To determine how many more people North Dakota had than Michigan, solve the equation 3784 + x = 8910.



**Solve Equations Using Addition or Subtraction** In an equation, the variable represents the number that satisfies the equation. To **solve an equation** means to find the value of the variable that makes the equation true.

The process of solving an equation involves isolating the variable (with a coefficient of 1) on one side of the equation. Each step in this process results in equivalent equations. **Equivalent equations** have the same solution.

Key (	Concept Addition Property of Equality	For Your			
Words	If an equation is true and the same number is added to each side of the equation, the resulting equivalent equation is also true.				
Symbols	For any real numbers $a, b$ , and $c$ , if $a = b$ , then $a + c = b$	+ c.			
Examples	$ \begin{array}{rrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrr$				

# EXAMPLE 1 Solve by Adding

Solve c - 22 = 54.

#### **Horizontal Method**

c - 22 = 54c - 22 + 22 = 54 + 22c = 76 Original equation Add 22 to each side. Simplify. Vertical Method c - 22 = 54

+22 = +22c = 76

To check that 76 is the solution, substitute 76 for *c* in the original equation.

**CHECK** c - 22 = 54 Original equation  $76 - 22 \stackrel{?}{=} 54$  Substitute 76 for c.  $54 = 54\checkmark$  Subtract.

# **Check Your Progress** Solve each equation.

**1A.** 113 = g - 25

**1B.** j - 87 = -3

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# Math Online

solve an equation

equivalent equations

# glencoe.com

Then

into equations.

(Lesson 2-1)

Now/

division.

You translated sentences

Solve equations by

 Solve equations by using multiplication and

New/Vocabulary/

using addition and subtraction.

- Extra Examples
- Personal TutorSelf-Check Ouiz
- Homework Help

Similar to the Addition Property of Equality, the **Subtraction Property of Equality** can also be used to solve equations.

# **Study**Tip

Subtraction Subtracting a value is equivalent to adding the opposite of the value.

Key (	Concept Subtract	ion Property of Equality	For Your
Words	If an equation is true and from each side of the eq equation is also true.	d the same number is subtracted juation, the resulting equivalent	
Symbols	For any real numbers a,	b, and c, if $a = b$ , then $a - c = b$	b — с.
Examples	87 = 87 87 - 17 = 87 - 17 70 = 70	$ \begin{array}{rcl} 13 &=& 13 \\ -28 &=& -28 \\ \hline -15 &=& -15 \end{array} $	

#### StudyTip

Solving an Equation When solving equations you can use either the horizontal method or the vertical method. Both methods will produce the same result.

## EXAMPLE 2 Solve by Subtracting

Solve $63 + m = 79$ .				
Horizontal Method		Vertic	al Method	
63 + m = 79	<b>Original equation</b>	63 +	m = 79	
63 - 63 + m = 79 - 63	Subtract 63 from each side.	-63	= -63	
m = 16	Simplify.		m = 16	

To check that 16 is the solution, replace m with 16 in the original equation.

 CHECK
 63 + m = 79 Original equation

  $63 + 16 \stackrel{?}{=} 79$  Substitution, m = 16 

  $79 = 79 \checkmark$  Simplify.

**Check Your Progress** Solve each equation.

**2A.** 
$$27 + k = 30$$

**2B.** 
$$-12 = p + 16$$

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**Solve Equations Using Multiplication or Division** In the equation  $\frac{x}{3} = 9$ , the variable *x* is divided by 3. To solve for *x*, undo the division by multiplying each side by 3. This is an example of the **Multiplication Property of Equality**.

Key	Concept Multiplication Property of Equality For Your
Words	If an equation is true and each side is multiplied by the same nonzero number, the resulting equation is equivalent.
Symbols	For any real numbers $a$ , $b$ , and $c$ , $c \neq 0$ , if $a = b$ , then $ac = bc$ .
Example	If $x = 5$ , then $3x = 15$ .
	Division Property of Equality
Words	If an equation is true and each side is divided by the same nonzero number, the resulting equation is equivalent.
Symbols	For any real numbers a, b, and c, $c \neq 0$ , if $a = b$ , then $\frac{a}{c} = \frac{b}{c}$ .
Example	If $x = -20$ , then $\frac{x}{5} = \frac{-20}{5}$ or $-4$ .

The reciprocal of a number can be used to solve equations.



EXAMPLE 3	Solve by Multiplying and Dividing
Solve each equ	ation.
<b>a.</b> $\frac{2}{3}q = \frac{1}{2}$	
$\frac{2}{3}q = \frac{1}{2}$	Original equation
$\frac{3}{2}\left(\frac{2}{3}\right)q = \frac{3}{2}\left(\frac{1}{2}\right)$	Multiply each side by $\frac{3}{2}$ , the reciprocal of $\frac{2}{3}$ .
$q = \frac{3}{4}$	Check the result.
<b>b.</b> $39 = -3r$	
39 = -3r	Original equation
$\frac{39}{-3} = \frac{-3r}{-3}$	Divide each side by $-3$ .
-13 = r	Check the result.
Check Your Pr	ogress
<b>3A.</b> $\frac{3}{5}k = 6$	<b>3B.</b> $-\frac{1}{4} = \frac{2}{3}b$

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We can also use reciprocals and properties of equality to solve real-world problems.



#### Real-World Link

Almost half of 10- to 18-year-olds in the U.S. use a cell phone. Of those, 53% play games on their phones, more than 33% download games, 52% use the calendar/organizer, and nearly all teens with camera phones snap pictures.

Source: Lexdon Business Library

# Real-World EXAMPLE 4 Solve by Multiplying

**SURVEYS** Of a group of 13- to 15-year-old girls surveyed, 225, or about  $\frac{9}{20}$  said they talk on the telephone while they watch television. About how many girls were surveyed?



About 500 girls were surveyed.

#### Check Your Progress

**4. STAINED GLASS** Allison is making a stained glass window. Her pattern requires that one fifth of the glass should be blue. She has 288 square inches of blue glass. If she intends to use all of her blue glass, how much glass will she need for the entire project?

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# Check Your Understanding

Examples 1 and 3	Solve each equation	. Check your solution.	
рр. 83–85	<b>1.</b> $g + 5 = 33$	<b>2.</b> $104 = y - 67$	<b>3.</b> $\frac{2}{3} + w = 1\frac{1}{2}$
	<b>4.</b> $-4 + t = -7$	<b>5.</b> $a + 26 = 35$	<b>6.</b> $-6 + c = 32$
	<b>7.</b> $1.5 = y - (-5.6)$	<b>8.</b> $3 + g = \frac{1}{4}$	<b>9.</b> $x + 4 = \frac{3}{4}$
	<b>10.</b> $\frac{t}{7} = -5$	<b>11.</b> $\frac{a}{36} = \frac{4}{9}$	<b>12.</b> $\frac{2}{3}n = 10$
	<b>13.</b> $\frac{8}{9} = \frac{4}{5}k$	<b>14.</b> $12 = \frac{x}{-3}$	<b>15.</b> $-\frac{r}{4} = \frac{1}{7}$

#### Example 4 p. 85

- **16. FUNDRAISING** The television show "Idol Gives Back" raised money for relief organizations. During this show, viewers could call in and vote for their favorite performer. The parent company contributed \$5 million for the 50 million votes cast. What did they pay for each vote?
- **17. SHOPPING** Hana decides to buy her cat a bed from an online fund that gives  $\frac{7}{8}$  of her purchase to shelters that care for animals. How much of Hana's money went to the animal shelter?



= Step-by-Step Solutions begin on page R12. Extra Practice begins on page 815.

# **Practice and Problem Solving**

Examples 1 and 3	Solve each equation. Check your solution.		
pp. 83–85	<b>18.</b> $v - 9 = 14$	<b>19.</b> $44 = t - 72$	<b>20.</b> $-61 = d + (-18)$
	<b>21.</b> $18 + z = 40$	<b>22.</b> $-4a = 48$	<b>23.</b> $12t = -132$
	<b>24.</b> $18 - (-f) = 91$	<b>25</b> $-16 - (-t) = -45$	<b>26.</b> $\frac{1}{3}v = -5$
	<b>27.</b> $\frac{u}{8} = -4$	<b>28.</b> $\frac{a}{6} = -9$	<b>29.</b> $-\frac{k}{5} = \frac{7}{5}$
	<b>30.</b> $\frac{3}{4} = w + \frac{2}{5}$	<b>31.</b> $-\frac{1}{2} + a = \frac{5}{8}$	<b>32.</b> $-\frac{t}{7} = \frac{1}{15}$
	<b>33.</b> $-\frac{5}{7} = y - 2$	<b>34.</b> $v + 914 = -23$	<b>35.</b> 447 + <i>x</i> = −261
	<b>36.</b> $-\frac{1}{7}c = 21$	<b>37.</b> $-\frac{2}{3}h = -22$	<b>38.</b> $\frac{3}{5}q = -15$
	<b>39.</b> $\frac{n}{8} = -\frac{1}{4}$	<b>40.</b> $\frac{c}{4} = -\frac{9}{8}$	<b>41.</b> $\frac{2}{3} + r = -\frac{4}{9}$

#### Example 4 p. 85

- **42. CATS** A domestic cat can run at speeds of 27.5 miles per hour when chasing prey. A cheetah can run 42.5 miles per hour faster when chasing prey. How fast can the cheetah go?
- **43. CARS** The average time *t* it takes to manufacture a car in the United States is 24.9 hours. This is 8.1 hours longer than the average time it takes to manufacture a car in Japan. Write and solve an equation to find the average time in Japan.

Solve each equation. Check your solution.



#### Write an equation for each sentence. Then solve the equation.

- **56.** Six times a number is 132.
- **57.** Two thirds equals negative eight times a number.
- **58.** Five elevenths times a number is 55.
- **59.** Four fifths is equal to ten sixteenths of a number.
- 60. Three and two thirds times a number equals two ninths.
  - Four and four fifths times a number is one and one fifth.
- **62. SHOPPING** Adelina is comparing prices for two brands of health and energy bars at the local grocery store. She wants to get the best price for each bar.
  - **a.** Write an equation to find the price for each bar of the Feel Great brand.
  - **b.** Write an equation to find the price of each bar for the Super Power brand.
  - c. Which bar should Adelina buy? Explain.
- **63. MEDIA** The world's largest passenger plane, the Airbus A380, was first used by Singapore Airlines in 2005. The following description appeared on a news Web site after the plane was introduced.

"That airline will see the A380 transporting some 555 passengers, 139 more than a similarly set-up 747." How many passengers will a similarly set-up 747 transport?

- **64. FUEL** In 2004, approximately 5 million cars and trucks were classified as flex-fuel, which means they could run on gasoline or ethanol. In 2006, that number increased to 7.5 million. How many more cars and trucks were flex-fuel in 2006?
- **65. CHEERLEADING** At a certain cheerleading competition, the maximum time per team, including the set up, is 3 minutes. The Ridgeview High School squad's performance time is 2 minutes and 34 seconds. How much time does the squad have left for their set up?
- **66. COMIC BOOKS** An X-Men #1 comic book in mint condition recently sold for \$45,000. An Action Comics #63 (Mile High), also in mint condition, sold for \$15,000. How much more did the X-Men comic book sell for than the Action Comics book?
- **67. MOVIES** A certain movie made \$1.6 million in ticket sales. Its sequel made \$0.8 million in ticket sales. How much more did the first movie make than the sequel?
- **68.** CAMERAS An electronics store sells a certain digital camera for \$126. This is  $\frac{2}{3}$  of the price that a photography store charges. What is the cost of the camera at the photography store?



#### Real-World Link

Ethanol is produced from corn and is considered energy efficient because it yields 25% more energy than the process to create it.

Source: U.S. Department of Energy \$ 18.00

\$ 21.75

Height (cm)

17.1

24.3

28.5

7.2

Base (cm)

3.8

5.4

6.3

1.6

Triangle

 $\triangle ABC$ 

 $\triangle MQP$ 

 $\triangle RST$ 

 $\triangle TRW$ 



#### Real-World Link

Schools have begun using an online voting system that allows students to log in and vote for homecoming king and queen.

Source: NewBay Media

- **BLOGS** In 2006, 57 million American adults read online blogs. However, 45 million fewer American adults say that they maintain their own blog. How many American adults maintain a blog?
- **70. SCIENCE CAREERS** According to the Bureau of Labor and Statistics, approximately 65,000,000 women were employed in the United States in 2004.
  - **a.** The number of women in the computer science fields times 26 is the number of working women. Write an equation to represent the number of women employed in the computer sciences in 2004. Then solve the equation.
  - **b.** The number of women in natural science fields is 2,266,000 less than the number of women in computer science fields. How many women are in natural science fields?
- **71. DANCES** Student Council has a budget of \$1000 for the homecoming dance. So far, they have spent \$350 dollars for music.
  - **a.** Write an equation to represent the amount of money left to spend. Then solve the equation.
  - **b.** They then spent \$225 on decorations. Write an equation to represent the amount of money left.
  - **c.** If the Student Council spent their entire budget, write an equation to represent how many \$6 tickets they must sell to make a profit.

# H.O.T. Problems Use Higher-Order Thinking Skills

**72.** WHICH ONE DOESN'T BELONG? Identify the equation that does not belong with the other three. Explain your reasoning.

$$n + 14 = 27$$
  $12 + n = 25$   $n - 16 = 29$   $n - 4 = 9$ 

- **73. OPEN ENDED** Write an equation involving addition and demonstrate two ways to solve it.
- **74. REASONING** For which triangle is the height not  $4\frac{1}{2}b$ , where *b* is the length of the base?
- **75. CHALLENGE** Determine whether each sentence is *sometimes, always,* or *never true.* Explain your reasoning.

**a.** x + x = x **b.** x + 0 = x

**76. REASONING** Determine the value for each statement below.

**a.** If x - 7 = 14, what is the value of x - 2?

- **b.** If t + 8 = -12, what is the value of t + 1?
- **77.** CHALLENGE Discuss why  $\frac{2}{2}b = 16$  and 48 = 2c have the same solution.
- **78.** WRITING IN MATH Consider the Multiplication Property of Equality and the Division Property of Equality. Explain why they can be considered the same property. Which one do you think is easier to use?

# **Standardized Test Practice**

- **79.** Which of the following best represents the equation w 15 = 33?
  - A Jake added *w* ounces of water to his bottle, which originally contained 33 ounces of water. How much water did he add?
  - **B** Jake added 15 ounces of water to his bottle, for a total of 33 ounces. How much water *w* was originally in the bottle?
  - **C** Jake drank 15 ounces of water from his bottle and 33 ounces were left. How much water *w* was originally in the bottle?
  - **D** Jake drank 15 ounces of water from his water bottle, which originally contained 33 ounces. How much water *w* was left?
- **80. SHORT RESPONSE** Charlie's company pays him for every mile that he drives on his trip. When he drives 50 miles, he is paid \$30. To the nearest tenth, how many miles did he drive if he was paid \$275?

**81.** The table shows the results of a survey given to 500 international travelers. Based on the data, which statement is true?

Vacation Plans		
Destination	Percent	
The Tropics	37	
Europe	19	
Asia	17	
Other	17	
No Vacation	10	

- **F** Fifty have no vacation plans.
- **G** Fifteen are going to Asia.
- H One third are going to the tropics.
- J One hundred are going to Europe.
- **82. GEOMETRY** The amount of water needed to fill a pool represents the pool's \_\_\_\_\_.
  - A volume
- C circumference D perimeter
- **B** surface area

# Spiral Review

#### Translate each sentence into an equation. (Lesson 2-1)

- **83.** The sum of twice *r* and three times *k* is identical to thirteen.
- **84.** The quotient of *t* and forty is the same as twelve minus half of *u*.
- **85.** The square of *m* minus the cube of *p* is sixteen.
- **86.** Two times *z* is equal to two times the sum of *v* and *x*.

#### Write each statement in if-then form. (Lesson 1-8)

- **87.** The trash is picked up on Monday.
- **88.** Vito will call after school.

- **89.** For x = 8,  $x^2 3x = 40$ .
- **90.** 4q + 6 > 42 when q > 9.

# **Skills Review**

- **91. COMMUNICATION** Sato communicates with his friends for a math project. In a week, he averages 5 hours using e-mail, 18 hours on the phone, and 12 hours meeting with them in person. Write and evaluate an expression to predict how many hours he will spend communicating with his friends over the next 12 weeks. (Lesson 1-4)
- **92. PETS** The Poochie Pet Supply Store has the following items on sale. Write and evaluate an expression to find the total cost of purchasing 1 collar, 2 T-shirts, 3 kerchiefs, 1 leash, and 4 flying disks. (Lesson 1-4)

ltem	Cost (\$)
studded collar	4.50
kerchief	3.00
doggy T-shirt	6.25
leash	5.50
flying disk	3.25

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 2-3
 Solving
 Math in Motion, Animation

 Multi-Step Equations
 Math in Motion, Animation

You can use algebra tiles to model solving multi-step equations.



Place 4 *x*-tiles and 3 positive 1-tiles on one side of the mat. Place 5 negative 1-tiles on the other side.



Group the tiles to form zero pairs and remove the zero pairs.



Since there are 3 positive 1-tiles with the *x*-tiles, add 3 negative 1-tiles to each side to form zero pairs.



Separate the remaining tiles into 4 equal groups to match the 4 *x*-tiles. Each *x*-tile is paired with 2 negative 1-tiles. The resulting equation is x = -2.

**3.** 5x - 7 = 8

**7.** 11 = 2x - 5

**4.** -7 = 3x + 8

**8.** 7 + 6x = -11

#### **Model** Use algebra tiles to solve each equation.

- 1. 3x 7 = -10
- **5.** 5 + 4x = -11
- **9.** What would be your first step in solving 8x 29 = 67?

**2.** 2x + 5 = 9

**6.** 3x + 1 = 7

- **10.** What steps would you use to solve 9x + 14 = -49?
- 90 Chapter 2 Linear Equations

# Then

You solved single-step equations. (Lesson 2-2)

#### Now

- Solve equations involving more than one operation.
- Solve equations involving consecutive integers.

#### New/Vocabulary/

multi-step equation consecutive integers number theory

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# **Solving Multi-Step Equations**

# Why?

The Tour de France is the premier cycling event in the world. The map shows the 2007 Tour de France course. If the length of the shortest portion of the race can be represented by k, the expression 4k + 20 is the length of the longest stage or 236 kilometers.

**Solve Multi-Step Equations** The situation above can be described by the equation 4k + 20 = 236. Because this equation requires more than one step to solve, it is called a **multi-step equation**. To solve this equation, we must undo each operation by working backward.



# **EXAMPLE 1** Solve Multi-Step Equations Solve each equation. **a.** 11x - 4 = 29

a.	11x - 4 - 29	
	11x - 4 = 29	Original equation
	11x - 4 + 4 = 29 + 4	Add 4 to each side.
	11x = 33	Simplify.
	$\frac{11x}{11} = \frac{33}{11}$	Divide each side by 11.
	x = 3	Simplify.
b.	$\frac{a+7}{8} = 5$	
	$\frac{a+7}{8} = 5$	Original equation
	$\frac{8}{8}\left(\frac{a+7}{8}\right) = 8(5)$	Multiply each side by 8.
	a + 7 = 40	Simplify.
	-7 = -7	Subtract 7 from each side
	a = 33	Simplify.

