Get Ready for Chapter 4

Diagnose Readiness You have two options for checking Prerequisite Skills.

*Quick*Check

Evaluate $3a^2 - 2ab + c$ for the values given. (Lesson 1-5)

1. $a = 2, b = 1, c = 5$

2. $a = -3, b = -2, c = 3$

3. $a = -1, b = 0, c = 11$

4. $a = 5, b = -3, c = -9$

5. **CAR RENTAL** The cost of renting a car is given by $49x + 0.3y$. Let x represent the number of days rented, and let y represent the number of miles driven. Find the cost for a five-day rental over 125 miles.

Solve each equation for the given variable. (Lesson 2-8)

6. $x + y = 5$ for y 7. $2x - 4y = 6$ for x

8. $y - 2 = x + 3$ for y 9. $4x - 3y = 12$ for x

10. **GEOMETRY** The formula for the perimeter of a rectangle is $P = 2w + 2\ell$, where w represents width and ℓ represents length. Solve for w.

Write the ordered pair for each point. (Lesson 1-6)

11. A

12. B

13. C

14. D

15. E

16. F
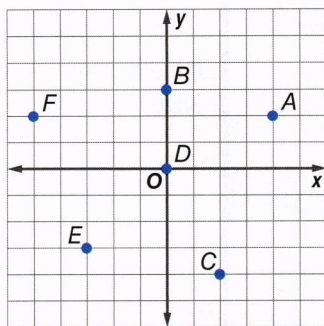
*Quick*Review

EXAMPLE 1

Evaluate $2(m - n)^2 + 3p$ for $m = 5, n = 2,$ and $p = -3$.

$2(m - n)^2 + 3p$	Original expression
$= 2(5 - 2)^2 + 3(-3)$	Substitute.
$= 2(3)^2 + 3(-3)$	Subtract.
$= 2(9) + 3(-3)$	Evaluate power.
$= 18 + (-9)$	Multiply.
$= 9$	Add.

EXAMPLE 2

Solve $5x + 15y = 9$ for x.

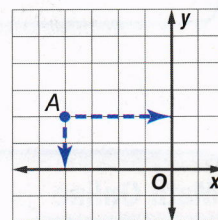|	
$5x + 15y = 9$	Original equation
$5x + 15y - 15y = 9 - 15y$	Subtract 15y from each side.
$5x = 9 - 15y$	Simplify.
$\dfrac{5x}{5} = \dfrac{9 - 15y}{5}$	Divide each side by 5.
$x = \dfrac{9}{5} - 3y$	Simplify.

EXAMPLE 3

Write the ordered pair for A.

Step 1 Begin at point A.

Step 2 Follow along a vertical line to the x-axis. The x-coordinate is -4.

Step 3 Follow along a horizontal line to the y-axis. The y-coordinate is 2.

The ordered pair for point A is $(-4, 2)$.

Get Started on Chapter 4

You will learn several new concepts, skills, and vocabulary terms as you study Chapter 4. To get ready, identify important terms and organize your resources. You may wish to refer to **Chapter 0** to review prerequisite skills.
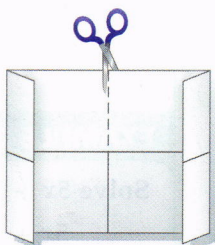
FOLDABLES® Study Organizer

Linear Functions Make this Foldable to help you organize your Chapter 4 notes about linear functions. Begin with one sheet of 11" by 17" paper.
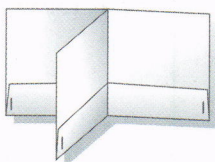
1 **Fold** each end of the paper in about 2 inches.
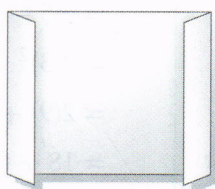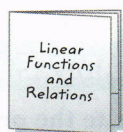
2 **Fold** along the width and the length. Unfold. Cut along the fold line from the top to the center.

3 **Fold** the top flaps down. Then fold in half and turn to form a folder. Staple the flaps down to form pockets.
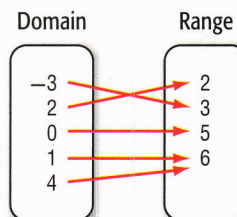
4 **Label** the front with the chapter title.

Linear Functions and Relations

Math Online ▶ glencoe.com

- Study the chapter online
- Explore **Math in Motion**
- Get extra help from your own **Personal Tutor**
- Use **Extra Examples** for additional help
- Take a **Self-Check Quiz**
- **Review Vocabulary** in fun ways

New Vocabulary

English		Español
slope-intercept form	• p. 214 •	forma pendiente-intersección
linear extrapolation	• p. 226 •	extrapolación lineal
point-slope form	• p. 231 •	forma punto-pendiente
parallel lines	• p. 237 •	rectas paralelas
perpendicular lines	• p. 238 •	rectas perpendiculares
scatter plot	• p. 245 •	gráfica de dispersión
line of fit	• p. 246 •	recta de ajuste
linear interpolation	• p. 247 •	interpolación lineal
best-fit line	• p. 253 •	recta de ajuste óptimo
linear regression	• p. 253 •	retroceso lineal
correlation coefficient	• p. 253 •	coeficiente de correlación
median-fit line	• p. 255 •	línea de mediana-ataque
step function	• p. 261 •	función etapa
piecewise function	• p. 261 •	función a intervalos
greatest integer function	• p. 261 •	función del máximo entero

Review Vocabulary

coefficient • p. 26 • coeficiente the numerical factor of a term

function • p. 45 • función a relation in which each element of the domain is paired with exactly one element of the range

ratio • p. 111 • razon a comparison of two numbers by division

Multilingual eGlossary glencoe.com
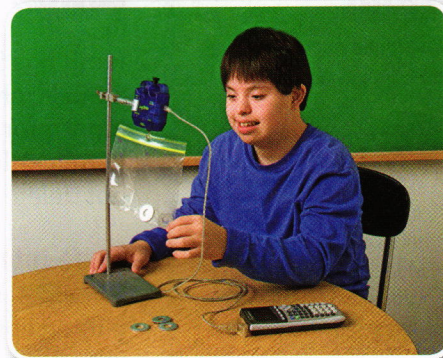
Objective
Use a graphing calculator to collect data and investigate slope-intercept form.

Set Up the Lab

- Cut a small hole in a top corner of a plastic sandwich bag. Hang the bag from the end of the force sensor.

- Connect the force sensor to your data collection device.

ACTIVITY

Step 1 Use the sensor to collect the weight with 0 washers in the bag. Record the data pair in the calculator.

Step 2 Place one washer in the plastic bag. Wait for the bag to stop swinging, then measure and record the weight.

Step 3 Repeat the experiment, adding different numbers of washers to the bag. Each time, record the number of washers and the weight.

Analyze the Results

1. The domain contains values of the independent variable, number of washers. The range contains values of the dependent variable, weight. Use the graphing calculator to create a scatter plot using the ordered pairs (washers, weight).

2. Write a sentence that describes the points on the graph.

3. Describe the position of the point on the graph that represents the trial with no washers in the bag.

4. The rate of change can be found by using the formula for slope.

$$\frac{\text{rise}}{\text{run}} = \frac{\text{change in weight}}{\text{change in number of washers}}$$

Find the rate of change in the weight as more washers are added.

5. Explain how the rate of change is shown on the graph.

Make a Conjecture

The graph shows sample data from a washer experiment. Describe the graph for each situation.

6. a bag that hangs weighs 0.8 N when empty and increases in weight at the rate of the sample

7. a bag that has the same weight when empty as the sample and increases in weight at a faster rate

8. a bag that has the same weight when empty as the sample and increases in weight at a slower rate
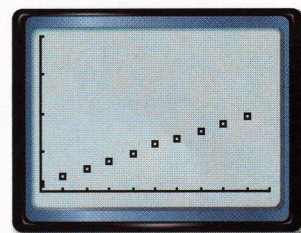
[0, 20] scl: 2 by [0, 1] scl: 0.25

Graphing Equations in Slope-Intercept Form

Why?

Jamil has 500 songs on his MP3 player. He joins a music club that lets him download 30 songs per month for a monthly fee. The number of songs that Jamil could eventually have in his player if he does not delete any songs is represented by $y = 30x + 500$.

Slope-Intercept Form An equation of the form $y = mx + b$, where m is the slope and b is the y-intercept, is in **slope-intercept form**. The variables m and b are called *parameters* of the equation. Changing either value changes the equation's graph.
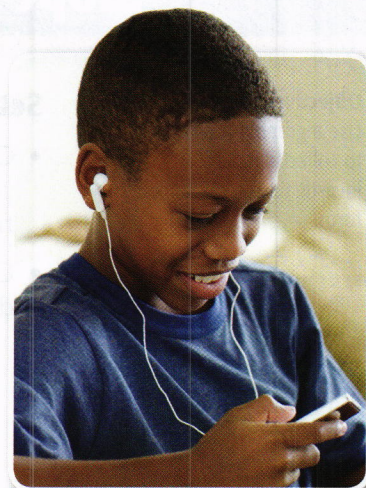
Key Concept — Slope-Intercept Form

For Your FOLDABLE

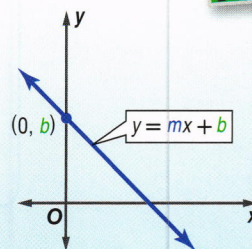**Words** The slope-intercept form of a linear equation is $y = mx + b$, where m is the slope and b is the y-intercept.

Example
$$y = mx + b$$
$$y = 2x + 6$$
slope ↑   ↑ y-intercept

$(0, b)$ $y = mx + b$

Math in Motion, BrainPOP® glencoe.com

EXAMPLE 1 Write and Graph an Equation

Write an equation in slope-intercept form for the line with a slope of $\frac{3}{4}$ and a y-intercept of -2. Then graph the equation.

$y = mx + b$ **Slope-intercept form**

$y = \frac{3}{4}x + (-2)$ **Replace m with $\frac{3}{4}$ and b with -2.**

$y = \frac{3}{4}x - 2$ **Simplify.**

Now graph the equation.

$y = \frac{3}{4}x - 2$

Step 1 Plot the y-intercept $(0, -2)$.

Step 2 The slope is $\frac{\text{rise}}{\text{run}} = \frac{3}{4}$. From $(0, -2)$, move up 3 units and right 4 units. Plot the point.
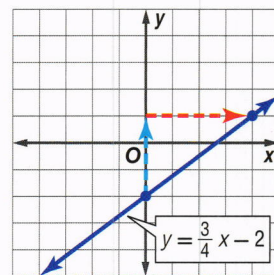
Step 3 Draw a line through the two points.

✓ Check Your Progress

Write an equation of a line in slope intercept form with the given slope and y-intercept. Then graph the equation.

1A. slope: $-\frac{1}{2}$, y-intercept: 3

1B. slope: -3, y-intercept: -8

▶ **Personal Tutor** glencoe.com

When an equation is not written in slope-intercept form, it may be easier to rewrite it before graphing.

EXAMPLE 2 **Graph Linear Equations**

Graph $3x + 2y = 6$.

Rewrite the equation in slope-intercept form.

$3x + 2y = 6$	**Original equation**
$3x + 2y - 3x = 6 - 3x$	**Subtract 3x from each side.**
$2y = 6 - 3x$	**Simplify.**
$2y = -3x + 6$	**6 − 3x = 6 + (−3x) or −3x + 6**
$\dfrac{2y}{2} = \dfrac{-3x + 6}{2}$	**Divide each side by 2.**
$y = -\dfrac{3}{2}x + 3$	**Slope-intercept form**

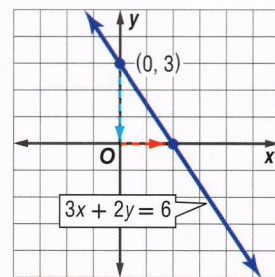Now graph the equation. The slope is $-\dfrac{3}{2}$, and the y-intercept is 3.

