Get Ready for the Chapter

Diagnose Readiness | You have two options for checking prerequisite skills.
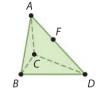
1 **Textbook Option** Take the Quick Check below. Refer to the Quick Review for help.

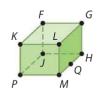| **Quick**Check	**Quick**Review

QuickCheck

Refer to the figure to identify each of the following.

1. How many planes are shown in this figure?

2. Name three points that are collinear.

3. Are points C and D coplanar? Explain.

4. **PHOTOGRAPHY** Tina is taking a picture of her friends. If she sets a tripod level on the ground, will the bottom of each of the three legs of the tripod be coplanar?
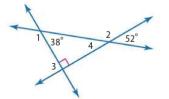
QuickReview

Example 1

Refer to the figure.

a. How many planes are shown in this figure?
Six: plane $FGLK$, plane $JHMP$, plane $FKPJ$, plane $GLMH$, plane $FGHJ$, and plane $KLMP$

b. Name three points that are collinear.
Points M, Q, and H are collinear.

c. Are points F, K, and J coplanar? Explain.
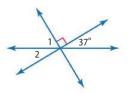Yes. Points F, K, and J all lie in plane $FKPJ$.

Find each angle measure.

5. $\angle 1$

6. $\angle 2$

7. $\angle 3$

8. $\angle 4$

Example 2

Find $m\angle 1$.

$$m\angle 1 + 37 + 90 = 180 \quad \text{Add.}$$
$$m\angle 1 = 53 \quad \text{Simplify.}$$

Find the value of x for the given values of a and b.

9. $a + 8 = -4(x - b)$, for $a = 8$ and $b = 3$

10. $b = 3x + 4a$, for $a = -9$ and $b = 12$

11. $\dfrac{a + 2}{b + 13} = 5x$, for $a = 18$ and $b = -1$

12. **MINIATURE GOLF** A miniature golf course offers a $1 ice cream cone with each round of golf purchased. If five friends each had a cone after golfing and spend a total of $30, how much does one round of golf cost?

Example 3

Find x in $a + 8 = b(x - 7)$ if $a = 12$ and $b = 10$.

$$a + 8 = b(x - 7) \quad \text{Write the equation.}$$
$$12 + 8 = 10(x - 7) \quad a = 12 \text{ and } b = 10$$
$$20 = 10x - 70 \quad \text{Simplify.}$$
$$90 = 10x \quad \text{Add.}$$
$$x = 9 \quad \text{Divide.}$$

2 **Online Option** Take an online self-check Chapter Readiness Quiz at **connectED.mcgraw-hill.com**.

Get Started on the Chapter

You will learn several new concepts, skills, and vocabulary terms as you study Chapter 3. To get ready, identify important terms and organize your resources. You may wish to refer to Chapter 0 to review prerequisite skills.

FOLDABLES StudyOrganizer

Parallel and Perpendicular Lines Make this Foldable to help you organize your Chapter 3 notes about relationships between lines. Begin with a sheet of 11″ × 17″ paper and six index cards.

1 **Fold** lengthwise about 3″ from the bottom.

2 **Fold** the paper in thirds.

3 **Open** and staple the edges on either side to form three pockets.

4 **Label** the pockets as shown. Place two index cards in each pocket.

NewVocabulary

| English		Español
parallel lines	p. 173	rectas paralelas
skew lines	p. 173	rectas alabeadas
parallel planes	p. 173	planos paralelos
transversal	p. 174	transversal
interior angles	p. 174	ángulos interiores
exterior angles	p. 174	ángulos externos
corresponding angles	p. 174	ángulos correspondientes
slope	p. 188	pendiente
rate of change	p. 189	tasa de cambio
slope-intercept form	p. 198	forma pendiente-intersección
point-slope form	p. 198	forma punto-pendiente
equidistant	p. 218	equidistante
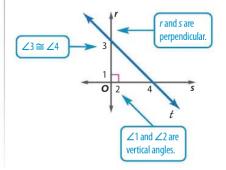
ReviewVocabulary

congruent angles **ángulos congruentes** two angles that have the same degree measure

perpendicular **perpendicular** two lines, segments, or rays that intersect to form right angles

vertical angles **ángulos opuestos por el vértice** two nonadjacent angles formed by intersecting lines

r and s are perpendicular.

$\angle 3 \cong \angle 4$

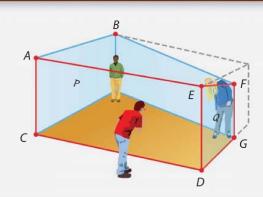$\angle 1$ and $\angle 2$ are vertical angles.

:: Then	:: Now	:: Why?

● You used angle and line segment relationships to prove theorems.

● **1** Identify the relationships between two lines or two planes.

● **2** Name angle pairs formed by parallel lines and transversals.

● An Ames room creates the illusion that a person standing in the right corner is much larger than a person standing in the left corner.

From a front viewing hole the front and back walls appear parallel, when in fact they are slanted. The ceiling and floor appear horizontal, but are actually tilted.

1 Relationships Between Lines and Planes The construction of the Ames room above makes use of intersecting, parallel, and skew lines, as well as intersecting and parallel planes, to create an optical illusion.
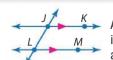
KeyConcepts Parallel and Skew

Parallel lines are coplanar lines that do not intersect.

Example $\overleftrightarrow{JK} \parallel \overleftrightarrow{LM}$

Arrows are used to indicate that lines are parallel.

Skew lines are lines that do not intersect and are not coplanar.

Example Lines ℓ and m are skew.

Parallel planes are planes that do not intersect.

Example Planes A and B are parallel.

$\overleftrightarrow{JK} \parallel \overleftrightarrow{LM}$ is read as *line JK is parallel to line LM*.

If segments or rays are contained within lines that are parallel or skew, then the segments or rays are parallel or skew.

🌐 Real-World Example 1 Identify Parallel and Skew Relationships

Identify each of the following using the wedge of cheese below.
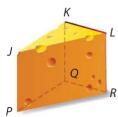
a. all segments parallel to $\overline{JP}$

$\overline{KQ}$ and $\overline{LR}$

b. a segment skew to $\overline{KL}$

$\overline{JP}$, $\overline{PQ}$, or $\overline{PR}$
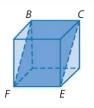
c. a plane parallel to plane PQR

Plane JKL is the only plane parallel to plane PQR.
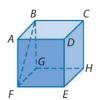
B   *C*

F   *E*

GuidedPractice

Identify each of the following using the cube shown.

1A. all segments skew to $\overleftrightarrow{BC}$

1B. a segment parallel to $\overleftrightarrow{EH}$

1C. all planes parallel to plane *DCH*

B   *C*

A   *D*

G

H

F   *E*

2 **Transversal Angle Pair Relationships** A line that intersects two or more coplanar lines at two different points is called a **transversal**. In the diagram below, line *t* is a transversal of lines *q* and *r*. Notice that line *t* forms a total of eight angles with lines *q* and *r*. These angles, and specific pairings of these angles, are given special names.

KeyConcept Transversal Angle Pair Relationships

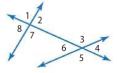|	
Four **interior angles** lie in the region between lines *q* and *r*.	$\angle 3, \angle 4, \angle 5, \angle 6$
Four **exterior angles** lie in the two regions that are not between lines *q* and *r*.	$\angle 1, \angle 2, \angle 7, \angle 8$
Consecutive interior angles are interior angles that lie on the same side of transversal *t*.	$\angle 4$ and $\angle 5$, $\angle 3$ and $\angle 6$
Alternate interior angles are nonadjacent interior angles that lie on opposite sides of transversal *t*.	$\angle 3$ and $\angle 5$, $\angle 4$ and $\angle 6$
Alternate exterior angles are nonadjacent exterior angles that lie on opposite sides of transversal *t*.	$\angle 1$ and $\angle 7$, $\angle 2$ and $\angle 8$
Corresponding angles lie on the same side of transversal *t* and on the same side of lines *q* and *r*.	$\angle 1$ and $\angle 5$, $\angle 2$ and $\angle 6$ $\angle 3$ and $\angle 7$, $\angle 4$ and $\angle 8$

Example 2 Classify Angle Pair Relationships

Refer to the figure below. Classify the relationship between each pair of angles as *alternate interior, alternate exterior, corresponding,* or *consecutive interior* angles.

a. $\angle 1$ and $\angle 5$
alternate exterior

b. $\angle 6$ and $\angle 7$
consecutive interior

c. $\angle 2$ and $\angle 4$
corresponding

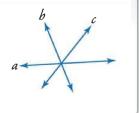**d.** $\angle 2$ and $\angle 6$
alternate interior

GuidedPractice

2A. $\angle 3$ and $\angle 7$ **2B.** $\angle 5$ and $\angle 7$ **2C.** $\angle 4$ and $\angle 8$ **2D.** $\angle 2$ and $\angle 3$

When more than one line can be considered a transversal, first identify the transversal for a given angle pair by locating the line that connects the vertices of the angles.

Example 3 Identify Transversals and Classify Angle Pairs

Identify the transversal connecting each pair of angles in the photo. Then classify the relationship between each pair of angles.
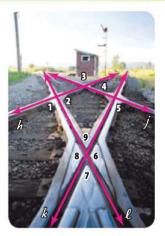
a. ∠1 and ∠3

   The transversal connecting ∠1 and ∠3 is line *h*. These are alternate exterior angles.

b. ∠5 and ∠6

   The transversal connecting ∠5 and ∠6 is line *k*. These are consecutive interior angles.

c. ∠2 and ∠6

   The transversal connecting ∠2 and ∠6 is line *ℓ*. These are corresponding angles.

▶ **Guided**Practice

3A. ∠3 and ∠5   **3B.** ∠2 and ∠8

3C. ∠5 and ∠7   **3D.** ∠2 and ∠9

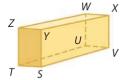---

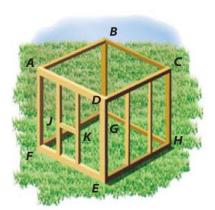**Check Your Understanding**   ⬤ = **Step-by-Step Solutions begin on page R14.**

Example 1   **Refer to the figure at the right to identify each of the following.**

1. a plane parallel to plane *ZWX*

2. a segment skew to $\overline{TS}$ that contains point *W*

3. all segments parallel to $\overline{SV}$

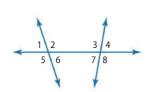4. **CONSTRUCTION** Use the diagram of the partially framed storage shed shown to identify each of the following.

   a. Name three pairs of parallel planes.

   b. Name three segments parallel to $\overline{DE}$.

   c. Name two segments parallel to $\overline{FE}$.

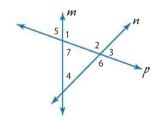   d. Name two pairs of skew segments.

Example 2   **Classify the relationship between each pair of angles as *alternate interior, alternate exterior, corresponding,* or *consecutive interior* angles.**

⑤ ∠1 and ∠8   6. ∠2 and ∠4

7. ∠3 and ∠6   8. ∠6 and ∠7

Example 3 Identify the transversal connecting each pair of angles. Then classify the relationship between each pair of angles.

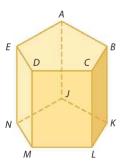**9.** ∠2 and ∠4   **10.** ∠5 and ∠6

11. ∠4 and ∠7   **12.** ∠2 and ∠7

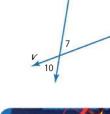## Practice and Problem Solving

Example 1 Refer to the figure to identify each of the following.

13. all segments parallel to $\overline{DM}$

14. a plane parallel to plane *ACD*

15 a segment skew to $\overline{BC}$

16. all planes intersecting plane *EDM*

17. all segments skew to $\overline{AE}$

18. a segment parallel to $\overline{EN}$

19. a segment parallel to $\overline{AB}$ through point *J*

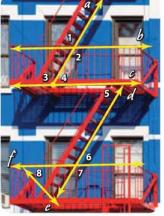**20.** a segment skew to $\overline{CL}$ through point *E*

Examples 2–3 **CCSS** **PRECISION** Identify the transversal connecting each pair of angles. Then classify the relationship between each pair of angles as *alternate interior, alternate exterior, corresponding,* or *consecutive interior* angles.

21. ∠4 and ∠9   **22.** ∠5 and ∠7

23. ∠3 and ∠5   **24.** ∠10 and ∠11

25. ∠1 and ∠6   **26.** ∠6 and ∠8

27. ∠2 and ∠3   **28.** ∠9 and ∠10

29. ∠4 and ∠11   **30.** ∠7 and ∠11
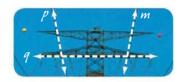
Example 3 **SAFETY** Identify the transversal connecting each pair of angles in the photo of a fire escape shown. Then classify the relationship between each pair of angles.

31. ∠1 and ∠2   **32.** ∠2 and ∠4

33. ∠4 and ∠5   **34.** ∠6 and ∠7

35. ∠7 and ∠8   **36.** ∠2 and ∠3

37. **POWER** Power lines are not allowed to intersect.

a. What must be the relationship between power lines *p* and *m*? Explain your reasoning.

b. What is the relationship between line *q* and lines *p* and *m*?

Describe the relationship between each pair of segments as *parallel*, *skew*, or *intersecting*.
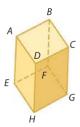
38. $\overline{FG}$ and $\overline{BC}$

39. $\overline{AB}$ and $\overline{CG}$

40. $\overline{DH}$ and $\overline{HG}$

41. $\overline{DH}$ and $\overline{BF}$

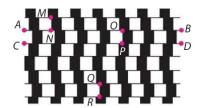**42.** $\overline{EF}$ and $\overline{BC}$
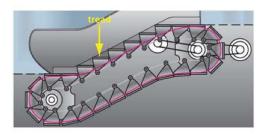
43. $\overline{CD}$ and $\overline{AD}$

44. **CCSS SENSE-MAKING** The illusion at the right is created using squares and straight lines.

a. How are $\overline{AB}$ and $\overline{CD}$ related? Justify your reasoning.

b. How are $\overline{MN}$ and $\overline{QR}$ related? $\overline{AB}$, $\overline{CD}$, and $\overline{OP}$?

45 **ESCALATORS** Escalators consist of steps on a continuous loop that is driven by a motor. At the top and bottom of the platform, the steps collapse to provide a level surface for entrance and exit.

a. What is the relationship between the treads of the ascending stairs?

b. What is the relationship between the treads of the two steps at the top of the incline?

c. How do the treads of the steps on the incline of the escalator relate to the treads of the steps on the bottom of the escalator?

H.O.T. Problems   Use Higher-Order Thinking Skills

46. **OPEN ENDED** Plane P contains lines a and b. Line c intersects plane P at point J. Lines a and b are parallel, lines a and c are skew, and lines b and c are not skew. Draw a figure based upon this description.

47. **CHALLENGE** Suppose points A, B, and C lie in plane P, and points D, E, and F lie in plane Q. Line m contains points D and F and does not intersect plane P. Line n contains points A and E.

a. Draw a diagram to represent the situation.

b. What is the relationship between planes P and Q?

c. What is the relationship between lines m and n?

REASONING Plane X and plane Y are parallel and plane Z intersects plane X. Line $\overleftrightarrow{AB}$ is in plane X, line $\overleftrightarrow{CD}$ is in plane Y, and line $\overleftrightarrow{EF}$ is in plane Z. Determine whether each statement is *always*, *sometimes*, or *never* true. Explain.
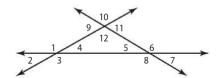
48. $\overleftrightarrow{AB}$ is skew to $\overleftrightarrow{CD}$.

49. $\overleftrightarrow{AB}$ intersects $\overleftrightarrow{EF}$.

50. **WRITING IN MATH** Can a pair of planes be described as skew? Explain.
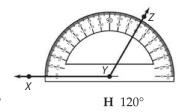
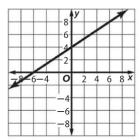**51.** Which of the following angle pairs are alternate exterior angles?

A $\angle 1$ and $\angle 5$ **C** $\angle 2$ and $\angle 10$

B $\angle 2$ and $\angle 6$ **D** $\angle 5$ and $\angle 9$

52. What is the measure of $\angle XYZ$?

F $30°$ **H** $120°$

G $60°$ **J** $150°$

53. SHORT RESPONSE Name the coordinates of the points representing the x- and y-intercepts of the graph shown below.

54. SAT/ACT Of the following, the one that is *not* equivalent to 485 is:

A $(3 \times 100) + (4 \times 10) + 145$

B $(3 \times 100) + (18 \times 10) + 5$

C $(4 \times 100) + (8 \times 10) + 15$

D $(4 \times 100) + (6 \times 10) + 25$

E $(4 \times 100) + (5 \times 10) + 35$

Spiral Review

Find the measure of each numbered angle. (Lesson 2-8)
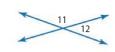
55. $m\angle 9 = 2x - 4$,
$m\angle 10 = 2x + 4$

56. $m\angle 11 = 4x$,
$m\angle 12 = 2x - 6$

57. $m\angle 19 = 100 + 20x$,
$m\angle 20 = 20x$
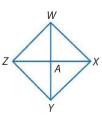
58. PROOF Prove the following. (Lesson 2-7)

Given: $\overline{WY} \cong \overline{ZX}$
A is the midpoint of $\overline{WY}$.
A is the midpoint of $\overline{ZX}$.
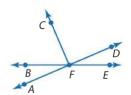
Prove: $\overline{WA} \cong \overline{ZA}$

ALGEBRA Use the figure at the right. (Lesson 1-5)
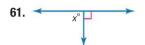
59. If $m\angle CFD = 12a + 45$, find a so that $\overrightarrow{FC} \perp \overrightarrow{FD}$.

60. If $m\angle AFB = 8x - 6$ and $m\angle BFC = 14x + 8$, find the value of x so that $\angle AFC$ is a right angle.

Skills Review

Find x.

61.

62.

63.
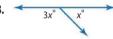
You can use The Geometer's Sketchpad® to explore the angles formed by two parallel lines and a transversal.