You can check your solutions by substituting the results back into the original equations.

#### Check Your Progress

Solve each equation. Check your solution.

**1A.** 
$$2a - 6 = 4$$

**1B.** 
$$\frac{n+1}{-2} = 15$$

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#### Real-World EXAMPLE 2 Write and Solve a Multi-Step Equation

**SHOPPING** Hiroshi is buying a pair of water skis that are on sale for  $\frac{2}{3}$  of the original price. After he uses a \$25 gift certificate, the total cost before taxes is \$115. What was the original price of the skis? Write an equation for the problem. Then solve the equation.



The original price of the skis was \$210.

#### **Check Your Progress**

- **2A. RETAIL** A music store has sold  $\frac{3}{5}$  of their hip-hop CDs, but 10 were returned. Now the store has 62 hip-hop CDs. How many were there originally?
- **2B. READING** Len read  $\frac{3}{4}$  of a graphic novel over the weekend. Monday, he read 22 more pages. If he has read 220 pages, how many pages does the book have?

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**Solve Consecutive Integer Problems** Consecutive integers are integers in counting order, such as 4, 5, and 6 or n, n + 1, and n + 2. Counting by two will result in *consecutive even integers* if the starting integer n is even and *consecutive odd integers* if the starting integer n is odd.

Concept Summary Consecutive Integers For Your FOLDABLE				
Туре	Words	Symbols	Example	
Consecutive Integers	Integers that come in counting order.	n, n + 1, n + 2,	, -2, -1, 0, 1, 2,	
Consecutive Even Integers	Even integer followed by the next even integer.	<i>n</i> , <i>n</i> + 2, <i>n</i> + 4,	, –2, 0, 2, 4,	
Consecutive Odd Integers	Odd integer followed by the next even integer.	n, n + 2, n + 4,	, –1, 1, –3, 5,	

**Number theory** is the study of numbers and the relationships between them.



#### Real-World Link

Shoppers in Shanghai, China, can pay for purchased items at a terminal that can match the buyers' fingerprints with their bank accounts.

Source: Shanghai Daily

#### EXAMPLE 3 Solve a Consecutive Integer Problem

**NUMBER THEORY** Write an equation for the following problem. Then solve the equation and answer the problem.

Find three consecutive odd integers with a sum of -51.

Let n = the least odd integer.

Then n + 2 = the next greater odd integer, and n + 4 = the greatest of the three integers.



n + 2 = -19 + 2 or -17 n + 4 = -19 + 4 or -15. The consecutive odd integers are -19, -17, and -15.

**CHECK** -19, -17, and -15 are consecutive odd integers. $<math>-19 + (-17) + (-15) = -51 \checkmark$ 

#### Check Your Progress

**3.** Write an equation for the following problem. Then solve the equation and answer the problem.

Find three consecutive integers with a sum of 21.

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# 🗹 Check Your Understanding

Example 1

Solve each equation. Check your solution.

p. 91

	3m + 4 = -1	
4.	$\frac{3}{2}a - 8 = 11$	

= -11	<b>2.</b> $12 = -7f - 7f$	9
= 11	<b>5.</b> $8 = \frac{x-5}{7}$	

3.	$-3 = 2 + \frac{a}{11}$
6.	$\frac{c+1}{-3} = -21$

Example 2 p. 92

- **7. NUMBER THEORY** Twelve decreased by twice a number equals -34. Write an equation for this situation and then find the number.
- **8. BASEBALL** Among the career home run leaders for Major League Baseball, Hank Aaron has 175 fewer than twice the number that Dave Winfield has. Hank Aaron hit 755 home runs. Write an equation for this situation. How many home runs did Dave Winfield hit in his career?

Example 3 p. 93

#### Write an equation and solve each problem.

- **9.** Find three consecutive odd integers with a sum of 75.
- **10.** Find three consecutive integers with a sum of -36.

#### Lesson 2-3 Solving Multi-Step Equations 93

**Representing Consecutive Integers** You can use the same expressions to represent either consecutive even integers or consecutive odd integers. It is the value of *n* (odd or

even) that differs

between the two

expressions.

StudyTip

# Practice and Problem Solving

Step-by-Step Solutions begin on page R12.
 Extra Practice begins on page 815.

**Example 1** Solve each equation. Check your solution.

<b>11.</b> $3t + 7 = -8$	<b>12.</b> $8 = 16 + 8n$	<b>13.</b> $-34 = 6m - 4$
<b>14.</b> $9x + 27 = -72$	<b>15.</b> $\frac{y}{5} - 6 = 8$	<b>16.</b> $\frac{f}{-7} - 8 = 2$
<b>17.</b> $1 + \frac{r}{9} = 4$	<b>18.</b> $\frac{k}{3} + 4 = -16$	<b>19.</b> $\frac{n-2}{7} = 2$
<b>20.</b> $14 = \frac{6+z}{-2}$	<b>21.</b> $-11 = \frac{a-5}{6}$	<b>22.</b> $\frac{22-w}{3} = -7$

#### Example 2 p. 92

p. 91

**FINANCIAL LITERACY** The Cell+ Cellular Phone store offers the plans shown in the table. Raul chose the business plan and has budgeted \$100 per month. Write an equation for this situation, and determine how many minutes per month he can use the phone and stay within budget.

Plan	Flat Monthly Fee	Anytime Minutes	Cost per Minute After Anytime Minutes
personal	\$29.99	250	\$0.20
business	\$49.99	650	\$0.15
executive	\$59.99	1200	\$0.10

Example 3 p. 93

#### Write an equation and solve each problem.

- 24. Fourteen less than three fourths of a number is negative eight. Find the number.
- 25. Seventeen is thirteen subtracted from six times a number. What is the number?
- **26.** Find three consecutive even integers with the sum of -84.
- **27.** Find three consecutive odd integers with the sum of 141.
- 28. Find four consecutive integers with the sum of 54.
- **29.** Find four consecutive integers with the sum of -142.

#### Solve each equation. Check your solution.

30.	-6m - 8 = 24	31.	45 = 7 - 5n
32.	$\frac{2b}{3} + 6 = 24$	33.	$\frac{5x}{9} - 11 = -51$
34.	$65 = \frac{3}{4}c - 7$	35.	$9 + \frac{2}{3}x = 81$
36.	$-\frac{5}{2} = \frac{3}{4}z + \frac{1}{2}$	37.	$\frac{5}{6}k + \frac{2}{3} = \frac{4}{3}$
38.	$-\frac{1}{5} - \frac{4}{9}a = \frac{2}{15}$	39.	$-\frac{3}{7} = \frac{3}{4} - \frac{b}{2}$

#### Write an equation and solve each problem.

- **40. FAMILY** The ages of three brothers are consecutive integers with the sum of 96. How old are the brothers?
- **41. VOLCANOES** Moving lava can build up and form beaches at the coast of an island. The growth of an island in a seaward direction may be modeled as 8y + 2 centimeters, where *y* represents the number of years that the lava flows. An island has expanded 60 centimeters seaward. How long has the lava flowed?

Solve each equation. Check your solution.

#### Real-World Link

The top amusement parks in the world for attendance are:

- Magic Kingdom Orlando, FL
- Disneyland Anaheim, CA
- Tokyo Disneyland Urayasu, Chiba, Japan
- Tokyo Disney Sea Urayasu, Chiba, Japan
- 5. Disneyland Paris Cedex, Marne La Valle, France

**44.** 0.6a + 9 = 14.4

**42.** -5x - 4.8 = 6.7

- **46.**  $9 = \frac{-6p (-3)}{-8}$
- **48.** If 7m 3 = 53, what is the value of 11m + 2?
- **49.** If 13y + 25 = 64, what is the value of 4y 7?
- **50.** If -5c + 6 = -69, what is the value of 6c 15?
- I. AMUSEMENT PARKS An amusement park offers a yearly membership of \$275 that allows for free parking and admission to the park. Members can also use the water park for an additional \$5 per day. Nonmembers pay \$6 for parking, \$15 for admission, and \$9 for the water park.

 $(43) \quad 3.7q + 26.2 = 111.67$ 

**45.**  $\frac{c}{2} - 4.3 = 11.5$ 

**47.** 3.6 - 2.4m = 12

- **a.** Write and solve an equation to find the number of visits it would take for the total cost to be the same for a member and a nonmember if they both use the water park at each visit.
- **b.** Make a table for the costs of members and nonmembers after 3, 6, 9, 12, and 15 visits to the park.
- **c.** Plot these points on a coordinate graph and describe things you notice from the graph.
- **52. SHOPPING** At The Family Farm, you can pick your own fruits and vegetables.
  - **a.** The cost of a bag of potatoes is \$1.50 less than  $\frac{1}{2}$  of
  - the price of apples. Write and solve an equation to find the cost of potatoes.
  - **b.** The price of each zucchini is 3 times the price of winter squash minus \$7. Write and solve an equation to find the cost of zucchini.
  - **c.** Write an equation to represent the cost of a pumpkin using the cost of the blueberries.

#### H.O.T. Problems

Use Higher-Order Thinking Skills

- **53. OPEN ENDED** Write a problem that can be modeled by the equation 2x + 40 = 60. Then solve the equation and explain the solution in the context of the problem.
- **54. REASONING** Describe the steps you can use to solve  $\frac{w+3}{5} 4 = 6$ .
- **55. CHALLENGE** To find the measure of an interior angle of a regular polygon, you can use the formula  $m = \frac{180(n-2)}{n}$ , where *m* represents the measure of each angle and *n* represents the number of sides in the polygon. If m = 156, how many sides does the polygon have?
- **56. CHALLENGE** Determine whether the following statement is *sometimes, always,* or *never* true. Explain your reasoning.

The sum of three consecutive odd integers equals an even integer.

**57.** WRITING IN MATH Write a paragraph explaining the order of the steps that you would take to solve a multi-step equation.

The Family Farm		
Fruit	Price (\$)	
Apples	6.99/bag	
Pumpkins	5.00 each	
Blueberries	2.99/qt	
Winter squash	2.99 each	

# **Standardized Test Practice**

**58.** Which is the best estimate for the number of minutes on the calling card advertised below?



**B** 20 min

**D** 200 min

**59. GRIDDED RESPONSE** The scale factor for two similar triangles is 2 : 3. The perimeter of the smaller triangle is 56 cm. What is the perimeter of the larger triangle in centimeters?

**60.** Mr. Morrison is draining his cylindrical pool. The pool has a radius of 10 feet and a standard height of 4.5 feet. If the pool water is pumped out at a constant rate of 5 gallons per minute, about how long will it take to drain the pool? (1  $\text{ft}^3 = 7.5 \text{ gal}$ )

F	37.8 min	Η	25.4 h
G	7 h	J	35.3 h

**61. STATISTICS** Look at the golf scores for the five players in the table.

Player	1	2	3	4	5
Score	80	91	103	79	78

Which of these is the range of the golf scores?

A	10	C	35
B	25	D	40

# Spiral Review

- **62. GAS MILEAGE** A midsize car with a 4-cylinder engine travels 34 miles on a gallon of gas. This is 10 miles more than a luxury car with an 8-cylinder engine travels on a gallon of gas. How many miles does a luxury car travel on a gallon of gas? (Lesson 2-2)
- **63. DEER** In a recent year, 1286 female deer were born in Clark County. That is 93 fewer than the number of male deer born. How many male deer were born that year? (Lesson 2-2)

Translate each equation into a verbal sentence. (Lesson 2-1)

<b>64.</b> $f - 15 = 6$	<b>65.</b> $3h + 7 = 20$	<b>66.</b> $k^2 + 18 = 54 - m$
<b>67.</b> $3p = 8p - r$	<b>68.</b> $\frac{3}{5}t + \frac{1}{3} = t$	<b>69.</b> $\frac{1}{2}v = \frac{2}{3}v + 4$

**70. GEOGRAPHY** The Pacific Ocean covers about 46% of Earth. If *P* represents the surface area of the Pacific Ocean and *E* represents the surface area of Earth, write an equation for this situation. (Lesson 2-1)

Find the value of n in each equation. Then name the property that is used. (Lesson 1-3)

<b>71.</b> 1.5 +	n = 1.5		72.	8n = 1
<b>73.</b> 4 – 7	n = 0		74.	1 = 2n

#### **Skills Review**

Evaluate each expression.		(Lesson 1-2)	
<b>75.</b> $5 + 3(4^2)$	76.	$\frac{38-12}{2\cdot 13}$	

**77.**  $[5(1+1)]^3$ 

**78.**  $[8(2) - 4^2] + 7(4)$ 

96 Chapter 2 Linear Equations

# Solving Equations with the Variable on Each Side

# Then

You solved multi-step equations. (Lesson 2-3)

# Now/

- Solve equations with the variable on each side.
- Solve equations involving grouping symbols.

New/ Vocabulary identity

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# Why?

The equation y = 1.3x + 19 represents the number of times Americans eat in their cars each year, where *x* is the number of years since 1985, and *y* is the number of times that they eat in their car. The equation y = -1.3x + 93 represents the number of times Americans eat in restaurants each year, where *x* is the number of years since 1985, and *y* is the number of times that they eat in a restaurant.

The equation 1.3x + 19 = -1.3x + 93 represents the year when the number of times Americans eat in their cars will equal the number of times Americans eat in restaurants.



**Variables on Each Side** To solve an equation that has variables on each side, use the Addition or Subtraction Property of Equality to write an equivalent equation with the variable terms on one side.

# EXAMPLE 1 Solve an Equation with Variables on Each Side

#### Solve 2 + 5k = 3k - 6. Check your solution.

2 + 5k = 3k - 6	Original equation
-3k = -3k	Subtract 3k from each side.
2 + 2k = -6	Simplify.
-2 = -2	Subtract 2 from each side.
2k = -8	Simplify.
$\frac{2k}{2} = \frac{-8}{2}$	Divide each side by 2.
k = -4	Simplify.
<b>CHECK</b> $2 + 5k = 3k - 6$	Original equation
$2 + 5(-4) \stackrel{?}{=} 3(-4) - 6$	Substitution, $k = -4$
$2 + -20 \stackrel{?}{=} -12 - 6$	Multiply.
$-18 = -18 \checkmark$	Simplify.

## Check Your Progress

Solve each equation. Check your solution.

**1A.** 3w + 2 = 7w**1C.**  $\frac{x}{2} + 1 = \frac{1}{4}x - 6$  **1B.** 5a + 2 = 6 - 7a**1D.** 1.3c = 3.3c + 2.8

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**Grouping Symbols** If equations contain grouping symbols such as parentheses or brackets, use the Distributive Property first to remove the grouping symbols.

#### StudyTip

Solving an Equation You may want to eliminate the terms with a variable from one side before eliminating a constant.

#### EXAMPLE 2 Solve an Equation with Grouping Symbols

Solve  $6(5m - 3) = \frac{1}{3}(24m + 12)$ .  $6(5m-3) = \frac{1}{3}(24m+12)$ **Original equation** 30m - 18 = 8m + 4**Distributive Property** 30m - 18 - 8m = 8m + 4 - 8mSubtract 8m from each side. 22m - 18 = 4Simplify. 22m - 18 + 18 = 4 + 18Add 18 to each side. 22m = 22Simplify.  $\frac{22m}{22} = \frac{22}{22}$ Divide each side by 22. m = 1Simplify.

## Check Your Progress

Solve each equation. Check your solution.

**2A.** 
$$8s - 10 = 3(6 - 2s)$$

**2B.** 7(n-1) = -2(3+n)

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Some equations may have no solution. That is, there is no value of the variable that will result in a true equation. Some equations are true for all values of the variables. These are called **identities**.

#### EXAMPLE 3 Find Special Solutions

Solve each equation. a. 5x + 5 = 3(5x - 4) - 10x 5x + 5 = 3(5x - 4) - 10x Original equation 5x + 5 = 15x - 12 - 10x Distributive Property 5x + 5 = 5x - 12 Simplify. -5x = -5x $5 \neq -12$  Subtract 5x from each side.

Since  $5 \neq -12$ , this equation has no solution.

```
b. 3(2b - 1) - 7 = 6b - 10

3(2b - 1) - 7 = 6b - 10

6b - 3 - 7 = 6b - 10

6b - 10 = 6b - 10

0 = 0

Original equation

Distributive Property

Simplify.

0 = 0

Subtract 6b - 10 from each side.
```

Since the expressions on each side of the equation are the same, this equation is an identity. It is true for all values of *b*.

#### Check Your Progress

**3A.** 7x + 5(x - 1) = -5 + 12x

**3B.** 
$$6(y-5) = 2(10+3y)$$

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# ReadingMath

**No Solution** The symbol that represents no solution is Ø.

The steps for solving an equation can be summarized as follows.

Concep	ot Summary Steps for Solving Equations For Your
Step 1	Simplify the expressions on each side. Use the Distributive Property as needed.
Step 2	Use the Addition and/or Subtraction Properties of Equality to get the variables on one side and the numbers without variables on the other side. Simplify.
Step 3	Use the Multiplication or Division Property of Equality to solve.
	Math in Motion, BrainPOP <sup>®</sup> glencoe.com

There are many situations in which variables are on both sides of the equation.

#### STANDARDIZED TEST EXAMPLE 4



#### **Read the Test Item**

The area of the first rectangle is 10x, and the area of the second is 6(3 + x). The equation 10x = 6(3 + x) represents this situation.

#### Solve the Test Item

A	10x = 6(3+x)	В	10x = 6(3+x)
	$10(3) \stackrel{?}{=} 6(3+3)$		$10(4.5) \stackrel{?}{=} 6(3 + 4.5)$
	$30 \stackrel{?}{=} 6(6)$		$45 \stackrel{?}{=} 6(7.5)$
	30 ≠ 36 <b>×</b>		45 = 45 <b>✓</b>

Since the value 4.5 results in a true statement, you do not need to check 6.5 and 7. The answer is B.

#### Check Your Progress

**4.** Find the value of *x* so that the figures have the same perimeter.



#### Test-TakingTip

**Choose a Method** There is often more than one way to solve a problem. In this example, you can write an algebraic equation and solve for *x*. Or you can substitute each answer choice into the formulas to find the correct answer.

# Check Your Understanding



**36.** 5[2p - 4(p + 5)] = 25

**35.** 3.2k - 4.3 = 12.6k + 14.5





#### Real-World Link

About 25% of American adults have Internet access and give time or money to charity. Of those, 56% have never visited the Web site of a charity, and only 7% say they have given online.

Source: Craver, Mathews, Smith & Co.

- **37. NUMBER THEORY** Three times the lesser of two consecutive even integers is 6 less than six times the greater number. Find the integers.
- **38. MONEY** Chris has saved twice the number of quarters that Nora saved plus 6. The number of quarters Chris saved is also five times the difference of the number of quarters and 3 that Nora has saved. Write and solve an equation to find the number of quarters they each have saved.
- 39 DVD A company that replicates DVDs spends \$1500 per day in building overhead plus \$0.80 per DVD in supplies and labor. If the DVDs sell for \$1.59 per disk, how many DVDs must the company sell each day before it makes a profit?
- **40. INTERNET ACCESS** The table shows the percent of households that have broadband Internet access and the average growth rates for two age groups. How long will it take for the percents to be the same?