Step 1 Plot the y-intercept $(0, 3)$.

Step 2 The slope is $\dfrac{\text{rise}}{\text{run}} = -\dfrac{3}{2}$. From $(0, 3)$, move down 3 units and right 2 units. Plot the point.

Step 3 Draw a line through the two points.

✔ **Check Your Progress**

Graph each equation.

2A. $3x - 4y = 12$ **2B.** $-2x + 5y = 10$

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Horizontal lines have a slope of 0. So, equations of horizontal lines can be written in slope intercept form as $y = 0x + b$ or $y = b$. Vertical lines have no slope. So, equations of vertical lines cannot be written in slope-intercept form.

EXAMPLE 3 **Graph Linear Equations**

Graph $y = -3$.

Step 1 Plot the y-intercept $(0, -3)$.

Step 2 The slope is 0. Draw a line through the points with y-coordinate -3.
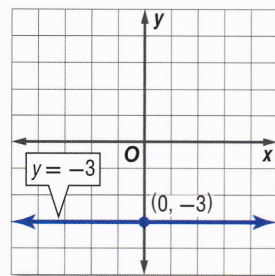
✔ **Check Your Progress**

Graph each equation.

3A. $y = 5$ **3B.** $2y = 1$

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Notice that the equations of horizontal lines do not have an x variable. The graph of a horizontal line does not cross the x-axis. The equation $y = 0$ lies on the x-axis.

There are times when you will need to write an equation when given a graph. To do this, locate the y-intercept and use the rise and run to find another point on the graph. Then write the equation in slope-intercept form.

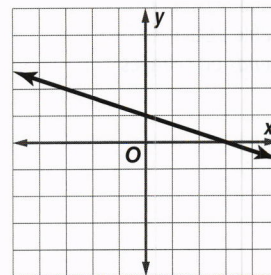### STANDARDIZED TEST EXAMPLE 4

Which of the following is an equation in slope-intercept form for the line shown?

A $y = -3x + 1$

B $y = -3x + 3$

C $y = -\frac{1}{3}x + 1$

D $y = -\frac{1}{3}x + 3$

Read the Test Item

You need to find the slope and y-intercept of the line to write the equation.

Solve the Test Item

Step 1 The line crosses the y-axis at $(0, 1)$, so the y-intercept is 1. The answer is either A or C.

Step 2 To get from $(0, 1)$ to $(3, 0)$, go down 1 unit and 3 units to the right. The slope is $-\frac{1}{3}$.

Step 3 Write the equation.

$y = mx + b$

$y = -\frac{1}{3}x + 1$

CHECK The graph also passes through $(-3, 2)$. If the equation is correct, this should be a solution.

$y = -\frac{1}{3}x + 1$

$2 \stackrel{?}{=} -\frac{1}{3}(-3) + 1$

$2 \stackrel{?}{=} 1 + 1$

$2 = 2$ ✔   The answer is C.

☑ Check Your Progress

4. Which of the following is an equation in slope-intercept form for the line shown?

F $y = \frac{1}{4}x - 1$

G $y = \frac{1}{4}x + 4$

H $y = 4x - 1$

J $y = 4x + 4$
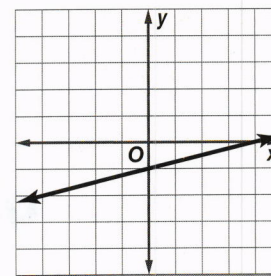
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Modeling Real-World Data Real-world data can be modeled by a linear equation if there is a constant rate of change. The rate of change represents the slope. The y-intercept is the point where the value of the independent variable is 0.

Real-World EXAMPLE 5   Write and Graph a Linear Equation

SPORTS Use the information at the left about high school sports.

a. Write a linear equation to find the number of girls in high school sports after 1997.

Words	Number of girls competing	equals	rate of change	times	number of years	plus	amount at start.
Variable	Let G = number of girls competing.			Let n = number of years since 1997.			
Equation	G	=	0.06	×	n	+	2.6

The equation is $G = 0.06n + 2.6$.

b. Graph the equation.

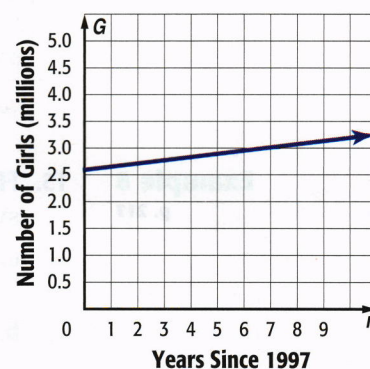The y-intercept is where the data begins. So, the graph passes through (0, 2.6).

The rate of change is the slope, so the slope is 0.06.

c. Estimate the number of girls competing in 2017.

The year 2017 is 20 years after 1997.

$$G = 0.06n + 2.6 \quad \text{Write the equation.}$$
$$= 0.06(20) + 2.6 \quad \text{Replace } n \text{ with 20.}$$
$$= 3.8 \quad \text{Simplify.}$$

There will be about 3.8 million girls competing in high school sports in 2017.

Graph: vertical axis "Number of Girls (millions)" labeled G from 0.5 to 5.0; horizontal axis "Years Since 1997" labeled n from 1 to 9.

✓ Check Your Progress

5. FUNDRAISERS The band boosters are selling sandwiches for $5 each. They bought $1160 in ingredients.

  A. Write an equation for the profit P made on n sandwiches.

  B. Graph the equation.

  C. Find the total profit if 1400 sandwiches are sold.

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✓ Check Your Understanding

Example 1
p. 214

Write an equation of a line in slope-intercept form with the given slope and y-intercept. Then graph the equation.

  1 slope: 2, y-intercept: 4   **2.** slope: -5, y-intercept: 3

  3. slope: $\frac{3}{4}$, y-intercept: -1 **4.** slope: $-\frac{5}{7}$, y-intercept: $-\frac{2}{3}$
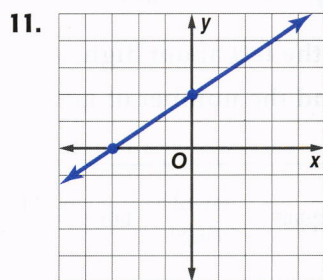
Examples 2 and 3
p. 215

Graph each equation.

  5. $-4x + y = 2$ **6.** $2x + y = -6$

  7. $-3x + 7y = 21$ **8.** $6x - 4y = 16$

  9. $y = -1$ **10.** $15y = 3$

Example 4
p. 216

Write an equation in slope-intercept form for each graph shown.

11.

12.

13.

14.
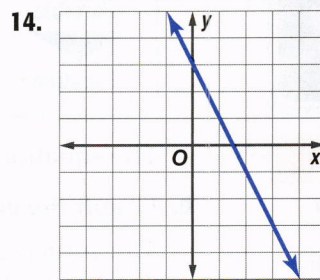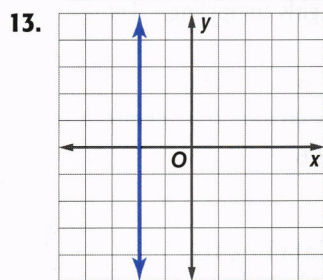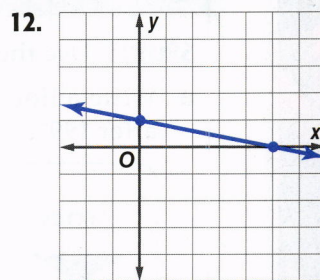
Example 5
p. 217

15. FINANCIAL LITERACY Rondell is buying a new stereo system for his car using a layaway plan.

  a. Write an equation for the total amount S that he has paid after w weeks.

  b. Graph the equation.

  c. Find out how much Rondell will have saved after 8 weeks.

Jack's Stereo Layaway Plan
$75 *down* and
$10 *each week*

16. TRAVEL Ana is driving from her home in Miami, Florida, to her grandmother's house in New York City. On the first day, she will travel 240 miles to Orlando, Florida, to pick up her cousin. Then they will travel 350 miles each day.

  a. Write an equation for the total number of miles m that Ana has traveled after d days.

  b. Graph the equation.

  c. It is 1293 miles from Miami to New York City. How long will the drive take?

Practice and Problem Solving

● = **Step-by-Step Solutions** begin on page R12.
Extra Practice begins on page 815.

Example 1
p. 214

Write an equation of a line in slope-intercept form with the given slope and y-intercept. Then graph the equation.

17 slope: 5, y-intercept: 8

18. slope: 3, y-intercept: 10

19. slope: -4, y-intercept: 6

20. slope: -2, y-intercept: 8

21. slope: 3, y-intercept: -4

22. slope: 4, y-intercept: -6

Examples 2 and 3
p. 215

Graph each equation.

23. $-3x + y = 6$

24. $-5x + y = 1$

25. $-2x + y = -4$

26. $y = 8x - 7$

27. $5x + 2y = 8$

28. $4x + 9y = 27$

29. $y = 7$

30. $y = -\frac{2}{3}$

31. $21 = 7y$

32. $3y - 6 = 2x$

Example 4
p. 216

Write an equation in slope-intercept form for each graph shown.

33.

34.

35.

36.
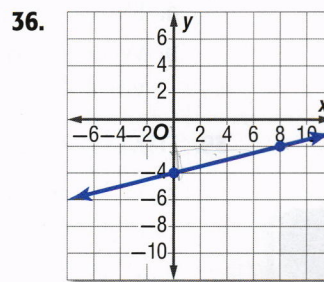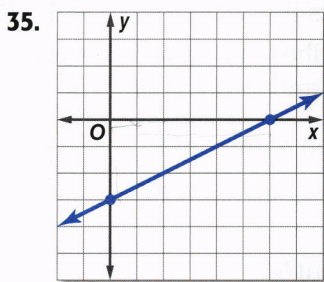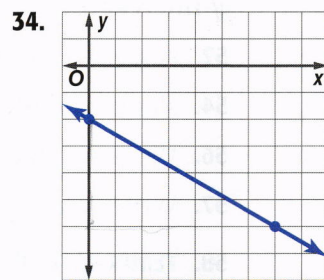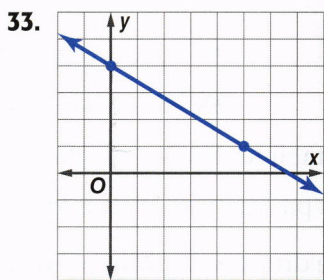
Example 5
p. 217

37 **MANATEES** In 1991, 1267 manatees inhabited Florida's waters. The manatee population has increased at a rate of 123 manatees per year.

a. Write an equation for the manatee population, P, t years since 1991.

b. Graph this equation.

c. In 2006, the manatee was removed from Florida's endangered species list. What was the manatee population in 2006?

Write an equation of a line in slope-intercept form with the given slope and y-intercept.

38. slope: $\frac{1}{2}$, y-intercept: -3

39. slope: $\frac{2}{3}$, y-intercept: -5

40. slope: $-\frac{5}{6}$, y-intercept: 5

41. slope: $-\frac{3}{7}$, y-intercept: 2

42. slope: 1, y-intercept: 4

43. slope: 0, y-intercept: 5

Graph each equation.

44. $y = \frac{3}{4}x - 2$

45. $y = \frac{5}{3}x + 4$

46. $3x + 8y = 32$

47. $5x - 6y = 36$

48. $-4x + \frac{1}{2}y = -1$

49. $3x - \frac{1}{4}y = 2$

50. TRAVEL A rental company charges $8 per hour for a mountain bike plus a $5 fee for a helmet.

a. Write an equation in slope-intercept form for the total rental cost C for a helmet and a bicycle for t hours.

b. Graph the equation.

c. What would the cost be for 2 helmets and 2 bicycles for 8 hours?