Activity   Parallel Lines and a Transversal

Step 1 **Draw a line.**

Draw and label points *F* and *G*. Then use the line tool to draw $\overleftrightarrow{FG}$.

Step 2 **Draw a parallel line.**

Draw a point that is not on $\overleftrightarrow{FG}$ and label it *J*. Select $\overleftrightarrow{FG}$ and point *J*, and then choose **Parallel Line** from the **Construct** menu. Draw and label a point *K* on this parallel line.
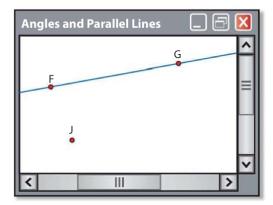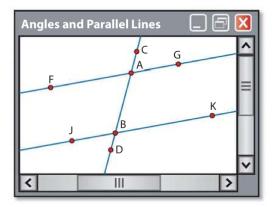
Step 3 **Draw a transversal.**

Draw and label point *A* on $\overleftrightarrow{FG}$ and point *B* on $\overleftrightarrow{JK}$. Select *A* and *B* and then choose **Line** from the **Construct** menu to draw transversal $\overleftrightarrow{AB}$. Then draw and label points *C* and *D* on $\overleftrightarrow{AB}$ as shown.

Step 4 **Measure each angle.**

Measure all eight angles formed by these lines. For example, select points *F*, *A*, then *C*, and choose **Angle** from the **Measure** menu to find *m∠FAC*.

Angles and Parallel Lines

Angles and Parallel Lines

Analyze the Results

1. Record the measures from Step 4 in a table like this one. Which angles have the same measure?

Angle	∠FAC	∠CAG	∠GAB	∠FAB	∠JBA	∠ABK	∠KBD	∠JBD
1st Measure								

2. Drag point *C* or *D* to move transversal $\overleftrightarrow{AB}$ so that it intersects the two parallel lines at a different angle. Add a row **2nd Measure** to your table and record the new measures. Repeat these steps until your table has 3rd, 4th, and 5th Measure rows of data.

3. Using the angles listed in the table, identify and describe the relationship between all angle pairs that have the following special names. Then write a conjecture in if-then form about each angle pair when formed by any two parallel lines cut by a transversal.

   a. corresponding   **b.** alternate interior   **c.** alternate exterior   **d.** consecutive interior

4. Drag point *C* or *D* so that the measure of any of the angles is 90.

   a. What do you notice about the measures of the other angles?

   b. Make a conjecture about a transversal that is perpendicular to one of two parallel lines.

Angles and Parallel Lines

1 Parallel Lines and Angle Pairs In the photo, line *t* is a transversal of lines *a* and *b*, and ∠1 and ∠2 are corresponding angles. Since lines *a* and *b* are parallel, there is a special relationship between corresponding angle pairs.

Postulate 3.1 Corresponding Angles Postulate

If two parallel lines are cut by a transversal, then each pair of corresponding angles is congruent.

Examples $\angle 1 \cong \angle 3$, $\angle 2 \cong \angle 4$, $\angle 5 \cong \angle 7$, $\angle 6 \cong \angle 8$

Example 1 Use Corresponding Angles Postulate

In the figure, $m\angle 5 = 72$. Find the measure of each angle. Tell which postulate(s) or theorem(s) you used.

a. ∠4
$\angle 4 \cong \angle 5$	Corresponding Angles Postulate
$m\angle 4 = m\angle 5$	Definition of congruent angles
$m\angle 4 = 72$	Substitution

b. ∠2
$\angle 2 \cong \angle 4$	Vertical Angles Theorem
$\angle 4 \cong \angle 5$	Corresponding Angles Postulate
$\angle 2 \cong \angle 5$	Transitive Property of Congruence
$m\angle 2 = m\angle 5$	Definition of congruent angles
$m\angle 2 = 72$	Substitution
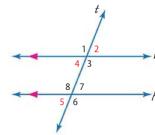
Guided Practice

In the figure, suppose that $m\angle 8 = 105$. Find the measure of each angle. Tell which postulate(s) or theorem(s) you used.

1A. ∠1 **1B.** ∠2 **1C.** ∠3

In Example 1, ∠2 and ∠5 are congruent alternate exterior angles. This and other examples suggest the following theorems about the other angle pairs formed by two parallel lines cut by a transversal.

StudyTip

Angle Relationships These theorems generalize the relationships between specific pairs of angles. If you get confused about the relationships, you can verify them with the methods you used in Example 1, using only corresponding, vertical, and supplementary angles.

Theorems Parallel Lines and Angle Pairs

3.1 Alternate Interior Angles Theorem If two parallel lines are cut by a transversal, then each pair of alternate interior angles is congruent.
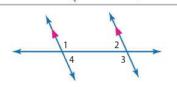
Examples $\angle 1 \cong \angle 3$ and $\angle 2 \cong \angle 4$
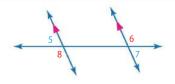
3.2 Consecutive Interior Angles Theorem If two parallel lines are cut by a transversal, then each pair of consecutive interior angles is supplementary.

Examples $\angle 1$ and $\angle 2$ are supplementary.
$\angle 3$ and $\angle 4$ are supplementary.

3.3 Alternate Exterior Angles Theorem If two parallel lines are cut by a transversal, then each pair of alternate exterior angles is congruent.

Examples $\angle 5 \cong \angle 7$ and $\angle 6 \cong \angle 8$

You will prove Theorems 3.2 and 3.3 in Exercises 30 and 35, respectively.

Since postulates are accepted without proof, you can use the Corresponding Angles Postulate to prove each of the theorems above.
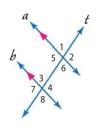
Proof Alternate Interior Angles Theorem

Given: $a \parallel b$
t is a transversal of a and b.

Prove: $\angle 4 \cong \angle 5$, $\angle 3 \cong \angle 6$

Paragraph Proof: We are given that $a \parallel b$ with a transversal t. By the Corresponding Angles Postulate, corresponding angles are congruent. So, $\angle 2 \cong \angle 4$ and $\angle 6 \cong \angle 8$. Also, $\angle 5 \cong \angle 2$ and $\angle 8 \cong \angle 3$ because vertical angles are congruent. Therefore, $\angle 5 \cong \angle 4$ and $\angle 3 \cong \angle 6$ since congruence of angles is transitive.

Real-World Example 2 Use Theorems about Parallel Lines
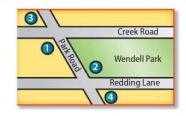
COMMUNITY PLANNING Redding Lane and Creek Road are parallel streets that intersect Park Road along the west side of Wendell Park. If $m\angle 1 = 118$, find $m\angle 2$.

|	
$\angle 2 \cong \angle 1$	Alternate Interior Angles Postulate
$m\angle 2 = m\angle 1$	Definition of congruent angles
$m\angle 2 = 118$	Substitution

Real-World Link

Some cities require that streets in newly planned subdivisions intersect at no less than a 60° angle.

Glowimages/Getty Images

▶ **Guided**Practice

COMMUNITY PLANNING Refer to the diagram above to find each angle measure. Tell which postulate(s) or theorem(s) you used.

2A. If $m\angle 1 = 100$, find $m\angle 4$.

2B. If $m\angle 3 = 70$, find $m\angle 4$.

2 Algebra and Angle Measures
The special relationships between the angles formed by two parallel lines and a transversal can be used to find unknown values.

Example 3 Find Values of Variables

ALGEBRA Use the figure at the right to find the indicated variable. Explain your reasoning.

a. If $m\angle 4 = 2x - 17$ and $m\angle 1 = 85$, find x.

$\angle 3 \cong \angle 1$	Vertical Angles Theorem
$m\angle 3 = m\angle 1$	Definition of congruent angles
$m\angle 3 = 85$	Substitution
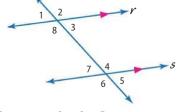
Since lines r and s are parallel, $\angle 4$ and $\angle 3$ are supplementary by the Consecutive Interior Angles Theorem.

$m\angle 3 + m\angle 4 = 180$	Definition of supplementary angles
$85 + 2x - 17 = 180$	Substitution
$2x + 68 = 180$	Simplify.
$2x = 112$	Subtract 68 from each side.
$x = 56$	Divide each side by 2.

b. Find y if $m\angle 3 = 4y + 30$ and $m\angle 7 = 7y + 6$.

$\angle 3 \cong \angle 7$	Alternate Interior Angles Theorem
$m\angle 3 = m\angle 7$	Definition of congruent angles
$4y + 30 = 7y + 6$	Substitution
$30 = 3y + 6$	Subtract 4y from each side.
$24 = 3y$	Subtract 6 from each side.
$8 = y$	Divide each side by 3.
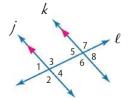
> **GuidedPractice**

3A. If $m\angle 2 = 4x + 7$ and $m\angle 7 = 5x - 13$, find x.

3B. Find y if $m\angle 5 = 68$ and $m\angle 3 = 3y - 2$.

A special relationship exists when the transversal of two parallel lines is a perpendicular line.
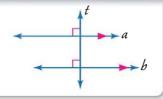
Theorem 3.4 Perpendicular Transversal Theorem

In a plane, if a line is perpendicular to one of two parallel lines, then it is perpendicular to the other.
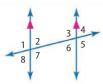
Examples If line $a \parallel$ line b and line $a \perp$ line t, then line $b \perp$ line t.
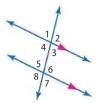
You will prove Theorem 3.4 in Exercise 37.

Example 1 In the figure, $m\angle 1 = 94$. Find the measure of each angle. Tell which postulate(s) or theorem(s) you used.

   1. $\angle 3$ **2.** $\angle 5$ **3.** $\angle 4$
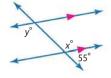
Example 2 In the figure, $m\angle 4 = 101$. Find the measure of each angle. Tell which postulate(s) or theorem(s) you used.

   4. $\angle 6$ **5.** $\angle 7$ **6.** $\angle 5$
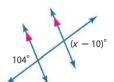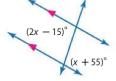
   7. ROADS In the diagram, the guard rail is parallel to the surface of the roadway and the vertical supports are parallel to each other. Find the measures of angles 2, 3, and 4.

Example 3 Find the value of the variable(s) in each figure. Explain your reasoning.

   8.   **9.**   **10.**

   $y°$ $(2x - 15)°$
   $x°$ $(x - 10)°$
   $55°$ $104°$ $(x + 55)°$
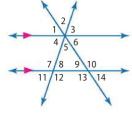
Examples 1–2 In the figure, $m\angle 11 = 62$ and $m\angle 14 = 38$. Find the measure of each angle. Tell which postulate(s) or theorem(s) you used.

   11. $\angle 4$ **12.** $\angle 3$ **13.** $\angle 12$

   14. $\angle 8$ **15.** $\angle 6$ **16.** $\angle 2$

   17. $\angle 10$ **18.** $\angle 5$ **19.** $\angle 1$

Example 3 **CCSS MODELING** A solar dish collects energy by directing radiation from the Sun to a receiver located at the focal point of the dish. Assume that the radiation rays are parallel. Determine the relationship between each pair of angles, and explain your reasoning.
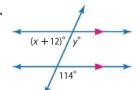
   20. $\angle 1$ and $\angle 2$ **㉑** $\angle 1$ and $\angle 3$ **22.** $\angle 4$ and $\angle 5$ **23.** $\angle 3$ and $\angle 4$
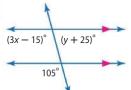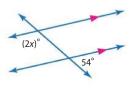
Find the value of the variable(s) in each figure. Explain your reasoning.

24.

$(x + 12)°$ $y°$

$114°$

25

$(3x - 15)°$ $(y + 25)°$

$105°$

26.

$(2x)°$

$54°$

27.

$96°$

$(2x)°$

$94°$ $(3y + 44)°$

28.

$(5y)°$ $(2x)°$

$120°$ $108°$

29.

$(3x - 70)°$

$(3y + 40)°$

$120°$

$x°$
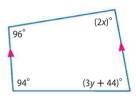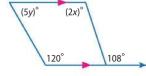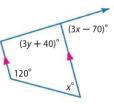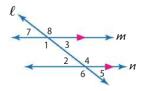
30. PROOF Copy and complete the proof of Theorem 3.2.

Given: $m \parallel n$; ℓ is a transversal.

Prove: ∠1 and ∠2 are supplementary;
∠3 and ∠4 are supplementary.

Proof:
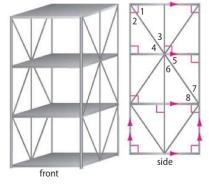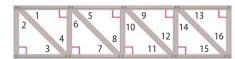
Statements	Reasons
a. ?	**a.** Given
b. ∠1 and ∠3 form a linear pair; ∠2 and ∠4 form a linear pair.	**b.** ?
c. ?	**c.** If two angles form a linear pair, then they are supplementary.
d. ∠1 ≅ ∠4, ∠2 ≅ ∠3	**d.** ?
e. $m∠1 = m∠4$, $m∠2 = m∠3$	**e.** Definition of Congruence
f. ?	**f.** ?

STORAGE When industrial shelving needs to be accessible from either side, additional support is provided on the side by transverse members. Determine the relationship between each pair of angles and explain your reasoning.

31. ∠1 and ∠8

32. ∠1 and ∠5

33. ∠3 and ∠6

34. ∠1 and ∠2

front

side

35. CCSS ARGUMENTS Write a two-column proof of the Alternate Exterior Angles Theorem. (Theorem 3.3)
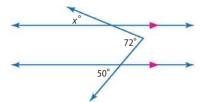
36. BRIDGES Refer to the diagram of the double decker Michigan Avenue Bridge in Chicago, Illinois, at the right. The two levels of the bridge, and its diagonal braces, are parallel.
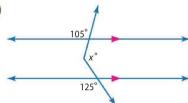
a. How are the measures of the odd-numbered angles related? Explain.

b. How are the measures of the even-numbered angles related? Explain.

c. How are any pair of angles in which one is odd and the other is even related?

d. What geometric term(s) can be used to relate the two roadways contained by the bridge?

37. PROOF In a plane, prove that if a line is perpendicular to one of two parallel lines, then it is perpendicular to the other. (Theorem 3.4)

CCSS TOOLS Find x. (*Hint:* **Draw an auxiliary line.**)

38.

$x°$
$72°$
$50°$

39

$105°$
$x°$
$125°$
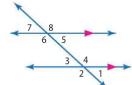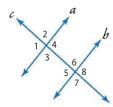
40. PROBABILITY Suppose you were to pick any two angles in the figure below.

a. How many possible angle pairings are there? Explain.
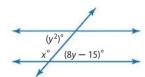
b. Describe the possible relationships between the measures of the angles in each pair. Explain.

c. Describe the likelihood of randomly selecting a pair of congruent angles. Explain your reasoning.

41. **MULTIPLE REPRESENTATIONS** In this problem, you will investigate the relationship between same-side exterior angles.

a. Geometry Draw five pairs of parallel lines, m and n, a and b, r and s, j and k, and x and y, cut by a transversal t, and measure the four angles on one side of t.

b. Tabular Record your data in a table.

c. Verbal Make a conjecture about the relationship between the pair of angles formed on the exterior of parallel lines and on the same side of the transversal.

d. Logical What type of reasoning did you use to form your conjecture? Explain.

e. Proof Write a proof of your conjecture.

H.O.T. Problems *Use Higher-Order Thinking Skills*

42. WRITING IN MATH If line a is parallel to line b and $\angle 1 \cong \angle 2$, describe the relationship between lines b and c. Explain your reasoning.
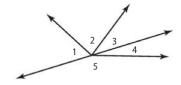
43. WRITING IN MATH Compare and contrast the Alternate Interior Angles Theorem and the Consecutive Interior Angles Theorem.

44. OPEN ENDED Draw a pair of parallel lines cut by a transversal and measure the two exterior angles on the same side of the transversal. Include the measures on your drawing. Based on the pattern you have seen for naming other pairs of angles, what do you think the name of the pair you measured would be?

45. CHALLENGE Find x and y.

$(y^2)°$
$x°$ $(8y - 15)°$

46. REASONING Determine the minimum number of angle measures you would have to know to find the measures of all the angles formed by two parallel lines cut by a transversal. Explain.
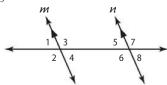
47. Suppose $\angle 4$ and $\angle 5$ form a linear pair. If $m\angle 1 = 2x$, $m\angle 2 = 3x - 20$, and $m\angle 3 = x - 4$, what is $m\angle 3$?

A 26° **C** 30°
B 28° **D** 32°

48. SAT/ACT A farmer raises chickens and pigs. If his animals have a total of 120 heads and a total of 300 feet, how many chickens does the farmer have?

F 60 **H** 80
G 70 **J** 90

49. SHORT RESPONSE If $m \parallel n$, then which of the following statements must be true?

   I. $\angle 3$ and $\angle 6$ are Alternate Interior Angles.
  II. $\angle 4$ and $\angle 6$ are Consecutive Interior Angles.
 III. $\angle 1$ and $\angle 7$ are Alternate Exterior Angles.

50. ALGEBRA If $-2 + x = -6$, then $-17 - x = ?$

A -13 **D** 13
B -4 **E** 21
C 9
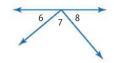
51. AVIATION Airplanes are assigned an altitude level based on the direction they are flying. If one airplane is flying northwest at 34,000 feet and another airplane is flying east at 25,000 feet, describe the type of lines formed by the paths of the airplanes. Explain your reasoning. (Lesson 3-1)

Use the given statement to find the measure of each numbered angle. (Lesson 2-8)

52. $\angle 1$ and $\angle 2$ form a linear pair and $m\angle 2 = 67$.

53. $\angle 6$ and $\angle 8$ are; complementary $m\angle 8 = 47$.

54. $m\angle 4 = 32$

55. TRAINS A train company wants to provide routes to New York City, Dallas, Chicago, Los Angeles, San Francisco, and Washington, D.C. An engineer draws lines between each pair of cities on a map. No three of the cities are collinear. How many lines did the engineer draw? (Lesson 2-5)

Simplify each expression.