Age Group	Percent with Broadband in Their Homes in 2006	Growth Rate Percentage per Year
18–49	52.5	42
50+	25.5	52

Source: Pew Internet & American Life Project

- **41. WULTIPLE REPRESENTATIONS** In this problem, you will explore 2x + 4 = -x - 2.
  - **a. GRAPHICAL** Make a table of values with five points for y = 2x + 4 and y = -x - 2. Graph the points from the tables.
  - **b.** ALGEBRAIC Solve 2x + 4 = -x 2.
  - **c. VERBAL** Explain how the solution you found in part **b** is related to the intersection point of the graphs in part **a**.

**H.O.T. Problems** Use Higher-Order Thinking Skills

- **42. REASONING** Solve the equation below. Describe each step. t = 2 - 2[2t - 3(1 - t)]
- **43.** CHALLENGE Write an equation with the variable on each side of the equals sign, at least one fractional coefficient, and a solution of -6. Discuss the steps you used.
- **44. OPEN ENDED** Create an equation with at least two grouping symbols for which there is no solution.
- **45. REASONING** Determine whether each solution is correct. If the solution is not correct, describe the error and give the correct solution.

a. 2(g + 5) = 222g = 172g = 8.5

- 5d = 2d 18C. -6z + 13 = 7z2g + 5 = 22 5d - 2d = 2d - 18 - 2d -6z + 13 - 6z = 7z - 6z 2g + 5 - 5 = 22 3d = -18 13 = zd = -6
- **46. CHALLENGE** Find the value of *k* for which each equation is an identity.

**a.** 
$$k(3x - 2) = 4 - 6x$$

$$15y - 10 + k = 2(ky - 1) - y$$

47. WRITING IN MATH Compare and contrast solving equations with variables on both sides of the equation to solving one-step or multi-step equations with a variable on one side of the equation.

b.

# **Standardized Test Practice**

- **48.** A hang glider 25 meters above the ground starts to descend at a constant rate of 2 meters per second. Which equation shows the height *h* after *t* seconds of descent?
  - **A** h = 25t + 2t
  - **B** h = -25t + 2
  - **C** h = 2t + 25
  - **D** h = -2t + 25
- **49. GEOMETRY** Two rectangular walls each with a length of 12 feet and a width of 23 feet need to be painted. It costs \$0.08 per square foot for paint. How much will it cost to paint the walls?

F	\$22.08	Η	\$34.50	
G	\$23.04	J	\$44.16	

**50. SHORT RESPONSE** Maddie works at Game Exchange. They are having a sale as shown.

Item	Price	Special
video games	\$20	Buy 2 get 1 Free
DVDs	\$15	Buy 1 get 1 Free

She purchases four video games and uses her employee discount of 15%. If sales tax is 7.25%, how much does she spend on the games?

**D** -10

**54.**  $\frac{1}{2}z + 7 = 16 - \frac{3}{5}z$ **57.**  $9 + \frac{y}{5} = 6$ 

**ZOO ADMISSION** 

Adults.....\$9.75

Children.....\$7.25

**51.** Solve 
$$\frac{4}{5}x + 7 = \frac{3}{15}x - 3$$
.  
**A**  $-16\frac{2}{3}$  **C**  $-6\frac{2}{3}$ 

**B**  $-14\frac{4}{2}$ 

Solve each equation. Check your solution. (Lesson 2-3)

52.	5n + 6 = -4	53.	-1 = 7 + 3c
55.	$\frac{2}{5}x + 6 = \frac{2}{3}x + 10$	56.	$\frac{a}{7} - 3 = -2$

**58.** WORLD RECORDS In 1998, Winchell's House of Donuts in Pasadena, California, made the world's largest donut. It weighed 5000 pounds and had a circumference of 298.3 feet. What was the donut's diameter to the nearest tenth? (*Hint:*  $C = \pi d$ ) (Lesson 2-2)

**59. ZOO** At a zoo, the cost of admission is posted on the sign. Find the cost of admission for two adults and two children. (Lesson 1-3)

Find the value of *n*. Then name the property used in each step. (Lesson 1-3)

60.	25n = 25	<b>61.</b> $n \cdot 1 = 2$	62.	$12 \cdot n = 12 \cdot 6$
63.	$n+0 = \frac{2}{3}$	<b>64.</b> $4 \cdot \frac{1}{4} = n$	65.	(10 - 8)(7) = 2(n)

# **Skills Review**

Translate each sentence into an equation. (Lesson 2-1)

- **66.** Twice a number *t* decreased by eight equals seventy.
- **67.** Five times the sum of *m* and *k* is the same as seven times *k*.
- **68.** Half of *p* is the same as *p* minus 3.

#### Evaluate each expression. (Lesson 0-3)

<b>69.</b> −9 − (−14)	<b>70.</b> -10 + (20)	<b>71.</b> -15 - 9
<b>72.</b> 5(14)	<b>73.</b> -55 ÷ (-5)	<b>74.</b> -25(-5)

**102** Chapter 2 Linear Equations

# Solving Equations Involving Absolute Value

# Then

You solved equations with the variable on each side. (Lesson 2-4)

## Now/

- Evaluate absolute value expressions.
- Solve absolute value equations.

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# Why?

In 2007, a telephone poll was conducted to determine the reading habits of Americans. People in this survey were allowed to select more than one type of book.

The survey had a margin of error of 3 percentage points. This means that the results could be three points higher or lower. So, the percent of people who read religious material could be as high as 69% or as low as 63%.



Source: CNN

**Absolute Value Expressions** Expressions with absolute values define an upper and lower range in which a value must lie. Expressions involving absolute value can be evaluated using the given value for the variable.

# **EXAMPLE 1** Expressions with Absolute Value Evaluate |m + 6| - 14 if m = 4.

|m + 6| - 14 = |4 + 6| - 14 Replace *m* with 4. = |10| - 14 4 + 6 = 10 = 10 - 14 |10| = 10 = -4 Simplify.

## Check Your Progress

**1.** Evaluate 
$$23 - |3 - 4x|$$
 if  $x = 2$ .

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**Absolute Value Equations** Looking at the example at the top of the page, we notice that the margin of error in the bar graph is an example of absolute value. The distance between 66 and 69 on a number line is the same as the distance between 63 and 66.



There are three types of open sentences involving absolute value, |x| = n, |x| < n, and |x| > n. In this lesson, we will consider only the first type. Look at the equation |x| = 4. This means that the distance between 0 and *x* is 4.



If |x| = 4, then x = -4 or x = 4. Thus, the solution set is  $\{-4, 4\}$ .

For each absolute value equation, we must consider both cases. To solve an absolute value equation, first isolate the absolute value on one side of the equals sign if it is not already by itself.

# ReadingMath

Absolute Values The expression |f + 5| is read the absolute value of the quantity f plus 5.

Key	Concept Abso	olute Value Ec	luations	For FOL
Words	When solving equati there are two cases	ons that involve to consider.	e absolute values,	
	<b>Case 1</b> : The expression or zero.	on inside the al	bsolute value symb	ool is posit
	Case 2: The expression	on inside the al	osolute value symb	ol is negat
Symbols	For any real number $a = -b$ .	rs a and b, if Ia	$I = b$ and $b \ge 0$ , t	hen $a = b$
Example	d  = 10, so $d = 10$	or $d = -10$ .		
FXAMPLE	2 Solve Absolute	Value Equation	IS	
Solve each	equation. Then grar	oh the solution	set.	
<b>a</b> . $ f + 5 $ :	= 17			
f + 5  =	= 17 Original equation			
J)   9 -	- 17 Oliginal equation			
Case 1			Case 2	
f +	5 = 17		f + 5 = -17	
<i>f</i> + 5 <b>–</b>	5 = 17 - 5 Subtract 5	from each side.	f + 5 - 5 = -17 - 5	- 5
	f = 12 S	implify.	f = -22	
	<mark>→   ●      </mark> -25-20-15-10 -	5 0 5 10 15	20 25	
<b>b</b> .   <i>b</i> − 1	= -3			
b - 1  b and 1 negative	= -3 means the distance of the solution is the solution is the e	nce between cannot be mpty set Ø.	<del></del>	0 1 2 3
Check Yo	ur Progress			
	- 1	<b>2B</b> 3	n-4  = -1	
<b>2A.</b>   <i>y</i> + 2	= 4		N	



SNAKES The temperature of an enclosure for a pet snake should be about 80°F, give or take 5°. Find the maximum and minimum temperatures.

You can use a number line to solve.



The distance from 80 to 75 is 5 units. The distance from 80 to 85 is 5 units.

The solution set is {75, 85}. The maximum and minimum temperatures are 85° and 75°.



#### Real-World Link

In 2001, the number of households in the U.S. that had either a turtle, snake, lizard, or other reptile as a pet was 1,678,000.

Source: American Veterinary Medical Association

# Check Your Progress

**3. ICE CREAM** Ice cream should be stored at 5°F with an allowance for 5°. Write and solve an equation to find the maximum and minimum temperatures at which the ice cream should be stored.

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When given two points on a graph, you can write an absolute value equation for the graph.

# StudyTip

Find the Midpoint To find the point midway between two points, add the values together and divide by 2. For Example 4, 11 +  $19 = 30, 30 \div 2 = 15$ . So 15 is the point halfway between 11 and 19.

#### EXAMPLE 4 Write an Absolute Value Equation

Write an equation involving absolute value for the graph.

Find the point that is the same distance from 11 and from 19. This is the midpoint between 11 and 19, which is 15.



The distance from 15 to 11 is 4 units. The distance from 15 to 19 is 4 units.

So an equation is |x - 15| = 4.

#### Check Your Progress

4. Write an equation involving absolute value for the graph.

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# Check Your Understanding

Example 1	Evaluate each expression if $f = 3$ , $g = -4$ , and $h = 5$ .				
p. 103	<b>1.</b> $ 3 - h  + 13$	<b>2.</b> $16 -  g + 9 $	<b>3.</b> $ f + g  - h$		
Example 2	Solve each equation. Then graph the solution set.				
р. 104	<b>4.</b> $ n+7  = 5$	<b>5.</b> $ 3z - 3  = 9$	<b>6.</b> $ 4n-1  = -6$		
	<b>7.</b> $ b+4  = 2$	<b>8.</b> $ 2t - 4  = 8$	<b>9.</b> $ 5h+2  = -8$		
Example 3 p. 104	<b>10. FINANCIAL LITERACY</b> For they will receive a 12% r equation to find the leas	<b>TERACY</b> For a company to invest in a product, they must believe ceive a 12% return on investment (ROI) plus or minus 3%. Write as find the least and the greatest ROI they believe they will receive.			

Example 4 p. 105

Write an equation involving absolute value for each graph.

-4 -2

0

2

6

# **Practice and Problem Solving**

= Step-by-Step Solutions begin on page R12.

Extra Practice begins on page 815.

Example 1	Evaluate each expression if $a = -2$ , $b = -3$ , $c = 2$ , $x = 2.1$ , $y = 3$ , and $z = -4.2$ .			
р. 103	13 $ 2x + z  + 2y$	<b>14.</b> $4a -  3b + 2c $	<b>15.</b> $- 5a + c  +  3y + 2z $	
	<b>16.</b> $-a +  2x - a $	<b>17.</b> $ y - 2z  - 3$	<b>18.</b> $3 3b - 8c  - 3$	
	<b>19.</b> $ 2x - z  + 6b$	<b>20.</b> $-3 z  + 2(a + y)$	<b>21.</b> $-4 c-3 +2 z-a $	
Example 2	Solve each equation. The	en graph the solution set.		
р. 104	<b>22.</b> $ n-3  = 5$	<b>23.</b> $ f + 10  = 1$	<b>24.</b> $ v-2  = -5$	
	<b>25.</b> $ 4t - 8  = 20$	<b>26.</b> $ 8w + 5  = 21$	<b>27.</b> $ 6y - 7  = -1$	
a	<b>28.</b> $\left \frac{1}{2}x+5\right  = -3$	<b>29.</b> $ -2y+6 =6$	<b>30.</b> $\left \frac{3}{4}a - 3\right  = 9$	
Example 3 p. 104	<b>31. SURVEY</b> The circle gray of a survey that asked be rich some day?" If is the range of the per- likely that they will be	ph at the right shows the rest, "How likely is it that you w the margin of error is $\pm 4\%$ , w cent of teens who say it is ver e rich?	alts Not Likely at All 2% Vhat Y Very Likely	
	<b>32. CHEERLEADING</b> For competition, the cheerleading team is preparing a dance routine that must last 4 minutes, with a variation of ±5 seconds.			
	<b>a.</b> Find the least and g the routine in minut	reatest possible times for tes and seconds.		
	<b>b.</b> Find the least and g	reatest possible times in seco	nds.	
Example 4 p. 105	Write an equation involv <b>33.</b> $-5-4-3-2-1$ 0 1 2 3	ing absolute value for each $34 10-8-6$	<b>graph.</b>	
	<b>35.</b>	<b>36.</b> <del>-</del> 7−6−5	-4-3-2-1 0 1 2 3	
	Solve each equation. The	n graph the solution set.		
	<b>37.</b> $\left  -\frac{1}{2}b - 2 \right  = 10$	<b>38.</b> $ -4d+6  = 12$	<b>39.</b> $ 5f-3  = 12$	
	<b>40.</b> $2 h  - 3 = 8$	<b>41.</b> $4 - 3 q  = 10$	<b>42.</b> $\frac{4}{ p } + 12 = 14$	
	<b>43. TRACK</b> The 4×400 rela or one lap around the	y is a race where 4 runners ta track.	ake turns running 400 meters,	

- **a.** If a runner runs the first leg in 52 seconds plus or minus 2 seconds, write an equation to find the fastest and slowest times.
- **b.** If the runners of the second and third legs run their laps in 53 seconds plus or minus 1 second, write an equation to find the fastest and slowest times.
- **c.** Suppose the runner of the fourth leg is the fastest on the team. If he runs an average of 50.5 seconds plus or minus 1.5 seconds, what are the team's fastest and slowest times?



#### Real-World Link

An increasing number of fashion designers are using computer-aided design (CAD). CAD allows designers to view designs on virtual models.

Source: Bureau of Labor Statistics

- **44. FASHION** To allow for a model's height, a designer is willing to use models that require him to change hems either up or down 2 inches. The length of the skirts is 20 inches.
  - **a.** Write an absolute value equation that represents the length of the skirts.
  - **b**. What is the range of the lengths of the skirts?
  - **c.** If a 20-inch skirt was fitted for a model that is 5 feet 9 inches tall, will the designer use a 6-foot-tall model?
- **45. CARS** Speedometer accuracy can be affected by many details such as tire diameter and axle ratio. For example, there is variation of  $\pm 3$  miles per hour when calibrated at 50 miles per hour.
  - a. What is the range of actual speeds of the car if calibrated at 50 miles per hour?
  - **b.** A speedometer calibrated at 45 miles per hour has an accepted variation of  $\pm 1$  mile per hour. What can we conclude from this?

#### Write an equation involving absolute value for each graph.



- **52. MUSIC** A CD will record an hour and a half of music plus or minus 3 minutes for time between tracks.
  - **a.** Write an absolute value equation that represents the recording time.
  - **b**. What is the range of time in minutes that the CD could run?
  - **c.** Graph the possible times on a number line.
- ACOUSTICS The Red Rocks Amphitheater located in the Red Rock Park near Denver, Colorado, is the only naturally occurring amphitheater. The acoustic qualities here are such that a maximum of 20,000 people, plus or minus 1000, can hear natural voices clearly.
  - **a.** Write an equation involving an absolute value that represents the number of people that can hear natural voices at Red Rocks Amphitheater.
  - **b.** Find the maximum and minimum number of people that can hear natural voices clearly in the amphitheater.
  - **c.** What is the range of people in part **b**?



#### Real-World Link

Each spring since 1997, Gallaudet University holds its National Academic Bowl for Deaf and Hard of Hearing High School Students. Here, teams of high school students compete in local, regional, and national events throughout the U.S.

Source: Galluadet University

- **54. BOOK CLUB** The members of a book club agree to read within ten pages of the last page of the chapter. The chapter ends on page 203.
  - **a.** Write an absolute value equation that represents the pages where club members could stop reading.
  - **b**. Write the range of the pages where the club members could stop reading.
- **SCHOOL** Washington High School and McKinley High School are competing in an academic challenge. The team with a correct response is awarded 10 points. An incorrect response has a point value of -10. There are 5 mathematics questions.
  - **a.** Write an equation that represents the scoring for the challenge.
  - **b.** Make a table of values for the possible points that a school could receive during the mathematics portion of the challenge.
  - c. Write about how absolute values can be used in classes other than math.

H.O.T. Problems

Use Higher-Order Thinking Skills

**56. OPEN ENDED** Describe a real-world situation that could be represented by the absolute value equation |x - 4| = 10.

# **REASONING** Determine whether the following statements are *sometimes*, *always*, or *never* true, if *c* is an integer. Explain your reasoning.

- **57.** The value of |x + 1| is greater than zero.
- **58.** The solution of |x + c| = 0 is greater than 0.
- **59.** The inequality |x| + c < 0 has no solution.
- **60.** The value of |x + c| + c is greater than zero.
- **61. REASONING** Explain why an absolute value can never be negative.
- **62. CHALLENGE** Use the sentence  $x = 7 \pm 4.6$ .
  - **a.** Describe the values of *x* that make the sentence true.
  - **b**. Translate the sentence into an equation involving absolute value.
- **63.** FIND THE ERROR Alex and Wesley are solving |x + 5| = -3. Is either of them correct? Explain your reasoning.

Alex |x+5|=3 ov |x+5|=-3 x+5=3 +5=-3 -5-5 -5 -5 x=-2 -5 -5x=-8



**64.** WRITING IN MATH Explain why there are either two, one, or no solutions for absolute value equations. Demonstrate an example of each possibility.
# **Standardized Test Practice**

- **65.** Which equation represents the second step of the solution process? Step 1: 4(2x + 7) - 6 = 3xStep 2: Step 3: 5x + 28 - 6 = 0Step 4: 5x = -22Step 5: x = -4.4A 4(2x - 6) + 7 = 3x**B** 4(2x + 1) = 3x**C** 8x + 7 - 6 = 3x**D** 8x + 28 - 6 = 3x**66. GEOMETRY** The area of a circle is  $25\pi$  square centimeters. What is the circumference? F  $625\pi$  cm **G** 50 $\pi$  cm H  $5\pi$  cm J  $10\pi$  cm
  - **Spiral Review**

Write and solve an equation for each sentence. (Lesson 2-4)

- 69. One half of a number increased by 16 is four less than two thirds of the number.
- **70.** The sum of one half of a number and 6 equals one third of the number.
- **71. SHOE** If  $\ell$  represents the length of a man's foot in inches, the expression  $2\ell 12$  can be used to estimate his shoe size. What is the approximate length of a man's foot if he wears a size 8? (Lesson 2-3)

# **Skills Review**

Write an equation for each problem. Then solve the equation. (Lesson 2-2)

- **72.** Seven times a number equals -84. What is the number?
- **73.** Two fifths of a number equals -24. Find the number.
- **74.** Negative 117 is nine times a number. Find the number.
- **75.** Twelve is one fifth of a number. What is the number?