51. COLLEGE TUITION For Illinois residents, the average tuition at Chicago State University is $157 per credit hour. Fees cost $218 per year.

a. Write an equation in slope-intercept form for the tuition T for c credit hours.

b. Find the cost for a student who is taking 32 credit hours.

Write an equation of a line in slope-intercept form with the given slope and y-intercept.

52. slope: -1, y-intercept: 0

53. slope: 0.5, y-intercept: 7.5

54. slope: 0, y-intercept: 7

55. slope: -1.5, y-intercept: -0.25

56. Write an equation of a horizontal line that crosses the y-axis at $(0, -5)$.

57. Write an equation of a line that passes through the origin and has a slope of 3.

58. **TEMPERATURE** The temperature dropped rapidly overnight. Starting at 80°F, the temperature dropped 3° per minute.

    a. Draw a graph that represents this drop from 0 to 8 minutes.

    b. Write an equation that describes this situation. Describe the meaning of each variable as well as the slope and y-intercept.

59. **FITNESS** Refer to the information at the right.

    a. Write an equation that represents the cost C of a membership for m months.

    b. What does the slope represent?

    c. What does the C-intercept represent?

    d. What is the cost of a two-year membership?

> **GET FIT GYM**
> startup fee **$145**
>
> **$45** monthly fee

60. **MAGAZINES** A teen magazine began with a circulation of 500,000 in its first year. Since then, the circulation has increased an average of 33,388 per year.

    a. Write an equation that represents the circulation c after y years.

    b. What does the slope represent?

    c. What does the y-intercept represent?

    d. If the magazine began in 1944, and this trend continues, in what year will the circulation reach 3,000,000?

61. **CELL PHONES** In 2007, 3.25 billion customers worldwide used a cell phone, and 1000 new customers signed up each minute.

    a. Write an equation for the number of cell phone customers in billions, C, t years since 2007. Graph this equation.

    b. How many cell phone customers will there be in 2011?

    c. During what year will the number of customers reach 10 billion?

H.O.T. Problems   Use **H**igher-**O**rder **T**hinking Skills

62. **OPEN ENDED** Draw a graph representing a real-world linear function and write an equation for the graph. Describe what the graph represents.

63. **REASONING** Determine whether the equation of a vertical line can be written in slope-intercept form. Explain your reasoning.

64. **CHALLENGE** Summarize the characteristics that the graphs $y = 2x + 3$, $y = 4x + 3$, $y = -x + 3$, and $y = -10x + 3$ have in common.

65. **WRITING IN MATH** If given an equation in standard form, explain how to determine the rate of change.

66. **WRITING IN MATH** Explain how you would use a given y-intercept and the slope to make predictions about what the y-value will be for a given x-value without graphing.

67. A music store has x CDs in stock. If 350 are sold and $3y$ are added to stock, which expression represents the number of CDs in stock?

A $350 + 3y - x$ **C** $x + 350 + 3y$

B $x - 350 + 3y$ **D** $3y - 350 - x$

68. PROBABILITY The table shows the result of a survey of favorite activities. What is the probability that a student's favorite activity is sports or drama club?

Extracurricular Activity	Students
art club	24
band	134
choir	37
drama club	46
mock trial	19
school paper	26
sports	314

F $\dfrac{3}{8}$ **G** $\dfrac{4}{9}$ **H** $\dfrac{3}{5}$ **J** $\dfrac{2}{3}$

69. A recipe for fruit punch calls for 2 ounces of orange juice for every 8 ounces of lemonade. If Jennifer uses 64 ounces of lemonade, which proportion can she use to find x, the number of ounces of orange juice needed?

A $\dfrac{2}{x} = \dfrac{64}{6}$ **C** $\dfrac{2}{8} = \dfrac{x}{64}$

B $\dfrac{8}{x} = \dfrac{64}{2}$ **D** $\dfrac{6}{2} = \dfrac{x}{64}$

70. EXTENDED RESPONSE The table shows the results of a canned food drive. 1225 cans were collected, and the 12th-grade class collected 55 more cans than the 10th-grade class. How many cans each did the 10th- and 12th-grade classes collect? Show your work.

Grade	Cans
9	340
10	x
11	280
12	y

For each arithmetic sequence, determine the related function. Then determine if the function is *proportional* or *nonproportional*. (Lesson 3-6)

71. 3, 7, 11, …   **72.** 8, 6, 4, …   **73.** 0, 3, 6, …   **74.** 1, 2, 3, …

75. GAME SHOWS Contestants on a game show win money by answering 10 questions. (Lesson 3-5)

   a. Find the value of the 10th question.

   b. If all questions are answered correctly, how much are the winnings?

10 QUESTIONS!

1. $3000   > + $2500
2.   > + $2500
3.   > + $2500
4.   > + $2500
5.

Suppose y varies directly as x. Write a direct variation equation that relates x and y. Then solve. (Lesson 3-4)

76. If $y = 10$ when $x = 5$, find y when $x = 6$.

77. If $y = -16$ when $x = 4$, find x when $y = 20$.

78. If $y = 6$ when $x = 18$, find y when $x = -12$.

79. If $y = 12$ when $x = 15$, find x when $y = -6$.

Find the slope of the line that passes through each pair of points. (Lesson 3-3)

80. $(2, 3), (9, 7)$ **81.** $(-3, 6), (2, 4)$ **82.** $(2, 6), (-1, 3)$ **83.** $(-3, 3), (1, 3)$

EXTEND

4-1

Graphing Technology Lab

The Family of Linear Graphs

Math Online ⟩ glencoe.com
• Other Calculator Keystrokes
• Graphing Technology Personal Tutor

A family of people is related by birth, marriage, or adoption. Often people in families share characteristics. The graphs in a family share at least one characteristic. Graphs in the linear family are all lines, with the simplest graph in the family being that of the parent function $y = x$.

Parent Graph

You can use a graphing calculator to investigate how changing the parameters m and b in $y = mx + b$ affects the graphs in the family of linear functions.
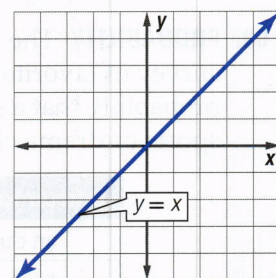
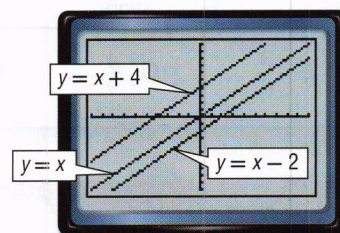## ACTIVITY 1   Changing *b* in $y = mx + b$

Graph $y = x$, $y = x + 4$, and $y = x - 2$ in the standard viewing window.

Enter the equations in the **Y=** list as **Y1, Y2,** and **Y3**. Then graph the equations.

KEYSTROKES: *Review graphing on pages 167 and 168.*

1A. How do the slopes of the graphs compare?

1B. Compare the graph of $y = x + 4$ and the graph of $y = x$. How would you obtain the graph of $y = x + 4$ from the graph of $y = x$?

1C. How would you obtain the graph of $y = x - 2$ from the graph of $y = x$?
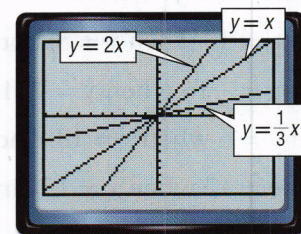

[−10, 10] scl: 1 by [−10, 10] scl: 1

Changing m in $y = mx + b$ affects the graphs in a different way than changing b. First, investigate positive values of m.

ACTIVITY 2   Changing *m* in $y = mx + b$, Positive Values

Graph $y = x$, $y = 2x$, and $y = \frac{1}{3}x$ in the standard viewing window.

Enter the equations in the **Y=** list and graph.

2A. How do the y-intercepts of the graphs compare?

2B. Compare the graph of $y = 2x$ and the graph of $y = x$.

2C. Which is steeper, the graph of $y = \frac{1}{3}x$ or the graph of $y = x$?

[−10, 10] scl: 1 by [−10, 10] scl: 1

Does changing m to a negative value affect the graph differently than changing it to a positive value?

Changing m in $y = mx + b$, Negative Values

Graph $y = x$, $y = -x$, $y = -3x$, and $y = -\frac{1}{2}x$ in the standard viewing window.

Enter the equations in the **Y=** list and graph.

3A. How are the graphs with negative values of m different than graphs with a positive m?

3B. Compare the graphs of $y = -x$, $y = -3x$, and $y = -\frac{1}{2}x$. Which is steepest?
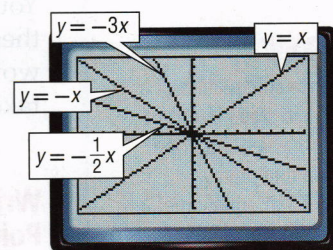
$[-10, 10]$ scl: 1 by $[-10, 10]$ scl: 1

Analyze the Results

Graph each set of equations on the same screen. Describe the similarities or differences.

1. $y = 2x$
$y = 2x + 3$
$y = 2x - 7$

2. $y = x + 1$
$y = 2x + 1$
$y = \frac{1}{4}x + 1$

3. $y = x + 4$
$y = 2x + 4$
$y = \frac{3}{4}x + 4$

4. $y = 0.5x + 2$
$y = 0.5x - 5$
$y = 0.5x + 4$

5. $y = -2x - 2$
$y = -4.2x - 2$
$y = -\frac{1}{3}x - 2$

6. $y = 3x$
$y = 3x + 6$
$y = 3x - 7$

7. Families of graphs have common characteristics. What do the graphs of all equations of the form $y = mx + b$ have in common?

8. How does the value of b affect the graph of $y = mx + b$?

9. What is the result of changing the value of m on the graph of $y = mx + b$ if m is positive?

10. How can you determine which graph is steepest by examining the following equations?
$y = 3x$, $y = -4x - 7$, $y = \frac{1}{2}x + 4$

11. Explain how knowing about the effects of m and b can help you sketch the graph of an equation.

12. The equation $y = k$ can also be a parent graph. Graph $y = 5$, $y = 2$, and $y = -4$ on the same screen. Describe the similarities or differences among the graphs.

Extension

Nonlinear functions can also be defined in terms of a family of graphs. Graph each set of equations on the same screen. Describe the similarities or differences.

13. $y = x^2$
$y = -3x^2$
$y = (-3x)^2$

14. $y = x^2$
$y = x^2 + 3$
$y = (x - 2)^2$

15. $y = x^2$
$y = 2x^2 + 4$
$y = (3x)^2 - 5$

16. Describe the similarities and differences in the classes of functions $f(x) = x^2 + c$ and $f(x) = (x + c)^2$, where c is any real number.

Writing Equations in Slope-Intercept Form

Why?

In 2000, Americans took 337.1 million vacations. In 2004, Americans took 375.4 million vacations. You can find the average rate of change for these data. Then you can write an equation that would model the average number of vacations taken per year.

Write an Equation Given the Slope and a Point
The next example shows how to write an equation of a line if you are given a slope and a point other than the *y*-intercept.
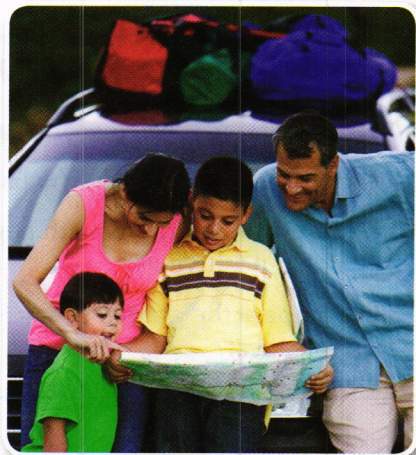
EXAMPLE 1   **Write an Equation Given the Slope and a Point**

Write an equation of the line that passes through (2, 1) with a slope of 3.

You are given the slope but not the *y*-intercept.