56. $\dfrac{6 - 5}{4 - 2}$

57. $\dfrac{-5 - 2}{4 - 7}$

58. $\dfrac{-11 - 4}{12 - (-9)}$

59. $\dfrac{16 - 12}{15 - 11}$

60. $\dfrac{10 - 22}{8 - 17}$

61. $\dfrac{8 - 17}{12 - (-3)}$

3-3

Investigating Slope

The rate of change of the steepness of a line is called the *slope*. Slope can be used to investigate the relationship between real-world quantities.

Set Up the Lab

- Connect a data collection device to a graphing calculator. Place the device on a desk or table so that it can read the motion of a walker.

- Mark the floor at distances of 1 meter and 6 meters from the device.

Activity

Step 1 Have one group member stand at the 1-meter mark. When another group member presses the button to begin collecting data, the walker should walk away from the device at a slow, steady pace.

Step 2 Stop collecting data when the walker passes the 6-meter mark. Save the data as Trial 1.

Step 3 Repeat the experiment, walking more quickly. Save the data as Trial 2.

Step 4 For Trial 3, repeat the experiment by slowly walking toward the data collection device.

Step 5 Repeat the experiment, walking quickly toward the device. Save the data as Trial 4.

Analyze the Results

1. Compare and contrast the graphs for Trials 1 and 2. How do the graphs for Trials 1 and 3 compare?

2. Use the **TRACE** feature of the calculator to find the coordinates of two points on each graph. Record the coordinates in a table like the one shown. Then use the points to find the slope of the line.

Trial	Point A (x_1, y_1)	Point B (x_2, y_2)	Slope = $\dfrac{y_2 - y_1}{x_2 - x_1}$
1			
2			
3			
4			

3. Compare and contrast the slopes for Trials 1 and 2. How do the slopes for Trials 1 and 2 compare to the slopes for Trials 3 and 4?

4. The slope of a line describes the rate of change of the quantities represented by the *x*- and *y*-values. What is represented by the rate of change in this experiment?

5. **MAKE A CONJECTURE** What would the graph look like if you were to collect data while the walker was standing still? Use the data collection device to test your conjecture.

3-3 Slopes of Lines

∴Then

- You used the properties of parallel lines to determine congruent angles.

∴Now

1 Find slopes of lines.
2 Use slope to identify parallel and perpendicular lines.

∴Why?

- Ski resorts assign ratings to their ski trails according to their difficulty. A primary factor in determining this rating is a trail's steepness or *slope gradient*. A trail with a 6% or $\frac{6}{100}$ grade falls 6 feet vertically for every 100 feet traveled horizontally.

  The easiest trails, labeled ●, have slopes ranging from 6% to 25%, while more difficult trails, labeled ◆ or ◆◆, have slopes of 40% or greater.

$$slope = \frac{vertical\ rise}{horizontal\ run}$$

vertical rise

horizontal run

ᵃᵇᶜ NewVocabulary
slope
rate of change

CCSS Common Core State Standards

Content Standards
G.GPE.5 Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).

Mathematical Practices
4 Model with mathematics.
7 Look for and make use of structure.
8 Look for and express regularity in repeated reasoning.

---

1 **Slope of a Line** The steepness or slope of a hill is described by the ratio of the hill's vertical rise to its horizontal run. In algebra, you learned that the slope of a line in the coordinate plane can be calculated using any two points on the line.
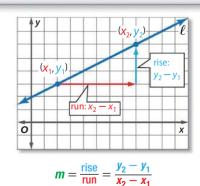
📋 KeyConcept Slope of a Line

In a coordinate plane, the **slope** of a line is the ratio of the change along the *y*-axis to the change along the *x*-axis between any two points on the line.

The slope *m* of a line containing two points with coordinates (x_1, y_1) and (x_2, y_2) is given by the formula

$$m = \frac{y_2 - y_1}{x_2 - x_1}, \text{ where } x_1 \neq x_2.$$

(x_2, y_2) ℓ

rise: $y_2 - y_1$

(x_1, y_1)

run: $x_2 - x_1$

$$m = \frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1}$$
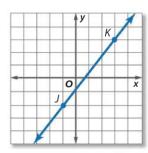
Example 1 Find the Slope of a Line

Find the slope of each line.
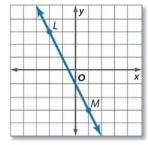
a.

Substitute $(-1, -2)$ for (x_1, y_1) and $(3, 3)$ for (x_2, y_2).

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad \text{Slope Formula}$$

$$= \frac{3 - (-2)}{3 - (-1)} \qquad \text{Substitution}$$

$$= \frac{5}{4} \qquad \text{Simplify.}$$

Allan Bard/Corbis
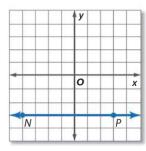
b.

Substitute $(-2, 3)$ for (x_1, y_1) and $(1, -3)$ for (x_2, y_2).

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad \text{Slope Formula}$$

$$= \frac{-3 - 3}{1 - (-2)} \qquad \text{Substitution}$$

$$= -2 \qquad \text{Simplify.}$$
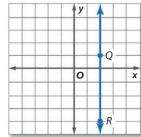
c.

Substitute $(-4, -3)$ for (x_1, y_1) and $(3, -3)$ for (x_2, y_2).

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad \text{Slope Formula}$$

$$= \frac{-3 - (-3)}{3 - (-4)} \qquad \text{Substitution}$$

$$= \frac{0}{7} \text{ or } 0 \qquad \text{Simplify.}$$

d.

Substitute $(2, 1)$ for (x_1, y_1) and $(2, -4)$ for (x_2, y_2).

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad \text{Slope Formula}$$

$$= \frac{-4 - 1}{2 - 2} \qquad \text{Substitution}$$

$$= \frac{-5}{0} \qquad \text{Simplify.}$$

This slope is **undefined**.
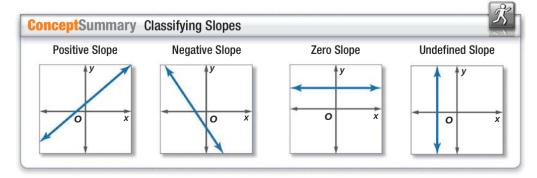
StudyTip

Dividing by 0 The slope $\frac{-5}{0}$ is undefined because there is no number that you can multiply by 0 and get -5. Since this is true for any number, all numbers divided by 0 will have an undefined slope. All vertical lines have undefined slopes.

▶ **Guided**Practice

1A. the line containing $(6, -2)$ and $(-3, -5)$

1B. the line containing $(8, -3)$ and $(-6, -2)$

1C. the line containing $(4, 2)$ and $(4, -3)$

1D. the line containing $(-3, 3)$ and $(4, 3)$

Example 1 illustrates the four different types of slopes.

ConceptSummary **Classifying Slopes**

Positive Slope	Negative Slope	Zero Slope	Undefined Slope

Slope can be interpreted as a **rate of change**, describing how a quantity y changes in relation to quantity x. The slope of a line can also be used to identify the coordinates of any point on the line.

TRAVEL A pilot flies a plane from Columbus, Ohio, to Orlando, Florida. After 0.5 hour, the plane reaches its cruising altitude and is 620 miles from Orlando. Half an hour later, the plane is 450 miles from Orlando. How far was the plane from Orlando 1.25 hours after takeoff?
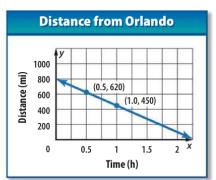
Understand Use the data given to graph the line that models the distance from Orlando y in miles as a function of time x in hours.

Assume that speed is constant. Plot the points (**0.5**, **620**) and (**1.0**, **450**), and draw a line through them.

You want to find the distance from Orlando after 1.25 hours.

From the graph we can estimate that after 1.25 hours, the distance was a little less than 400 miles.

Distance from Orlando

Plan Find the slope of the line graphed. Use this rate of change in the plane's distance from Orlando per hour to find the distance from Orlando after 1.25 hours.

Solve Use the Slope Formula to find the slope of the line.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(450 - 620)\text{ miles}}{(1.0 - 0.5)\text{ hours}} = \frac{-170\text{ miles}}{0.5\text{ hour}} \text{ or } -\frac{340\text{ miles}}{1\text{ hour}}$$

The plane traveled at an average speed of 340 miles per hour. The negative sign indicates a *decrease* in distance over time.

Use the slope of the line and one known point on the line to calculate the distance y when the time x is 1.25.

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad \text{Slope Formula}$$

$$-340 = \frac{y_2 - 620}{1.25 - 0.5} \qquad m = -340, x_1 = 0.5, y_1 = 620, \text{ and } x_2 = 1.25$$

$$-340 = \frac{y_2 - 620}{0.75} \qquad \text{Simplify.}$$

$$-255 = y_2 - 620 \qquad \text{Multiply each side by 0.75.}$$

$$365 = y_2 \qquad \text{Add 620 to each side.}$$

Thus, the distance from Orlando after 1.25 hours is 365 miles.

Check Since 365 is close to the estimate, our answer is reasonable. ✔

▶ **Guided**Practice

2. **DOWNLOADS** In 2006, 500 million songs were legally downloaded from the Internet. In 2004, 200 million songs were legally downloaded.

A. Use the data given to graph the line that models the number of songs legally downloaded y as a function of time x in years.

B. Find the slope of the line, and interpret its meaning.

C. If this trend continues at the same rate, how many songs will be legally downloaded in 2020?

Real-World Career

Flight Attendants Flight attendants check tickets, assist passengers with boarding and carry-ons, and provide an overview of emergency equipment and procedures. A high school diploma is required, but airlines increasingly favor bi- or multi-lingual candidates with college degrees.

2 **Parallel and Perpendicular Lines** You can use the slopes of two lines to determine whether the lines are parallel or perpendicular. Lines with the same slope are parallel.
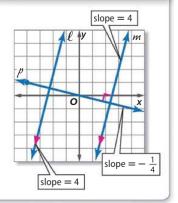

Postulates Parallel and Perpendicular Lines

3.2 Slopes of Parallel Lines Two nonvertical lines have the same slope if and only if they are parallel. All vertical lines are parallel.

Example Parallel lines ℓ and m have the same slope, 4.

3.3 Slopes of Perpendicular Lines Two nonvertical lines are perpendicular if and only if the product of their slopes is -1. Vertical and horizontal lines are perpendicular.

Example line $m \perp$ line p
product of slopes $= 4 \cdot -\dfrac{1}{4}$ or -1

slope = 4

slope = $-\dfrac{1}{4}$

slope = 4

Example 3 Determine Line Relationships

Determine whether $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are *parallel*, *perpendicular*, or *neither* for $A(1, 1)$, $B(-1, -5)$, $C(3, 2)$, and $D(6, 1)$. Graph each line to verify your answer.

Step 1 Find the slope of each line.

$$\text{slope of } \overleftrightarrow{AB} = \frac{-5 - 1}{-1 - 1} = \frac{-6}{-2} \text{ or } 3 \qquad \text{slope of } \overleftrightarrow{CD} = \frac{1 - 2}{6 - 3} \text{ or } \frac{-1}{3}$$
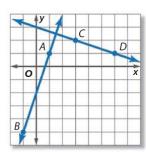
Step 2 Determine the relationship, if any, between the lines.

The two lines do not have the same slope, so they are *not* parallel. To determine if the lines are perpendicular, find the product of their slopes.

$$3\left(-\frac{1}{3}\right) = -1 \qquad \text{Product of slopes for } \overleftrightarrow{AB} \text{ and } \overleftrightarrow{CD}$$

Since the product of their slopes is -1, $\overleftrightarrow{AB}$ is perpendicular to $\overleftrightarrow{CD}$.

CHECK When graphed, the two lines appear to intersect and form four right angles. ✔

StudyTip

Slopes of Perpendiculars
If a line ℓ has a slope of $\frac{a}{b}$, then the slope of a line perpendicular to line ℓ is the opposite reciprocal, $-\frac{b}{a}$, since $\frac{a}{b}\left(-\frac{b}{a}\right) = -1$.

▶ **Guided**Practice

Determine whether $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are *parallel*, *perpendicular*, or *neither*. Graph each line to verify your answer.

3A. $A(14, 13)$, $B(-11, 0)$, $C(-3, 7)$, $D(-4, -5)$

3B. $A(3, 6)$, $B(-9, 2)$, $C(5, 4)$, $D(2, 3)$

Example 4 Use Slope to Graph a Line

Graph the line that contains $A(-3, 0)$ and is perpendicular to $\overleftrightarrow{CD}$ with $C(-2, -3)$ and $D(2, 0)$.

The slope of $\overleftrightarrow{CD}$ is $\dfrac{0 - (-3)}{2 - (-2)}$ or $\dfrac{3}{4}$.

Since $\dfrac{3}{4}\left(\dfrac{4}{-3}\right) = -1$, the slope of the line perpendicular to $\overleftrightarrow{CD}$ through A is $-\dfrac{4}{3}$ or $\dfrac{-4}{3}$.
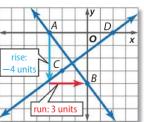
To graph the line, start at point A. Move down 4 units and then right 3 units. Label the point B and draw $\overleftrightarrow{AB}$.

rise: −4 units

run: 3 units

▶ **Guided**Practice

4. Graph the line that contains $P(0, 1)$ and is perpendicular to $\overleftrightarrow{QR}$ with $Q(-6, -2)$ and $R(0, -6)$.
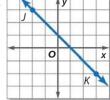
Check Your Understanding
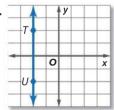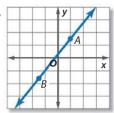
= Step-by-Step Solutions begin on page R14.

Example 1 Find the slope of each line.

1.

2.

3.
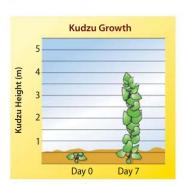
Example 2 **4. BOTANY** Kudzu is a fast-growing vine found in the southeastern United States. An initial measurement of the length of a kudzu vine was 0.5 meter. Seven days later the plant was 4 meters long.

a. Graph the line that models the length of the plant over time.

b. What is the slope of your graph? What does it represent?

c. Assuming that the growth rate of the plant continues, how long will the plant be after 15 days?

Kudzu Growth

Kudzu Height (m)

Day 0   Day 7
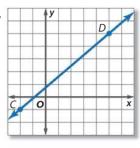
Example 3 Determine whether $\overleftrightarrow{WX}$ and $\overleftrightarrow{YZ}$ are *parallel*, *perpendicular*, or *neither*. Graph each line to verify your answer.
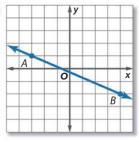
5 $W(2, 4), X(4, 5), Y(4, 1), Z(8, -7)$ **6.** $W(1, 3), X(-2, -5), Y(-6, -2), Z(8, 3)$

7. $W(-7, 6), X(-6, 9), Y(6, 3), Z(3, -6)$ **8.** $W(1, -3), X(0, 2), Y(-2, 0), Z(8, 2)$

Example 4 Graph the line that satisfies each condition.

9. passes through $A(3, -4)$, parallel to $\overleftrightarrow{BC}$ with $B(2, 4)$ and $C(5, 6)$

10. slope $= 3$, passes through $A(-1, 4)$

11. passes through $P(7, 3)$, perpendicular to $\overleftrightarrow{LM}$ with $L(-2, -3)$ and $M(-1, 5)$

Example 1   Find the slope of each line.
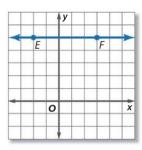
12.
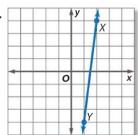
13.
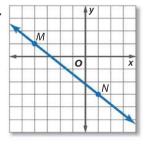
14.
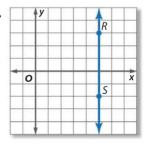
15.

16.

17.

Determine the slope of the line that contains the given points.

18. $C(3, 1)$, $D(-2, 1)$

19. $E(5, -1)$, $F(2, -4)$

20. $G(-4, 3)$, $H(-4, 7)$

21. $J(7, -3)$, $K(-8, -3)$

22. $L(8, -3)$, $M(-4, -12)$

23. $P(-3, -5)$, $Q(-3, -1)$

24. $R(2, -6)$, $S(-6, 5)$

25. $T(-6, -11)$, $V(-12, -10)$

Example 2   26. **CCSS MODELING** In 2004, 8 million Americans over the age of 7 participated in mountain biking, and in 2006, 8.5 million participated.

    a. Create a graph to show the number of participants in mountain biking based on the change in participation from 2004 to 2006.

    b. Based on the data, what is the growth per year of the sport?

    c. If participation continues at the same rate, what will be the participation in 2013 to the nearest 10,000?

27. **FINANCIAL LITERACY** Suppose an MP3 player cost $499 in 2003 and $249.99 in 2009.

    a. Graph a trend line to predict the price of the MP3 player for 2003 through 2009.

    b. Based on the data, how much does the price drop per year?

   **c.** If the trend continues, what will be the cost of an MP3 player in 2013?

Example 3   Determine whether $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are *parallel*, *perpendicular*, or *neither*. Graph each line to verify your answer.

28. $A(1, 5)$, $B(4, 4)$, $C(9, -10)$, $D(-6, -5)$

29. $A(-6, -9)$, $B(8, 19)$, $C(0, -4)$, $D(2, 0)$

30. $A(4, 2)$, $B(-3, 1)$, $C(6, 0)$, $D(-10, 8)$

31. $A(8, -2)$, $B(4, -1)$, $C(3, 11)$, $D(-2, -9)$

32. $A(8, 4)$, $B(4, 3)$, $C(4, -9)$, $D(2, -1)$

33. $A(4, -2)$, $B(-2, -8)$, $C(4, 6)$, $D(8, 5)$

Example 4

Graph the line that satisfies each condition.