**67.** Tanya makes \$5 an hour and 15% commission of the total dollar value on cosmetics she sells. Suppose Tanya's commission is increased to 17%. How much money will she make if she sells \$300 worth of product and works 30 hours?

A	\$201	С	\$255
B	\$226	D	\$283

- **68. EXTENDED RESPONSE** John's mother has agreed to take him driving every day for two weeks. On the first day, John drives for 20 minutes. Each day after that, John drives 5 minutes more than the day before.
  - **a.** Write an expression for the *n*th term. Explain how you found the expression.
  - **b.** For how many minutes will John drive on the last day? Show your work.
  - **c.** John's driver's education teacher requires that each student drive for 30 hours with an adult outside of class. Will John's sessions with his mother fulfill this requirement?

# Mid-Chapter Quiz

Translate each sentence into an equation. (Lesson 2-1)

- 1. The sum of three times *a* and four is the same as five times *a*.
- **2.** One fourth of *m* minus six is equal to two times the sum of *m* and 9.
- **3.** The product of five and *w* is the same as *w* to the third power.
- **4. MARBLES** Drew has 50 red, green, and blue marbles. He has six more red marbles than blue marbles and four fewer green marbles than blue marbles. Write and solve an equation to determine how many blue marbles Drew has. (Lesson 2-1)

Solve each equation. Check your solution. (Lesson 2-2)

- **5.** p + 8 = 13
- **6.** -26 = b 3
- **7.**  $\frac{t}{6} = 3$
- **8. MULTIPLE CHOICE** Solve the equation  $\frac{3}{5}a = \frac{1}{4}$ . (Lesson 2-2)
  - **A**  $\frac{3}{20}$  **B** 2 **C**  $\frac{5}{12}$ **D** -3

Solve each equation. Check your solution. (Lesson 2-3)

- **9.** 2x + 5 = 13
- **10.** -21 = 7 4y
- **11.**  $\frac{m}{6} 3 = 8$
- **12.**  $-4 = \frac{d+3}{5}$
- **13. FISH** The average length of a yellow-banded angelfish is 12 inches. This is 4.8 times as long as an average common goldfish. (Lesson 2-3)
  - **a.** Write an equation you could use to find the length of the average common goldfish.
  - **b.** What is the length of an average common goldfish?

#### Write an equation and solve each problem. (Lesson 2-3)

- **14.** Three less than three fourths of a number is negative 9. Find the number.
- **15.** Thirty is twelve added to six times a number. What is the number?
- **16.** Find four consecutive integers with a sum of 106.

#### Solve each equation. Check your solution. (Lesson 2-4)

- **17.** 8p + 3 = 5p + 9
- **18.**  $\frac{3}{4}w + 6 = 9 \frac{1}{4}w$ **19.**  $\frac{z+6}{3} = \frac{2z}{4}$
- **20. PERIMETER** Find the value of *x* so that the triangles have the same perimeter. (Lesson 2-4)



**21. PRODUCTION** ABC Sporting Goods Company produces baseball gloves. Their fixed monthly production cost is \$8000 with a per glove cost of \$5. XYZ Sporting Goods Company also produces baseball gloves. Their fixed monthly production cost is \$10,000 with a per glove cost of \$3. Find the value of x, the number of gloves produced monthly, so that the total monthly production cost is the same for both companies. (Lesson 2-4)

Evaluate each expression if x = -4, y = 7, and z = -9. (Lesson 2-5)

- **22.** |3x 2| + 2y
- **23.** |-4y+2z|-7z

**24.** MULTIPLE CHOICE Solve |6m - 3| = 9. (Lesson 2-5)

<b>F</b> {2}	H {-3, 6}
<b>G</b> {-1, 2}	J {-3, 3}

**25. COFFEE** Some say to brew an excellent cup of coffee, you must have a brewing temperature of 200° F, plus or minus 5 degrees. Write and solve an equation describing the maximum and minimum brewing temperatures for an excellent cup of coffee.

# 2-6

# Then

You evaluated percents by using a proportion. (Lesson 0-5)

# Now/

- Compare ratios.
- Solve proportions.

#### New Vocabulary

ratio proportion means extremes rate unit rate scale scale model

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# **Ratios and Proportions**

# Why?

Ratios allow us to compare many items by using a common reference. The table below shows the number of a certain popular fast food restaurants, per 10,000 people, in the United States as well as other countries. This allows us to compare the number of these restaurants using an equal reference.



Countries	United States	New Zealand	Canada	Australia	Japan	Singapore
Number of Restaurants per 10,000 People	0.433	0.369	0.352	0.349	0.282	0.273

**Ratios and Proportions** The comparison between the number of restaurants and the number of people is a ratio. A **ratio** is a comparison of two numbers by division. The ratio of x to y can be expressed in the following ways.



Suppose you wanted to determine the number of restaurants per 100,000 people in Australia. Notice that this ratio is equal to the original ratio.



An equation stating that two ratios are equal is called a **proportion**. So, we can state that  $\frac{0.349}{10,000} = \frac{3.49}{100,000}$  is a proportion.

# **EXAMPLE 1** Determine Whether Ratios Are Equivalent

Determine whether  $\frac{2}{3}$  and  $\frac{16}{24}$  are equivalent ratios. Write *yes* or *no*. Justify your answer.



When expressed in simplest form, the ratios are equivalent.

# Check Your Progress

Determine whether each pair of ratios are equivalent ratios. Write *yes* or *no*. Justify your answer.

**IA.** 
$$\frac{6}{10}, \frac{2}{5}$$

**1B.** 
$$\frac{1}{6}, \frac{5}{30}$$

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### **Study**Tip

#### Means and Extremes To solve a proportion using cross products, write an equation that sets the product of the extremes equal to the product of the means.

There are special names for the terms in a proportion.



# Key Concept

For Your

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# **Means-Extremes Property of Proportion**

Words	In a proportion, the product of the extremes is equal to the product of the means.
Symbols	If $\frac{a}{b} = \frac{c}{d}$ and $b, d \neq 0$ , then $ad = bc$ .
Example	Since $\frac{2}{4} = \frac{1}{2}$ , 2(2) = 4(1) or 4 = 4.

Another way to determine whether two ratios form a proportion is to use cross products. If the cross products are equal, then the ratios form a proportion.

This is the same as multiplying the means, and multiplying the extremes.

# EXAMPLE 2 Cross Products Use cross products to determine whether each pair of ratios forms a proportion. **a.** $\frac{2}{3.5}, \frac{8}{14}$ $\frac{2}{3.5} \stackrel{?}{=} \frac{8}{14}$ **Original proportion** $2(14) \stackrel{?}{=} 3.5(8)$ **Cross products** $28 = 28 \checkmark$ Simplify. The cross products are equal, so the ratios form a proportion. **b.** $\frac{0.3}{1.5}, \frac{0.5}{2.0}$ $\frac{0.3}{1.5} \stackrel{?}{=} \frac{0.5}{2.0}$ **Original proportion** $0.3(2.0) \stackrel{?}{=} 1.5(0.5)$ **Cross products** $0.6 \neq 0.75 \times$ Simplify. The cross products are not equal, so the ratios do not form a proportion. Check Your Progress **2A.** $\frac{0.2}{1.8}, \frac{1}{0.9}$ **2B.** $\frac{15}{36}, \frac{35}{42}$

**112** Chapter 2 Linear Equations

Solve Proportions To solve proportions, use cross products.



The ratio of two measurements having different units of measure is called a **rate**. For example, a price of \$9.99 per 10 songs is a rate. A rate that tells how many of one item is being compared to 1 of another item is called a **unit rate**.



Real-World Career

#### **Retail Buyer**

A retail buyer purchases goods for stores, primarily from wholesalers, for resale to the general public. Buyers use math to determine the amount of each product to order. A bachelor's degree with an emphasis on business studies is usually required.

#### Real-World EXAMPLE 4 **Rate of Growth**

**RETAIL** In the past two years, a retailer has opened 232 stores. If the rate of growth remains constant, how many stores will the retailer open in the next 3 years?

**Understand** Let *r* represent the number of retail stores.

Plan	Write a proportion for the problem.				
	$\frac{232 \text{ retail stores}}{2 \text{ years}} = \frac{r \text{ retail}}{3 \text{ years}}$	il stores			
Solve	$\frac{232}{2} = \frac{r}{3}$	Original proportion			
	232(3) = 2r	Find the cross products.			
	696 = 2r	Simplify.			
	$\frac{696}{2} = \frac{2r}{2}$	Divide each side by 2.			
	348 = r	Simplify.			

The retailer will open 348 stores in 3 years.

**Check** If the clothing retailer continues to open 232 stores every 2 years, then in the next 3 years, it will open 348 stores.

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#### Check Your Progress

**4. EXERCISE** It takes 7 minutes for Isabella to walk around the gym track twice. At this rate, how many times can Isabella walk around the track in a half hour?

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A **scale** is used when making a model of something that is too large or too small to be convenient at actual size. The scale compares the model to the actual size of the object using a proportion. A **scale model** is a three-dimensional reproduction of an item that has been reduced or increased in size proportionally.



**MOUNTAIN TRAIL** The scale on a map of the Great Smoky Mountains National Park is 3 inches = 10 miles. The length of the Ramsey Cascades Trail is about  $1\frac{1}{9}$  inches on the map. What is the actual length of the trail?

Let  $\ell$  represent the actual length.

scale  $\longrightarrow$   $\frac{3}{10} = \frac{1\frac{1}{8}}{\ell}$  scale actual  $3(\ell) = 1\frac{1}{8}$  (10) Find the cross products.  $3\ell = \frac{45}{4}$  Simplify.  $3\ell \div 3 = \frac{45}{4} \div 3$  Divide each side by 3.  $\ell = \frac{15}{4}$  or  $3\frac{3}{4}$  Simplify. The actual length is about  $3\frac{3}{4}$  miles.

#### Check Your Progress

**5. AIRPLANES** On a model airplane, the scale is 5 centimeters = 2 meters. If the wingspan of the scale model is 28.5 centimeters, what is the actual wingspan?

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# Check Your Understanding

Examples 1 and 2	Determine whether each 1. $\frac{3}{7}, \frac{9}{14}$	ach pair of ratios are eq	uivalent ratios. Write <i>yes</i> or <i>no</i> .
pp. 111-112		2. $\frac{7}{8'}\frac{42}{48}$	$\frac{2.8}{4.4}, \frac{1.4}{2.1}$
Examples 3	Solve each proportion	<b>1.</b> If necessary, round to	nearest hundredth.
p. 113	4. $\frac{n}{9} = \frac{6}{27}$	<b>5.</b> $\frac{4}{u} = \frac{28}{35}$	6. $\frac{3}{8} = \frac{b}{10}$
Example 4	<b>7. RACE</b> Jennie ran th maintain the same	ne first 6 miles of a marat	hon in 58 minutes. If she is able to
p. 113		pace, how long will it ta	ake her to finish the 26.2 miles?
Example 5	<b>8. MAPS</b> On a map of apart. If the scale i	f North Carolina, Raleig	h and Asheville are about 8 inches
p. 114		s 1 inch = 12 miles, how	far apart are the cities?



The Great Smoky Mountains National Park in Tennessee is home to several waterfalls. The Ramsey Cascades is 100 feet tall. It is the tallest in

Real-World Link

Source: National Park Service

the park.

Math *in Motion,* Interactive Lab glencoe.com

# **Practice and Problem Solving**

= Step-by-Step Solutions begin on page R12. Extra Practice begins on page 815.

11.  $\frac{8.4}{9.2}, \frac{8.8}{9.6}$ 

**14.**  $\frac{39.68}{60.14}, \frac{6.4}{9.7}$ 

Examples 1 and 2 Determine whether each pair of ratios are equivalent ratios. Write yes or no. pp. 111-112 **10.**  $\frac{3}{7}, \frac{18}{42}$ 9.  $\frac{9}{11}, \frac{81}{99}$ **13.**  $\frac{29.2}{10.4}, \frac{7.3}{2.6}$ 12.  $\frac{4}{3}, \frac{6}{8}$ 

**Example 3** p. 113 Solve each proportion. If necessary, round to the nearest hundredth.

- 15.  $\frac{3}{8} = \frac{15}{a}$ **16.**  $\frac{t}{2} = \frac{6}{12}$ 17.  $\frac{4}{9} = \frac{13}{a}$ **18.**  $\frac{15}{35} = \frac{g}{7}$ **19.**  $\frac{7}{10} = \frac{m}{14}$ **20.**  $\frac{8}{13} = \frac{v}{21}$ **22.**  $\frac{1}{0.19} = \frac{12}{n}$ **23.**  $\frac{2}{0.21} = \frac{8}{n}$ **21.**  $\frac{w}{2} = \frac{4.5}{6.8}$ **26.**  $\frac{7}{1.066} = \frac{z}{9.65}$ **24.**  $\frac{2.4}{3.6} = \frac{k}{1.8}$ **25**  $\frac{t}{0.3} = \frac{1.7}{0.9}$ **29.**  $\frac{10}{15} = \frac{4}{r-5}$ **27.**  $\frac{x-3}{5} = \frac{6}{10}$ **28.**  $\frac{7}{r+9} = \frac{21}{36}$
- **Example 4** p. 113

p. 114

Example 5

- **30.** CAR WASH The B-Clean Car Wash washed 128 cars in 3 hours. At that rate, how many cars can they wash in 8 hours?
- **31.** MENU On Monday, a restaurant made \$545 from selling 110 hamburgers. If they sold 53 hamburgers on Tuesday, how much did they make?
- 32. MODELS An artist used interlocking building blocks to build a scale model of Kennedy Space Center, Florida. In the model, 1 inch equals 1.67 feet of an actual space shuttle. The model is 110.3 inches tall. How tall is the actual space shuttle? Round to the nearest tenth.
- **33. GEOGRAPHY** On a map of Florida, the distance between Jacksonville and Tallahassee is 7.5 centimeters. If 2 centimeters = 40 miles, what is the distance between the two cities?



Solve each proportion. If necessary, round to the nearest hundredth.

34.	$\frac{6}{14} = \frac{7}{x-3}$	35.	$\frac{7}{4} = \frac{f-4}{8}$	36.	$\frac{3-y}{4} = \frac{1}{9}$
37.	$\frac{4v+7}{15} = \frac{6v+2}{10}$	38.	$\frac{9b-3}{9} = \frac{5b+5}{3}$	39.	$\frac{2n-4}{5} = \frac{3n+3}{10}$

- **40.** ATHLETES At Piedmont High School, 3 out of every 8 students are athletes. If there are 1280 students at the school, how many are not athletes?
- 41. BRACES Two out of five students in the ninth grade have braces. If there are 325 students in the ninth grade, how many have braces?
- 42. PAINT Joel used a half gallon of paint to cover 84 square feet of wall. He has 932 square feet of wall to paint. How many gallons of paint should he purchase?



#### Real-World Link

At a drive-in theater, the film is projected onto a large outdoor screen while the moviegoers sit in their cars. The sound for the film is broadcast on a radio station. In 1987, there were over 2000 drive-in theaters. By 1989, half of them had closed.

Source: North American Theater Owners

#### **MOVIE THEATERS** Use the table at the right.

- **a.** Write a ratio of the number of indoor theaters to the total number of theaters for each year.
- **b.** Do any two of the ratios you wrote for part a form a proportion? If so, explain the real-world meaning of the proportion.
- **44. DIARIES** In a survey, 36% of the students said that they kept an electronic diary. There were 900 students who kept an electronic diary. How many students were in the survey?

Year	Indoor	Drive-In	Total
2000	35,567	683	36,250
2001	34,490	683	35,173
2002	35,170	666	35,836
2003	35,361	634	35,995
2004	36,012	640	36,652
2005	37,092	648	37,740
2006	37,776	649	38,425

Source: North American Theater Owners

- **45. MULTIPLE REPRESENTATIONS** In this problem, you will explore how changing the lengths of the sides of a shape by a factor changes the perimeter of that shape.
  - **a. GEOMETRIC** Draw a square *ABCD*. Measure and label the sides. Draw a second square *MNPQ* with sides twice as long as *ABCD*. Draw a third square *FGHJ* with sides half as long as *ABCD*.
  - **b. TABULAR** Complete the table below using the appropriate measures.

ABCD	MNPQ	FGHJ
Side length	Side length	Side length
Perimeter	Perimeter	Perimeter

**c. VERBAL** Make a conjecture about the change in the perimeter of a square if the side length is increased or decreased by a factor.

#### H.O.T. Problems

Use Higher-Order Thinking Skills

- **46. OPEN ENDED** Write a real-life example of a ratio.
- **47. REASONING** Compare and contrast ratios and rates.
- **48.** CHALLENGE If  $\frac{a+1}{b-1} = \frac{5}{1}$  and  $\frac{a-1}{b+1} = \frac{1}{1}$ , find the value of  $\frac{b}{a}$ . (*Hint:* Choose different values of *a* and *b* for which the proportions are true and evaluate the expression  $\frac{b}{a}$ .)
- **49. FIND THE ERROR** Tim and Aisha are solving the following problem. Is either of them correct? Explain.

Two years ago, 78 women were enrolled in a dance class, while 162 men were enrolled. This year 193 men enrolled, while the ratio of women to men did not change. How many women enrolled this year?



**50.** WRITING IN MATH Describe how businesses can use ratios. Write about a real-world situation in which a business would use a ratio.

# **Standardized Test Practice**

**51.** In the figure, x : y = 2 : 3 and y : z = 3 : 5. If x = 10, find the value of *z*.



- **A** 15
- **B** 20
- C 25
- **D** 30
- **52. GRIDDED RESPONSE** A race car driver records the finishing times for recent practice trials.

Trial	Time (seconds)
1	5.09
2	5.10
3	4.95
4	4.91
5	5.05

What is the mean time, in seconds, for the trials?



**54.** Which equation below illustrates the Commutative Property?

A 
$$(3x + 4y) + 2z = 3x + (4y + 2z)$$

$$\mathbf{B} \quad 7(x+y) = 7x + 7y$$

- **C** xyz = yxz
- $\mathbf{D} \ x + 0 = x$

# **Spiral Review**

Solve each equation. (Lesson 2-5)

- **55.** |x+5| = -8
- **57.** |2p 3| = 17

56. |b + 9| = 2
58. |5c - 8| = 12

**61.** 7g - 14 = -63

**63.**  $\frac{t}{8} - 6 = -12$ 

**59. HEALTH** When exercising, a person's pulse rate should not exceed a certain limit. This maximum rate is represented by the expression 0.8(220 - a), where *a* is age in years. Find the age of a person whose maximum pulse rate is 152. (Lesson 2-4)

Solve each equation. Check your solution. (Lesson 2-3)

- **60.** 15 = 4a 5
- **62.**  $9 + \frac{y}{5} = 6$
- **64. GEOMETRY** Find the area of  $\triangle ABC$  if each small triangle has a base of 5.2 inches and a height of 4.5 inches. (Lesson 1-3)

Evaluate each expression. (Lesson 1-2)

**65.**  $3 + 16 \div 8 \cdot 5$ 

**66.**  $4^2 \cdot 3 - 5(6+3)$ 

# **Skills Review**

Solve each equation. (Lesson 2-2)

**67.** 4*p* = 22

**68.** 5*h* = 33

**69.** 1.25y = 4.375

С

**70.** 9.8*m* = 30.87

B

# Spreadsheet Lab Financial Ratios

You can use a spreadsheet to investigate the debt-to-income ratio in mortgage lending.