Step 1   Find the *y*-intercept.

$y = mx + b$	Slope-intercept form
$1 = 3(2) + b$	Replace *m* with 3, *y* with 1, and *x* with 2.
$1 = 6 + b$	Simplify.
$1 - 6 = 6 + b - 6$	Subtract 6 from each side.
$-5 = b$	Simplify.

Step 2   Write the equation in slope-intercept form.

$y = mx + b$	Slope-intercept form
$y = 3x - 5$	Replace *m* with 3 and *b* with −5.

Therefore, the equation of the line is $y = 3x - 5$.

✓ Check Your Progress

Write an equation of a line that passes through the given point and has the given slope.

1A. (−2, 5), slope 3   **1B.** (4, −7), slope −1

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Write an Equation Given Two Points
If you are given two points through which a line passes, you can use them to find the slope first. Then follow the steps in Example 1 to write the equation.

EXAMPLE 2   Write an Equation Given Two Points

Write an equation of the line that passes through each pair of points.

a. (3, 1) and (2, 4)

Step 1   Find the slope of the line containing the given points.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$ Slope Formula

$$= \frac{4 - 1}{2 - 3}$$ $(x_1, y_1) = (3, 1)$ and $(x_2, y_2) = (2, 4)$

$$= \frac{3}{-1} \text{ or } -3$$ Simplify.

Step 2   Use either point to find the y-intercept.

$$y = mx + b$$ Slope-intercept form

$$4 = (-3)(2) + b$$ Replace m with -3, x with 2, and y with 4.

$$4 = -6 + b$$ Simplify.

$$4 - (-6) = -6 + b - (-6)$$ Subtract -6 from each side.

$$10 = b$$ Simplify.

Step 3   Write the equation in slope-intercept form.

$$y = mx + b$$ Slope-intercept form

$$y = -3x + 10$$ Replace m with -3 and b with 10.

Therefore, the equation is $y = -3x + 10$.

b. (−4, −2) and (−5, −6)

Step 1   Find the slope of the line containing the given points.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$ Slope Formula

$$= \frac{-6 - (-2)}{-5 - (-4)}$$ $(x_1, y_1) = (-4, -2)$ and $(x_2, y_2) = (-5, -6)$

$$= \frac{-4}{-1} \text{ or } 4$$ Simplify.

Step 2   Use either point to find the y-intercept.

$$y = mx + b$$ Slope-intercept form

$$-2 = 4(-4) + b$$ Replace m with 4, x with -4, and y with -2.

$$-2 = -16 + b$$ Simplify.

$$-2 - (-16) = -16 + b - (-16)$$ Subtract -16 from each side.

$$14 = b$$ Simplify.

Step 3   Write the equation in slope-intercept form.

$$y = mx + b$$ Slope-intercept form

$$y = 4x + 14$$ Replace m with 4 and b with 14.

Therefore, the equation is $y = 4x + 14$.

✓ Check Your Progress

Write an equation of the line that passes through each pair of points.

2A. $(-1, 12), (4, -8)$ **2B.** $(5, -8), (-7, 0)$
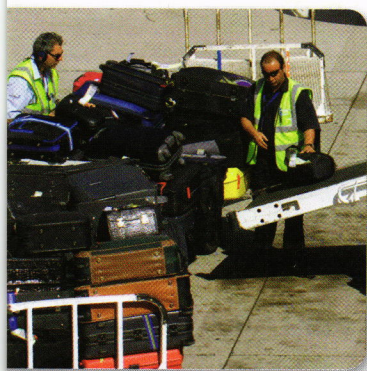
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🌐 Real-World EXAMPLE 3   Use Slope-Intercept Form

AIR FARES The table shows the average fares for domestic flights in certain years. Write an equation that could be used to predict air fares if fares continue to increase at this rate.

Year	Cost ($)
2004	354
2005	366
2006	378
2007	390

Understand You know the air fares for the years listed.

Plan Let x represent the number of years since 2000, and let y represent the air fare. Write an equation of the line that passes through $(5, 366)$ and $(6, 378)$.

Solve Find the slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad \text{Slope Formula}$$

$$= \frac{378 - 366}{6 - 5} \qquad \text{Let } (x_1, y_1) = (5, 366) \text{ and } (x_2, y_2) = (6, 378).$$

$$= \frac{12}{1} \text{ or } 12 \qquad \text{Simplify.}$$

Choose $(6, 378)$ and find the y-intercept of the line.

$y = mx + b$ — **Slope-intercept form**
$378 = 12(6) + b$ — **Replace m with 12, x with 6, and y with 378.**
$378 = 72 + b$ — **Simplify.**
$306 = b$ — **Subtract 72 from each side.**

Write the equation using $m = 12$ and $b = 306$.

$y = mx + b$ — **Slope-intercept form**
$y = 12x + 306$ — **Replace m with 12 and b with 306.**

Check Check your result by using the coordinates of the other point.

$y = 12x + 306$ — **Original equation**
$366 \overset{?}{=} 12(5) + 306$ — **Replace x with 5 and y with 366.**
$366 = 366$ ✔ — **Simplify.**

✅ Check Your Progress

3. FINANCIAL LITERACY In addition to his weekly salary, Ethan is paid $16 per delivery. Last week, he made 5 deliveries, and his total pay was $215. Write a linear equation to find Ethan's total weekly pay T if he makes d deliveries.

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You can use a linear equation to make predictions about values that are beyond the range of the data. This process is called **linear extrapolation**.

🌐 Real-World EXAMPLE 4   Predict from Slope-Intercept Form

AIR FARES Use the equation from Example 3 to estimate the cost of airfares in 2010.

$y = 12x + 306$ — **Original equation**
$ = 12(10) + 306$ — **Replace x with 10.**
$ = 426$ — An estimate of the average air fares is $426.

✅ Check Your Progress

4. MONEY Use the equation in Check Your Progress 3 to predict how much money Ethan will earn in a week if he makes 8 deliveries.

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✔ Check Your Understanding

Write an equation of the line that passes through the given point and has the given slope.

1. (3, −3), slope 3   **2.** (2, 4), slope 2

3. (1, 5), slope −1   **4.** (−4, 6), slope −2
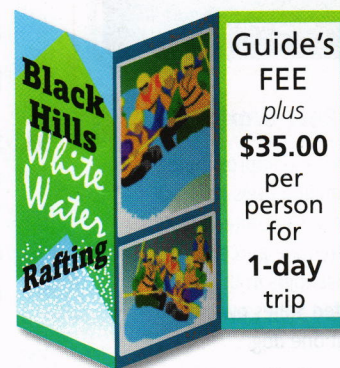
Write an equation of the line that passes through each pair of points.

5. (4, −3), (2, 3)   **6.** (−7, −3), (−3, 5)

7. (−1, 3), (0, 8)   **8.** (−2, 6), (0, 0)

9. **WHITEWATER RAFTING** Ten people from a local youth group went to Black Hills Whitewater Rafting Tour Company for a one-day rafting trip. The group paid $425.

Black Hills White Water Rafting

Guide's FEE *plus* **$35.00** per person for **1-day** trip

 a. Write an equation in slope-intercept form to find the total cost C for p people.

 b. How much would it cost for 15 people?

= **Step-by-Step Solutions** begin on page R12.
Extra Practice begins on page 815.

Practice and Problem Solving

Write an equation of the line that passes through the given point and has the given slope.

10. (3, 1), slope 2   **11** (−1, 4), slope −1   **12.** (1, 0), slope 1

13. (7, 1), slope 8   **14.** (2, 5), slope −2   **15.** (2, 6), slope 2

Write an equation of the line that passes through each pair of points.

16. (9, −2), (4, 3)   **17.** (−2, 5), (5, −2)   **18.** (−5, 3), (0, −7)

19. (3, 5), (2, −2)   **20.** (−1, −3), (−2, 3)   **21.** (−2, −4), (2, 4)

22. **RC CAR** Greg is driving a remote control car at a constant speed. He starts the timer when the car is 5 feet away. After 2 seconds the car is 35 feet away.

 a. Write a linear equation to find the distance d of the car from Greg.

 b. Estimate the distance the car has traveled after 10 seconds.

23. **TRAVEL** Refer to the beginning of the lesson.

 a. Write a linear equation to find the number of vacations (in millions) y after x years. Let x be the number of years since 2000.

 b. Estimate the number of vacations that will be taken in 2012.

Problem-Solving Tip

▶ **Determine Reasonable Answers** Deciding whether an answer is reasonable is useful when an exact answer is not neccessary.

24. **BOOKS** In 1904, a dictionary cost 30¢. Since then the cost of a dictionary has risen an average of 6¢ per year.

 a. Write a linear equation to find the cost C of a dictionary y years after 2004.

 b. If this trend continues, what will the cost of a dictionary be in 2020?

Write an equation of the line that passes through the given point and has the given slope.

25. (4, 2), slope $\frac{1}{2}$ **26.** (3, −2), slope $\frac{1}{3}$ **27.** (6, 4), slope $-\frac{3}{4}$

28. (2, −3), slope $\frac{2}{3}$ **29.** (2, −2), slope $\frac{2}{7}$ **30.** (−4, −2), slope $-\frac{3}{5}$

31. DOGS In 2001, there were about 56.1 thousand golden retrievers registered in the United States. In 2002, the number was 62.5 thousand.

  a. Write a linear equation to find the number of golden retrievers G that will be registered in year t, where $t = 0$ is the year 2000.

  b. Graph the equation.

  c. Estimate the number of golden retrievers that will be registered in 2012.

32. GYM MEMBERSHIPS A local recreation center offers a yearly membership for $265. The center offers aerobics classes for an additional $5 per class.

  a. Write an equation that represents the total cost of the membership.

  b. Carly spent $500 one year. How many aerobics classes did she take?

33. SUBSCRIPTION A magazine offers an online subscription that allows you to view up to 25 archived articles free. To view 30 archived articles, you pay $49.15. To view 33 archived articles, you pay $57.40.

  a. What is the cost of each archived article for which you pay a fee?

  b. What is the cost of the magazine subscription?

Write an equation of the line that passes through the given points.

34. $(5, -2), (7, 1)$ **35** $(5, -3), (2, 5)$ **36.** $\left(\frac{5}{4}, 1\right), \left(-\frac{1}{4}, \frac{3}{4}\right)$ **37.** $\left(\frac{5}{12}, -1\right), \left(-\frac{3}{4}, \frac{1}{6}\right)$

Determine whether the given point is on the line. Explain why or why not.

38. $(3, -1); y = \frac{1}{3}x + 5$ **39.** $(6, -2); y = \frac{1}{2}x - 5$

For Exercises 40–42, determine which equation best represents each situation. Explain the meaning of each variable.

| **A** $y = -\frac{1}{3}x + 72$ | **B** $y = 2x + 225$ | **C** $y = 8x + 4$ |

40. CONCERTS Tickets to a concert cost $8 each plus a processing fee of $4 per order.

41. FUNDRAISING The freshman class has $225. They sell raffle tickets at $2 each to raise money for a field trip.

42. POOLS The current water level of a swimming pool in Tucson, Arizona, is 6 feet. The rate of evaporation is $\frac{1}{3}$ inch per day.

43. ENVIRONMENT A manufacturer implemented a program to reduce waste. In 1998 they sent 946 tons of waste to landfills. Each year after that, they reduced their waste by an average 28.4 tons.

  a. How many tons were sent to the landfill in 2010?

  b. In what year will it become impossible for this trend to continue? Explain.

44. ⊞ **MULTIPLE REPRESENTATIONS** In this problem, you will explore the slopes of perpendicular lines.

  a. GRAPHICAL On a coordinate plane, graph $y = \frac{3}{4}x + 1$.

  b. PICTORIAL Use a straightedge and a protractor to draw a line that is perpendicular to the line you graphed.

  c. ALGEBRAIC Find the equation of the line that is perpendicular to the original line. Describe which method you used to write the equation.

  d. ANALYTICAL Compare the slopes of the lines. Describe the relationship, if any, between the two values.