34. passes through $A(2, -5)$, parallel to $\overleftrightarrow{BC}$ with $B(1, 3)$ and $C(4, 5)$

35. slope $= -2$, passes through $H(-2, -4)$

36. passes through $K(3, 7)$, perpendicular to $\overleftrightarrow{LM}$ with $L(-1, -2)$ and $M(-4, 8)$

37. passes through $X(1, -4)$, parallel to $\overleftrightarrow{YZ}$ with $Y(5, 2)$ and $Z(-3, -5)$

38. slope $= \frac{2}{3}$, passes through $J(-5, 4)$

39. passes through $D(-5, -6)$, perpendicular to $\overleftrightarrow{FG}$ with $F(-2, -9)$ and $G(1, -5)$

40. **STADIUMS** Before it was demolished, the RCA Dome was home to the Indianapolis Colts. The attendance in 2001 was 450,746, and the attendance in 2005 was 457,373.

   a. What is the approximate rate of change in attendance from 2001 to 2005?

   b. If this rate of change continues, predict the attendance for 2012.

   c. Will the attendance continue to increase indefinitely? Explain.

   d. The Colts have now built a new, larger stadium. Do you think their decision was reasonable? Why or why not?

Determine which line passing through the given points has a steeper slope.

41. Line 1: $(0, 5)$ and $(6, 1)$

    Line 2: $(-4, 10)$ and $(8, -5)$

42. Line 1: $(0, -4)$ and $(2, 2)$

    Line 2: $(0, -4)$ and $(4, 5)$

43. Line 1: $(-9, -4)$ and $(7, 0)$

    Line 2: $(0, 1)$ and $(7, 4)$

44. Line 1: $(-6, 7)$ and $(9, -3)$

    Line 2: $(-9, 9)$ and $(3, 5)$

45. **CCSS MODELING** Michigan provides habitat for two endangered species, the bald eagle and the gray wolf. The graph shows the Michigan population of each species in 1992 and 2006.

   a. Which species experienced a greater rate of change in population?

   b. Make a line graph showing the growth of both populations.

   c. If both species continue to grow at their respective rates, what will the population of each species be in 2012?

Michigan Bald Eagle and Gray Wolf Populations

Find the value of x or y that satisfies the given conditions. Then graph the line.

46. The line containing $(4, -1)$ and $(x, -6)$ has a slope of $-\frac{5}{2}$.

47. The line containing $(-4, 9)$ and $(4, 3)$ is parallel to the line containing $(-8, 1)$ and $(4, y)$.

48. The line containing $(8, 7)$ and $(7, -6)$ is perpendicular to the line containing $(2, 4)$ and $(x, 3)$.

49. The line containing $(1, -3)$ and $(3, y)$ is parallel to the line containing $(5, -6)$ and $(9, y)$.

50. **SCHOOLS** In 2000, Jefferson High School had 1125 students. By 2006, the student body had increased to 1425 students. When Fairview High School was built in 2001, it had 1275 students. How many students did Fairview High School have in 2006 if the student body grew at the same rate as Jefferson High School?

51 **MUSIC** Maggie and Mikayla want to go to the music store near Maggie's house after school. They can walk 3.5 miles per hour and ride their bikes 10 miles per hour.

a. Create a table to show how far Maggie and Mikayla can travel walking and riding their bikes. Include distances for 0, 1, 2, 3, and 4 hours.

b. Create a graph to show how far Maggie and Mikayla can travel based on time for both walking and riding their bikes. Be sure to label the axes of your graph.

c. What does the slope represent in your graph?

d. Maggie's mom says they can only go if they can make it to the music store and back in less than two hours. If they want to spend at least 30 minutes in the music store and it is four miles away, can they make it? Should they walk or ride their bikes? Explain your reasoning.

H.O.T. Problems   Use Higher-Order Thinking Skills

52. WRITE A QUESTION A classmate says that all lines have positive or negative slope. Write a question that would challenge his conjecture.

53. ERROR ANALYSIS Terrell and Hale calculated the slope of the line passing through the points $Q(3, 5)$ and $R(-2, 2)$. Is either of them correct? Explain your reasoning.

<table>
<tr><td>

Terrell

$$m = \frac{5 - 2}{3 - (-2)}$$

$$= \frac{3}{5}$$

</td><td>

Hale

$$m = \frac{5 - 2}{-2 - 3}$$

$$= -\frac{3}{5}$$

</td></tr>
</table>

54. CCSS REASONING Draw a square $ABCD$ with opposite vertices at $A(2, -4)$ and $C(10, 4)$.

a. Find the other two vertices of the square and label them B and D.

b. Show that $\overline{AD} \parallel \overline{BC}$ and $\overline{AB} \parallel \overline{DC}$.

c. Show that the measure of each angle inside the square is equal to 90.

55. WRITING IN MATH Describe the slopes of the Sears Tower and the Leaning Tower of Pisa.

56. CHALLENGE In this lesson you learned that $m = \dfrac{y_2 - y_1}{x_2 - x_1}$. Use an algebraic proof to show that the slope can also be calculated using the equation $m = \dfrac{y_1 - y_2}{x_1 - x_2}$.

Sears Tower

Leaning Tower of Pisa

57. WRITING IN MATH Find two additional points that lie along the same line as $X(3, -1)$ and $Y(-1, 7)$. Generalize a method you can use to find additional points on the line from any given point.

58. What is the slope of a line perpendicular to the line through the points $(-1, 6)$ and $(3, -4)$?

A $m = -\dfrac{5}{2}$

B $m = -1$

C $m = -\dfrac{2}{5}$

D $m = \dfrac{2}{5}$

59. SHORT RESPONSE A set of 25 cards is randomly placed face down on a table. 15 cards have only the letter A written on the face, and 10 cards have only the letter B. Patrick turned over 1 card. What is the probability of this card having the letter B written on its face?

60. ALGEBRA Jamie is collecting money to buy an $81 gift for her teacher. She has already contributed $24. She will collect $3 from each contributing student. How many other students must contribute?

F 3 students

G 9 students

H 12 students

J 19 students

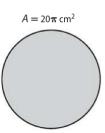**61. SAT/ACT** The area of a circle is 20π square centimeters. What is its circumference?

$A = 20\pi$ cm^2

A $\sqrt{5}\pi$ cm

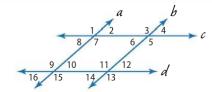**B** $2\sqrt{5}\pi$ cm
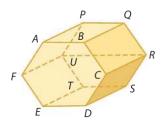
C $4\sqrt{5}\pi$ cm

D 20π cm

E 40π cm

Spiral Review

In the figure, $a \parallel b$, $c \parallel d$, and $m\angle 4 = 57$.
Find the measure of each angle. (Lesson 3-2)

62. $\angle 5$

63. $\angle 1$

64. $\angle 8$

65. $\angle 10$

Refer to the diagram at the right. (Lesson 3-1)

66. Name all segments parallel to $\overline{TU}$.

67. Name all planes intersecting plane BCR.

68. Name all segments skew to $\overline{DE}$.

Determine whether the stated conclusion is valid based on the given information. If not, write *invalid*. Explain your reasoning. (Lesson 2-4)

69. Given: $\angle B$ and $\angle C$ are vertical angles.

Conclusion: $\angle B \cong \angle C$

70. Given: $\angle W \cong \angle Y$

Conclusion: $\angle W$ and $\angle Y$ are vertical angles.

71. CONSTRUCTION There are four buildings on the Mansfield High School Campus, no three of which stand in a straight line. How many sidewalks need to be built so that each building is directly connected to every other building? (Lesson 1-1)

Skills Review

Solve for y.
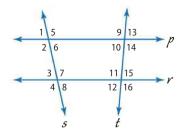
72. $3x + y = 5$

73. $4x + 2y = 6$

74. $4y - 3x = 5$
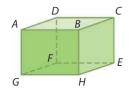
Identify the transversal connecting each pair of angles. Then classify the relationship between each pair of angles as *alternate interior, alternate exterior, corresponding,* or *consecutive interior* angles. (Lesson 3-1)
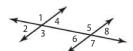

1. $\angle 6$ and $\angle 3$

2. $\angle 1$ and $\angle 14$

3. $\angle 10$ and $\angle 11$

4. $\angle 5$ and $\angle 7$

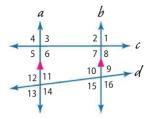Refer to the figure to identify each of the following. (Lesson 3-1)

5. a plane parallel to plane *ABCD*

6. a segment skew to $\overline{GH}$ that contains point *D*

7. all segments parallel to $\overline{HE}$
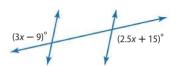
8. MULTIPLE CHOICE Which term best describes $\angle 4$ and $\angle 8$? (Lesson 3-1)

A corresponding

B alternate exterior

C alternate interior

D consecutive interior

In the figure, $m\angle 4 = 104$, $m\angle 14 = 118$. Find the measure of each angle. Tell which postulate(s) or theorem(s) you used. (Lesson 3-2)

9. $\angle 2$

10. $\angle 9$

11. $\angle 10$

12. $\angle 7$
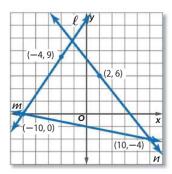

13. Find *x*. (Lesson 3-2)

14. MODEL TRAINS Amy is setting up two parallel train tracks so that a third track runs diagonally across the first two. To properly place a switch, she needs the angle between the diagonal and the top right portion of the second track to be twice as large as the angle between the diagonal and bottom right portion of the first track. What is the measure of the angle between the diagonal and the top right portion of the second track? (Lesson 3-2)

Determine whether $\overleftrightarrow{AB}$ and $\overleftrightarrow{XY}$ are *parallel, perpendicular,* or *neither*. Graph each line to verify your answer. (Lesson 3-3)

15. $A(2, 0)$, $B(4, -5)$, $X(-3, 3)$, $Y(-5, 8)$

16. $A(1, 1)$, $B(6, -9)$, $X(4, -10)$, $Y(7, -4)$

Find the slope of each line. (Lesson 3-3)

17. line ℓ

18. a line parallel to m

19. a line perpendicular to n

20. SALES The 2008 and 2011 sales figures for Vaughn Electronics are in the table below. (Lesson 3-3)

Year	Approximate Sales ($)
2008	240,000
2011	330,000

a. What is the rate of change in approximate sales from 2008 to 2011?

b. If this rate of change continues, predict the approximate sales for the year 2015.

3-4 Equations of Lines

:·Then	:·Now	:·Why?
● You found the slopes of lines.	● **1** Write an equation of a line given information about the graph. **2** Solve problems by writing equations.	● On an interstate near Lauren's hometown, the minimum fine for speeding ten or fewer miles per hour over the speed limit of 65 miles per hour is $42.50. There is an additional charge of $2 for each mile per hour over this initial ten miles per hour. The total charge, not including court costs, can be represented by the equation $C = 42.5 + 2m$.

Cost of Speeding

NewVocabulary
slope-intercept form
point-slope form

Common Core State Standards

Content Standards
G.GPE.5 Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).

Mathematical Practices
4 Model with mathematics.
8 Look for and express regularity in repeated reasoning.

1 Write Equations of Lines You may remember from algebra that an equation of a nonvertical line can be written in different but equivalent forms.

KeyConcept Nonvertical Line Equations

The **slope-intercept form** of a linear equation is $y = mx + b$, where m is the slope of the line and b is the y-intercept.

$$y = mx + b \qquad y = 3x + 8$$

The **point-slope form** of a linear equation is $y - y_1 = m(x - x_1)$, where (x_1, y_1) is any point on the line and m is the slope of the line.

point on line (3, 5)

$$y - 5 = -2(x - 3)$$

When given the slope and either the y-intercept or a point on a line, you can use these forms to write the equation of the line.
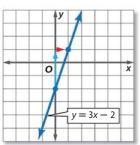
Example 1 Slope and y-intercept

Write an equation in slope-intercept form of the line with slope 3 and y-intercept of −2. Then graph the line.

$y = mx + b$	Slope-intercept form
$y = 3x + (-2)$	$m = 3, b = -2$
$y = 3x - 2$	Simplify.

Plot a point at the y-intercept, −2. Use the slope of 3 or $\frac{3}{1}$ to find another point 3 units up and 1 unit to the right of the y-intercept. Then draw the line through these two points.

▶ **Guided**Practice

1. Write an equation in slope-intercept form of the line with slope $-\frac{1}{2}$ and y-intercept of 8. Then graph the line.

Example 2 Slope and a Point on the Line

Write an equation in point-slope form of the line with slope $-\frac{3}{4}$ that contains $(-2, 5)$. Then graph the line.
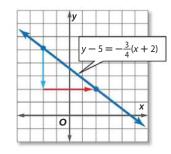
$y - y_1 = m(x - x_1)$ ⟶ Point-Slope form

$y - 5 = -\frac{3}{4}[x - (-2)]$ ⟶ $m = -\frac{3}{4}, (x_1, y_1) = (-2, 5)$

$y - 5 = -\frac{3}{4}(x + 2)$ ⟶ Simplify.

Graph the given point $(-2, 5)$. Use the slope $-\frac{3}{4}$ or $\frac{-3}{4}$ to find another point **3** units down and **4** units to the right. Then draw the line through these two points.

$y - 5 = -\frac{3}{4}(x + 2)$

▶ **Guided**Practice

2. Write an equation in point-slope form of the line with slope 4 that contains $(-3, -6)$. Then graph the line.

When the slope of a line is not given, use two points on the line to calculate the slope. Then use the point-slope or slope-intercept form to write an equation of the line.

Example 3 Two Points

Write an equation of the line through each pair of points in slope-intercept form.

a. $(0, 3)$ and $(-2, -1)$

Step 1 Find the slope of the line through the points.

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1 - 3}{-2 - 0} = \frac{-4}{-2}$ or **2** ⟶ Use the Slope Formula.

Step 2 Write an equation of the line.

$y = mx + b$ ⟶ Slope-Intercept form

$y = 2x + 3$ ⟶ $m = 2$; $(0, 3)$ is the y-intercept.

b. $(-7, 4)$ and $(9, -4)$

Step 1 $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-4 - 4}{9 - (-7)} = \frac{-8}{16}$ or $-\frac{1}{2}$ ⟶ Use the Slope Formula.

Step 2 $y - y_1 = m(x - x_1)$ ⟶ Point-Slope form

$y - 4 = -\frac{1}{2}[x - (-7)]$ ⟶ $m = -\frac{1}{2}, (x_1, y_1) = (-7, 4)$

$y - 4 = -\frac{1}{2}(x + 7)$ ⟶ Simplify.

$y - 4 = -\frac{1}{2}x - \frac{7}{2}$ ⟶ Distribute.

$y = -\frac{1}{2}x + \frac{1}{2}$ ⟶ Add 4 to each side: $\frac{7}{2} + 4 = -\frac{7}{2} + \frac{8}{2}$
$= \frac{1}{2}$

▶ **Guided**Practice

3A. $(-2, 4)$ and $(8, 10)$ **3B.** $(-1, 3)$ and $(7, 3)$

Example 4 Horizontal Line

Write an equation of the line through $(-2, 6)$ and $(5, 6)$ in slope-intercept form.

Step 1 $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{6 - 6}{5 - (-2)} = \dfrac{0}{7}$ or 0 This is a horizontal line.

Step 2 $y - y_1 = m(x - x_1)$ Point-Slope form

$\quad\quad y - 6 = 0[x - (-2)]$ $m = -\dfrac{1}{2}, (x_1, y_1) = (-2, 6)$

$\quad\quad y - 6 = 0$ Simplify.

$\quad\quad\quad\quad y = 6$ Add 6 to each side.

▶ **Guided**Practice

4. Write an equation of the line through $(5, 0)$ and $(-1, 0)$ in slope-intercept form.

Math History Link

Gaspard Monge
(1746–1818) Monge presented the point-slope form of an equation of a line in a paper published in 1784.

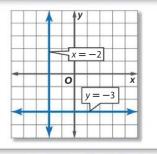The equations of horizontal and vertical lines involve only one variable.

KeyConcepts **Horizontal and Vertical Line Equations**

The equation of a horizontal line is $y = b$, where b is the y-intercept of the line.

Example $y = -3$

The equation of a vertical line is $x = a$, where a is the x-intercept of the line.

Example $x = -2$

Parallel lines that are not vertical have equal slopes. Two nonvertical lines are perpendicular if the product of their slope is -1. Vertical and horizontal lines are always perpendicular to one another.

Example 5 Write Equations of Parallel or Perpendicular Lines

Write an equation in slope-intercept form for a line perpendicular to $y = -3x + 2$ containing $(4, 0)$.

The slope of $y = -3x + 2$ is -3, so the slope of a line perpendicular to it is $\dfrac{1}{3}$.

$y = mx + b$ Slope-Intercept form

$0 = \dfrac{1}{3}(4) + b$ $m = \dfrac{1}{3}$ and $(x, y) = (4, 0)$

$0 = \dfrac{4}{3} + b$ Simplify.

$-\dfrac{4}{3} = b$ Subtract $\dfrac{4}{3}$ from each side.

So, the equation is $y = \dfrac{1}{3}x + \left(-\dfrac{4}{3}\right)$ or $y = \dfrac{1}{3}x - 1\dfrac{1}{3}$.

▶ **Guided**Practice

5. Write an equation in slope-intercept form for a line parallel to $y = -\dfrac{3}{4}x + 3$ containing $(-3, 6)$.

2 Write Equations to Solve Problems

Many real-world situations can be modeled using a linear equation.

Real-World Example 6 Write Linear Equations

FINANCIAL LITERACY Benito's current wireless phone plan, Plan X, costs $39.95 per month for unlimited calls and $0.05 per text message. He is considering switching to Plan Y, which costs $35 per month for unlimited calls plus $0.10 for each text message. Which plan offers him the better rate?

Understand Plan X costs $39.95 per month plus $0.05 per text message. Plan Y costs $35 per month plus $0.10 per text message. You want to compare the two plans to determine when the cost of one plan is less than the other.
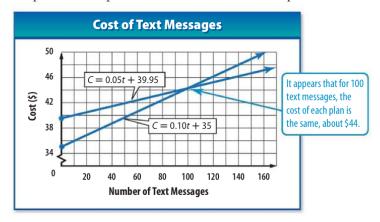
Plan Write an equation to model the total monthly cost C of each plan for t text messages sent or received. Then graph the equations in order to compare the two plans.