# ACTIVITY

Dorrie is thinking about buying a house. She has the following expenses: rent of \$650, credit card monthly bills of \$320, a car payment of \$410, and a student loan payment of \$115. Dorrie has a yearly salary of \$46,500. You can use a spreadsheet to find Dorrie's debt-to-income ratio.

**Step 1** Enter Dorrie's debts in column B.

- **Step 2** Add her debts using a function in cell B6. Go to Insert and then Function. Then choose Sum. The resulting sum of 1495 should appear in B6.
- **Step 3** Now insert Dorrie's salary in column C. Remember to find her monthly salary by dividing the yearly salary by 12.

A mortgage company will use her debt-to-income ratio in part to determine if Dorrie qualifies for a mortgage loan. The **debt-to-income ratio** is calculated as *how much she owes per month* divided by *how much she earns each month*.

**Step 4** Enter a formula to find the debt-to-income ratio in cell C6. In the formula bar, enter =B6/C2.

The ratio of about 0.39 appears. An ideal ratio would be 0.36 or less. A ratio higher than 0.36 would cause an increased interest rate or may require a higher down payment.

The spreadsheet shows a debt-to-income ratio of about 0.39. Dorrie should try to eliminate or reduce some debts or try to earn more money in order to lower her debt-to-income ratio.

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Lab 2-6 B Spreadsheet.xls 🛛 🖃 🔀					
$\diamond$	А	В	С	^	
1	Type of Debt	Expenses	Salary		
2	Rent	650	3875		
3	Credit Cards	320			
4	Car Payment	410			
5	Student Loan	115			
6		1495	0.385806		
7					
<	111			>	

### **Exercises**

- 1. If Dorrie waits until she pays off her credit card bills to buy a house, what would be her new debt-to-income ratio?
- **2.** Dorrie decides to reduce her monthly credit card payments to \$160 per month, and she sells her car. How would her debt-to-income ratio change?
- 3. How could Dorrie improve her debt-to-income ratio?
- **4.** How would your spreadsheet be different if Dorrie had income other than her monthly salary?

# Then

You solved proportions. (Lesson 2-6)

### Now/

- Find the percent of change.
- Solve problems involving percent of change.

#### New Vocabulary

percent of change percent of increase percent of decrease

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# **Percent of Change**

# Why?

Every year, millions of people volunteer their time to improve their community. The difference in the number of volunteers from one year to the next can be used to determine a percent to represent the increase or decrease in volunteers.

**Percent of Change Percent of change** is the ratio of the change in an amount to the original amount expressed as a percent. If the new number is greater than the original number, the percent of change is a **percent of increase.** If the new number is less than the original number, the percent of change is a **percent of decrease.** 





Source: Bureau of Labor and Statistics

### EXAMPLE 1 Percent of Change

Determine whether each percent of change is a percent of *increase* or a percent of *decrease*. Then find the percent of change.

**a**. original: 20 final: 23

Subtract the original amount from the final amount to find the amount of change: 23 - 20 = 3.

Since the new amount is greater than the original, this is a percent of increase.

Use the original number, 20, as the base.

change  $\rightarrow$   $\frac{3}{20} = \frac{r}{100}$ original amount  $\rightarrow$   $\frac{3}{20} = r(20)$  3(100) = r(20) 300 = 20r  $\frac{300}{20} = \frac{20r}{20}$ 15 = r

The percent of increase is 15%.

#### Check Your Progress

- **1A.** original: 66 new: 30
- **1C.** original: 24 new: 40

# **b.** original: 25 final: 17

Subtract the original amount from the final amount to find the amount of change: 17 - 25 = -8.

Since the new amount is less than the original, this is a percent of decrease.

Use the original number, 25, as the base.

change —> original amount —>	$\frac{-8}{25} = \frac{r}{100}$
-80	(100) = r(25)
-	-800 = 25r
-	$\frac{-800}{25} = \frac{25r}{25}$
	-32 = r

The percent of decrease is 32%.

- **1B.** original: 9.8 new: 12.1**1D.** original: 500
  - new: 131

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#### Real-World Link

Over 11.8 million people around the world went on a cruise in 2006. Source: M&L Research, Inc.

# Real-World EXAMPLE 2 Percent of Change

**CRUISE** The number of cruise ships in North America increased 18% from 2000 to 2005. If there were 192 ships in 2005, how many were there in 2000?

Let c = the number of cruise ships in 2000. Since 18% is a percent of increase, the number of cruise ships in 2000 is less than the number of ships in 2005.

change —> original amount —>	$\frac{192 - c}{c} = \frac{18}{100}$	Percent proportion
(192 -	(-c)100 = 18c	Find the cross products.
19,200	-100c = 18c	<b>Distibutive Property</b>
19,200 — 100 <i>c</i>	+ 100c = 18c + 100c	Add 100c to each side.
	19,200 = 118c	Simplify.
	$\frac{19,200}{118} = \frac{118c}{118}$	Divide each side by 118.
	$163 \approx c$	Simplify.

There were approximately 163 cruise ships in 2000.

#### Check Your Progress

**2. TUITION** A recent percent of increase in tuition at Northwestern University, in Evanston, Illinois, was 5.4%. If the new cost is \$33,408 per year, find the original cost per year.

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**Solve Problems** Two applications of percent of change are sales tax and discounts. Sales tax is an example of a percent of increase. Discount is an example of a percent of decrease.

### EXAMPLE 3 Sales Tax

**SHOPPING** Marta is purchasing wire and beads to make jewelry. Her merchandise is \$28.62 before tax. If the tax is 7.25% of the total sales, what is the final cost?

Step 1 Find the tax.

The tax is 7.25% of the price of the merchandise.

7.25% of  $$28.62 = 0.0725 \times 28.62$  **7.25% = 0.0725** 

= 2.07495 Use a calculator.

#### Step 2 Find the cost with tax.

Round \$2.07495 to \$2.07 since tax is always rounded to the nearest cent. Add this amount to the original price: \$28.62 + \$2.07 = \$30.69.

The total cost of Marta's jewelry supplies is \$30.69.

#### Check Your Progress

3. SHOPPING A new DVD costs \$24.99. If the sales tax is 6.85%, what is the total cost?

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To find a discounted amount, you will follow similar steps to those for sales tax.

#### **Study**Tip

#### **Key Words**

When translating a problem from word sentences to math sentences, the word "is" translates to =, and the word "of" translates to  $\times$ .

#### EXAMPLE 4 Discounts

**DISCOUNT** Since Tyrell has earned good grades in school, he qualifies for the Good Student Discount on his car insurance. His monthly payment without the discount is \$85. If the discount is 20%, what will he pay each month?

#### Step 1 Find the discount.

The discount is 20% of the original payment.

 $20\% \text{ of } \$85 = 0.20 \times 85$  **20% = 0.20** 

Use a calculator.

**Step 2** Find the cost after discount.

= 17

Subtract \$17 from the original payment: \$85 - \$17 = \$68.

With the Good Student Discount, Tyrell will pay \$68 per month.

#### Check Your Progress

**4. SALES** A picture frame originally priced at \$14.89 is on sale for 40% off. What is the discounted price?

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# Check Your Understanding

Example 1 p. 119 State whether each percent of change is a percent of *increase* or a percent of *decrease*. Then find the percent of change. Round to the nearest whole percent.

#### original: 78 new: 125

**2.** original: 41 new: 24

**3.** original: 6 candles new: 8 candles

**4.** original: 35 computers new: 32 computers

#### Example 2 p. 120

**5. GEOGRAPHY** The distance from Phoenix to Tucson is 120 miles. The distance from Phoenix to Flagstaff is about 21.7% longer. To the nearest mile, what is the distance from Phoenix to Flagstaff?

Example 3 p. 120

#### Find the total price of each item.

**6.** dress: \$22.50 sales tax: 7.5%

**7.** video game: \$35.99 sales tax: 6.75%

- **8. PROM** A limo costs \$85 to rent for 3 hours plus a 7% sales tax. What is the total cost to rent a limo for 6 hours?
- **9. GAMES** A computer game costs \$49.95 plus a 6.25% sales tax. What is the total cost of the game?

Example 4 p. 121

#### Find the discounted price of each item.

- **10.** guitar: \$95.00 discount: 15%
   **11.** DVD: \$22.95 discount: 25%
- **12. SKATEBOARD** A skateboard costs \$99.99. If you have a coupon for 20% off, how much will you save?
- **13. TICKETS** Tickets to the county fair are \$8 for an adult and \$5 for a child. If you have a 15% discount card, how much will 2 adult tickets and 2 child tickets cost?

# **Practice and Problem Solving**

State whether each percent of change is a percent of increase or a percent of Example 1 decrease. Then find the percent of change. Round to the nearest whole percent. p. 119

- 14. original: 35 new: 40
- 16. original: 27 new: 73
- **18.** original: 21.2 grams new: 10.8 grams
- **20.** original: \$68 new: \$76
- 22. GASOLINE The average cost of regular gasoline in North Carolina increased by 73% from 2006 to 2007. If the average cost of a gallon of gas in 2006 was \$2.069,
- **23.** CARS Beng is shopping for a car. The cost of a new car is \$15,500. This is 25%

Example 3 p. 120

Example 2

p. 120

- tax: 5.5% tax: 7.25%
- **27.** book: \$25.95 tax: 5.75% tax: 5.25%

#### **Example 4**

p. 121



#### Real-World Link

Softball became an Olympic event in 1996. The American women, winning the gold medal in 2004, capped a 79-game winning streak. Source: USA TODAY

#### 122 Chapter 2 Linear Equations

- what was the average cost in 2007? Round to the nearest cent.
- greater than the cost of a used car. What is the cost of the used car?
- Find the total price of each item.
- **24.** messenger bag: \$28.00 **25.** software: \$45.00

#### Find the discounted price of each item.

- **31.** CD player: \$89.99 **30.** computer: \$1099.00 discount: 25% discount: 15%
- **33.** jeans: \$24.50 34. jacket: \$125.00 discount: 25% discount: 33%

#### Find the final price of each item.

- **37.** printer: \$60.00 **36.** sweater: \$14.99 discount: 15% discount: 25% discount: 12% tax: 6.25% tax: 6.75% tax: 7.5%
- **39.** CONSUMER PRICE INDEX An *index* measures the percent change of a value from a base year. An index of 115 means that there was a 15% increase from the base year. In 2000, the consumer price index of dairy products was 160.7. In 2005, it was 182.4. Determine the percent of change.
- **40. FINANCIAL LITERACY** The current price of each share of a technology company is \$135. If this represents a 16.2% increase over the past year, what was the price per share a year ago?
- **41. SHOPPING** A group of girls are shopping for dresses to wear to the spring dance. One finds a dress priced \$75 with a 20% discount. A second girl finds a dress priced \$85 with a 30% discount.

a. Find the amount of discount for each dress.

- **b.** Which girl is getting the better price for the dress?
- 42. RECREATIONAL SPORTS In 1995, there were 73,567 youth softball teams. By 2007, there were 86,049. Determine the percent of increase.

- 15 original: 16 new: 10
- **17.** original: 92 new: 21
- **19.** original: 11 feet new: 25 feet
- 21. original: 21 hours new: 40 hours

- **28.** magazine: \$3.50
- tax: 6.25% 29. pillow: \$9.99

26. vase: \$5.50

- tax: 6.75%
- **32.** athletic shoes: \$59.99 discount: 40%
- 35. belt: \$14.99 discount: 20%

**38.** board game: \$25.00

**GROCERIES** Which grocery item had the greatest percent increase in cost from 2000 to 2005?

Average Retail Prices of Selected Grocery Items				
Grocery Item	Cost in 2000 (\$ per pound)	Cost in 200 (\$ per pound		
milk (gallon)	2.79	3.24		
eggs (dozen)	0.96	1.35		
chicken (whole)	1.08	1.06		
ground beef	1.63	2.30		
apples	0.82	0.97		
iceberg lettuce	0.85	0.85		
peanut butter	1.96	1.70		

Source: Statistical Abstract of the United States

44. S MULTIPLE REPRESENTATIONS In this problem, you will explore patterns in percentages.

**a. TABULAR** Copy and complete the following table.

1% of	500	is 5.	100% of	is 20.	% of 80 is 20.
2% of		is 5.	50% of	is 20.	% of 40 is 20.
4% of		is 5.	25% of	is 20.	% of 20 is 20.
8% of		is 5.	12.5% of	is 20.	% of 10 is 20.

- **b. VERBAL** Describe the patterns in the second and fifth columns.
- **c. ANALYTICAL** Use the patterns to write the fifth row of the table.



#### Real-World Link

In a recent year, the amount of sales tax varied across the United States from 0% to 7.25%.

Source: Federation of Tax Administrators

# H.O.T. Problems

Use Higher-Order Thinking Skills

- 45. OPEN ENDED Write a real-world problem to find the total price of an item including sales tax.
- **46. REASONING** If you have 75% of a number *n*, what percent of decrease is it from the number *n*? If you have 40% of a number *a*, what percent of decrease do you have from the number *a*? What pattern do you notice? Is this always true?
- 47. FIND THE ERROR Maddie and Xavier are solving for the percent change if the original amount was \$25 and the new amount is \$28. Is either of them correct? Explain your reasoning.

Maddie  $\frac{3}{28} = \frac{r}{100}$ 3(100) = 28r300 = 28r10.7 = r

Xavier  $r = \frac{r}{100}$ 25 3(100) = 25r300 = 25r12 = r

- 48. CHALLENGE Determine whether the following statement is sometimes, always, or never true. The percent of change is less than 100%.
- 49. WRITING IN MATH Explain how to find a percent of change between two values and how to determine whether the change is a percent of increase or decrease.

# **Standardized Test Practice**

**50. GEOMETRY** The rectangle has a perimeter of *P* centimeters. Which equation could be used to find the length  $\ell$  of the rectangle?



**51. SHORT RESPONSE** Henry is painting a room with four walls that are 12 feet by 14 feet. A gallon of paint costs \$18 and covers 350 square feet. If he uses two coats of paint, how much will it cost him to paint the room?

- **52.** The number of students at Franklin High School increased from 840 to 910 over a 5-year period. What was the percent of increase?
  - **F** 8.3%
  - **G** 14.0%
  - H 18.5%
  - J 92.3%

**53. PROBABILITY** Two dice are rolled. What is the probability that the sum is 10?

**A** 
$$\frac{1}{6}$$
 **B**  $\frac{1}{3}$  **C**  $\frac{1}{12}$  **D**  $\frac{1}{36}$ 

**57.** |y + x| - z + 4

# **Spiral Review**

**54. TRAVEL** The Chan's minivan requires 5 gallons of gasoline to travel 120 miles. How many gallons of gasoline will they need to travel 360 miles? (Lesson 2-6)

Evaluate each expression if x = -2, y = 6, and z = 4. (Lesson 2-5)

**55.** |3 - x| + 7 **56.** 12 - |z + 9|

Solve each equation. Round to the nearest hundredth. Check your solution. (Lesson 2-4)

58.	1.03p - 4 = -2.15p + 8.72	59.	18 - 3.8t = 7.36 - 1.9t
60.	5.4w + 8.2 = 9.8w - 2.8	61.	2[d + 3(d - 1)] = 18

Solve each equation. Check your solution. (Lesson 2-3)

62.	5n + 6 = -4	63.	-11 = 7 + 3c
64.	15 = 4a - 5	65.	-14 + 7g = -63

**66. RIVERS** The Congo River in Africa is 2900 miles long. That is 310 miles longer than the Niger River, which is also in Africa. (Lesson 2-2)

**a.** Write an equation you could use to find the length of the Niger River.

**b.** What is the length of the Niger River?

alate each equation into a contance (losson )

**67. GEOMETRY** Two perpendicular lines meet to form four right angles. Write two different if-then statements for this definition. (Lesson 1-8)

# **Skills Review**

IId	issiale each equation into a sen		
<b>68</b> .	d - 14 = 5	<b>69.</b> $2f + 6 = 19$	<b>70.</b> $y - 12 = y + 8$
71.	3a + 5 = 27 - 2a	<b>72.</b> $-6c^2 - 4c = 25$	<b>73.</b> $d^4 + 64 = 3d^3 + 77$

**124** Chapter 2 Linear Equations

# Algebra Lab Percentiles

**Objective** Use percentiles to represent data. A **percentile** is a measure that is often used to report test data, such as standardized test scores. It tells us what percent of the total scores were below a given score.

- Percentiles measure rank from the bottom.
- There is no 0 percentile rank. The lowest score is at the 1st percentile.
- There is no 100th percentile rank. The highest score is at the 99th percentile.

# ACTIVITY

A talent show was held for the fifteen finalists in the Teen Idol contest. Each performer received a score from 0 through 30 with 30 being the highest. What is Victor's percentile rank?

Name	Score	Name	Score
Arnold	17	Malik	10
Benito	9	Natalie	26
Carmen	21	Pearl	4
Delia	29	Twyla	6
Fernando	15	Victor	28
Horatio	5	Warren	22
Ingrid	11	Yolanda	18
Ishi	27		

**Step 1** Write one score on each of 15 slips of paper.

**Step 2** Arrange the slips vertically from greatest to least score.

**Step 3** Find Victor's percentile rank.

Victor had a score of 28. There are 13 scores below his score. To find his percentile rank, use the following formula:

 $\frac{\text{number of scores below 28}}{\text{total number of scores}} \cdot 100 = \frac{13}{15} \cdot 100 \text{ or about 87.}$ 

Victor scored at the 87th percentile in the contest.

### **Analyze the Results**

- 1. Find the median, lower quartile, and upper quartile of the scores.
- **2.** Which performer was at the 50th percentile? Which performer was at the 25th percentile? the 75th percentile?
- **3.** Compare and contrast the values for the median, lower quartile, and upper quartile and the scores for the 25th, 50th, and 75th percentiles.
- **4.** While Victor scored at the 87th percentile, what percent of the 30 possible points did he score?
- 5. Compare and contrast the percentile rank and the percent score.
- **6.** Are there any outliers in the data that could alter the results of our computations?

# Then

You solved equations with variables on each side. (Lesson 2-4)

#### Now/

- Solve equations for given variables.
- Use formulas to solve real-world problems.

#### New Vocabulary

literal equation dimensional analysis unit analysis

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# Literal Equations and Dimensional Analysis

# Why?

Each year, more people use credit cards to make everyday purchases. If the entire balance is not paid by the due date, compound interest is applied. The formula for computing the balance of an account with compound interest added annually is  $A = P(1 + r)^t$ .

- *A* represents the amount of money in the account including the interest,
- *P* is the amount in the account before interest is added,
- *r* is the interest rate written as a decimal,
- *t* is the time in years.



**Solve for a Specific Variable** Some equations such as the one above contain more than one variable. At times, you will need to solve these equations for one of the variables.