Real-World Link

There are approximately 73 million dogs kept as pets in the United States. Thirty-nine percent of households in the United States own at least one dog.

Source: The Humane Society of the United States

45 **CONCERT TICKETS** Jackson is ordering tickets for a concert online. There is a processing fee for each order, and the tickets are $52 each. Jackson ordered 5 tickets and the cost was $275.

   a. Determine the processing fee. Write a linear equation to represent the total cost C for t tickets.

   b. Make a table of values for at least three other numbers of tickets.

   c. Graph this equation. Predict the cost of 8 tickets.

46. **MUSIC** A music store is offering a Frequent Buyers Club membership. The membership costs $22 per year, and then a member can buy CDs at a reduced price. If a member buys 17 CDs in one year, the cost is $111.25.

   a. Determine the cost of each CD for a member.

   b. Write a linear equation to represent the total cost y of a one year membership, if x CDs are purchased.

   c. Graph this equation.

H.O.T. Problems   Use **H**igher-**O**rder **T**hinking Skills

47. **FIND THE ERROR** Tess and Jacinta are writing an equation of the line through $(3, -2)$ and $(6, 4)$. Is either of them correct? Explain your reasoning.

| Tess	Jacinta
$m = \dfrac{4-(-2)}{6-3} = \dfrac{6}{3}$ or 2	$m = \dfrac{4-(-2)}{6-3} = \dfrac{6}{3}$ or 2
$y = mx + b$	$y = mx + b$
$6 = 2(4) + b$	$-2 = 2(3) + b$
$6 = 8 + b$	$-2 = 6 + b$
$-2 = b$	$-8 = b$
$y = 2x - 2$	$y = 2x - 8$

48. **CHALLENGE** Consider three points, $(3, 7)$, $(-6, 1)$ and $(9, p)$, on the same line. Find the value of p and explain your steps.

49. **REASONING** Consider the standard form of a linear equation, $Ax + By = C$.

   a. Rewrite the equation in slope-intercept form.

   b. What is the slope?

   c. What is the y-intercept?

   d. Is this true for all real values of A, B, and C?

50. **OPEN ENDED** Create a real-world situation that fits the graph at the right. Define the two quantities and describe the functional relationship between them. Write an equation to represent this relationship and describe what the slope and y-intercept mean.

51. **WRITING IN MATH** Linear equations are useful in predicting future events. Describe some factors in real-world situations that might affect the reliability of the graph in making any predictions.

52. **WRITING IN MATH** What information is needed to write the equation of a line? Explain.
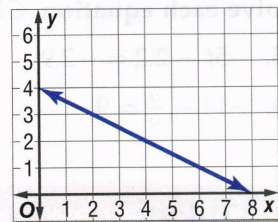
53. Which equation *best* represents the graph?

A $y = 2x$

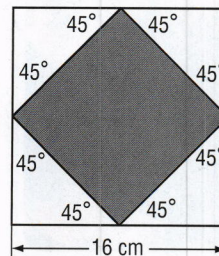B $y = -2x$

C $y = \frac{1}{2}x$

D $y = -\frac{1}{2}x$
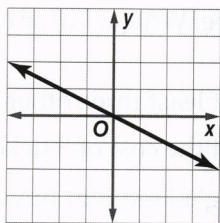
54. Roberto receives an employee discount of 12%. If he buys a $355 item at the store, what is his discount to the nearest dollar?

F $3   H $30

G $4   J $43

55. GEOMETRY The midpoints of the sides of the large square are joined to form a smaller square. What is the area of the smaller square?

A 64 cm^2

B 128 cm^2

C 248 cm^2

D 256 cm^2

56. SHORT RESPONSE If $\frac{5(x+4)}{2} + 7 = 37$, what is the value of $3x - 9$?

Graph each equation. (Lesson 4-1)
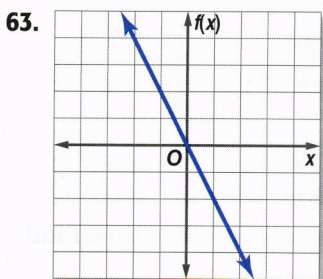
57. $y = 3x + 2$

58. $y = -4x + 2$

59. $3y = 2x + 6$

60. $y = \frac{1}{2}x + 6$

61. $3x + y = -1$

62. $2x + 3y = 6$

Write an equation in function notation for each relation. (Lesson 3-6)

63.

64.
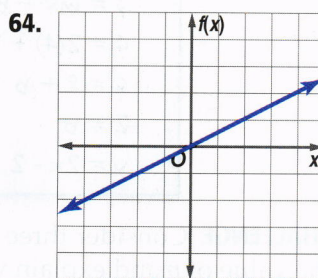
65. METEOROLOGY The distance d in miles that the sound of thunder travels in t seconds is given by the equation $d = 0.21t$. (Lesson 3-1)

a. Graph the equation.

b. Use the graph to estimate how long it will take you to hear thunder from a storm 3 miles away.

Solve each equation. Check your solution. (Lesson 2-3)

66. $-5t - 2.2 = -2.9$

67. $-5.5a - 43.9 = 77.1$

68. $4.2r + 7.14 = 12.6$

69. $-14 - \frac{n}{9} = 9$

70. $\frac{-8b - (-9)}{-10} = 17$

71. $9.5x + 11 - 7.5x = 14$

Find the value of r so the line through each pair of points has the given slope. (Lesson 3-3)

72. $(6, -2), (r, -6), m = 4$

73. $(8, 10), (r, 4), m = 6$

74. $(7, -10), (r, 4), m = -3$

75. $(6, 2), (9, r), m = -1$

76. $(9, r), (6, 3), m = -\frac{1}{3}$

77. $(5, r), (2, -3), m = \frac{4}{3}$

Writing Equations in Point-Slope Form

Why?

Most humane societies have foster homes for newborn puppies, kittens, and injured or ill animals. During the spring and summer, a large shelter can place 3000 animals in homes each month.

If a shelter had 200 animals in foster homes at the beginning of spring, the number of animals in foster homes at the end of the summer could be represented by $y = 3000x + 200$, where x is the number of months and y is the number of animals.
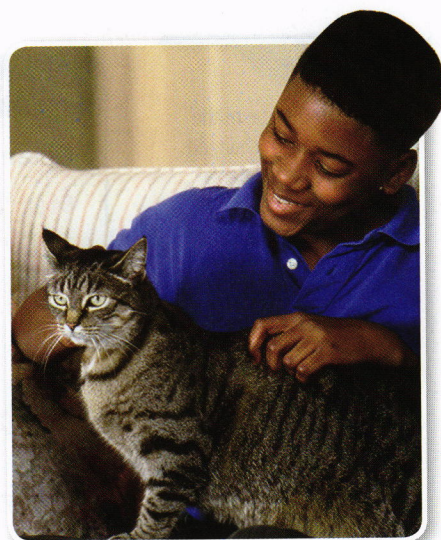
Point-Slope Form An equation of a line can be written in **point-slope form** when given the coordinates of one known point on a line and the slope of that line.
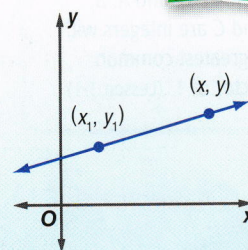
Key Concept — Point-Slope Form

For Your FOLDABLE

Words	The linear equation $y - y_1 = m(x - x_1)$ is written in point-slope form, where (x_1, y_1) is a given point on a nonvertical line and m is the slope of the line.
Symbols	$y - y_1 = m(x - x_1)$

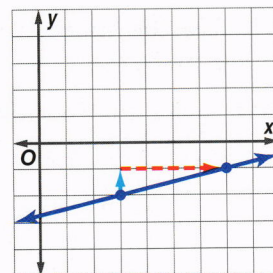### EXAMPLE 1 Write and Graph an Equation in Point-Slope Form

Write an equation in point-slope form for the line that passes through $(3, -2)$ with a slope of $\frac{1}{4}$. Then graph the equation.

$y - y_1 = m(x - x_1)$ **Point-slope form**

$y - (-2) = \frac{1}{4}(x - 3)$ $(x_1, y_1) = (3, -2), m = \frac{1}{4}$

$y + 2 = \frac{1}{4}(x - 3)$ **Simplify.**

Plot the point at $(3, -2)$ and use the slope to find another point on the line. Draw a line through the two points.

✓ Check Your Progress

1. Write an equation in point-slope form for the line that passes through $(-2, 1)$ with a slope of -6. Then graph the equation.

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Forms of Linear Equations If you are given the slope and the coordinates of one or two points, you can write the linear equation in the following ways.

Concept Summary | **Writing Equations** | For Your **FOLDABLE**

Given the Slope and One Point

Step 1 Substitute the value of m and let the x and y coordinates be (x_1, y_1). Or, substitute the values of m, x, and y into the slope-intercept form and solve for b.

Step 2 Rewrite the equation in the needed form.

Given Two Points

Step 1 Find the slope.

Step 2 Choose one of the two points to use.

Step 3 Follow the steps for writing an equation given the slope and one point.

EXAMPLE 2 **Standard Form**

Write $y - 1 = -\frac{2}{3}(x - 5)$ in standard form.

$y - 1 = -\frac{2}{3}(x - 5)$	Original equation
$3(y - 1) = 3\left(-\frac{2}{3}\right)(x - 5)$	Multiply each side by 3 to eliminate the fraction.
$3(y - 1) = -2(x - 5)$	Simplify.
$3y - 3 = -2x + 10$	Distributive Property
$3y = -2x + 13$	Add 3 to each side.
$2x + 3y = 13$	Add 2x to each side.

✔ **Check Your Progress**

2. Write $y - 1 = 7(x + 5)$ in standard form.

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To find the y-intercept of an equation, rewrite the equation in slope-intercept form.

EXAMPLE 3 **Slope-Intercept Form**

Write $y + 3 = \frac{3}{2}(x + 1)$ in slope-intercept form.

$y + 3 = \frac{3}{2}(x + 1)$	Original equation
$y + 3 = \frac{3}{2}x + \frac{3}{2}$	Distributive Property
$y = \frac{3}{2}x - \frac{3}{2}$	Subtract 3 from each side.

✔ **Check Your Progress**

3. Write $y + 6 = -3(x - 4)$ in slope-intercept form.

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Being able to use a variety of forms of linear equations can be useful in other subjects as well.

EXAMPLE 4 **Point-Slope Form and Standard Form**

GEOMETRY The figure shows square $RSTU$.

a. Write an equation in point-slope form for the line containing side $\overline{TU}$.

Step 1 Find the slope of $\overline{TU}$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad \text{Slope Formula}$$

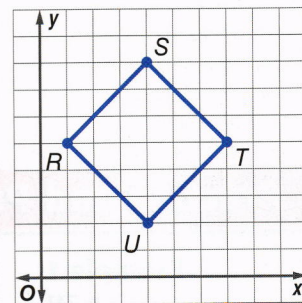$$= \frac{5-2}{7-4} \text{ or } 1 \qquad (x_1, y_1) = (4, 2) \text{ and } (x_2, y_2) = (7, 5)$$

Step 2 You can select either point for (x_1, y_1) in the point-slope form.

$$y - y_1 = m(x - x_1) \qquad \text{Point-slope form}$$

$$y - 2 = 1(x - 4) \qquad (x_1, y_1) = (4, 2)$$

$$y - 5 = 1(x - 7) \qquad (x_1, y_1) = (7, 5)$$

b. Write an equation in standard form for the same line.