Solve The rates of increase, or slopes m, in the total costs are 0.05 for Plan X and 0.10 for Plan Y. When the number of text messages is 0, the total charge is just the monthly fee. So, the y-intercept b is 39.95 for Plan X and 35 for Plan Y.

Plan X		**Plan Y**
$C = mt + b$	Slope-intercept form	$C = mt + b$
$C = 0.05t + 39.95$	Substitute for m and b.	$C = 0.10t + 35$

Graph the two equations on the same coordinate plane.

Cost of Text Messages

$C = 0.05t + 39.95$

$C = 0.10t + 35$

It appears that for 100 text messages, the cost of each plan is the same, about $44.

From the graph, it appears that if Benito sends or receives less than about 100 text messages, Plan Y offers the lower rate. For more than 100 messages, Plan X is lower.

Check Check your estimate. For 100 text messages, Plan X costs $0.05(100) + 39.95$ or 44.95, and Plan Y costs $0.1(100) + 35$ or 45. Adjusting our estimate, we find that when the number of messages is 99, both plans cost $44.90. ✔

Guided Practice

6. Suppose the rate for Plan Y was $44 a month and $0.02 per text message. Which plan would offer Benito the better rate? Justify your answer.

Example 1　Write an equation in slope-intercept form of the line having the given slope and *y*-intercept. Then graph the line.

1. m: 4, y-intercept: -3　　　**2.** m: $\frac{1}{2}$, y-intercept: -1　　　**3.** m: $-\frac{2}{3}$, y-intercept: 5

Example 2　Write an equation in point-slope form of the line having the given slope that contains the given point. Then graph the line.

4. $m = 5$, $(3, -2)$　　　**5.** $m = \frac{1}{4}$, $(-2, -3)$　　　**6.** $m = -4.25$, $(-4, 6)$
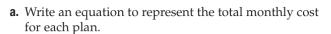
Examples 3–4　Write an equation of the line through each pair of points in slope-intercept form.

7.

x	*y*
0	−1
4	4

8.

x	*y*
4	3
1	−6

9.

x	*y*
6	5
−1	−4

Example 5　**10.** Write an equation in slope-intercept form for a line perpendicular to $y = -2x + 6$ containing $(3, 2)$.

11. Write an equation in slope-intercept form for a line parallel to $y = 4x - 5$ containing $(-1, 5)$.

Example 6　**12.** **CCSS MODELING** Kameko currently subscribes to Ace Music, an online music service, but she is considering switching to another online service, Orange Tunes. The plan for each online music service is shown.

ACE MUSIC — Subscription fee $5/mo — $0.79 per song downloaded

Subscription fee $10/mo — ORANGE TUNES — 40 downloads per month

a. Write an equation to represent the total monthly cost for each plan.

b. Graph the equations.

c. If Kameko downloads 15 songs per month, should she keep her current plan, or change to the other plan? Explain.

Example 1　Write an equation in slope-intercept form of the line having the given slope and *y*-intercept or points. Then graph the line.

13. m: -5, y-intercept: -2　　　**14.** m: -7, b: -4　　　**15.** m: 9, b: 2

16. m: 12, y-intercept: $\frac{4}{5}$　　　**17.** m: $-\frac{3}{4}$, $(0, 4)$　　　**18.** m: $\frac{5}{11}$, $(0, -3)$

Example 2　Write an equation in point-slope form of the line having the given slope that contains the given point. Then graph the line.

19. $m = 2$, $(3, 11)$　　　**20.** $m = 4$, $(-4, 8)$　　　**21.** $m = -7$, $(1, 9)$

22. $m = \frac{5}{7}$, $(-2, -5)$　　　**23.** $m = -\frac{4}{5}$, $(-3, -6)$　　　**24.** $m = -2.4$, $(14, -12)$

Examples 3–4　Write an equation of the line through each pair of points in slope-intercept form.
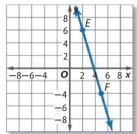
25. $(-1, -4)$ and $(3, -4)$　　　　**26.** $(2, -1)$ and $(2, 6)$

27. $(-3, -2)$ and $(-3, 4)$　　　　**28.** $(0, 5)$ and $(3, 3)$

29. $(-12, -6)$ and $(8, 9)$　　　　**30.** $(2, 4)$ and $(-4, -11)$

Write an equation in slope-intercept form for each line shown or described.

31.

32.

33.

x	−1	3
y	−2	4

34.

x	−4	−8
y	−5	−13

35. x-intercept $= 3$, y-intercept $= -2$

36. x-intercept $= -\frac{1}{2}$, y-intercept $= 4$

Example 5 **Write an equation in slope-intercept form for each line described.**

37. passes through $(-7, -4)$, perpendicular to $y = \frac{1}{2}x + 9$

38. passes through $(-1, -10)$, parallel to $y = 7$

39. passes through $(6, 2)$, parallel to $y = -\frac{2}{3}x + 1$

40. passes through $(-2, 2)$, perpendicular to $y = -5x - 8$
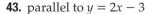
Example 6 **41** **PLANNING** Karen is planning a graduation party for the senior class. She plans to rent a meeting room at the convention center that costs $400. There is an additional fee of $5.50 for each person who attends the party.

    a. Write an equation to represent the cost y of the party if x people attend.

    b. Graph the equation.

    c. There are 285 people in Karen's class. If $\frac{2}{3}$ of these people attend, how much will the party cost?

    d. If the senior class has raised $2000 for the party, how many people can attend?

42. **CCSS MODELING** Victor is saving his money to buy a new satellite radio for his car. He wants to save enough money for the radio and one year of satellite radio service before he makes the purchase. He started saving for the radio with $50 that he got for his birthday. Since then, he has been adding $15 every week after he cashes his paycheck.

    a. Write an equation to represent Victor's savings y after x weeks.

    b. Graph the equation.

    c. How long will it take Victor to save $150?

    d. A satellite radio costs $180. Satellite radio service costs $10 per month. If Victor started saving two weeks ago, how much longer will it take him to save enough money? Explain.
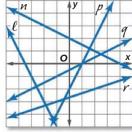
Name the line(s) on the graph shown that match each description.

43. parallel to $y = 2x - 3$

44. perpendicular to $y = \frac{1}{2}x + 7$

45. intersecting, but not perpendicular to $y = \frac{1}{2}x - 5$

Determine whether the lines are *parallel*, *perpendicular*, or *neither*.

46. $y = 2x + 4$, $y = 2x - 10$

47. $y = -\frac{1}{2}x - 12$, $y = 2x + 7$

48. $y - 4 = 3(x + 5)$, $y + 3 = -\frac{1}{3}(x + 1)$

49. $y - 3 = 6(x + 2)$, $y + 3 = -\frac{1}{3}(x - 4)$

50. Write an equation in slope-intercept form for a line containing (4, 2) that is parallel to the line $y - 2 = 3(x + 7)$.

51 Write an equation for a line containing $(-8, 12)$ that is perpendicular to the line containing the points $(3, 2)$ and $(-7, 2)$.

52. Write an equation in slope-intercept form for a line containing (5, 3) that is parallel to the line $y + 11 = \frac{1}{2}(4x + 6)$.

53. POTTERY A community center offers pottery classes. A $40 enrollment fee covers supplies and materials, including one bag of clay. Extra bags of clay cost $15 each. Write an equation to represent the cost of the class and x bags of clay.

54. MULTIPLE REPRESENTATIONS In Algebra 1, you learned that the solution of a system of two linear equations is an ordered pair that is a solution of both equations. Consider lines q, r, s, and t with the equations given.

line q: $y = 3x + 2$ line r: $y = 0.5x - 3$ line s: $2y = x - 6$ line t: $y = 3x - 3$

a. Tabular Make a table of values for each equation for $x = -3, -2, -1, 0, 1, 2,$ and 3. Which pairs of lines appear to represent a system of equations with one solution? no solution? infinitely many solutions? Use your tables to explain your reasoning.

b. Graphical Graph the equations on the same coordinate plane. Describe the geometric relationship between each pair of lines, including points of intersection.

c. Analytical How could you have determined your answers to part **a** using only the equations of the lines?

d. Verbal Explain how to determine whether a given system of two linear equations has one solution, no solution, or infinitely many solutions using a table, a graph, or the equations of the lines.

H.O.T. Problems   Use Higher-Order Thinking Skills

55. CHALLENGE Find the value of n so that the line perpendicular to the line with the equation $-2y + 4 = 6x + 8$ passes through the points at $(n, -4)$ and $(2, -8)$.

56. REASONING Determine whether the points at $(-2, 2)$, $(2, 5)$, and $(6, 8)$ are collinear. Justify your answer.

57. OPEN ENDED Write equations for two different pairs of perpendicular lines that intersect at the point at $(-3, -7)$.

58. CCSS CRITIQUE Mark and Josefina wrote an equation of a line with slope -5 that passes through the point $(-2, 4)$. Is either of them correct? Explain your reasoning.
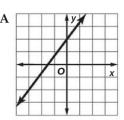
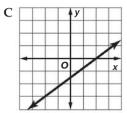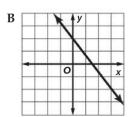| Mark	Josefina
$y - 4 = -5(x - (-2))$	$y - 4 = -5(x - (-2))$
$y - 4 = -5(x + 2)$	$y - 4 = -5(x + 2)$
$y - 4 = -5x - 10$	
$y = -5x - 6$	

59. WRITING IN MATH When is it easier to use the point-slope form to write an equation of a line and when is it easier to use the slope-intercept form?
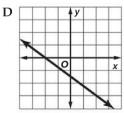
60. Which graph best represents a line passing through the point $(-2, -3)$?

A

C

B

D

61. Which equation describes the line that passes through the point at $(-2, 1)$ and is perpendicular to the line $y = \frac{1}{3}x + 5$?

F $y = 3x + 7$ **H** $y = -3x - 5$

G $y = \frac{1}{3}x + 7$ **J** $y = -\frac{1}{3}x - 5$

62. GRIDDED RESPONSE At Jefferson College, 80% of students have cell phones. Of the students who have cell phones, 70% have computers. What percent of the students at Jefferson College have both a cell phone and a computer?

63. SAT/ACT Which expression is equivalent to $4(x - 6) - \frac{1}{2}(x^2 + 8)$?

A $4x^2 + 4x - 28$ **D** $3x - 20$

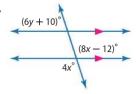**B** $-\frac{1}{2}x^2 + 4x - 20$ **E** $-\frac{1}{2}x^2 + 4x - 28$

C $-\frac{1}{2}x^2 + 6x - 24$
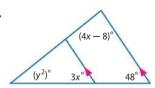
Spiral Review

Determine the slope of the line that contains the given points. (Lesson 3-3)

64. $J(4, 3), K(5, -2)$ **65.** $X(0, 2), Y(-3, -4)$ **66.** $A(2, 5), B(5, 1)$

Find x and y in each figure. (Lesson 3-2)

67.

68.

69. DRIVING Lacy's home is located at the midpoint between Newman's Gas Station and Gas-O-Rama. Newman's Gas Station is a quarter mile away from Lacy's home. How far away is Gas-O-Rama from Lacy's home? How far apart are the two gas stations? (Lesson 1-3)
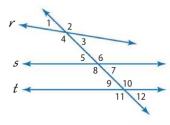
Skills Review

Determine the relationship between each pair of angles.

70. $\angle 1$ and $\angle 12$

71. $\angle 7$ and $\angle 10$

72. $\angle 4$ and $\angle 8$

73. $\angle 2$ and $\angle 11$

3-4

Geometry Lab
Equations of Perpendicular Bisectors

You can apply what you have learned about slope and equations of lines to geometric figures on a plane.
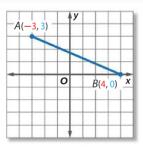
Activity

Find the equation of a line that is a perpendicular bisector of a segment AB with endpoints $A(-3, 3)$ and $B(4, 0)$.

Step 1 A segment bisector contains the midpoint of the segment. Use the Midpoint Formula to find the midpoint M of $\overline{AB}$.

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = M\left(\frac{-3 + 4}{2}, \frac{3 + 0}{2}\right)$$

$$= M\left(\frac{1}{2}, \frac{3}{2}\right)$$

Step 2 A perpendicular bisector is perpendicular to the segment through the midpoint. In order to find the slope of the bisector, first find the slope of $\overline{AB}$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad \text{Slope Formula}$$

$$= \frac{0 - 3}{4 - (-3)} \qquad x_1 = -3, x_2 = 4, y_1 = 3, y_2 = 0$$

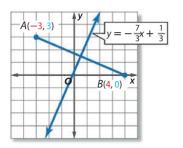$$= -\frac{3}{7} \qquad \text{Simplify.}$$

Step 3 Now use the point-slope form to write the equation of the line. The slope of the bisector is $\frac{7}{3}$ since $-\frac{3}{7}\left(\frac{7}{3}\right) = -1$

$$y - y_1 = m(x - x_1) \qquad \text{Point-slope form}$$

$$y - \frac{3}{2} = \frac{7}{3}\left(x - \frac{1}{2}\right) \qquad m = \frac{7}{3}, (x_1, y_1) = \left(\frac{1}{2}, \frac{3}{2}\right)$$

$$y - \frac{3}{2} = \frac{7}{3}x - \frac{7}{6} \qquad \text{Distributive Property}$$

$$y = \frac{7}{3}x + \frac{1}{3} \qquad \text{Add } \frac{3}{2} \text{ to each side.}$$

Exercises

Find the equation of a line that is the perpendicular bisector of $\overline{PQ}$ for the given endpoints.

1. $P(5, 2), Q(7, 4)$

2. $P(-3, 9), Q(-1, 5)$

3. $P(-6, -1), Q(8, 7)$

4. $P(-2, 1), Q(0, -3)$

5. $P(0, 1.6), Q(0.5, 2.1)$

6. $P(-7, 3), Q(5, 3)$

7. CHALLENGE Find the equations of the lines that contain the sides of $\triangle XYZ$ with vertices $X(-2, 0), Y(1, 3),$ and $Z(3, -1)$.

Proving Lines Parallel

- You used slopes to identify parallel and perpendicular lines.

1. Recognize angle pairs that occur with parallel lines.

2. Prove that two lines are parallel.

- When you see a roller coaster track, the two sides of the track are always the same distance apart, even though the track curves and turns. The tracks are carefully constructed to be parallel at all points so that the car is secure on the track.

Common Core State Standards

Content Standards

G.CO.9 Prove theorems about lines and angles.

G.CO.12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.).
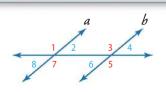
Mathematical Practices

1 Make sense of problems and persevere in solving them.

3 Construct viable arguments and critique the reasoning of others.

1 Identify Parallel Lines The two sides of the track of a roller coaster are parallel, and all of the supports along the track are also parallel. Each of the angles formed between the track and the supports are corresponding angles. We have learned that corresponding angles are congruent when lines are parallel. The converse of this relationship is also true.

Postulate 3.4 **Converse of Corresponding Angles Postulate**

If two lines are cut by a transversal so that corresponding angles are congruent, then the lines are parallel.

Examples If $\angle 1 \cong \angle 3$, $\angle 2 \cong \angle 4$, $\angle 5 \cong \angle 7$, $\angle 6 \cong \angle 8$, then $a \parallel b$.
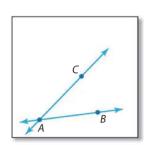

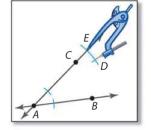The Converse of the Corresponding Angles Postulate can be used to construct parallel lines.

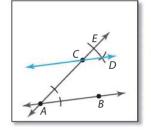**Construction** Parallel Line Through a Point Not on the Line

Step 1 Use a straightedge to draw $\overleftrightarrow{AB}$. Draw a point C that is not on $\overleftrightarrow{AB}$. Draw $\overleftrightarrow{CA}$.

Step 2 Copy $\angle CAB$ so that C is the vertex of the new angle. Label the intersection points D and E.

Step 3 Draw CD. Because $\angle ECD \cong \angle CAB$ by construction and they are corresponding angles, $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$.

The construction establishes that there is *at least* one line through C that is parallel to $\overleftrightarrow{AB}$. The following postulate guarantees that this line is the *only* one.

Postulate 3.5 Parallel Postulate

If given a line and a point not on the line, then there exists exactly one line through the point that is parallel to the given line.

Parallel lines that are cut by a transversal create several pairs of congruent angles. These special angle pairs can also be used to prove that a pair of lines are parallel.

Theorems Proving Lines Parallel

3.5 Alternate Exterior Angles Converse If two lines in a plane are cut by a transversal so that a pair of alternate exterior angles is congruent, then the two lines are parallel.	If $\angle 1 \cong \angle 3$, then $p \parallel q$.
3.6 Consecutive Interior Angles Converse If two lines in a plane are cut by a transversal so that a pair of consecutive interior angles is supplementary, then the lines are parallel.	If $m\angle 4 + m\angle 5 = 180$, then $p \parallel q$.
3.7 Alternate Interior Angles Converse If two lines in a plane are cut by a transversal so that a pair of alternate interior angles is congruent, then the lines are parallel.	If $\angle 6 \cong \angle 8$, then $p \parallel q$.
3.8 Perpendicular Transversal Converse In a plane, if two lines are perpendicular to the same line, then they are parallel.	If $p \perp r$ and $q \perp r$, then $p \parallel q$.

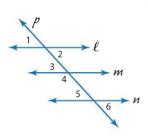You will prove Theorems 3.5, 3.6, 3.7, and 3.8 in Exercises 6, 23, 31, and 30, respectively.

Example 1 Identify Parallel Lines

Given the following information, determine which lines, if any, are parallel. State the postulate or theorem that justifies your answer.

a. $\angle 1 \cong \angle 6$

$\angle 1$ and $\angle 6$ are alternate exterior angles of lines ℓ and n.