# EXAMPLE 1 Solve for a Specific Variable

Solve 4m - 3n = 8 for m. 4m - 3n = 8 Original equation 4m - 3n + 3n = 8 + 3n Add 3n to each side. 4m = 8 + 3n Simplify.  $\frac{4m}{4} = \frac{8 + 3n}{4}$  Divide each side by 4.  $m = \frac{8}{4} + \frac{3}{4}n$  Simplify.  $m = 2 + \frac{3}{4}n$  Simplify.

### Check Your Progress

Solve each equation for the variable indicated.

**1A.** 
$$15 = 3n + 6p$$
, for  $n$ **1B.**  $\frac{k-2}{5} = 11j$ , for  $k$ **1C.**  $28 = t(r+4)$ , for  $t$ **1D.**  $a(q-8) = 23$ , for  $q$ 

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Sometimes we need to solve equations for a variable that is on both sides of the equation. When this happens, you must get all terms with that variable onto one side of the equation. It is then helpful to use the Distributive Property to isolate the variable for which you are solving.

#### StudyTip

**Solving for a Specific** Variable When an equation has more than one variable, it can be helpful to highlight the variable for which you are solving on your paper.

# EXAMPLE 2 Solve for a Specific Variable



Add 2y to each side. Subtract xz from each side. **Distributive Property** Divide each side by 3 - z.

Since division by 0 is undefined,  $3 - z \neq 0$  so  $z \neq 3$ .

#### **Check Your Progress**

#### Solve each equation for the variable indicated.

**2A.** d + 5c = 3d - 1, for d

**2B.** 6q - 18 = qr + t, for q

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**Use Formulas** A formula or equation that involves several variables is called a **literal** equation. To solve a literal equation, apply the process of solving for a specific variable.

#### Real-World EXAMPLE 3 **Use Literal Equations**

**YO-YOS** Use the information about the largest yo-yo at the left. The formula for the circumference of a circle is  $C = 2\pi r$ , where C represents circumference and r represents radius.

**a**. Solve the formula for *r*.

$$C = 2\pi r$$
Formula for circumference $\frac{C}{2\pi} = \frac{2\pi r}{2\pi}$ Divide each side by  $2\pi$ . $\frac{C}{2\pi} = r$ Simplify.

**b.** Find the radius of the yo-yo.

$$\frac{C}{2\pi} = r$$
Formula for radius $\frac{32.7}{2\pi} = r$  $C = 32.7$  $5.2 \approx r$ Use a calculator.

The yo-yo has a radius of about 5.2 feet.

#### **Check Your Progress**

- **3. GEOMETRY** The formula for the volume of a rectangular prism is  $V = \ell w h$ , where  $\ell$  is the length, *w* is the width, and *h* is the height.
  - **A.** Solve the formula for *w*.
  - **B.** Find the width of a rectangular prism that has a volume of 79.04 cubic centimeters, a length of 5.2 centimeters, and a height of 4 centimeters.

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Real-World Link

The largest yo-yo in the world is 32.7 feet in circumference. It was launched by crane from a height of 189 feet.

Source: Guinness Book of World Records

When using formulas, you may want to use dimensional analysis. **Dimensional analysis** or **unit analysis** is the process of carrying units throughout a computation.

# **Study**Tip

Rewriting Equations with Exponents

When rewriting an equation to solve for a specific variable, the exponents of a variable are moved with that variable. They are not separated.

#### **EXAMPLE 4** Use Dimensional Analysis

**RUNNING** A 10K run is 10 kilometers long. If 1 meter = 1.094 yards, use dimensional analysis to find the length of the race in miles. (*Hint*: 1 mi = 1760 yd)

Since the given conversion relates meters to yards, first convert 10 kilometers to meters. Then multiply by the conversion factor such that the unit meters

are divided out. To convert from yards to miles, multiply by  $\frac{1 \text{ Im}}{1760 \text{ yd}}$ .

length of run	×	kilometers to meters	×	meters to yards	×	yards to miles
10 km	×	<u>1000 m</u> 1 km	×	1.094 yd 1 m	×	<u>1 mi</u> 1760 yd

Notice how the units cancel, leaving the unit to which you are converting.

$$10 \text{ km} \times \frac{1000 \text{ sm}}{1 \text{ km}} \times \frac{1.094 \text{ yd}}{1 \text{ sm}} \times \frac{1 \text{ mi}}{1760 \text{ yd}} = \frac{10940}{1760} \text{ mi}$$
$$= \frac{10,940}{1760}$$
$$\approx 6.2 \text{ mi}$$

A 10K race is approximately 6.2 miles.

#### Check Your Progress

**4.** A car travels a distance of 100 feet in about 2.8 seconds. What is the velocity of the car in miles per hour? Round to the nearest whole number.

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# Check Your Understanding

Examples 1 and 2	Solve each equation or formula	for the variable indicated.
рр. 126–127	1) $5a + c = -8a$ , for a	<b>2.</b> $7h + f = 2h + g$ , for g
	<b>3.</b> $\frac{k+m}{-7} = n$ , for k	<b>4.</b> $q = p(r + s)$ , for $p$
Example 3 p. 127	<b>5. PACKAGING</b> A soap company container to hold their new l	wants to use a cylindrical iquid soap.
	<b>a.</b> Solve the formula for <i>h</i> .	·
	<b>b.</b> What is the height of a consistence of the base o	tainer if the volume is 56.52 cubic 5 inches? Round to the nearest tenth. $V = \pi r^2 h$
<b>Example 4</b> p. 128	<b>6. SHOPPING</b> Scott found a rare 35 Australian dollars. If the the cost of the game in Unite	video game on an online auction site priced at exchange rate is \$1 U.S. = \$1.24 Australian, find ed States dollars. Round to the nearest cent.
	<b>7. PHOTOGRAPHY</b> A fisheye len 1 centimeter is equal in leng range of the lens in feet?	s has a minimum focus range of 13.5 centimeters. If th to about 0.39 inches, what is the minimum focus

# **Practice and Problem Solving**

= Step-by-Step Solutions begin on page R12. Extra Practice begins on page 815.

**Examples 1 and 2** pp. 126-127 Solve each equation or formula for the variable indicated.

9 x = b - cd, for c **8.** u = vw + z, for v **10.** fg - 9h = 10j, for g **11.** 10m - p = -n, for m **13.**  $\frac{5}{9}v + w = z$ , for v **12.**  $r = \frac{2}{2}t + v$ , for t **15.**  $\frac{df + 10}{6} = g$ , for f14.  $\frac{10ac - x}{11} = -3$ , for a

#### Example 3 p. 127

- **16.** FITNESS The formula to compute a person's body mass index is  $B = 703 \cdot \frac{w}{h^2}$ . *B* represents the body mass index, *w* is the person's weight in pounds, and *h* represents the person's height in inches.
  - **a.** Solve the formula for *w*.
  - **b.** What is the weight to the nearest pound of a person who is 64 inches tall and has a body mass index of 21.45?
- 17. PHYSICS Acceleration is the measure of how fast a velocity is changing. The

formula for acceleration is  $a = \frac{v_f - v_i}{t}$ . *a* represents the acceleration rate,

- $v_f$  is the final velocity,  $v_i$  is the initial velocity, and t represents the time in seconds.
- **a.** Solve the formula for  $v_f$ .
- **b.** What is the final velocity of a runner who is accelerating at 2 feet per second squared for 3 seconds with an initial velocity of 4 feet per second?
- **18.** SWIMMING If each lap in a pool is 100 meters long, how many laps equal one mile? Round to the nearest tenth. (*Hint*: 1 foot  $\approx 0.3048$  meter)
- **19. GASOLINE** How many liters of gasoline are needed to fill a 13.2-gallon tank? There are about 1.06 quarts per 1 liter. Round to the nearest tenth.

Solve each equation or formula for the variable indicated.

<b>20.</b> $-14n + q = rt - 4n$ , for <i>n</i>	<b>21.</b> $18t + 11v = w - 13t$ , for t
<b>22.</b> $ax + z = aw - y$ , for a	<b>23.</b> $10c - f = -13 + cd$ , for c

Select an appropriate unit from the choices below and convert the rate to that unit.

- mph
- mm/s

#### km/s

- **24.** a car traveling at 36 ft/s **25.** a snail moving at 3.6 m/h
- **26.** a person walking at 3.4 mph
- **27.** a satellite moving at 234,000 m/min
- **28. DANCING** The formula  $P = \frac{1.2W}{H^2}$  represents the amount of pressure exerted on the floor by a ballroom dancer's heel. In this formula, *P* is the pressure in pounds per square inch, W is the weight of a person wearing the shoe in pounds, and H is the width of the heel of the shoe in inches.
  - **a.** Solve the formula for *W*.
  - **b.** Find the weight of the dancer if the heel is 3 inches wide and the pressure exerted is 30 pounds per square inch.

#### Real-World Link

In most dance competitions, dancers compete as couples, or formation teams. At "Jack & Jill" competitions, individuals are paired by random draw.

Source: USA Dance

#### Example 4 p. 128



Write an equation and solve for the variable indicated.

- **29.** Seven less than a number *t* equals another number *r* plus 6. Solve for *t*.
- **30.** Ten plus eight times a number *a* equals eleven times another number *d* minus six. Solve for *a*.
- **31.** Nine tenths of a number *g* is the same as seven plus two thirds of another number *k*. Solve for *k*.
- **32.** Three fourths of a number *p* less two is five sixths of another number *r* plus five. Solve for *r*.
- **GIFTS** Ashley has 214 square inches of paper to wrap a gift box. The surface area *S* of the box can be found by using the formula  $S = 2w(\ell + h) + 2\ell h$ , where *w* is the width of the box,  $\ell$  is the length of the box, and *h* is the height. If the length of the box is 7 inches and the width is 6 inches, how tall can Ashley's box be?
- **34.** Solution 34. The surface area of cylinder can be found by the formula  $S = 2\pi rh + 2\pi r^2$ .
  - **a. ALGEBRAIC** Solve for *h*. Rewrite the solution using 3.14 for  $\pi$  and 2500 for *S*.
  - **b. TABULAR** Make a table of values using your new formula to find *h* if r = 20, 15, 10, 5, and 0. Round to the nearest hundredth.
  - **c. VERBAL** What do we know about the domain (possible values of *r*)?

### H.O.T. Problems Use Higher-Order Thinking Skills

- **35. CHALLENGE** The circumference of an NCAA women's basketball is 29 inches, and the rubber coating is  $\frac{3}{16}$  inch thick. Use the formula  $v = \frac{4}{3}\pi r^3$ , where *v* represents the volume and *r* is the radius of the inside of the ball, to determine the volume of the air inside the ball. Round to the nearest whole number.
- **36. REASONING** Select an appropriate unit to describe the highway speed of a car and the speed of a caterpillar crawling on a tree. Can the same unit be used for both situations? Explain.
- **37. FIND THE ERROR** Sandrea and Fernando are solving 4a 5b = 7 for *b*. Is either of them correct? Explain.

Sandrea	Fernando
4a - 5b = 7	4a-5b=7
$ \frac{-5b}{-5} = 7 - 4a \\ \frac{-5b}{-5} = \frac{7 - 4a}{-5} \\ b = \frac{7 - 4a}{-5} $	$5b = 7 - 4a$ $\frac{5b}{5} = \frac{7 - 4a}{5}$ $b = \frac{7 - 4a}{5}$

- **38. OPEN ENDED** Write a formula for *A*, the area of a geometric figure such as a triangle or rectangle. Then solve the formula for a variable other than *A*.
- 39. CHALLENGE Solve each equation or formula for the variable indicated.

**a.** 
$$n = \frac{x + y - 1}{xy}$$
 for x

**b.** 
$$\frac{x+y}{x-y} = \frac{1}{2}$$
 for y

**40.** WRITING IN MATH Explain what a literal equation is and how to solve one.



#### Real-World Link

In 2007, Louisiana State University lost to the University of Connecticut by a score of 71 to 72, ending a 43-game winning streak, an NCAA Women's Basketball record.

Source: National Collegiate Athletic Association

# **Standardized Test Practice**

41. Eula is investing \$6000, part at 4.5% interest and the rest at 6% interest. If *d* represents the amount invested at 4.5%, which expression represents the amount of interest earned in one year by the amount paying 6%?

A	0.06d	С	0.06(d + 6000)	
B	0.06(d - 6000)	D	0.06(6000 - d)	

42. Todd drove from Boston to Cleveland, a distance of 616 miles. His breaks, gasoline, and food stops took 2 hours. If his trip took 16 hours altogether, what was Todd's average speed?

F	38.5 mph	Н	44 mph
G	40 mph	J	47.5 mph

- **43.** SHORT RESPONSE Brian has 3 more books than Erika. Jasmine has triple the number of books that Brian has. Altogether Brian, Erika, and Jasmine have 22 books. How many books does Jasmine have?
- 44. **GEOMETRY** Which of the following best describes a plane?
  - A a location having neither size nor shape
  - **B** a flat surface made up of points having no depth
  - **C** made up of points and has no thickness or width
  - D a boundless, three-dimensional set of all points

# **Spiral Review**

#### Find the final price of each item. (Lesson 2-7)

45. lamp: \$120.00 discount: 20% tax: 6%

discount: 30% tax: 7%

46. dress: \$70.00

47. camera: \$58.00 discount: 25% tax: 6.5%

48. jacket: \$82.00 discount: 15% tax: 6%

- **49.** comforter: \$67.00 discount: 20% tax: 6.25%
- **50.** lawnmower: \$720.00
- discount: 35% tax: 7%

#### Solve each proportion. If necessary, round to the nearest hundredth. (Lesson 2-6)

- **52.**  $\frac{2}{0.36} = \frac{7}{p}$ **53.**  $\frac{m}{9} = \frac{2.8}{4.9}$ **51.**  $\frac{3}{45} = \frac{x}{25}$
- 54. JOBS Laurie mows lawns to earn extra money. She can mow at most 30 lawns in one week. She profits \$15 on each lawn she mows. Identify a reasonable domain and range for this situation and draw a graph. (Lesson 1-6)
- **55. ENTERTAINMENT** Each member of the pit orchestra is selling tickets for the school musical. The trombone section sold 50 floor tickets and 90 balcony tickets. Write and evaluate an expression to find how much money the trombone section collected. (Lesson 1-2)



#### **Skills Review**

Solve each equation. (Lesson 2-5)

**56.** 8k + 9 = 7k + 6

**59.**  $\frac{1}{4} - \frac{2}{3}y = \frac{3}{4} - \frac{1}{3}y$ 

**57.** 3 - 4q = 10q + 10

**60.** 4(2a-1) = -10(a-5)

**58.**  $\frac{3}{4}n + 16 = 2 - \frac{1}{8}n$ **61.** 2(w-3) + 5 = 3(w-1)

# 2-9

# Then

You translated sentences into equations. (Lesson 2-1)

# Now/

- Solve mixture problems.
- Solve uniform motion problems.

#### New Vocabulary

weighted average mixture problem uniform motion problem rate problem

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# Weighted Averages

# Why?

Baseball players' performance is measured in large part by statistics. Slugging average (SLG) is a weighted average that measures the power of a hitter. The slugging average is calculated by using the following formula.

 $SLG = \frac{1B + (2 \times 2B) + (3 \times 3B) + (4 \times HR)}{at \text{ bats}}$ 



**Weighted Averages** The batter's slugging percentage is an example of a weighted average. The **weighted average** *M* of a set of data is the sum of the product of the number of units and the value per unit divided by the sum of the number of units.

**Mixture problems** are problems in which two or more parts are combined into a whole. They are solved using weighted averages. In a mixture problem, the units are usually the number of gallons or pounds and the value is the cost, value, or concentration per unit.

# Real-World EXAMPLE 1 Mixture Problem

**RETAIL** A tea company sells blended tea for \$25 per pound. To make blackberry tea, dried blackberries that cost \$10.50 per pound are blended with black tea that costs \$35 per pound. How many pounds of black tea should be added to 5 pounds of dried blackberries to make blackberry tea?

**Step 1** Let *w* be the weight of the black tea. Make a table to organize the information.

$\frac{1}{2}$	Number of Units (lb)	Price per Unit (\$)	Total Price (price)(units)
Dried Blackberries	5	10.50	10.50(5)
Black Tea	W	35	35w
Blackberry Tea	5 + W	25	25(5 + w)

Write an equation using the information in the table.

Price of				price of
blackberries	plus	price of tea	equals	blackberry tea.
10.50(5)	+	35w	=	25(5+w)

**Step 2** Solve the equation.

10.50(5) + 35w = 25(5 + w) 52.5 + 35w = 125 + 25w 52.5 + 35w - 25w = 125 + 25w - 25w 52.5 + 10w = 125 52.5 - 52.5 + 10w = 125 - 52.5 10w = 72.5w = 7.25

Original equation Distributive Property Subtract 25*w* from each side. Simplify. Subtract 52.5 from each side. Simplify. Divide each side by 10.

#### **Study**Tip

#### **Mixture Problems**

When you organize the information in mixture problems, remember that the final mixture must contain the sum of the parts in the correct quantities and at the correct percents. To make the blackberry tea, 7.25 pounds of black tea will need to be added to the dried blackberries.

#### Check Your Progress

1. **COFFEE** How many pounds of Premium coffee beans should be mixed with 2 pounds of Supreme coffee to make the Blend coffee?



Sometimes mixture problems are expressed in terms of percents.

#### Real-World EXAMPLE 2 Percent Mixture Problem

**FRUIT PUNCH** Mrs. Matthews has 16 cups of punch that is 3% pineapple juice. She also has a punch that is 33% pineapple juice. How many cups of the 33% punch will she need to add to the 3% punch to obtain a punch that is 20% pineapple juice?

**Step 1** Let x = the amount of 33% solution to be added. Make a table.

	Amount of Punch (cups)	Amount of Pineapple Juice
3% Punch	16	0.03(16)
33% Punch	X	0.33 <i>x</i>
20% Punch	16 + <i>x</i>	0.20(16 + x)

Write an equation using the information in the table.

Amount of pineapple		amount of pineapple		amount of pineapple
juice in 3% punch	plus	juice in 33% punch	equals	juice in 20% punch.
0.03(16)	+	0.33x	=	0.20(16 + x)

**Step 2** Solve the equation.

0.03(16) + 0.33x = 0.20(16 + x)	Original equation
0.48 + 0.33x = 3.2 + 0.20x	Simplify.
0.48 + 0.33x - 0.20x = 3.2 + 0.20x - 0.20x	Subtract 0.20x from each side.
0.48 + 0.13x = 3.2	Simplify.
0.48 - 0.48 + 0.13x = 3.2 - 0.48	Subtract 0.48 from each side.
0.13x = 2.72	Simplify.
$\frac{0.13x}{0.13} = \frac{2.72}{0.13}$	Divide each side by 0.13.
$x \approx 20.9$	Round to the nearest tenth.

Mrs. Matthews should add about 20.9 cups of the 33% punch to the 16 cups of the 3% punch.

#### Check Your Progress

**2. ANTIFREEZE** One type of antifreeze is 40% glycol, and another type of antifreeze is 60% glycol. How much of each kind should be used to make 100 gallons of antifreeze that is 48% glycol?

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#### Problem-SolvingTip

Make a Table Using a table is a great way to organize the given information. It also helps you understand how to write an equation to solve for the missing value.

#### Uniform Motion Problems Uniform motion problems or rate problems are

problems in which an object moves at a certain speed or rate. The formula d = rt is used to solve these problems. In the formula, d represents distance, r represents rate, and t represents time.