$y - 2 = 1(x - 4)$	**Original equation**	$y - 5 = 1(x - 7)$
$y - 2 = 1x - 4$	**Distributive Property**	$y - 5 = 1x - 7$
$y = 1x - 2$	**Add to each side.**	$y = 1x - 2$
$-1x + y = -2$	**Subtract 1x from each side.**	$-1x + y = -2$
$x - y = 2$	**Multiply each side by −1.**	$x - y = 2$

✓ **Check Your Progress**

4A. Write an equation in point-slope form of the line containing side $\overline{ST}$.

4B. Write an equation in standard form of the line containing $\overline{ST}$.

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✓ **Check Your Understanding**

Example 1
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Write an equation in point-slope form for the line that passes through the given point with the slope provided. Then graph the equation.

① $(-2, 5)$, slope -6 **2.** $(-2, -8)$, slope $\frac{5}{6}$ **3.** $(4, 3)$, slope $-\frac{1}{2}$

Example 2
p. 232

Write each equation in standard form.

4. $y + 2 = \frac{7}{8}(x - 3)$ **5.** $y + 7 = -5(x + 3)$ **6.** $y + 2 = \frac{5}{3}(x + 6)$

Example 3
p. 232

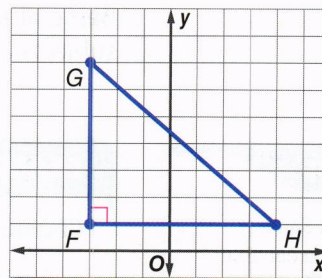Write each equation in slope-intercept form.

7. $y - 10 = 4(x + 6)$ **8.** $y - 7 = -\frac{3}{4}(x + 5)$ **9.** $y - 9 = x + 4$

10. GEOMETRY Use right triangle *FGH*.

   a. Write an equation in point-slope form for the line containing $\overline{GH}$.

   b. Write the standard form of the line containing $\overline{GH}$.

Practice and Problem Solving

Write an equation in point-slope form for the line that passes through each point with the given slope. Then graph the equation.

11. $(5, 3), m = 7$ **12.** $(2, -1), m = -3$ **13.** $(-6, -3), m = -1$

14. $(-7, 6), m = 0$ **15.** $(-2, 11), m = \frac{4}{3}$ **16.** $(-7, -8), m = -\frac{3}{7}$

17. Write an equation in point-slope form for a line that passes through $(-2, -9)$ with a slope of $-\frac{7}{5}$.

18. Write an equation in point-slope form for a horizontal line that passes through $(-6, 0)$.

Write each equation in standard form.

19. $y - 10 = 2(x - 8)$ **20.** $y - 6 = -3(x + 2)$

21. $y - 9 = -6(x + 9)$ **22.** $y + 4 = \frac{2}{3}(x + 7)$

23. $y + 7 = \frac{9}{10}(x + 3)$ **24.** $y + 7 = -\frac{3}{2}(x + 1)$

25. $2y + 3 = -\frac{1}{3}(x - 2)$ **26.** $4y - 5x = 3(4x - 2y + 1)$

Write each equation in slope-intercept form.

27. $y - 6 = -2(x - 7)$ **28.** $y - 11 = 3(x + 4)$

29. $y + 5 = -6(x + 7)$ **30.** $y - 1 = \frac{4}{5}(x + 5)$

31. $y + 2 = \frac{1}{6}(x - 4)$ **32.** $y + 6 = -\frac{3}{4}(x + 8)$

33. $y + 3 = -\frac{1}{3}(2x + 6)$ **34.** $y + 4 = 3(3x + 3)$

35. MOVIE RENTALS The number of copies of a movie rented at a video store decreased at a constant rate of 5 copies per week. The 6th week after the movie was released, 4 copies were rented. How many copies were rented during the second week?

36. CABLE A company offers premium cable for $39.95 per month plus a one-time setup fee. The total cost for setup and 6 months of service is $264.70.

   a. Write an equation in point-slope form to find the total price *y* for any number of months *x*. (*Hint*: The point (6, 264.70) is a solution to the equation.)

   b. Write the equation in slope-intercept form.

   c. What is the setup fee?

Write each equation in standard form.

37. $y + 8 = -\frac{11}{12}(x - 14)$ **38.** $y - 3 = 2.5(x + 1)$ **39.** $y + 2.1 = 1.4(x - 5)$

Write an equation in point-slope form for each line.
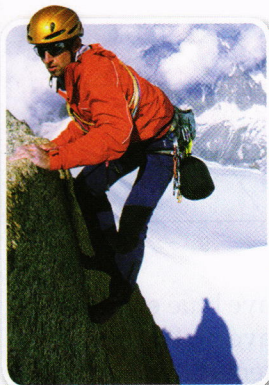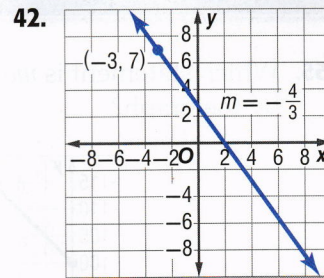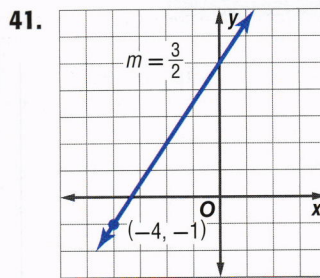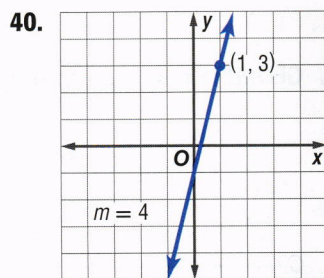
40.

(1, 3)
$m = 4$

41.

$m = \dfrac{3}{2}$
$(-4, -1)$

42.

$(-3, 7)$
$m = -\dfrac{4}{3}$

Write each equation in slope-intercept form.

43 $y + \dfrac{3}{5} = x - \dfrac{2}{5}$

44. $y - \dfrac{7}{2} = \dfrac{1}{2}(x - 4)$

45. $y + \dfrac{1}{3} = \dfrac{5}{6}\left(x + \dfrac{2}{5}\right)$

46. Write an equation in point-slope form, slope-intercept form, and standard form for a line that passes through $(-2, 8)$ with slope $\dfrac{8}{5}$.

47. Line ℓ passes through $(-9, 4)$ with slope $\dfrac{4}{7}$. Write an equation in point-slope form, slope-intercept form, and standard form for line ℓ.

48. WEATHER Barometric pressure is a linear function of altitude. The barometric pressure is 598 millimeters of mercury (mmHg) at an altitude of 1.8 kilometers. The pressure is 577 millimeters of mercury at 2.1 kilometers.

 a. Write a formula for the barometric pressure as a function of the altitude.

 b. What is the altitude if the pressure is 657 millimeters of mercury?

H.O.T. Problems   Use **H**igher-**O**rder **T**hinking Skills

49. WHICH ONE DOESN'T BELONG? Identify the equation that does not belong. Explain your reasoning.

$y - 5 = 3(x - 1)$	$y + 1 = 3(x + 1)$	$y + 4 = 3(x + 1)$	$y - 8 = 3(x - 2)$

50. FIND THE ERROR Juana and Sabrina wrote an equation in point-slope form for the line that passes through $(3, -7)$ and $(-6, 4)$. Is either of them correct? Explain.

Juana
$y - 7 = -\dfrac{11}{9}(x + 3)$

Sabrina
$y - 4 = -\dfrac{9}{11}(x + 6)$

51. OPEN ENDED Describe a real-life scenario that has a constant rate of change and a value of y for a particular value of x. Represent this situation using an equation in point-slope form and an equation in slope-intercept form.

52. REASONING Write an equation for the line that passes through $(-4, 8)$ and $(3, -7)$. What is the slope? Where does the line intersect the x-axis? the y-axis?

53. CHALLENGE Write an equation in point-slope form for the line that passes through the points (f, g) and (h, j).

54. WRITING IN MATH Demonstrate how you can use the Slope Formula to write the point-slope form of an equation of a line.

55. Which statement is *most* strongly supported by the graph?
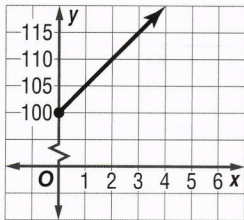
    A You have $100 and spend $5 weekly.
    B You have $100 and save $5 weekly.
    C You need $100 for a new CD player and save $5 weekly.
    D You need $100 for a new CD player and spend $5 weekly.

56. SHORT RESPONSE A store offers customers a $5 gift certificate for every $75 they spend. How much would a customer have to spend to earn $35 worth of gift certificates?

57. GEOMETRY Which triangle is similar to $\triangle ABC$?
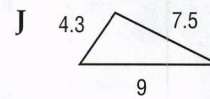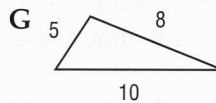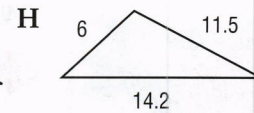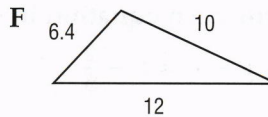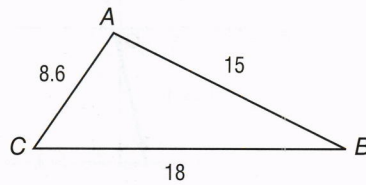
58. In a class of 25 students, 6 have blue eyes, 15 have brown hair, and 3 have blue eyes and brown hair. How many students have neither blue eyes nor brown hair?

    A 4   **C** 10
    B 7   **D** 22

Spiral Review

Write an equation of the line that passes through each pair of points. (Lesson 4-2)

59. $(4, 2), (-2, -4)$
60. $(3, -2), (6, 4)$
61. $(-1, 3), (2, -3)$

62. $(2, -2), (3, 2)$
63. $(7, -2), (-4, -2)$
64. $(0, 5), (-3, 5)$

Write an equation in slope-intercept form of the line with the given slope and y-intercept. (Lesson 4-1)

65. slope: -2, y-intercept: 6
66. slope: 3, y-intercept: -5
67. slope: $\frac{1}{2}$, y-intercept: 3

68. slope: $-\frac{3}{5}$, y-intercept: 12
69. slope: 0, y-intercept: 3
70. slope: -1, y-intercept: 0

71. THEATER The Coral Gables Actors' Playhouse has 7 rows of seats in the orchestra section. The number of seats in the rows forms an arithmetic sequence, as shown in the table. On opening night, 368 tickets were sold for the orchestra section. Was the section oversold? (Lesson 3-5)

Rows	Number of Seats
7	76
6	68
5	60

Skills Review

Solve each equation or formula for the variable specified. (Lesson 2-7)

72. $y = mx + b$, for m

73. $v = r + at$, for a

74. $km + 5x = 6y$, for m

75. $4b - 5 = -t$, for b

Parallel and Perpendicular Lines

Why?

Notice the squares, rectangles and lines in the piece of art shown at the right. Some of the lines intersect forming right angles. Other lines do not intersect at all.
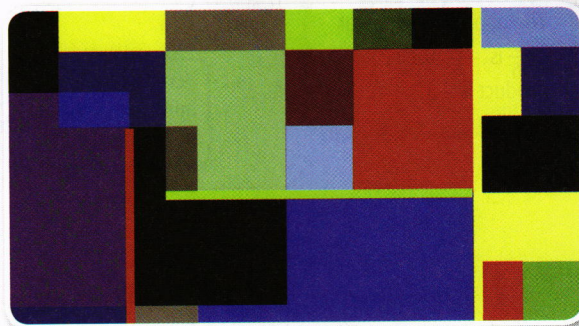
Parallel Lines Lines in the same plane that do not intersect are called **parallel lines**. Nonvertical parallel lines have the same slope.

All vertical lines are parallel.