Since $\angle 1 \cong \angle 6$, $\ell \parallel n$ by the Converse of the Alternate Exterior Angles Theorem.

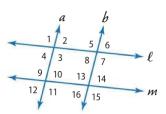**b.** $\angle 2 \cong \angle 3$

$\angle 2$ and $\angle 3$ are alternate interior angles of lines ℓ and m.

Since $\angle 2 \cong \angle 3$, $\ell \parallel m$ by the Converse of the Alternate Interior Angles Theorem.

GuidedPractice

1A. $\angle 2 \cong \angle 8$ **1B.** $\angle 3 \cong \angle 11$

1C. $\angle 12 \cong \angle 14$ **1D.** $\angle 1 \cong \angle 15$

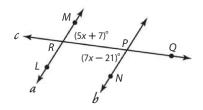**1E.** $m\angle 8 + m\angle 13 = 180$ **1F.** $\angle 8 \cong \angle 6$

Angle relationships can be used to solve problems involving unknown values.

Standardized Test Example 2   Use Angle Relationships

OPEN ENDED Find $m\angle MRQ$ so that $a \parallel b$.
Show your work.

Read the Test Item

From the figure, you know that $m\angle MRQ = 5x + 7$ and $m\angle RPN = 7x - 21$. You are asked to find the measure of $\angle MRQ$.

Solve the Test Item

$\angle MRQ$ and $\angle RPN$ are alternate interior angles. For lines a and b to be parallel, alternate interior angles must be congruent, so $\angle MRQ \cong \angle RPN$. By the definition of congruence, $m\angle MRQ = m\angle RPN$. Substitute the given angle measures into this equation and solve for x.

$m\angle MRQ = m\angle RPN$	Alternate interior angles
$5x + 7 = 7x - 21$	Substitution
$7 = 2x - 21$	Subtract 5x from each side.
$28 = 2x$	Add 21 to each side.
$14 = x$	Divide each side by 2.

Now, use the value of x to find $\angle MRQ$.

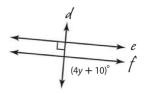|	
$m\angle MRQ = 5x + 7$	Substitution
$= 5(14) + 7$	$x = 14$
$= 77$	Simplify.

CHECK Check your answer by using the value of x to find $m\angle RPN$.

$m\angle RP = 7x - 21$
$= 7(14) - 21$ or 77 ✔

Since $m\angle MRQ = m\angle RPN$, $\angle MRQ \cong \angle RPN$ and $a \parallel b$. ✔

> **Study**Tip
>
> **Finding What Is Asked For**
> Be sure to reread test questions carefully to be sure you are answering the question that was asked. In Example 2, a common error would be to stop after you have found the value of x and say that the solution of the problem is 14.

GuidedPractice

2. Find y so that $e \parallel f$. Show your work.

2 Prove Lines Parallel The angle pair relationships formed by a transversal can be used to prove that two lines are parallel.

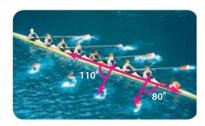**Real-World Example 3** Prove Lines Parallel

HOME FURNISHINGS In the ladder shown, each rung is perpendicular to the two rails. Is it possible to prove that the two rails are parallel and that all of the rungs are parallel? If so, explain how. If not, explain why not.

Since both rails are perpendicular to each rung, the rails are parallel by the Perpendicular Transversal Converse. Since any pair of rungs is perpendicular to the rails, they are also parallel.

GuidedPractice

3. ROWING In order to move in a straight line with maximum efficiency, rower's oars should be parallel. Refer to the photo at the right. Is it possible to prove that any of the oars are parallel? If so, explain how. If not, explain why not.
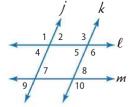
110°
80°

Check Your Understanding ● = Step-by-Step Solutions begin on page R14.
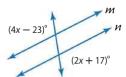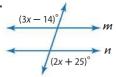
Example 1 Given the following information, determine which lines, if any, are parallel. State the postulate or theorem that justifies your answer.
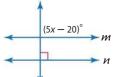
1. $\angle 1 \cong \angle 3$ **2.** $\angle 2 \cong \angle 5$

3 $\angle 3 \cong \angle 10$ **4.** $m\angle 6 + m\angle 8 = 180$

j k
1 2 3 ℓ
4 5 6
7 8 m
9 10

Example 2 **5. SHORT RESPONSE** Find x so that $m \parallel n$. Show your work.

m
n
$(4x - 23)°$
$(2x + 17)°$

Example 3 **6. PROOF** Copy and complete the proof of Theorem 3.5.

Given: $\angle 1 \cong \angle 2$
Prove: $\ell \parallel m$
Proof:

1 ℓ
3
2 m

Statements	Reasons
a. $\angle 1 \cong \angle 2$	**a.** Given
b. $\angle 2 \cong \angle 3$	**b.** ____?____
c. $\angle 1 \cong \angle 3$	**c.** Transitive Property
d. ____?____	**d.** ____?____

7. RECREATION Is it possible to prove that the backrest and footrest of the lounging beach chair are parallel? If so, explain how. If not, explain why not.

135°
135°

Practice and Problem Solving

Example 1 Given the following information, determine which lines, if any, are parallel. State the postulate or theorem that justifies your answer.

8. $\angle 1 \cong \angle 2$

9. $\angle 2 \cong \angle 9$

10. $\angle 5 \cong \angle 7$

11. $m\angle 7 + m\angle 8 = 180$

12. $m\angle 3 + m\angle 6 = 180$

13. $\angle 3 \cong \angle 5$

14. $\angle 3 \cong \angle 7$

15. $\angle 4 \cong \angle 5$

Example 2 Find x so that $m \parallel n$. Identify the postulate or theorem you used.

16.
$(3x - 14)°$
m
n
$(2x + 25)°$

17.
$(5x - 20)°$
m
n

18.
m n
$(21 + 2x)°$
$(x + 84)°$

19
m
$(7x - 2)°$
$(10 - 3x)°$
n

20.
m n
$(2x + 45)°$
$(3x)°$

21.
m n
$(6x - 144)°$
$(2x)°$
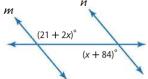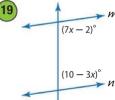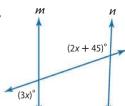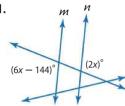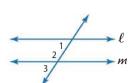
22. CCSS SENSE-MAKING Wooden picture frames are often constructed using a miter box or miter saw. These tools allow you to cut at an angle of a given size. If each of the four pieces of framing material is cut at a 45° angle, will the sides of the frame be parallel? Explain your reasoning.

Example 3 **23. PROOF** Copy and complete the proof of Theorem 3.6.
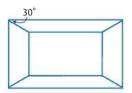
Given: $\angle 1$ and $\angle 2$ are supplementary.
Prove: $\ell \parallel m$

ℓ
1
2
3
m

Proof:

Statements	Reasons
a. _____?_____	**a.** Given
b. $\angle 2$ and $\angle 3$ form a linear pair.	**b.** _____?_____
c. _____?_____	**c.** _____?_____
d. $\angle 1 \cong \angle 3$	**d.** _____?_____
e. $\ell \parallel m$	**e.** _____?_____

24. CRAFTS Jacqui is making a stained glass piece. She cuts the top and bottom pieces at a 30° angle. If the corners are right angles, explain how Jacqui knows that each pair of opposite sides are parallel.
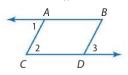
PROOF Write a two-column proof for each of the following.

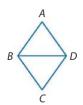**25. Given:** $\angle 1 \cong \angle 3$
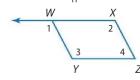
$\overline{AC} \parallel \overline{BD}$

Prove: $\overline{AB} \parallel \overline{CD}$

26. Given: $\overline{WX} \parallel \overline{YZ}$

$\angle 2 \cong \angle 3$

Prove: $\overline{WY} \parallel \overline{XZ}$

27. Given: $\angle ABC \cong \angle ADC$

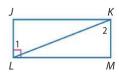$m\angle A + m\angle ABC = 180$

Prove: $\overline{AB} \parallel \overline{CD}$

28. Given: $\angle 1 \cong \angle 2$
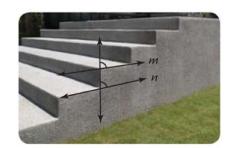
$\overline{LJ} \perp \overline{ML}$

Prove: $\overline{KM} \perp \overline{ML}$

29 MAILBOXES Mail slots are used to make the organization and distribution of mail easier. In the mail slots shown, each slot is perpendicular to each of the sides. Explain why you can conclude that the slots are parallel.
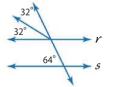
30. PROOF Write a paragraph proof of Theorem 3.8.

31. PROOF Write a two-column proof of Theorem 3.7.

32. CCSS REASONING Based upon the information given in the photo of the staircase at the right, what is the relationship between each step? Explain your answer.
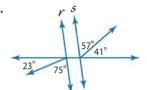
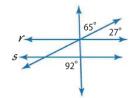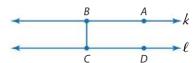Determine whether lines r and s are parallel. Justify your answer.

33.

34.

35.

36. **MULTIPLE REPRESENTATIONS** In this problem, you will explore the shortest distance between two parallel lines.

a. Geometric Draw three sets of parallel lines k and ℓ, s and t, and x and y. For each set, draw the shortest segment $\overline{BC}$ and label points A and D as shown below.

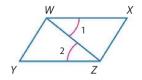**b. Tabular** Copy the table below, measure $\angle ABC$ and $\angle BCD$, and complete the table.

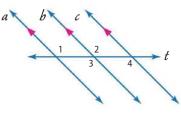| Set of Parallel Lines	$m\angle ABC$	$m\angle BCD$
k and ℓ		
s and t		
x and y		

c. Verbal Make a conjecture about the angle the shortest segment forms with both parallel lines.

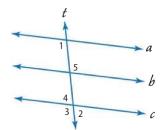**H.O.T. Problems**   Use Higher-Order Thinking Skills

37. **ERROR ANALYSIS** Sumi and Daniela are determining which lines are parallel in the figure at the right. Sumi says that since $\angle 1 \cong \angle 2$, $\overline{WY} \parallel \overline{XZ}$. Daniela disagrees and says that since $\angle 1 \cong \angle 2$, $\overline{WX} \parallel \overline{YZ}$. Is either of them correct? Explain.

38. **CCSS REASONING** Is Theorem 3.8 still true if the two lines are not coplanar? Draw a figure to justify your answer.

39. **CHALLENGE** Use the figure at the right to prove that two lines parallel to a third line are parallel to each other.

40. **OPEN ENDED** Draw a triangle ABC.

a. Construct the line parallel to $\overline{BC}$ through point A.

b. Use measurement to justify that the line you constructed is parallel to $\overline{BC}$.

c. Use mathematics to justify this construction.

41. **CHALLENGE** Refer to the figure at the right.

a. If $m\angle 1 + m\angle 2 = 180$, prove that $a \parallel c$.

b. Given that $a \parallel c$, if $m\angle 1 + m\angle 3 = 180$, prove that $t \perp c$.

42. **WRITING IN MATH** Summarize the five methods used in this lesson to prove that two lines are parallel.
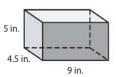
43. **WRITING IN MATH** Can a pair of angles be supplementary and congruent? Explain your reasoning.

44. Which of the following facts would be sufficient to prove that line d is parallel to $\overline{XZ}$?

A $\angle 1 \cong \angle 3$ **C** $\angle 1 \cong \angle Z$

B $\angle 3 \cong \angle Z$ **D** $\angle 2 \cong \angle X$

45. ALGEBRA The expression $\sqrt{52} + \sqrt{117}$ is equivalent to

F 13 **H** $6\sqrt{13}$

G $5\sqrt{13}$ **J** $13\sqrt{13}$

46. What is the approximate surface area of the figure?

A 101.3 in^2 **C** 202.5 in^2

B 108 in^2 **D** 216 in^2

47. SAT/ACT If $x^2 = 25$ and $y^2 = 9$, what is the greatest possible value of $(x - y)^2$?

F 4 **J** 64

G 16 **K** 70

H 58

Write an equation in slope-intercept form of the line having the given slope and y-intercept. (Lesson 3-4)

48. m: 2.5, y-intercept: 0.5

49. m: $\frac{4}{5}$, y-intercept: -9

50. m: $-\frac{7}{8}$, y-intercept: $-\frac{5}{6}$

51. ROAD TRIP Anne is driving 400 miles to visit Niagara Falls. She manages to travel the first 100 miles of her trip in two hours. If she continues at this rate, how long will it take her to drive the remaining distance? (Lesson 3-3)

Find a counterexample to show that each conjecture is false. (Lesson 2-1)

52. Given: $\angle 1$ and $\angle 2$ are complementary angles.

Conjecture: $\angle 1$ and $\angle 2$ form a right angle.
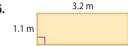
53. Given: points W, X, Y, and Z

Conjecture: W, X, Y, and Z are noncollinear.

Find the perimeter or circumference and area of each figure. Round to the nearest tenth. (Lesson 1-6)

54.

55.

56.

57. Find x and y so that $\overline{BE}$ and $\overline{AD}$ are perpendicular.

Perpendiculars and Distance

- You proved that two lines are parallel using angle relationships.

1 Find the distance between a point and a line.

2 Find the distance between parallel lines.

- A *plumb bob* is made of string with a specially designed weight. When the weight is suspended and allowed to swing freely, the point of the bob is precisely below the point to which the string is fixed.

  The plumb bob is useful in establishing what is the true vertical or *plumb* when constructing a wall or when hanging wallpaper.

NewVocabulary

equidistant

Common Core State Standards

Content Standards

G.CO.12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.).

G.MG.3 Apply geometric methods to solve problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios). ★

Mathematical Practices

2 Reason abstractly and quantitatively.

4 Model with mathematics.

1 **Distance From a Point to a Line** The plumb bob also indicates the shortest distance between the point at which it is attached on the ceiling and a level floor below. This perpendicular distance between a point and a line is the shortest in all cases.
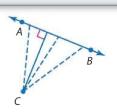
KeyConcept **Distance Between a Point and a Line**

Words: The distance between a line and a point not on the line is the length of the segment perpendicular to the line from the point.
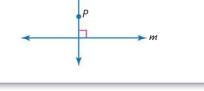
Model

The construction of a line perpendicular to an existing line through a point not on the existing line in Extend Lesson 1-5 establishes that there is *at least one* line through a point P that is perpendicular to a line AB. The following postulate states that this line is the *only* line through P perpendicular to $\overleftrightarrow{AB}$.

Postulate 3.6 **Perpendicular Postulate**

Words: If given a line and a point not on the line, then there exists exactly one line through the point that is perpendicular to the given line.

Model

LANDSCAPING A landscape architect notices that one part of a yard does not drain well. She wants to tap into an existing underground drain represented by line *m*. Construct and name the segment with the length that represents the shortest amount of pipe she will need to lay to connect this drain to point *A*.

The distance from a line to a point not on the line is the length of the segment perpendicular to the line from the point. Locate points *B* and *C* on line *m* equidistant from point *A*.

Locate a third point on line *m* equidistant from *B* and *C*. Label this point *D*. Then draw $\overleftrightarrow{AD}$ so that $\overleftrightarrow{AD} \perp \overleftrightarrow{BC}$.

The measure of $\overline{AD}$ represents the shortest amount of pipe the architect will need to lay to connect the drain to point *A*.

▶ **Guided**Practice

1. Copy the figure. Then construct and name the segment that represents the distance from *Q* to $\overleftrightarrow{PR}$.
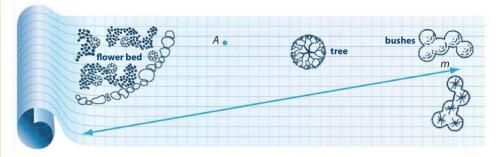

Real-World Career

Landscape Architect
Landscape architects enjoy working with their hands and possess strong analytical skills. Creative vision and artistic talent are also desirable qualities. Typically, a bachelor's degree is required of landscape architects, but a master's degree may be required for specializations such as golf course design.
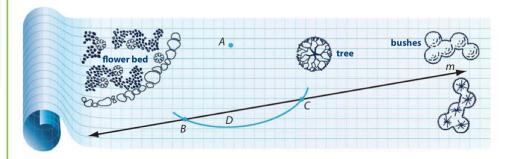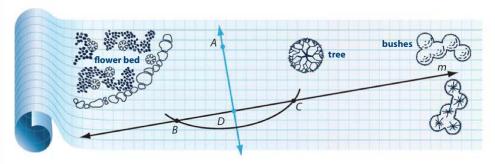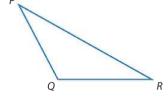
StudyTip

Drawing the Shortest Distance You can use tools like the corner of a piece of paper to help you draw a perpendicular segment from a point to a line, but only a compass and a straightedge can be used to construct this segment.

Ocean/Corbis

Example 2 Distance from a Point to a Line on Coordinate Plane

COORDINATE GEOMETRY Line ℓ contains points at $(-5, 3)$ and $(4, -6)$. Find the distance between line ℓ and point $P(2, 4)$.

Step 1 Find the equation of the line ℓ.

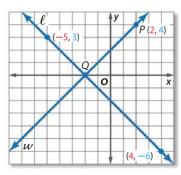Begin by finding the slope of the line through points $(-5, 3)$ and $(4, -6)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-6 - 3}{4 - (-5)} = \frac{-9}{9} \text{ or } -1$$

Then write the equation of this line using the point $(4, -6)$ on the line.

$y = mx + b$	Slope-intercept form
$-6 = -1(4) + b$	$m = -1, (x, y) = (4, -6)$
$-6 = -4 + b$	Simplify.
$-2 = b$	Add 4 to each side.

The equation of line ℓ is $y = -x + (-2)$ or $y = -x - 2$.

Step 2 Write an equation of the line w perpendicular to line ℓ through $P(2, 4)$.