#### Real-World Link

In-line skating is the fourth most popular recreational activity in the U.S.

**Source:** Statistical Abstract of the United States

Math in Motion, Animation glencoe.com

#### Real-World EXAMPLE 3 Speed of One Vehicle

**INLINE SKATING** It took Travis and Tony 40 minutes to skate 5 miles. The return trip took them 30 minutes. What was their average speed for the trip?

- **Understand** We know that the boys did not travel the same amount of time on each portion of their trip. So, we will need to find the weighted average of their speeds. We are asked to find their average speed for both portions of the trip.
  - **Plan** First find the rate of the going portion, and then the return portion of the trip. Because the rate is in miles per hour we convert 40 minutes to about 0.667 hours and 30 minutes to 0.5 hours.

Going		
$r = \frac{d}{t}$		Formula for rate
$\approx \frac{5 \text{ miles}}{0.667 \text{ hour}} \text{ or a}$	about 7.5 miles per hour	Substitution $d = 5$ mi, $t = 0.667$ h
Return		
$r = \frac{d}{t}$		Formula for rate
$=\frac{5 \text{ miles}}{0.5 \text{ hour}} \text{ or } 10$	miles per hour	Substitution $d = 5$ mi, $t = 0.5$ h

Because we are looking for a weighted average we cannot just average their speeds. We need to find the weighted average for the round trip.

Solve	$M = \frac{(\text{rate of going})(\text{time of going}) + 1}{(1 + 1)^2}$	me of going) + (rate of return)(time of return)		
	time of going	+ time of return		
	$\approx \frac{(7.5)(0.667) + (10)(0.5)}{0.667 + 0.5}$	Substitution		
	$\approx \frac{10.0025}{1.167}$ or about 8.6	Simplify.		

Their average speed was about 8.6 miles per hour.

**Check** Our solution of 8.6 miles per hour is between the going portion rate, 7.5 miles per hour, and the return rate, 10 miles per hour. So, we know that our answer is reasonable.

#### Check Your Progress

**3. EXERCISE** Austin jogged 2.5 miles in 16 minutes and then walked 1 mile in 10 minutes. What was his average speed?

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The formula d = rt can also be used to solve real-world problems involving two vehicles in motion.

#### **Study**Tip

#### **Draw a Diagram**

Drawing a diagram is not just for geometry problems. You can use diagrams to visualize many problem situations that can be represented by equations.

#### 🕙 Real-World EXAMPLE 4 **Speeds of Two Vehicles**

FREIGHT TRAINS Two trains are 550 miles apart heading toward each other on parallel tracks. Train A is traveling east at 35 miles per hour, while Train B travels west at 45 miles per hour. When will the trains pass each other?

#### Step 1 Draw a diagram.



**Step 2** Let t = the number of hours until the trains pass each other. Make a table.

i ile tab	r	t	d = rt
Train A	35	t	35t
Train B	45	t	45 <i>t</i>

#### **Step 3** Write and solve an equation.

Distance traveled by		distance traveled		
Train A	plus	by Train B	equals	550 miles.
35 <i>t</i>	+	45 <i>t</i>	=	550
35t + 45t = 550	Original equa	ation		
80t = 550	Simplify.			
$\frac{80t}{80} = \frac{550}{80}$	Divide each s	side by 80.		
t = 6.875	Simplify.			

The trains will pass each other in about 6.875 hours.

#### **Check Your Progress**

4. CYCLING Two cyclists begin traveling in opposite directions on a circular bike trail that is 5 miles long. One cyclist travels 12 miles per hour, and the other travels 18 miles per hour. How long will it be before they meet?

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# **Check Your Understanding**

eck roun	Understanding		
Example 1 p. 132	<b>FOOD</b> Tasha ordered soup and salad for lunch. If Tasha ordered 10 ounces of soup for lunch and the total cost was \$3.30, how many ounces of salad did Tasha order?		
Example 2 p. 133	<b>2. CHEMISTRY</b> Margo has 40 milliliters of 25% solution. How many milliliters of 60% solution should she add to obtain the required 30% solution?	15¢/ounce	20¢/our
Example 3 p. 134	<b>3. TRAVEL</b> A boat travels 16 miles due north in 2 hours and 2 hours. What is the average speed of the boat?	24 miles due	west in
	<b>4. EXERCISE</b> Felisa jogged 3 miles in 25 minutes and then jog 30 minutes. What was her average speed in miles per min	ged 3 more nute?	miles in
Example 4 p. 135	<b>5. CYCLING</b> A cyclist begins traveling 18 miles per hour. At t the same starting point, an inline skater follows the cyclist traveling 6 miles per hour. After how much time will the	he same tim st's path and y be 24 miles	e and at begins apart?



20¢/ounce

Lesson 2-9 Weighted Averages 135

# Practice and Problem Solving

Example 1 p. 132 **6. CANDY** A candy store wants to create a mix using two hard candies. One is priced at \$5.45 per pound, and the other is priced at \$7.33 per pound. How many pounds of the \$7.33 candy should be mixed with 11 pounds of the \$5.45 candy to sell the mixture for \$6.14 per pound?

**BUSINESS** Party Supplies Inc. sells metallic balloons for \$2 each and helium balloons for \$3.50 per bunch. Yesterday, they sold 36 more metallic balloons than the number of bunches of helium balloons. The total sales for both types of balloons were \$281. Let *b* represent the number of metallic balloons sold.

a. Copy and complete the table representing the problem.

a constraint of	Number	Price	Total Price
Metallic Balloons	Ь		
Bunches of Helium Balloons	<i>b</i> — 36		

- **b.** Write an equation to represent the problem.
- c. How many metallic balloons were sold?
- d. How many bunches of helium balloons were sold?
- **8. FINANCIAL LITERACY** Lakeisha spent \$4.57 on color and black-and-white copies for her project. She made 7 more black-and-white copies than color copies. How many color copies did she make?

Type of Copy	Cost per Page
color	\$0.44
black-and-white	\$0.07

= Step-by-Step Solutions begin on page R12.

Extra Practice begins on page 815.

#### Example 2 p. 133

- **9. FISH** Rosamaria is setting up a 20-gallon saltwater fish tank that needs to have a salt content of 3.5%. If Rosamaria has water that has 2.5% salt and water that has 3.7% salt, how many gallons of the water with 3.7% salt content should Rosamaria use?
- **10. CHEMISTRY** Hector is performing a chemistry experiment that requires 160 milliliters of 40% sulfuric acid solution. He has a 25% sulfuric acid solution and a 50% sulfuric acid solution. How many millimeters of each solution should he mix to obtain the needed solution?
- 11. TRAVEL A boat travels 36 miles in 1.5 hours and then 14 miles in 0.75 hour. What is the average speed of the boat?
  - **12. RUNNING** A runner ran 1.5 miles in 28 minutes and then 1.2 more miles in 10 minutes. What was the average speed in miles per minute?
  - **13. AIRLINERS** Two airliners are 1600 miles apart and heading toward each other at different altitudes. The first plane is traveling north at 620 miles per hour, while the second is traveling south at 780 miles per hour. When will the planes pass each other?

#### Example 4 p. 135

- **14. SAILING** A ship is sailing due east at 20 miles per hour when it passes the lighthouse. At the same time a ship is sailing due west at 15 miles per hour when it passes a point. The lighthouse and the point are 175 miles apart. When will these ships pass each other?
  - **15. CHEMISTRY** A lab technician has 40 gallons of a 15% iodine solution. How many gallons of a 40% iodine solution must he add to make a 20% iodine solution?

Example 3 p. 134



#### Real-World Link

Different countries have individual grading scales. For example, French schools give number grades ranging from 0 to 20, rather than letter grades like those in the U.S.

Source: Morris

**16. GRADES** At Westbridge High School, a student's grade point average (GPA) is based on the student's grade and the class credit rating. Brittany's grades for this quarter are shown. Find Brittany's GPA if a grade of A equals 4 and a B equals 3.

Class	Credit Rating	Grade
Algebra 1	1	А
Science	1	А
English	1	В
Spanish	1	А
Music	$\frac{1}{2}$	В

- **17. SPORTS** In a triathlon, Steve swam 0.5 mile in 15 minutes, biked 20 miles in 90 minutes, and ran 4 miles in 30 minutes. What was Steve's average speed for the triathlon in miles per hour?
- **18. MUSIC** Amalia has 10 songs on her MP3 player. If 3 songs are 5 minutes long, 3 are 4 minutes long, 2 are 2 minutes long, and 2 are 3.5 minutes long, what is the average length of the songs?
- 9 **DISTANCE** Garcia is driving to Florida for vacation. The trip is a total of 625 miles.
  - a. How far can he drive in 6 hours at 65 miles per hour?
  - **b.** If Garcia maintains a speed of 65 miles per hour, how long will it take him to drive to Florida?

**20. TRAVEL** Two buses leave Smithville at the same time, one traveling north and the other traveling south. The northbound bus travels at 50 miles per hour, and the southbound bus travels at 65 miles per hour. Let *t* represent the amount of time since their departure.

a. Copy and complete the table representing the situation.

ອອກອຸ່ມແຜ່ນແຮງ	r	t	<i>d</i> = <i>rt</i>
Northbound bus	?	?	?
Southbound bus	?	?	?

- **b**. Write an equation to find when the buses will be 345 miles apart.
- c. Solve the equation. Explain how you found your answer.
- **21. TRAVEL** A subway travels 60 miles per hour from Glendale to Midtown. Another subway, traveling at 45 miles per hour, takes 11 minutes longer for the same trip. How far apart are Glendale and Midtown?

#### H.O.T. Problems

Use Higher-Order Thinking Skills

- 22. OPEN ENDED Write a problem that depicts motion in opposite directions.
- **23. REASONING** Describe the conditions so that adding a 50% solution to a 100% solution would produce a 75% solution.
- **24. CHALLENGE** Find five consecutive odd integers from least to greatest in which the sum of the first and the fifth is one less than three times the fourth.
- **25. CHALLENGE** Describe a situation involving mixtures that could be represented by 1.00x + 0.15(36) = 0.50(x + 36).
- **26.** WRITING IN MATH Describe how a gallon of 25% solution is added to an unknown amount of 10% solution to get a 15% solution.

# **Standardized Test Practice**

**27.** If 2x + y = 5, what is the value of 4x?

**A** 10 - y **B** 10 - 2y **C**  $\frac{5 - y}{2}$ **D**  $\frac{10 - y}{2}$ 

**28.** Which expression is equivalent to  $7x^23x^{-4}$ ?

- **F**  $21x^{-8}$
- **G**  $21x^2$
- **H**  $21x^{-6}$
- J  $21x^{-2}$

**29. GEOMETRY** What is the base of the triangle if the area is 56 square meters?

**A** 4 m

- **B** 8 m
- **C** 16 m
- **D** 28 m



**30. SHORT RESPONSE** Brianne makes blankets for a baby store. She works on the blankets 30 hours per week. The store pays her \$9.50 per hour plus 30% of the profit. If her hourly rate is increased by \$0.75 and her commission is raised to 40%, how much will she earn for a \$300 profit?

### **Spiral Review**

Solve each equation or formula for *x*. (Lesson 2-8)

**31.** 2bx - b = -5

**32.** 3x - r = r(-3 + x)

**33.**  $A = 2\pi r^2 + 2\pi rx$ 

**34. SKIING** Yuji is registering for ski camp. The cost of the camp is \$1254, but there is a sales tax of 7%. What is the total cost of the camp including tax? (Lesson 2-7)

Translate each equation into a sentence. (Lesson 2-1)

**35.**  $\frac{n}{-6} = 2n + 1$  **36.** 18 - 5h = 13h

#### Refer to the graph.

- **38.** Name the ordered pair at point *A* and explain what it represents. (Lesson 1-6)
- **39.** Name the ordered pair at point *B* and explain what it represents. (Lesson 1-6)
- **40.** Identify the independent and dependent variables for the function. (Lesson 1-6)
- **41. BASEBALL** Tickets to a baseball game cost \$18.95, \$12.95, or \$9.95. A hot dog and soda combo costs \$5.50. The Madison family is having a reunion. They buy 10 tickets in each price category and plan to buy 30 combos. What is the total cost for the tickets and meals? (Lesson 1-3)

#### **Skills Review**

Solve each equation. (Lesson 2-4)

**42.** a - 8 = 15

**45.** 5 - 8y = 61

**43.** 9m - 11 = -29**46.**  $7 = \frac{h}{2} + 3$ 

**37.**  $2x^2 + 3 = 21$ 



**44.** 18 - 2k = 24**47.**  $\frac{n}{6} + 1 = 5$ 

- · STUDYTO GO
- Vocabulary Review

# **Chapter Summary**

# **Key Concepts**

## Writing Equations (Lesson 2-1)

• Identify the unknown you are looking for and assign a variable to it. Then, write the sentence as an equation.

#### Solving Equations (Lessons 2-2 to 2-4)

- Addition and Subtraction Properties of Equality: If an equation is true and the same number is added to or subtracted from each side, the resulting equation is true.
- Multiplication and Division Properties of Equality: If an equation is true and each side is multiplied or divided by the same nonzero number, the resulting equation is true.
- Steps for Solving Equations:
  - **Step 1** Simplify the expression on each side. Use the Distributive Property as needed.
  - **Step 2** Use the Addition and/or Subtraction Properties of Equality to get the variables on one side and the numbers without variables on the other side.
  - **Step 3** Use the Multiplication or Division Property of Equality to solve.

#### Absolute Value Equations (Lesson 2-5)

• For any real numbers *a* and *b*, if |a| = b, and  $b \neq 0$ , then a = b or a = -b.

#### Ratios and Proportions (Lesson 2-6)

• The Means-Extremes Property of Proportion states that in a proportion, the product of the extremes is equal to the product of the means.

# FOLDABLES Study Organizer

Be sure the Key Concepts are noted in your Foldable.



# **Key Vocabulary**

consecutive integers (p. 92)	percent of decrease (p. 119)
<b>dimensional analysis</b> (p. 128)	percent of increase (p. 119)
equivalent equations (p. 83)	proportion (p. 111)
extremes (p. 112)	rate (p. 113)
ormula (p. 76)	ratio (p. 111)
dentity (p. 98)	scale (p. 114)
iteral equation (p. 127)	scale model (p. 114)
<b>neans</b> (p. 112)	solve an equation (p. 83)
nulti-step equations (p. 91)	unit analysis (p. 128)
number theory (p. 92)	unit rate (p. 113)
percent of change (p. 119)	weighted average (p. 132)

# **Vocabulary Check**

State whether each sentence is *true* or *false*. If *false*, replace the underlined term to make a true sentence.

- 1. In order to write an equation to solve a problem, identify the unknown for which you are looking and assign a(n) <u>number</u> to it.
- **2.** To <u>solve an equation</u> means to find the value of the variable that makes the equation true.
- **3.** The numbers 10, 12, and 14 are an example of <u>consecutive even integers</u>.
- **4.** The <u>absolute value</u> of any number is simply the distance the number is away from zero on a number line.
- **5.** A(n) <u>equation</u> is a comparison of two numbers by division.
- **6.** An equation stating that two ratios are equal is called a(n) <u>proportion</u>.
- **7.** If the new number is less than the original number, the percent of change is a percent of <u>increase</u>.
- **8.** The <u>weighted average</u> of a set of data is the sum of the product of the number of units and the value per unit divided by the sum of the number of units.

# **Lesson-by-Lesson Review**

#### Writing Equations (pp. 75-80)

#### Translate each sentence into an equation.

- **9.** The sum of five times a number *x* and three is the same as fifteen.
- **10.** Four times the difference of *b* and six is equal to *b* squared.
- 11. One half of *m* cubed is the same as four times *m* minus nine.

#### Translate each equation into a sentence.

**12.** 
$$3p + 8 = 20$$

2-1

- **13.**  $h^2 5h + 6 = 0$ **14.**  $\frac{3}{4}w^2 + \frac{2}{3}w - \frac{1}{5} = 2$
- **15. FENCING** Adrianne wants to create an outdoor rectangular kennel. The length will be three feet more than twice the width. Write and use an equation to find the length and the width of the kennel if Adrianne has 54 feet of fencing.

### EXAMPLE 1

Translate the following sentence into an equation.

Six times the sum of a number n and four is the same as the difference between two times n to the second power and ten.

 $6(n+4) = 2n^2 - 10$ 

#### EXAMPLE 2

Translate  $3d^2 - 9d + 8 = 4(d + 2)$  into a sentence.

Three times a number d squared minus nine times d increased by eight is equal to four times the sum of d and two.

### 2-2 Solving One-Step Equations (pp. 83–89)

Solve each equation. Check your solution.

**16.** x - 9 = 4 **17.** -6 + g = -11 **18.**  $\frac{5}{9} + w = \frac{7}{9}$  **19.** 3.8 = m + 1.7 **20.**  $\frac{a}{12} = 5$  **21.** 8y = 48 **22.**  $\frac{2}{5}b = -4$ **23.**  $-\frac{t}{16} = -\frac{7}{8}$ 

**24. AGE** Max is four years younger than his sister Brenda. The total of their ages is 16. Write and solve an equation to find their ages.

#### **EXAMPLE 3**

Solve $x - 13 = 9$ . Che	ck your solution.
x - 13 = 9	Original equation
x - 13 + 13 = 9 + 13	Add 13 to each side.
x = 22	-13 + 13 = 0 and $9 + 13 = 22$
To check that 22 is the $x$ in the original equat	solution, substitute 22 for ion.

**CHECK** x - 13 = 9 Original equation  $22 - 13 \stackrel{?}{=} 9$  Substitute 22 for x.  $9 = 9 \checkmark$  Subtract.

-	7	ł
4	2	5

#### Solving Multi-Step Equations (pp. 91–96)

Solve each equation. Check your solution.

**25.** 2d - 4 = 8**26.** -9 = 3t + 6**27.** 14 = -8 - 2k **28.**  $\frac{n}{4} - 7 = -2$  **29.**  $\frac{r+4}{3} = 7$  **30.**  $-18 = \frac{9-a}{2}$ **31.** 6g - 3.5 = 8.5 **32.** 0.2c + 4 = 6**33.**  $\frac{f}{3} - 9.2 = 3.5$  **34.**  $4 = \frac{-3u - (-7)}{-8}$ 

- **35. CONSECUTIVE INTEGERS** Find three consecutive odd integers with a sum of 63.
- **36. CONSECUTIVE INTEGERS** Find three consecutive integers with a sum of -39.

#### **EXAMPLE 4** Solve 7y - 9 = 33. Check your solution. 7y - 9 = 33**Original equation** 7y - 9 + 9 = 33 + 9Add 9 to each side. $\frac{7y}{7y} = \frac{42}{7}$ Simplify.

y = 6

 $7(6) - 9 \stackrel{?}{=} 33$ 

 $42 - 9 \stackrel{?}{=} 33$ 

**CHECK** 7y - 9 = 33

33 = 33 🖌	Subtract.

#### Solving Equations with the Variable on Each Side (pp. 97-102)

Solve each equation. Check your solution.