If two nonvertical lines in a plane have the same slope, then they are parallel.
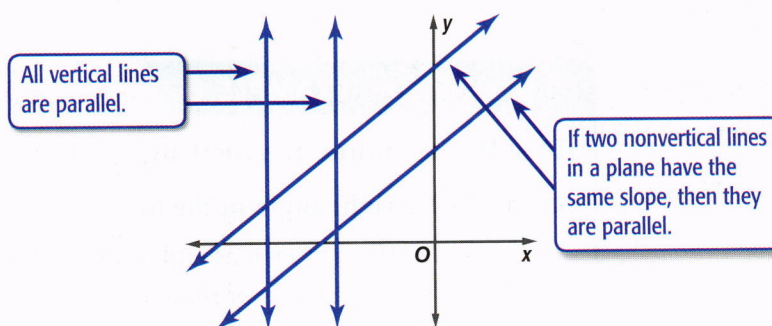
You can write an equation of a line parallel to a given line if you know a point on the line and an equation of the given line. First find the slope of the given line. Then, substitute the point provided and the slope from the given line into the point-slope form.

EXAMPLE 1 **Parallel Line Through a Given Point**

Write an equation in slope-intercept form for the line that passes through $(-3, 5)$ and is parallel to the graph of $y = 2x - 4$.

Step 1 The slope of the line with equation $y = 2x - 4$ is 2. The line parallel to $y = 2x - 4$ has the same slope, 2.

Step 2 Find the equation in slope-intercept form.

$y - y_1 = m(x - x_1)$	**Point-slope form**
$y - 5 = 2[x - (-3)]$	**Replace m with 2 and (x_1, y_1) with $(-3, 5)$.**
$y - 5 = 2(x + 3)$	**Simplify.**
$y - 5 = 2x + 6$	**Distributive Property**
$y - 5 + 5 = 2x + 6 + 5$	**Add 5 to each side.**
$y = 2x + 11$	**Write the equation in slope-intercept form.**

✔ **Check Your Progress**

1. Write an equation in point-slope form for the line that passes through $(4, -1)$ and is parallel to the graph of $y = \frac{1}{4}x + 7$.
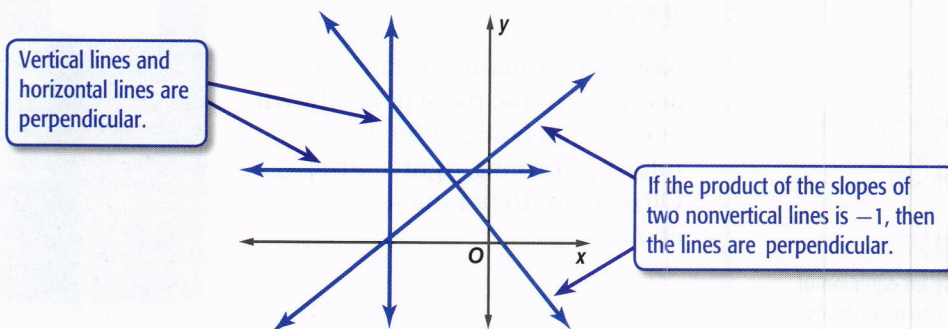
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Perpendicular Lines Lines that intersect at right angles are called **perpendicular lines**. The slopes of nonvertical perpendicular lines are opposite reciprocals. That is, if the slope of a line is 4, the slope of the line perpendicular to it is $-\frac{1}{4}$.

Vertical lines and horizontal lines are perpendicular.

If the product of the slopes of two nonvertical lines is -1, then the lines are perpendicular.

You can use slope to determine whether two lines are perpendicular.

🌐 Real-World EXAMPLE 2   Slopes of Perpendicular Lines

DESIGN The outline of a company's new logo is shown on a coordinate plane.

a. Is ∠DFE a right angle in the logo?

If $\overline{BE}$ and $\overline{AD}$ are perpendicular, then ∠DFE is a right angle. Find the slopes of $\overline{BE}$ and $\overline{AD}$.

slope of $\overline{BE}$: $m = \frac{1-3}{7-2}$ or $-\frac{2}{5}$

slope of $\overline{AD}$: $m = \frac{6-1}{4-2}$ or $\frac{5}{2}$

The line segments are perpendicular because $-\frac{2}{5} \times \frac{5}{2} = -1$. Therefore, ∠DFE is a right angle.

b. Is each pair of opposite sides parallel?

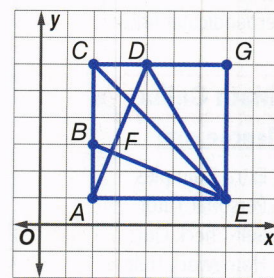If a pair of opposite sides are parallel, then they have the same slope.

slope of $\overline{AC}$: $m = \frac{6-1}{2-2}$ or undefined

Since $\overline{AC}$ and $\overline{GE}$ are both parallel to the y-axis, they are vertical and are therefore parallel.

slope of $\overline{CG}$: $m = \frac{6-6}{7-2}$ or 0

Since $\overline{CG}$ and $\overline{AE}$ are both parallel to the x-axis, they are horizontal and are therefore parallel.

✔️ Check Your Progress

2. CONSTRUCTION On the plans for a treehouse, a beam represented by $\overline{QR}$ has endpoints $Q(-6, 2)$ and $R(-1, 8)$. A connecting beam represented by $\overline{ST}$ has endpoints $S(-3, 6)$ and $T(-8, 5)$. Are the beams perpendicular? Explain.

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You can determine whether the graphs of two linear equations are parallel or perpendicular by comparing the slopes of the lines.
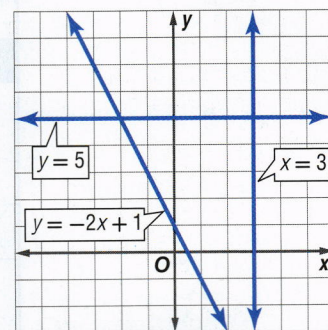
EXAMPLE 3   Parallel or Perpendicular Lines

Determine whether the graphs of $y = 5$, $x = 3$, and $y = -2x + 1$ are *parallel* or *perpendicular*. Explain.

Graph each line on a coordinate plane.

From the graph, you can see that $y = 5$ is parallel to the x-axis and $x = 3$ is parallel to the y-axis. Therefore, they are perpendicular. None of the lines are parallel.

✓ **Check Your Progress**

3. Determine whether the graphs of $6x - 2y = -2$, $y = 3x - 4$, and $y = 4$ are *parallel* or *perpendicular*. Explain.

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You can write the equation of a line perpendicular to a given line if you know a point on the line and the equation of the given line.

EXAMPLE 4   Perpendicular Line Through a Given Point

Write an equation in slope-intercept form for the line that passes through $(-4, 6)$ and is perpendicular to the graph of $2x + 3y = 12$.

Step 1   Find the slope of the given line by solving the equation for y.

$$2x + 3y = 12 \qquad \text{Original equation}$$

$$2x - 2x + 3y = -2x + 12 \qquad \text{Subtract 2x from each side.}$$

$$3y = -2x + 12 \qquad \text{Simplify.}$$

$$\frac{3y}{3} = \frac{-2x + 12}{3} \qquad \text{Divide each side by 3.}$$

$$y = -\frac{2}{3}x + 4 \qquad \text{Simplify.}$$

The slope is $-\frac{2}{3}$.

Step 2   The slope of the perpendicular line is the opposite reciprocal of $-\frac{2}{3}$ or $\frac{3}{2}$. Find the equation of the perpendicular line.

$$y - y_1 = m(x - x_1) \qquad \text{Point-slope form}$$

$$y - 6 = \frac{3}{2}(x - (-4)) \qquad (x_1, y_1) = (-4, 6) \text{ and } m = \frac{3}{2}$$

$$y - 6 = \frac{3}{2}(x + 4) \qquad \text{Simplify.}$$

$$y - 6 = \frac{3}{2}x + 6 \qquad \text{Distributive Property}$$

$$y - 6 + 6 = \frac{3}{2}x + 6 + 6 \qquad \text{Add 6 to each side.}$$

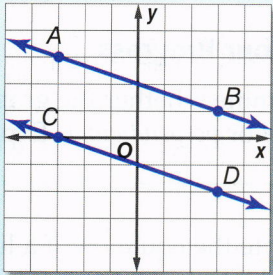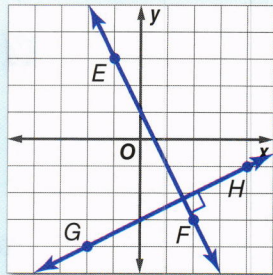$$y = \frac{3}{2}x + 12 \qquad \text{Simplify.}$$

✓ **Check Your Progress**

4. Write an equation in slope-intercept form for the line that passes through $(4, 7)$ and is perpendicular to the graph of $y = \frac{2}{3}x - 1$.

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Concept Summary — Parallel and Perpendicular Lines

For Your FOLDABLE

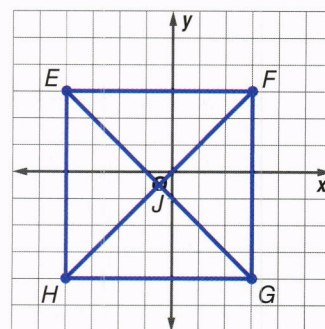|	Parallel Lines	Perpendicular Lines
Words	Two nonvertical lines are parallel if they have the same slope.	Two nonvertical lines are perpendicular if the product of their slopes is −1.
Symbols	$\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$	$\overleftrightarrow{EF} \perp \overleftrightarrow{GH}$
Models		

Check Your Understanding

Example 1
p. 237

Write an equation in slope-intercept form for the line that passes through the given point and is parallel to the graph of the given equation.

1. $(-1, 2)$, $y = \frac{1}{2}x - 3$

2. $(0, 4)$, $y = -4x + 5$

Example 2
p. 238

3. GARDENS A garden is in the shape of a quadrilateral with vertices $A(-2, 1)$, $B(3, -3)$, $C(5, 7)$, and $D(-3, 4)$. Two paths represented by $\overline{AC}$ and $\overline{BD}$ cut across the garden. Are the paths perpendicular? Explain.

4. GEOMETRY A square is a quadrilateral that has opposite sides parallel, consecutive sides that are perpendicular, and diagonals that are perpendicular. Determine whether the quadrilateral is a square. Explain.

Example 3
p. 239

Determine whether the graphs of the following equations are *parallel* or *perpendicular*. Explain.

5 $y = -2x$, $2y = x$, $4y = 2x + 4$

6. $y = \frac{1}{2}x$, $3y = x$, $y = -\frac{1}{2}x$

Example 4
p. 239

Write an equation in slope-intercept form for the line that passes through the given point and is perpendicular to the graph of the equation.

7. $(-2, 3)$, $y = -\frac{1}{2}x - 4$

8. $(-1, 4)$, $y = 3x + 5$

9. $(2, 3)$, $2x + 3y = 4$

10. $(3, 6)$, $3x - 4y = -2$

Practice and Problem Solving

● = **Step-by-Step Solutions** begin on page R12.
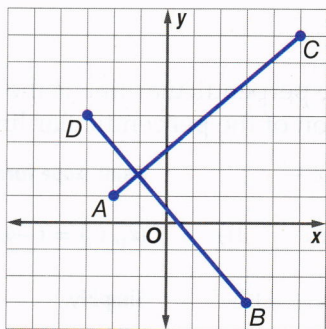**Extra Practice** begins on page 815.