Since the slope of line ℓ is -1, the slope of a line p is 1. Write the equation of line w through $P(2, 4)$ with slope 1.

$y = mx + b$	Slope-intercept form
$4 = 1(2) + b$	$m = -1, (x, y) = (2, 4)$
$4 = 2 + b$	Simplify.
$2 = b$	Subtract 2 from each side.

The equation of line w is $y = x + 2$.

Step 3 Solve the system of equations to determine the point of intersection.

line ℓ: $y = -x - 2$

line w: $(+) \; y = x + 2$

$2y = 0$	Add the two equations.
$y = 0$	Divide each side by 2.

Solve for x.

$0 = x + 2$	Substitute 0 for *y* in the second equation.
$-2 = x$	Subtract 2 from each side.

The point of intersection is $(-2, 0)$. Let this be point Q.

Step 4 Use the Distance Formula to determine the distance between $P(2, 4)$ and $Q(-2, 0)$.

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	Distance formula
$= \sqrt{(-2 - 2)^2 + (0 - 4)^2}$	$x_2 = -2, x_1 = 2, y_2 = 0, y_1 = 4$
$= \sqrt{32}$	Simplify.

The distance between the point and the line is $\sqrt{32}$ or about 5.66 units.
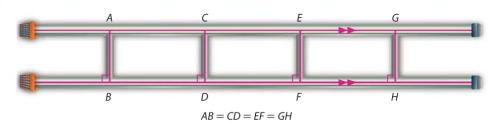
2. Line ℓ contains points at $(1, 2)$ and $(5, 4)$. Construct a line perpendicular to ℓ through $P(1, 7)$. Then find the distance from P to ℓ.

2 **Distance Between Parallel Lines** By definition, parallel lines do not intersect. An alternate definition states that two lines in a plane are parallel if they are everywhere **equidistant**. Equidistant means that the distance between two lines measured along a perpendicular line to the lines is always the same.
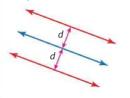
$$AB = CD = EF = GH$$

This leads to the definition of the distance between two parallel lines.

KeyConcept Distance Between Parallel Lines

The distance between two parallel lines is the perpendicular distance between one of the lines and any point on the other line.
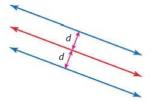

Recall from Lesson 1-1 that a *locus* is the set of all points that satisfy a given condition. Parallel lines can be described as the locus of points in a plane equidistant from a given line.

Theorem3.9 Two Lines Equidistant from a Third

In a plane, if two lines are each equidistant from a third line, then the two lines are parallel to each other.

You will prove Theorem 3.9 in Exercise 30.
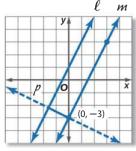
Example 3 Distance Between Parallel Lines

Find the distance between the parallel lines ℓ and m with equations $y = 2x + 1$ and $y = 2x - 3$, respectively.

You will need to solve a system of equations to find the endpoints of a segment that is perpendicular to both ℓ and m. From their equations, we know that the slope of line ℓ and line m is 2.

Sketch line p through the y-intercept of line m, $(0, -3)$, perpendicular to lines m and ℓ.

Step 1 Write an equation of line p. The slope of p is the opposite reciprocal of 2, or $-\frac{1}{2}$. Use the y-intercept of line m, $(0, -3)$, as one of the endpoints of the perpendicular segment.

$$(y - y_1) = m(x - x_1) \qquad \text{Point-slope form}$$

$$[y - (-3)] = -\frac{1}{2}(x - 0) \qquad x_1 = 0, y_1 = 3, \text{ and } m = -\frac{1}{2}$$

$$y + 3 = -\frac{1}{2}x \qquad \text{Simplify.}$$

$$y = -\frac{1}{2}x - 3 \qquad \text{Subtract 3 from each side.}$$

Step 2 Use a system of equations to determine the point of intersection of lines ℓ and p.

$$\ell: \ y = 2x + 1$$

$$p: \ y = -\frac{1}{2}x - 3$$

$$2x + 1 = -\frac{1}{2}x - 3 \qquad \text{Substitute } 2x + 1 \text{ for } y \text{ in the second equation.}$$

$$2x + \frac{1}{2}x = -3 - 1 \qquad \text{Group like terms on each side.}$$

$$\frac{5}{2}x = -4 \qquad \text{Simplify on each side.}$$

$$x = -\frac{8}{5} \qquad \text{Multiply each side by } \frac{2}{5}.$$

$$y = -\frac{1}{2}\left(-\frac{8}{5}\right) - 3 \qquad \text{Substitute } -\frac{8}{5} \text{ for } x \text{ in the equation for } p.$$

$$= -\frac{11}{5} \qquad \text{Simplify.}$$

The point of intersection is $\left(-\frac{8}{5}, -\frac{11}{5}\right)$ or $(-1.6, -2.2)$.

Step 3 Use the Distance Formula to determine the distance between $(0, -3)$ and $(-1.6, -2.2)$.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad \text{Distance Formula}$$

$$= \sqrt{(-1.6 - 0)^2 + [-2.2 - (-3)]^2} \qquad x_2 = -1.6, x_1 = 0, y_2 = -2.2, \text{ and } y_1 = -3$$

$$\approx 1.8 \qquad \text{Simplify using a calculator.}$$

The distance between the lines is about 1.8 units.

> **Study**Tip
>
> **Substitution Method** To review **solving systems of equations** using the substitution method, see p. P17.

▶ **Guided**Practice
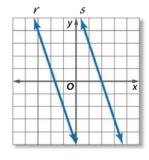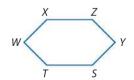
3A. Find the distance between the parallel lines r and s whose equations are $y = -3x - 5$ and $y = -3x + 6$, respectively.

3B. Find the distance between parallel lines a and b with equations $x + 3y = 6$ and $x + 3y = -14$, respectively.

Example 1   Copy each figure. Construct the segment that represents the distance indicated.
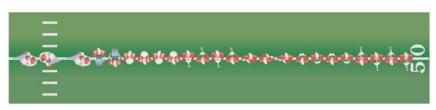
1. Y to $\overleftrightarrow{TS}$

2. C to $\overleftrightarrow{AB}$

3. **CCSS STRUCTURE** After forming a line, every even member of a marching band turns to face the home team's end zone and marches 5 paces straight forward. At the same time, every odd member turns in the opposite direction and marches 5 paces straight forward. Assuming that each band member covers the same distance, what formation should result? Justify your answer.

Example 2   **COORDINATE GEOMETRY** Find the distance from P to ℓ.

4. Line ℓ contains points $(4, 3)$ and $(-2, 0)$. Point P has coordinates $(3, 10)$.

5. Line ℓ contains points $(-6, 1)$ and $(9, -4)$. Point P has coordinates $(4, 1)$.

6. Line ℓ contains points $(4, 18)$ and $(-2, 9)$. Point P has coordinates $(-9, 5)$.

Example 3   Find the distance between each pair of parallel lines with the given equations.
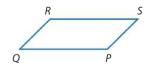
7 $y = -2x + 4$
$y = -2x + 14$

8. $y = 7$
$y = -3$

Example 1   Copy each figure. Construct the segment that represents the distance indicated.
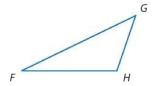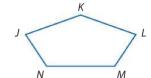
9. Q to $\overline{RS}$

10. A to $\overline{BC}$
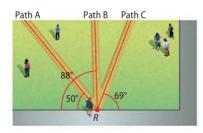
11. H to $\overline{FG}$

12. K to $\overline{LM}$

13. DRIVEWAYS In the diagram at the right, is the driveway shown the shortest possible one from the house to the road? Explain why or why not.

14. CCSS MODELING Rondell is crossing the courtyard in front of his school. Three possible paths are shown in the diagram at the right. Which of the three paths shown is the shortest? Explain your reasoning.

Example 2   **COORDINATE GEOMETRY** Find the distance from P to ℓ.

15 Line ℓ contains points $(0, -3)$ and $(7, 4)$. Point P has coordinates $(4, 3)$.

16. Line ℓ contains points $(11, -1)$ and $(-3, -11)$. Point P has coordinates $(-1, 1)$.

17. Line ℓ contains points $(-2, 1)$ and $(4, 1)$. Point P has coordinates $(5, 7)$.

18. Line ℓ contains points $(4, -1)$ and $(4, 9)$. Point P has coordinates $(1, 6)$.

19. Line ℓ contains points $(1, 5)$ and $(4, -4)$. Point P has coordinates $(-1, 1)$.

20. Line ℓ contains points $(-8, 1)$ and $(3, 1)$. Point P has coordinates $(-2, 4)$.

Example 3   **Find the distance between each pair of parallel lines with the given equations.**

21. $y = -2$
$y = 4$

22. $x = 3$
$x = 7$

23. $y = 5x - 22$
$y = 5x + 4$

24. $y = \frac{1}{3}x - 3$
$y = \frac{1}{3}x + 2$

25. $x = 8.5$
$x = -12.5$

26. $y = 15$
$y = -4$

27. $y = \frac{1}{4}x + 2$
$4y - x = -60$

28. $3x + y = 3$
$y + 17 = -3x$

29. $y = -\frac{5}{4}x + 3.5$
$4y + 10.6 = -5x$

30. PROOF Write a two-column proof of Theorem 3.9.

Find the distance from the line to the given point.

31. $y = -3$, $(5, 2)$

32. $y = \frac{1}{6}x + 6$, $(-6, 5)$

33. $x = 4$, $(-2, 5)$

34. POSTERS Alma is hanging two posters on the wall in her room as shown. How can Alma use perpendicular distances to confirm that the posters are parallel?

35 **SCHOOL SPIRIT** Brock is decorating a hallway bulletin board to display pictures of students demonstrating school spirit. He cuts off one length of border to match the width of the top of the board, and then uses that strip as a template to cut a second strip that is exactly the same length for the bottom.
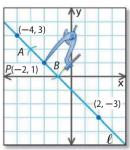
When stapling the bottom border in place, he notices that the strip he cut is about a quarter of an inch too short. Describe what he can conclude about the bulletin board. Explain your reasoning.

DON'T MESS WITH THE BEST!

CONSTRUCTION Line ℓ contains points at $(-4, 3)$ and $(2, -3)$. Point P at $(-2, 1)$ is on line ℓ. Complete the following construction.
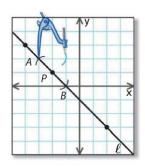
Step 1

Graph line ℓ and point P, and put the compass at point P. Using the same compass setting, draw arcs to the left and right of P. Label these points A and B.
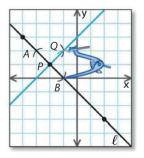
Step 2

Open the compass to a setting greater than AP. Put the compass at point A and draw an arc above line ℓ.

Step 3

Using the same compass setting, put the compass at point B and draw an arc above line ℓ. Label the point of intersection Q. Then draw $\overleftrightarrow{PQ}$.
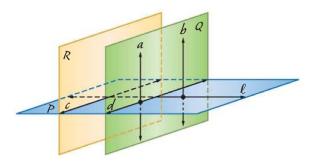
36. What is the relationship between line ℓ and $\overleftrightarrow{PQ}$? Verify your conjecture using the slopes of the two lines.

37. Repeat the construction above using a different line and point on that line.

38. **CCSS** **SENSE-MAKING** $\overline{AB}$ has a slope of 2 and midpoint $M(3, 2)$. A segment perpendicular to $\overline{AB}$ has midpoint $P(4, -1)$ and shares endpoint B with $\overline{AB}$.

  a. Graph the segments.

  b. Find the coordinates of A and B.

39. **MULTIPLE REPRESENTATIONS** In this problem, you will explore the areas of triangles formed by points on parallel lines.

  a. **Geometric** Draw two parallel lines and label them as shown.

  b. **Verbal** Where would you place point C on line m to ensure that triangle ABC would have the largest area? Explain your reasoning.

  c. **Analytical** If $AB = 11$ inches, what is the maximum area of $\triangle ABC$?

40. PERPENDICULARITY AND PLANES Make a copy of the diagram below to answer each question, marking the diagram with the given information.

a. If two lines are perpendicular to the same plane, then they are coplanar. If both line a and line b are perpendicular to plane P, what must also be true?

b. If a plane intersects two parallel planes, then the intersections form two parallel lines. If planes R and Q are parallel and they intersect plane P, what must also be true?

c. If two planes are perpendicular to the same line, then they are parallel. If both plane Q and plane R are perpendicular to line ℓ, what must also be true?

H.O.T. Problems   Use Higher-Order Thinking Skills

41 ERROR ANALYSIS Han draws the segments $\overline{AB}$ and $\overline{CD}$ shown below using a straightedge. He claims that these two lines, if extended, will never intersect. Shenequa claims that they will. Is either of them correct? Justify your answer.

42. CHALLENGE Describe the locus of points that are equidistant from two intersecting lines, and sketch an example.

43. CHALLENGE Suppose a line perpendicular to a pair of parallel lines intersects the lines at the points $(a, 4)$ and $(0, 6)$. If the distance between the parallel lines is $\sqrt{5}$, find the value of a and the equations of the parallel lines.

44. REASONING Determine whether the following statement is *sometimes, always,* or *never* true. Explain.

The distance between a line and a plane can be found.

45. OPEN ENDED Draw an irregular convex pentagon using a straightedge.

a. Use a compass and straightedge to construct a line between one vertex and a side opposite the vertex.

b. Use measurement to justify that the line constructed is perpendicular to the side chosen.

c. Use mathematics to justify this conclusion.

46. CCSS SENSE-MAKING Rewrite Theorem 3.9 in terms of two planes that are equidistant from a third plane. Sketch an example.

47. WRITING IN MATH Summarize the steps necessary to find the distance between a pair of parallel lines given the equations of the two lines.

48. EXTENDED RESPONSE Segment AB is perpendicular to segment CD. Segment AB and segment CD bisect each other at point X.

   a. Draw a figure to represent the problem.

   b. Find $\overline{BD}$ if $AB = 12$ and $CD = 16$.

   c. Find $\overline{BD}$ if $AB = 24$ and $CD = 18$.

49. A city park is square and has an area of 81,000 square feet. Which of the following is the closest to the length of one side of the park?

   A 100 ft   **C** 300 ft

   B 200 ft   **D** 400 ft

50. ALGEBRA Pablo bought a sweater on sale for 25% off the original price and another 40% off the discounted price. If the sweater originally cost $48, what was the final price of the sweater?

   F $14.40   **H** $31.20

   G $21.60   **J** $36.00

51. SAT/ACT After N cookies are divided equally among 8 children, 3 remain. How many would remain if $(N + 6)$ cookies were divided equally among the 8 children?
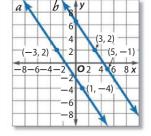
   A 0   **C** 2   **E** 6

   B 1   **D** 4

52. Refer to the figure at the right. Determine whether $a \parallel b$. Justify your answer. (Lesson 3-5)
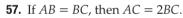

Write an equation in point-slope form of the line having the given slope that contains the given point. (Lesson 3-4)

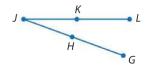**53.** $m: \dfrac{1}{4}, (3, -1)$

54. $m: 0, (-2, 6)$

55. $m: -1, (-2, 3)$

56. $m: -2, (-6, -7)$

Prove the following. (Lesson 2-7)
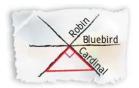
57. If $AB = BC$, then $AC = 2BC$.

58. Given: $\overline{JK} \cong \overline{KL}, \overline{HJ} \cong \overline{GH}, \overline{KL} \cong \overline{HJ}$

   Prove: $\overline{GH} \cong \overline{JK}$

59. MAPS Darnell sketched a map for his friend of the cross streets nearest to his home. Describe two different angle relationships between the streets. (Lesson 1-5)

Use the Distance Formula to find the distance between each pair of points.

60. $A(0, 0), B(15, 20)$

61. $O(-12, 0), P(-8, 3)$

62. $C(11, -12), D(6, 2)$

63. $R(-2, 3), S(3, 15)$

64. $M(1, -2), N(9, 13)$

65. $Q(-12, 2), T(-9, 6)$

Study Guide and Review

Study Guide

KeyConcepts

Transversals (Lessons 3-1 and 3-2)

- When a transversal intersects two lines, the following types of angles are formed: exterior, interior, consecutive interior, alternate interior, alternate exterior, and corresponding.
- If two parallel lines are cut by a transversal, then:
  - each pair of corresponding angles is congruent,
  - each pair of alternate interior angles is congruent,
  - each pair of consecutive interior angles is supplementary, and
  - each pair of alternate exterior angles is congruent.

Slope (Lessons 3-3 and 3-4)

- The slope m of a line containing two points with coordinates (x_1, y_1) and (x_2, y_2) is $m = \dfrac{y_2 - y_1}{x_2 - x_1}$, where $x_1 \neq x_2$.

Proving Lines Parallel (Lesson 3-5)

- If two lines in a plane are cut by a transversal so that any one of the following is true, then the two lines are parallel:
  - a pair of corresponding angles is congruent,
  - a pair of alternate exterior angles is congruent,
  - a pair of alternate interior angles is congruent, or
  - a pair of consecutive interior angles is supplementary.
- In a plane, if two lines are perpendicular to the same line, then they are parallel.

Distance (Lesson 3-6)

- The distance from a line to a point not on the line is the length of the segment perpendicular to the line from the point.
- The distance between two parallel lines is the perpendicular distance between one of the lines and any point on the other line.

FOLDABLES StudyOrganizer

Be sure the Key Concepts are noted in your Foldable.

KeyVocabulary

alternate exterior angles (p. 174)

alternate interior angles (p. 174)

consecutive interior angles (p. 174)

corresponding angles (p. 174)

equidistant (p. 218)

parallel lines (p. 173)

parallel planes (p. 173)

point-slope form (p. 198)

rate of change (p. 189)
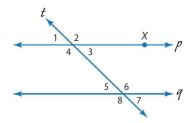
skew lines (p. 173)

slope (p. 188)

slope-intercept form (p. 198)

transversal (p. 174)

VocabularyCheck

State whether each sentence is *true* or *false*. If *false*, replace the underlined word or number to make a true sentence.