**37.** 
$$8m + 7 = 5m + 16$$

**38.** 
$$2h - 14 = -5h$$

**39.** 
$$21 + 3j = 9 - 3$$

**40.**  $\frac{x-3}{4} = \frac{x}{2}$ 

2-4

**41.** 
$$\frac{6r-7}{10} = \frac{r}{4}$$

- **42.** 3(p + 4) = 33
- **43.** -2(b-3) 4 = 18
- **44.** 4(3w 2) = 8(2w + 3)

#### Write an equation and solve each problem.

- 45. Find the sum of three consecutive odd integers if the sum of the first two integers is equal to twenty-four less than four times the third integer.
- 46. TRAVEL Mr. Jones drove 480 miles to a business meeting. His travel time to the meeting was 8 hours and from the meeting was 7.5 hours. Find his rate of travel for each leg of the trip.

EXAMPLE 5	Tims solusia
Solve $9w - 24 = 6w + 18$ .	
9w - 24 = 6w + 18	Original equation
9w - 24 - 6w = 6w + 18 - 6w	Subtract 6 <i>w</i> from each side.
3w - 24 = 18	Simplify.
3w - 24 + 24 = 18 + 24	Add 24 to each side.
3w = 42	Simplify.
$\frac{3w}{3} = \frac{42}{3}$	Divide each side by 3.
w = 14	Simplify.

Divide each side by 7.

**Original equation** 

Substitute 6 for y.

Simplify.

Multiply.

#### **EXAMPLE 6**

Write an equation to find three consecutive integers such that three times the sum of the first two integers is the same as thirteen more than four times the third integer.

Let x, x + 1, and x + 2 represent the three consecutive integers.

3(x + x + 1) = 4(x + 2) + 13

#### CHAPTER

# **Study Guide and Review**

2-5

#### Solving Equations Involving Absolute Value (pp. 103–109)

Evaluate each expression if m = -8, n = 4, and p = -12.

- **47.** |3*m* − *n*|
- **48.** |-2p + m| 3n
- **49.** -3|6n-2p|
- **50.** 4|7m+3p|+4n

Solve each equation. Then graph the solution set.

**51.** |x - 6| = 11 **52.** |-4w + 2| = 14 **53.**  $\left|\frac{1}{3}d - 6\right| = 15$ **54.**  $\left|\frac{2b}{3} + 8\right| = 20$ 

EXAMPLE 7	vé each equation. Che.	
Solve $ y - 9  = 16$ . The set.	en graph the solution	
Case 1		
y - 9 = 16	Original equation	
<i>y</i> - 9 <b>+ 9</b> = 16 <b>+ 9</b>	Add 9 to each side.	
y = 25	Simplify.	
Case 2		
y - 9 = -16	Original equation	
y - 9 + 9 = -16 + 9	Add 9 to each side.	
y = -7	Simplify.	
The solution set is $\{-7, 25\}$ .		
Graph the points on a 1	number line.	

-10 -5	0	5	10	15	20	25	30

#### 2-6

#### Ratios and Proportions (pp. 111–117)

Determine whether each pair of ratios are equivalent ratios. Write *yes* or *no*.

**55.**  $\frac{27}{45}, \frac{3}{5}$ 

**56.**  $\frac{18}{32}, \frac{3}{4}$ 

Solve each proportion. If necessary, round to the nearest hundredth.

**57.** 
$$\frac{4}{9} = \frac{a}{45}$$
  
**58.**  $\frac{3}{8} = \frac{21}{t}$   
**59.**  $\frac{9}{12} = \frac{g}{16}$ 

**60. CONSTRUCTION** A new gym is being built at Greenfield Middle School. The length of the gym as shown on the builder's blueprints is 12 inches. Find the actual length of the new gym.



### **EXAMPLE 8**

Determine whether  $\frac{7}{9}$  and  $\frac{42}{54}$  are equivalent ratios. Write *yes* or *no*. Justify your answer.

First, simplify each ratio.  $\frac{7}{9}$  is already in simplest form.

$$\frac{42}{54} = \frac{42 \div 6}{54 \div 6} = \frac{7}{9}$$

When expressed in simplest form, the ratios are equivalent. The answer is yes.

# EXAMPLE 9

Solve  $\frac{r}{8} = \frac{3}{4}$ . If necessary, round to the nearest hundredth.

$\frac{r}{8} = \frac{3}{4}$	Original equation
r(4) = 3(8)	Find the cross products.
4r = 24	Simplify.
$\frac{4r}{4} = \frac{24}{4}$	Divide each side by 4.
r = 6	Simplify.

# 2-7 Percent of Change (pp. 119–124)

State whether each percent of change is a percent of *increase* or a percent of *decrease*. Then find the percent of change. Round to the nearest whole percent.

**61.** original: 40, new: 50

**62.** original: 36, new: 24

**63.** original: \$72, new: \$60

#### Find the total price of each item.

**64.** boots: \$64, tax: 7%

**65.** video game: \$49, tax: 6.5%

**66.** hockey skates: \$199, tax: 5.25%

#### Find the discounted price of each item.

**67.** MP3 player: \$69.00, discount: 20%

68. jacket: \$129, discount: 15%

- **69.** backpack: \$45, discount: 25%
- **70. ATTENDANCE** An amusement park recorded attendance of 825,000 one year. The next year, the attendance increased to 975,000. Determine the percent of increase in attendance.

#### EXAMPLE 10

State whether the percent of change is a percent of *increase* or a percent of *decrease*. Then find the percent of change. Round to the nearest whole percent.

original: 80 final: 60

Subtract the original amount from the final amount to find the amount of change: 60 - 80 = -20. Since the new amount is less than the original, this is a percent of decrease.

Use the original number, 80, as the base.

$$\frac{\text{change}}{\text{original amount}} \xrightarrow{20} \frac{20}{80} = \frac{r}{100}$$

$$20(100) = r(80)$$

$$2000 = 80r$$

$$\frac{2000}{80} = \frac{80r}{80}$$

$$25 = r$$

The percent of decrease is 25%.

#### **2-8** Literal Equations and Dimensional Analysis (pp. 126–131)

indicated. Solve $6p - 8n = 12$ for $p$ .	
<b>71.</b> $3x + 2y = 9$ , for y $6p - 8n = 12$ <b>Original equation</b>	
<b>72.</b> $P = 2\ell + 2w$ , for $\ell$ 6p - 8n + 8n = 12 + 8n Add 8n to each side.	
<b>73.</b> $-5m + 9n = 15$ , for m $6p = 12 + 8n$ Simplify.	
<b>74.</b> $14w + 15x = y - 21w$ , for $w$ $\frac{6p}{12 + 8n}$ Divide each side by 6	
<b>75.</b> $m = \frac{2}{5}y + n$ , for y	
<b>76.</b> $7d - 3c = f + 2d$ , for $d$ $\frac{3p}{6} = \frac{12}{6} + \frac{8}{6}n$ Simplify.	
<b>77. GEOMETRY</b> The formula for the area of a $p = 2 + \frac{4}{3}n$ Simplify.	
trapezoid is $A = \frac{1}{2}h(a + b)$ , where h	
represents the height and <i>a</i> and <i>b</i> represent	

# 2-9 Weighted Averages (pp. 132–138)

- **78. CANDY** Michael is mixing two types of candy for a party. The chocolate pieces cost \$0.40 per ounce, and the hard candy costs \$0.20 per ounce. Michael purchases 20 ounces of the chocolate pieces, and the total cost of his candy was \$11. How many ounces of hard candy did he purchase?
- **79. TRAVEL** A car travels 100 miles east in 2 hours and 30 miles north in half an hour. What is the average speed of the car?
- **80. FINANCIAL LITERACY** A candle supply store sells votive wax and low-shrink wax. How many pounds of low-shrink wax should be mixed with 8 pounds of votive wax to obtain a blend that sells for \$0.98 a pound?



#### **EXAMPLE 12**

**METALS** An alloy of metals is 25% copper. Another alloy is 50% copper. How much of each should be used to make 1000 grams of an alloy that is 45% copper?

Let x = the amount of the 25% copper alloy. Write and solve an equation.

5x + 0.50(1000 - x) = 0.45(1000) Original Equation	
0.25x + 500 - 0.50x = 450 Distributive Property	
-0.25x + 500 = 450 Simplify.	
-0.25x + 500 - 500 = 450 - 500 Subtract 500 from each side	2.
-0.25x = -50 Simplify.	
$\frac{-0.25x}{-0.25} = \frac{-50}{-0.25}$ Divide each side by -0.25.	
x = 2.00 Simplify.	

200 grams of the 25% alloy and 800 grams of the 50% alloy should be used.
### Translate each sentence into an equation.

- 1. The sum of six and four times *d* is the same as *d* minus nine.
- **2.** Three times the difference of two times *m* and five is equal to eight times *m* to the second power increased by four.

### Solve each equation. Check your solutions.

**3.** 
$$x - 5 = -11$$
  
**4.**  $\frac{2}{3} = w + \frac{1}{4}$   
**5.**  $\frac{t}{6} = -3$ 

Solve each equation. Check your solution.

**6.** 2a - 5 = 13 **7.**  $\frac{p}{4} - 3 = 9$ 

- **8. MULTIPLE CHOICE** At Mama Mia Pizza, the price of a large pizza is determined by P = 9 + 1.5x, where *x* represents the number of toppings added to a cheese pizza. Daniel spent \$13.50 on a large pizza. How many toppings did he get?
  - **A** 0
  - **B** 1
  - **C** 3
  - **D** 5

Solve each equation. Check your solution.

**9.** 5y - 4 = 9y + 8

**10.** 3(2k-2) = -2(4k-11)

**11. GEOMETRY** Find the value of *x* so that the figures have the same perimeter.



**12.** Evaluate the expression |3t - 2u| + 5v if t = 2, u = -5, and v = -3.

Solve each equation. Then graph the solution set.  $17 + 10 = 10^{-10}$ 

**13.** |p - 4| = 6**14.** |2b + 5| = 9 Solve each proportion. If necessary, round to the nearest hundredth.

**15.** 
$$\frac{a}{3} = \frac{16}{24}$$
  
**16.**  $\frac{9}{k+3} = \frac{3}{5}$ 

- **17. MULTIPLE CHOICE** Akiko uses 2 feet of thread for every three squares that she sews for her quilt. How many squares can she sew if she has 38 feet of thread?
  - **F** 19

**G** 57

- H 76
- J 228
- **18.** State whether the percent of change is a percent of *increase* or a percent of *decrease*. Then find the percent of change. Round to the nearest whole percent.

original: 54 new: 45

- **19.** Find the total price of a sweatshirt that is priced at \$48 and taxed at 6.5%.
- **20. SHOPPING** Kirk wants to purchase a wide-screen TV. He sees an advertisement for a TV that was originally priced at \$3200 and is 20% off. Find the discounted price of the TV.
- **21.** Solve 5x 3y = 9 for *y*.

**22.** Solve 
$$A = \frac{1}{2}bh$$
 for *h*.

- **23. CHEMISTRY** Deon has 12 milliliters of a 5% solution. He also has a solution that has a concentration of 30%. How many milliliters of the 30% solution does Deon need to add to the 5% solution to obtain a 20% solution?
- **24. BICYCLING** Shanee bikes 5 miles to the park in 30 minutes and 3 miles to the library in 45 minutes. What was her average speed?
- **25. MAPS** On a map of North Carolina, the distance between Charlotte and Wilmington is 14.75 inches. If 2 inches equals 24 miles, what is the approximate distance between the two cities?

CHAPTER

# **Preparing for Standardized Tests**

# **Gridded Response Questions**

In addition to multiple-choice, short-answer, and extended-response questions, you will likely encounter gridded-response questions on standardized tests. For gridded-response questions, you must print your answer on an answer sheet and mark in the correct circles on the grid to match your answer.

### **Strategies for Solving Gridded Response Questions**

#### Step 1

Read the problem carefully.

- Ask yourself: "What information is given?" "What do I need to find?" "How do I solve this type of problem?"
- Solve the Problem: Use the information given in the problem to solve.
- **Check your answer:** If time permits, check your answer to make sure you have solved the problem correctly.

#### Step 2

Write your answer in the answer boxes.

- Print only one digit or symbol in each answer box.
- Do not write any digits or symbols outside the answer boxes.
- You may write your answer with the first digit in the left answer box, or with the last digit in the right answer box. You may leave blank any boxes you do not need on the right or the left side of your answer.

# Step 3

Fill in the grid.

- Fill in only one bubble for every answer box that you have written in. Be sure not to fill in a bubble under a blank answer box.
- Fill in each bubble completely and clearly.

	1		1. 671	51 E	65			-
3	/	5				3	/	5
		$\bigcirc$				$\bigcirc$	0	
$\bigcirc$	$\bigcirc$		$\bigcirc$		$\bigcirc$	$\bigcirc$	$\bigcirc$	$\bigcirc$
	0	0	0			0	0	0
1	1	1	1		1	1	1	1
2	2	2	2		2	2	2	2
•	3	3	3		3	•	3	3
4	4	4	4		4	4	4	4
5	5	•	5		5	5	5	•
6	6	6	6		6	6	6	6
D	7	D	D	11.	D	D	D	D
8	8	8	8		8	8	8	8
9	9	9	9		9	9	9	9

#### EXAMPLE

Read the problem. Identify what you need to know. Then use the information in the problem to solve.

**GRIDDED RESPONSE** Ashley is 3 years older than her sister, Tina. Combined, the sum of their ages is 27 years. How old is Ashley?

Read the problem carefully. You are told that Ashley is 3 years older than her sister and that their ages combined equal 27 years. You need to find Ashley's age.

#### **Solve the Problem**

#### Fill in the Grid



Solve the equation for *a*.

a + (a - 3) = 27	Original equation.
2a - 3 = 27	Add like terms.
2a = 30	Add 3 to each side.
a = 15	Divide each side by 2.

5 1 6 66 6  $D \overline{D} \overline{D} \overline{D}$ 8888 9 9 9 9 9

Since we let *a* represent Ashley's age, we know that she is 15 years old.

## **Exercises**

Read each problem. Identify what you need to know. Then use the information in the problem to solve. Copy and complete an answer grid on your paper.

- 1. Orlando has \$1350 in the bank. He wants to increase his balance to a total of \$2550 by depositing \$40 each week from his paycheck. How many weeks will he need to save in order to reach his goal?
- **2.** Fourteen less than three times a number is equal to 40. Find the number.
- **3.** The table shows the regular prices and sale prices of certain items at a department store this week. What is the percent of discount during the sale?

	Item	Regular Price (\$)	Sale Price (\$)		
nl	pillows	25	20		
090	sweaters	30	24		
	entertainment center	125	100		

- 4. Maureen is driving from Raleigh, North Carolina, to Charlotte, North Carolina, to visit her brother at college. If she averages 65 miles per hour on the trip, then the equation  $\frac{d}{2.65} = 65$  can be solved for the distance *d*. What is the distance to the nearest mile from Raleigh to Charlotte?
- 5. Find the value of *x* so that the figures below have the same area.



6. The sum of three consecutive whole numbers is 18. What is the greatest of the numbers?

**B** 1

# **Standardized Test Practice**

Cumulative, Chapters 1 and 2

# **Multiple Choice**

Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.

1. Which point on the number line best represents the position of  $\sqrt{8}$ ?



**2.** Find the value of *x* so that the figures have the same area.



3. The elevation of Black Mountain is 27 feet more than 16 times the lowest point in the state. If the elevation of the lowest point in the state is 257 feet, what is the elevation of Black Mountain?

A	4,085 feet	<b>C</b> 4,139 feet
B	4,103 feet	<b>D</b> 4,215 feet

- 4. The expression  $(3x^2 + 5x 12) 2(x^2 + 4x + 9)$ is equivalent to which of the following?
  - **F**  $x^2 3x 30$
  - **G**  $x^2 + 13x + 6$

**H** 
$$5x^2 + x - 18$$

 $\int x^2 + 3x - 21$ 

5. The amount of soda, in fluid ounces, dispensed from a machine must satisfy the equation |a - 0.4| = 20. Which of the following graphs shows the acceptable minimum and maximum amounts that can be dispensed from the machine?

- **6.** If *a* and *b* represent integers, ab = ba is an example of which property?
  - **F** Associative Property
  - **G** Commutative Property
  - **H** Distributive Property
  - J Closure Property
- 7. The sum of one fifth of a number and three is equal to half of the number. What is the number?

Α	5	<b>C</b> 15
B	10	<b>D</b> 20

- 8. Aaron charges \$15 to mow the lawn and \$10 per hour for other gardening work. Which expression represents his earnings?
  - **F** 10h
  - **G** 15h

**H** 15h + 10

J 15 + 10h

### Test-TakingTip

Question 2 Use the figures and the formula for area to set up an equation. The product of the length and width of each figure should be equal.

### Short Response/Gridded Response

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

**9.** The formula for the lateral area of a cylinder is  $A = 2\pi rh$ , where *r* is the radius and *h* is the height. Solve the equation for *h*.



- **10. GRIDDED RESPONSE** Solve the proportion  $\frac{x}{18} = \frac{7}{21}$ .
- **11. GRIDDED RESPONSE** The table shows the cost of renting a moving van. If Miguel budgeted \$75, how many miles could he drive the van and maintain his budget?

Moving V	an Rentals
Flat Fee	\$50 for up to 300 miles
Variable Fee	\$0.20 per mile over 300

- **12.** Find the height of a soup can if the area of the label is 302 square centimeters and the radius of the can is 4 centimeters. Round to the nearest whole number.
- **13. GRIDDED RESPONSE** Lara's car needed a particular part that costs \$75. The mechanic charges \$50 per hour to install the part. If the total cost was \$350, how many hours did it take to install the part?

14. Lucinda is buying a set of patio furniture that is on sale for  $\frac{4}{5}$  of the original price. After she uses a \$50 gift certificate, the total cost before sales tax is \$222. What was the original price of the patio furniture?

### **Extended Response**

# Record your answers on a sheet of paper. Show your work.

- 15. The city zoo offers a yearly membership that costs \$120. A yearly membership includes free parking. Members can also purchase a ride pass for an additional \$2 per day that allows them unlimited access to the rides in the park. Nonmembers pay \$12 for admission to the park, \$5 for parking, and \$5 for a ride pass.
  - **a.** Write an equation that could be solved for the number of visits it would take for the total cost to be the same for a member and a nonmember if they both purchase a ride pass each day. Solve the equation.
  - **b.** What would the total cost be for members and nonmembers after this number of visits?
  - **c.** Georgena is deciding whether or not to purchase a yearly membership. Explain how she could use the results above to help make her decision.

Need Extra Help?															
If you missed Question	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Go to Lesson or Page	0-2	2-5	2-2	1-4	2-5	1-3	2-4	1-1	2-8	2-6	2-3	2-8	2-3	2-3	2-4

# **Linear Functions**

# Then

In Chapter 2, you solved linear equations algebraically.

CHAPTER

### Now/

In Chapter 3, you will:

- Identify linear equations, intercepts, and zeros.
- Graph and write linear equations.
- Use rate of change to solve problems.

# Why?

AMUSEMENT PARKS The Magic Kingdom in Orlando, Florida, is one of the most popular amusement parks in the world. Yearly attendance figures increase steadily each year. Quantities like populations that change with respect to time can be described using rate of change. Often you can represent these situations with linear functions.



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