1. If $\angle 1 \cong \angle 5$, then lines p and q are <u>skew</u> lines.

2. Angles 4 and 6 are <u>alternate</u> interior angles.

3. Angles 1 and 7 are alternate <u>exterior</u> angles.

4. If lines p and q are parallel, then angles 3 and 6 are <u>congruent</u>.

5. The distance from point X to line q is the length of the segment <u>perpendicular</u> to line q from X.

6. Line t is called the <u>transversal</u> for lines p and q.

7. If $p \parallel q$, then $\angle 2$ and $\angle 8$ are <u>supplementary</u>.
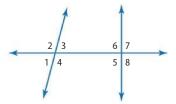
8. Angles 4 and 8 are <u>corresponding</u> angles.

Lesson-by-Lesson Review
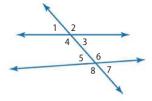
3-1 Parallel Lines and Transversals

Classify the relationship between each pair of angles as *alternate interior, alternate exterior, corresponding,* or *consecutive interior* angles.

9. $\angle 1$ and $\angle 5$ **10.** $\angle 4$ and $\angle 6$

11. $\angle 2$ and $\angle 8$ **12.** $\angle 4$ and $\angle 5$

13. BRIDGES The Roebling Suspension Bridge extends over the Ohio River connecting Cincinnati, Ohio, to Covington, Kentucky. Describe the type of lines formed by the bridge and the river.
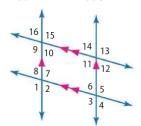
Example 1

Refer to the figure below. Classify the relationship between each pair of angles as *alternate interior, alternate exterior, corresponding,* or *consecutive interior* angles.

a. $\angle 3$ and $\angle 6$
consecutive interior

b. $\angle 2$ and $\angle 6$
corresponding

c. $\angle 1$ and $\angle 7$
alternate exterior

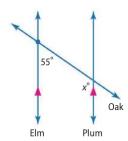**d.** $\angle 3$ and $\angle 5$
alternate interior

3-2 Angles and Parallel Lines

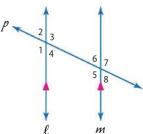In the figure, $m\angle 1 = 123$. Find the measure of each angle. Tell which postulate(s) or theorem(s) you used.

14. $\angle 5$ **15.** $\angle 14$ **16.** $\angle 16$

17. $\angle 11$ **18.** $\angle 4$ **19.** $\angle 6$

20. MAPS The diagram shows the layout of Elm, Plum, and Oak streets. Find the value of x.

Example 2

ALGEBRA If $m\angle 5 = 7x - 5$ and $m\angle 4 = 2x + 23$, find x. Explain your reasoning.

$$m\angle 4 + m\angle 5 = 180 \qquad \text{Def. of Supp. } \angle s$$
$$(2x + 23) + (7x - 5) = 180 \qquad \text{Substitution}$$
$$9x + 18 = 180 \qquad \text{Simplify.}$$
$$9x = 162 \qquad \text{Subtract.}$$
$$x = 18 \qquad \text{Divide.}$$

Since lines ℓ and m are parallel, $\angle 4$ and $\angle 5$ are supplementary by the Consecutive Interior Angles Theorem.

3-3 Slopes of Lines

Determine whether $\overleftrightarrow{AB}$ and $\overleftrightarrow{XY}$ are *parallel, perpendicular,* or *neither.* Graph each line to verify your answer.

21. $A(5, 3)$, $B(8, 0)$, $X(-7, 2)$, $Y(1, 10)$

22. $A(-3, 9)$, $B(0, 7)$, $X(4, 13)$, $Y(-5, 7)$

23. $A(8, 1)$, $B(-2, 7)$, $X(-6, 2)$, $Y(-1, -1)$

Graph the line that satisfies each condition.

24. contains $(-3, 4)$ and is parallel to $\overleftrightarrow{AB}$ with $A(2, 5)$ and $B(9, 2)$

25. contains $(1, 3)$ and is perpendicular to $\overleftrightarrow{PQ}$ with $P(4, -6)$ and $Q(6, -1)$

26. **AIRPLANES** Two Oceanic Airlines planes are flying at the same altitude. Using satellite imagery, each plane's position can be mapped onto a coordinate plane. Flight 815 was mapped at (23, 17) and (5, 11) while Flight 44 was mapped at (3, 15) and (9, 17). Determine whether their paths are *parallel, perpendicular,* or *neither.*
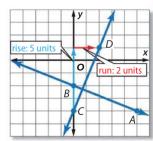
Example 3

Graph the line that contains $C(0, -4)$ and is perpendicular to $\overleftrightarrow{AB}$ with $A(5, -4)$ and $B(0, -2)$.

The slope of $\overleftrightarrow{AB}$ is $\dfrac{-2 - (-4)}{0 - 5}$ or $-\dfrac{2}{5}$.

Since $-\dfrac{2}{5}\left(\dfrac{5}{2}\right) = -1$, the slope of the line perpendicular to $\overleftrightarrow{AB}$ through C is $\dfrac{5}{2}$.

To graph the line, start at C. Move up 5 units and then right 2 units. Label the point D and draw $\overleftrightarrow{CD}$.

3-4 Equations of Lines

Write an equation in point-slope form of the line having the given slope that contains the given point.

27. $m = 2$, $(4, -9)$

28. $m = -\dfrac{3}{4}$, $(8, -1)$

Write an equation in slope-intercept form of the line having the given slope and *y*-intercept.

29. m: 5, *y*-intercept: -3

30. m: $\dfrac{1}{2}$, *y*-intercept: 4

Write an equation in slope-intercept form for each line.

31. $(-3, 12)$ and $(15, 0)$ **32.** $(-7, 2)$ and $(5, 8)$

33. **WINDOW CLEANING** Ace Window Cleaning Service charges $50 for the service call and $20 for each hour spent on the job. Write an equation in slope-intercept form that represents the total cost C in terms of the number of hours h.

Example 4

Write an equation of the line through (2, 5) and (6, 3) in slope-intercept form.

Step 1 Find the slope of the line through the points.

$m = \dfrac{y_2 - y_1}{x_2 - x_1}$ Slope Formula

$= \dfrac{3 - 5}{6 - 2}$ $x_1 = 2$, $y_1 = 5$, $x_2 = 6$, and $y_2 = 3$

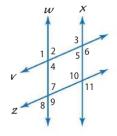$= \dfrac{-2}{4}$ or $-\dfrac{1}{2}$ Simplify.

Step 2 Write an equation of the line.

$y - y_1 = m(x - x_1)$ Point-slope form

$y - 5 = -\dfrac{1}{2}[x - (2)]$ $m = -\dfrac{1}{2}$, $(x_1, y_1) = (2, 5)$

$y - 5 = -\dfrac{1}{2}x + 1$ Simplify.

$y = -\dfrac{1}{2}x + 6$ Add 5 to each side.

3-5 Proving Lines Parallel
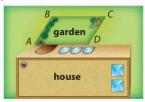
Given the following information, determine which lines, if any, are parallel. State the postulate or theorem that justifies your answer.

34. $\angle 7 \cong \angle 10$

35. $\angle 2 \cong \angle 10$

36. $\angle 1 \cong \angle 3$

37. $\angle 3 \cong \angle 11$

38. Find x so that $p \parallel q$. Identify the postulate or theorem you used.

$(5x + 21)° \quad (12x + 6)°$
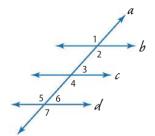
39. **LANDSCAPING** Find the measure needed for $m\angle ADC$ that will make $\overline{AB} \parallel \overline{CD}$ if $m\angle BAD = 45$.

Example 5

Given the following information, determine which lines, if any, are parallel. State the postulate or theorem that justifies your answer.

a. $\angle 1 \cong \angle 7$

$\angle 1$ and $\angle 7$ are alternate exterior angles of lines b and d.

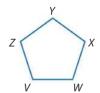Since $\angle 1 \cong \angle 7$, $b \parallel d$ by the Converse of the Alternate Exterior Angles Theorem.

b. $\angle 4 \cong \angle 5$

$\angle 4$ and $\angle 5$ are alternate interior angles of lines c and d.

Since $\angle 4 \cong \angle 5$, $c \parallel d$ by the Converse of the Alternate Interior Angles Theorem.
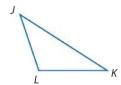
3-6 Perpendiculars and Distance

Copy each figure. Draw the segment that represents the distance indicated.
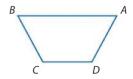
40. X to $\overline{VW}$

41. L to $\overline{JK}$

42. **HOME DÉCOR** Scott wants to hang two rows of framed pictures in parallel lines on his living room wall. He first spaces the nails on the wall in a line for the top row. Next, he hangs a weighted plumb line from each nail and measures an equal distance below each nail for the second row. Why does this ensure that the two rows of pictures will be parallel?
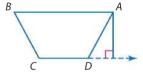
Example 6

Copy the figure. Draw the segment that represents the distance from point A to $\overline{CD}$.

The distance from a line to a point not on the line is the length of the segment perpendicular to the line that passes through the point.

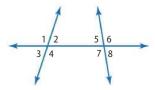Extend $\overline{CD}$ and draw the segment perpendicular to $\overline{CD}$ from A.

Classify the relationship between each pair of angles as *alternate interior*, *alternate exterior*, *corresponding*, or *consecutive interior* angles.
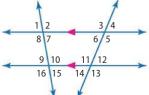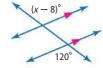
1. $\angle 6$ and $\angle 3$

2. $\angle 4$ and $\angle 7$

3. $\angle 5$ and $\angle 4$

Determine the slope of the line that contains the given points.

4. $G(8, 1)$, $H(8, 6)$ 5. $A(0, 6)$, $B(4, 0)$

6. $E(6, 3)$, $F(-6, 3)$ 7. $E(5, 4)$, $F(8, 1)$

In the figure, $m\angle 8 = 96$ and $m\angle 12 = 42$. Find the measure of each angle. Tell which postulate(s) or theorem(s) you used.

8. $\angle 9$

9. $\angle 11$

10. $\angle 6$

11. Find the value of x in the figure below.

12. **FITNESS** You would like to join a fitness center. Fit-N-Trim charges $80 per month. Fit-For-Life charges a one-time membership fee of $75 and $55 per month.
    a. Write and graph two equations in slope-intercept form to represent the cost y to attend each fitness center for x months.
    b. Are the lines you graphed in part **a** parallel? Explain why or why not.
    c. Which fitness center offers the better rate? Explain.

Write an equation in slope-intercept form for each line described.

13. passes through $(-8, 1)$, perpendicular to $y = 2x - 17$

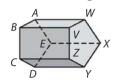14. passes through $(0, 7)$, parallel to $y = 4x - 19$

15. passes through $(-12, 3)$, perpendicular to $y = -\frac{2}{3}x - 11$

Find the distance between each pair of parallel lines with the given equations.

16. $y = x - 11$
    $y = x - 7$

17. $y = -2x + 1$
    $y = -2x + 16$

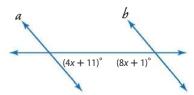18. **MULTIPLE CHOICE** Which segment is skew to $\overline{CD}$?

    A $\overline{ZY}$ C $\overline{DE}$

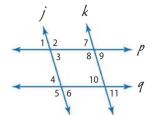   B $\overline{AB}$ D $\overline{VZ}$

19. Find x so that $a \parallel b$. Identify the postulate or theorem you used.

COORDINATE GEOMETRY Find the distance from P to ℓ.

20. Line ℓ contains points $(-4, 2)$ and $(3, -5)$. Point P has coordinates $(1, 2)$.

21. Line ℓ contains points $(6, 5)$ and $(2, 3)$. Point P has coordinates $(2, 6)$.

Given the following information, determine which lines, if any, are parallel. State the postulate or theorem that justifies your answer.

22. $\angle 4 \cong \angle 10$

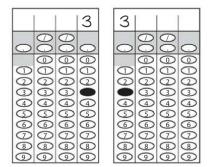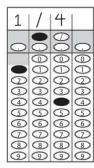23. $\angle 9 \cong \angle 6$

24. $\angle 7 \cong \angle 11$

25. **JOBS** Hailey works at a gift shop. She is paid $10 per hour plus a 15% commission on merchandise she sells. Write an equation in slope-intercept form that represents her earnings in a week if she sold $550 worth of merchandise.

Preparing for Standardized Tests

Gridded Response Questions

In addition to multiple-choice, short-answer, and extended-response questions, you will likely encounter gridded-response questions on standardized tests. After solving a gridded-response question, you must print your answer on an answer sheet and mark in the correct circles on the grid to match your answer. Answers to gridded-response questions may be whole numbers, decimals, or fractions.

Whole Numbers   **Decimals**   **Fractions**

Strategies for Solving Gridded-Response Questions

Step 1

Read the problem carefully and solve.

- Be sure your answer makes sense.

- If time permits, check your answer.

Step 2

Print your answer in the answer boxes.

- Print only one digit or symbol in each answer box.

- Do not write any digits or symbols outside the answer boxes.

- Write answer as a whole number, decimal, or fraction.
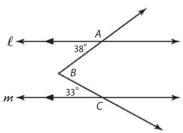
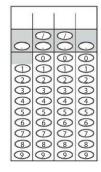| Step 3 |
| --- |

Fill in the grid.

- Fill in only one bubble for every answer box that you have written in. Be sure not to fill in a bubble under a blank answer box.

- Fill in each bubble completely and clearly.

Standardized Test Example

Read the problem. Identify what you need to know. Then use the information in the problem to solve.
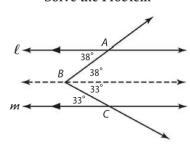
GRIDDED RESPONSE In the figure below, ∠ABC is intersected by parallel lines ℓ and m. What is the measure of ∠ABC? Express your answer in degrees.

Redraw the figure and add a third line parallel to lines ℓ and m through point B. Find the angle measures using alternate interior angles.

Solve the Problem

$m\angle ABC = 38 + 33 = 71$

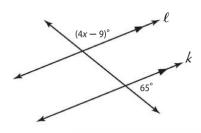Print your answer in the answer box and fill in the grid.

Fill in the Grid

Exercises

Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.

1. **GRIDDED RESPONSE** What is the slope of the line that contains the points $R(-2, 1)$ and $S(10, 6)$? Express your answer as a fraction.

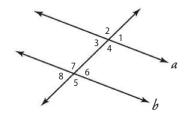2. **GRIDDED RESPONSE** Solve for x in the figure below.

Multiple Choice

Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.
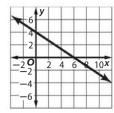
1. If $a \parallel b$ in the diagram below, which of the following may *not* be true?

A $\angle 1 \cong \angle 3$ **C** $\angle 2 \cong \angle 5$

B $\angle 4 \cong \angle 7$ **D** $\angle 8 \cong \angle 2$

2. Which of the following best describes a *counterexample* to the assertion below?

The sum of two odd numbers is odd.

F $3 + 3 = 6$ **H** $6 + 2 = 8$

G $5 + 4 = 9$ **J** $4 + 9 = 13$

3. What is the slope of the line?

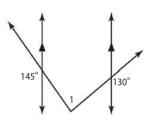**A** $-\dfrac{2}{3}$ **C** $-\dfrac{2}{5}$

B $-\dfrac{1}{2}$ **D** $-\dfrac{1}{6}$

4. Line k contains points at $(4, 1)$ and $(-5, -5)$. Find the distance between line k and point $F(-4, 0)$.

F 3.3 units   **H** 4.0 units

G 3.6 units   **J** 4.2 units

5. The globe has a diameter of 22 inches. What is the volume of the globe?

A 1520.5 in^3 **C** 5575.3 in^3

B 1741.4 in^3 **D** 6014.8 in^3

6. What is $m\angle 1$ in the figure below?

F 85   **H** 95

G 90   **J** 100

7. Jason is saving money to buy a car stereo. He has $45 saved, and he can save $15 per week. If the stereo that he wants is $210, how many weeks will it take Jason to buy the stereo?

A 10   **C** 12

B 11   **D** 13

> **Test-Taking Tip**
>
> **Question 6** *Drawing a diagram* can help you solve problems. Draw a third parallel line through the vertex of angle 1. Then use the properties of parallel lines and transversals to solve the problem.
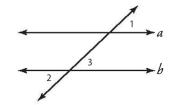
Short Response/Gridded Response

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

8. GRIDDED RESPONSE For a given line and a point not on the line, how many lines exist that pass through the point and are parallel to the given line?

9. GRIDDED RESPONSE Find the slope of the line that contains the points $(4, 3)$ and $(-2, -5)$.

10. Complete the proof.

Given: $\angle 1 \cong \angle 2$

Prove: $a \parallel b$

Proof:

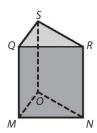| Statements	Reasons
1. $\angle 1 \cong \angle 2$	1. Given
2. $\angle 2 \cong \angle 3$	2. ___?___
3. $\angle 1 \cong \angle 3$	3. Transitive Prop.
4. $a \parallel b$	4. If corresponding angles are congruent, then the lines are parallel.

11. Write the contrapositive of the statement.

If a figure is a square, then the figure is a parallelogram.
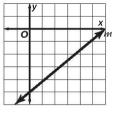
Extended Response

Record your answers on a sheet of paper. Show your work.

12. Refer to the figure to identify each of the following.

   a. all segments parallel to $\overline{MQ}$

   b. all planes intersecting plane SRN

   c. a segment skew to $\overline{ON}$

13. Use this graph to answer each question.

   a. What is the equation of line m?

   b. What is the slope of a line that is parallel to line m?

   c. What is the slope of a line that is perpendicular to *line m*?

Need ExtraHelp?

If you missed Question...	1	2	3	4	5	6	7	8	9	10	11	12	13
Go to Lesson...	3-2	2-1	3-3	3-6	1-7	3-2	3-4	3-6	3-3	3-1	2-3	3-1	3-4