Example 1
p. 237

Write an equation in slope-intercept form for the line that passes through the given point and is parallel to the graph of the given equation.
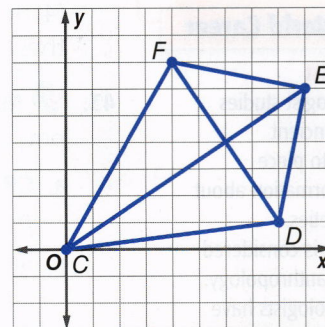
11. $(3, -2), y = x + 4$ **12.** $(4, -3), y = 3x - 5$ **13.** $(0, 2), y = -5x + 8$

14. $(-4, 2), y = -\frac{1}{2}x + 6$ **15.** $(-2, 3), y = -\frac{3}{4}x + 4$ **16.** $(9, 12), y = 13x - 4$

Example 2
p. 238

17. GEOMETRY A trapezoid is a quadrilateral that has exactly one pair of parallel opposite sides. Is *ABCD* a trapezoid? Explain your reasoning.
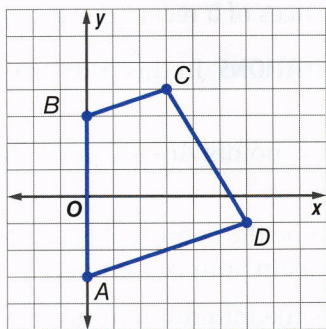
18. GEOMETRY *CDEF* is a kite. Are the diagonals of the kite perpendicular? Explain your reasoning.

19. Determine whether the graphs of $y = -6x + 4$ and $y = \frac{1}{6}x$ are perpendicular. Explain.

20. MAPS On a map, Elmwood Drive passes through $R(4, -11)$ and $S(0, -9)$, and Taylor Road passes through $J(6, -2)$ and $K(4, -5)$. If they are straight lines, are the two streets perpendicular? Explain.

Example 3
p. 239

Determine whether the graphs of the following equations are *parallel* or *perpendicular*. Explain.

21. $2x - 8y = -24, 4x + y = -2, x - 4y = 4$

22. $3x - 9y = 9, 3y = x + 12, 2x - 6y = 12$

Example 4
p. 239

Write an equation in slope-intercept form for the line that passes through the given point and is perpendicular to the graph of the equation.

23 $(-3, -2), y = -2x + 4$ **24.** $(-5, 2), y = \frac{1}{2}x - 3$ **25.** $(-4, 5), y = \frac{1}{3}x + 6$

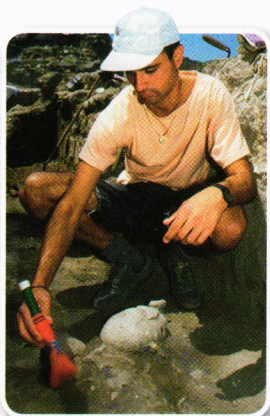**26.** $(2, 6), y = -\frac{1}{4}x + 3$ **27.** $(3, 8), y = 5x - 3$ **28.** $(4, -2), y = 3x + 5$

Write an equation in slope-intercept form for a line perpendicular to the graph of the equation that passes through the *x*-intercept of that line.

29. $y = -\frac{1}{2}x - 4$ **30.** $y = \frac{2}{3}x - 6$ **31.** $y = 5x + 3$

32. Write an equation in slope-intercept form for the line that is perpendicular to the graph of $3x + 2y = 8$ and passes through the *y*-intercept of that line.

Determine whether the graphs of each pair of equations are *parallel*, *perpendicular*, or *neither*.

33. $y = 4x + 3$
$4x + y = 3$

34. $y = -2x$
$2x + y = 3$

35. $3x + 5y = 10$
$5x - 3y = -6$

36. $-3x + 4y = 8$
$-4x + 3y = -6$

37. $2x + 5y = 15$
$3x + 5y = 15$

38. $2x + 7y = -35$
$4x + 14y = -42$

39. Write an equation of the line that is parallel to the graph of $y = 7x - 3$ and passes through the origin.

40. **EXCAVATION** Scientists excavating a dinosaur mapped the site on a coordinate plane. If one bone lies from $(-5, 8)$ to $(10, -1)$ and a second bone lies from $(-10, -3)$ to $(-5, -6)$, are the bones parallel? Explain.

41 **ARCHAEOLOGY** In the ruins of an ancient civilization, an archaeologist found pottery at $(2, 6)$ and hair accessories at $(4, -1)$. A pole is found with one end at $(7, 10)$ and the other end at $(14, 12)$. Is the pole perpendicular to the line through the pottery and the hair accessories? Explain.

42. **GRAPHICS** To create a design on a computer, Andeana must enter the coordinates for points on the design. One line segment she drew has endpoints of $(-2, 1)$ and $(4, 3)$. The other coordinates that Andeana entered are $(2, -7)$ and $(8, -3)$. Could these points be the vertices of a rectangle? Explain.

43. **MULTIPLE REPRESENTATIONS** In this problem, you will explore parallel and perpendicular lines.

a. GRAPHICAL Graph the points $A(-3, 3)$, $B(3, 5)$, and $C(-4, 0)$ on a coordinate plane.

b. ANALYTICAL Determine the coordinates of a fourth point D that would form a parallelogram. Explain your reasoning.

c. ANALYTICAL What is the minimum number of points that could be moved to make the parallelogram a rectangle? Describe which points should be moved, and explain why.

H.O.T. Problems Use **H**igher-**O**rder **T**hinking Skills

44. **CHALLENGE** If the line through $(-2, 4)$ and $(5, d)$ is parallel to the graph of $y = 3x + 4$, what is the value of d?

45. **REASONING** Is a horizontal line perpendicular to a vertical line *sometimes*, *always*, or *never*? Explain your reasoning.

46. **OPEN ENDED** Graph a line that is parallel and a line that is perpendicular to $y = 2x - 1$.

47. **FIND THE ERROR** Carmen and Chase are finding an equation of the line that is perpendicular to the graph of $y = \frac{1}{3}x + 2$ and passes through the point $(-3, 5)$. Is either of them correct? Explain your reasoning.
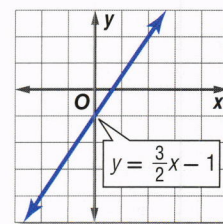
Carmen	Chase
$y - 5 = -3[x - (-3)]$	$y - 5 = 3[x - (-3)]$
$y - 5 = -3(x + 3)$	$y - 5 = 3(x + 3)$
$y = -3x - 9 + 5$	$y = 3x + 9 + 5$
$y = -3x - 4$	$y = -3x + 14$

48. **WRITING IN MATH** Illustrate how you can determine whether two lines are parallel or perpendicular. Write an equation for the graph that is parallel and an equation for the graph that is perpendicular to the line shown. Explain your reasoning.

$y = \frac{3}{2}x - 1$

49. Which of the following is an algebraic translation of the following phrase?

5 less than the quotient of a number and 8

A $5 - \frac{n}{8}$ **C** $5 - \frac{8}{n}$

B $\frac{n}{8} - 5$ **D** $\frac{8}{n} - 5$

50. A line through which two points would be parallel to a line with a slope of $\frac{3}{4}$?

F $(0, 5)$ and $(-4, 2)$ **H** $(0, 0)$ and $(0, -2)$

G $(0, 2)$ and $(-4, 1)$ **J** $(0, -2)$ and $(-4, -2)$

51. Which equation best fits the data in the table?

x	y
1	5
2	7
3	9
4	11

A $y = x + 4$
B $y = 2x + 3$
C $y = 7$
D $y = 4x - 5$

52. SHORT RESPONSE Tyler is filling his 6000-gallon pool at a constant rate. After 4 hours, the pool contained 800 gal. How many total hours will it take to completely fill the pool?

Spiral Review

Write each equation in standard form. (Lesson 4-3)

53. $y - 13 = 4(x - 2)$ **54.** $y - 5 = -2(x + 2)$ **55.** $y + 3 = -5(x + 1)$

56. $y + 7 = \frac{1}{2}(x + 2)$ **57.** $y - 1 = \frac{5}{6}(x - 4)$ **58.** $y - 2 = -\frac{2}{5}(x - 8)$

59. CANOE RENTAL Latanya and her friends rented a canoe for 3 hours and paid a total of $45. (Lesson 4-2)

a. Write a linear equation to find the total cost C of renting the canoe for h hours.

b. How much would it cost to rent the canoe for 8 hours?

Canoe Rentals
Daily rates
plus $10 per hour

Write an equation of the line that passes through each point with the given slope. (Lesson 4-2)

60. $(5, -2), m = 3$ **61.** $(-5, 4), m = -5$ **62.** $(3, 0), m = -2$

63. $(3, 5), m = 2$ **64.** $(-3, -1), m = -3$ **65.** $(-2, 4), m = -5$

Simplify each expression. If not possible, write *simplified*. (Lesson 1-4)

66. $13m + m$ **67.** $14a^2 + 13b^2 + 27$ **68.** $3(x + 2x)$

69. FINANCIAL LITERACY At a Farmers' Market, merchants can rent a small table for $5.00 and a large table for $8.50. One time, 25 small and 10 large tables were rented. Another time, 35 small and 12 large were rented. (Lesson 1-2)

a. Write an expression to show the total amount of money collected.

b. Evaluate the expression.

Skills Review

Express each relation as a graph. Then determine the domain and range. (Lesson 1-6)
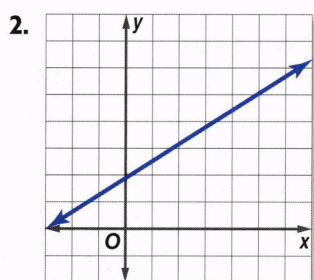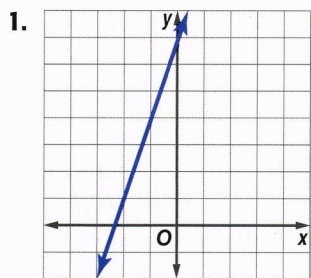
70. $\{(3, 8), (3, 7), (2, -9), (1, -9), (-5, -3)\}$

71. $\{(3, 4), (4, 3), (2, 2), (5, -4), (-4, 5)\}$

72. $\{(0, 2), (-5, 1), (0, 6), (-1, 9), (-4, -5)\}$

73. $\{(-7, 6), (-3, -4), (4, -5), (-2, 6), (-3, 2)\}$

Write an equation in slope-intercept form for each graph shown. (Lesson 4-1)

1.

2.

Graph each equation. (Lesson 4-1)

3. $y = 2x + 3$

4. $y = \frac{1}{3}x - 2$

5. BOATS Write an equation in slope-intercept form for the total rental cost C for a pontoon boat used for t hours. (Lesson 4-1)

Pontoon Boat Rentals
$60 per hour
plus
$20 cleaning fee

Write an equation of the line with the given conditions. (Lesson 4-2)

6. $(2, 5)$; slope 3

7. $(-3, -1)$, slope $\frac{1}{2}$

8. $(-3, 4)$, $(1, 12)$

9. $(-1, 6)$, $(2, 4)$

10. $(2, 1)$, slope 0

11. MULTIPLE CHOICE Write an equation of the line that passes through the point $(0, 0)$ and has slope -4. (Lesson 4-2)

A $y = x - 4$ **C** $y = -4x$

B $y = x + 4$ **D** $y = 4 - x$

Write an equation in point-slope form for the line that passes through each point with the given slope. (Lesson 4-3)

12. $(1, 4)$, $m = 6$ **13.** $(-2, -1)$, $m = -3$

14. Write an equation in point-slope form for the line that passes through the point $(8, 3)$, $m = -2$. (Lesson 4-3)

15. Write $y + 3 = \frac{1}{2}(x - 5)$ in standard form. (Lesson 4-3)

16. Write $y + 4 = -7(x - 3)$ in slope-intercept form. (Lesson 4-3)

Write each equation in standard form. (Lesson 4-3)

17. $y - 5 = -2(x - 3)$ **18.** $y + 4 = \frac{2}{3}(x - 3)$

Write each equation in slope-intercept form. (Lesson 4-3)

19. $y - 3 = 4(x + 3)$ **20.** $y + 1 = \frac{1}{2}(x - 8)$

21. MULTIPLE CHOICE Determine whether the graphs of the pair of equations are *parallel*, *perpendicular*, or *neither*. (Lesson 4-4)

$$y = -6x + 8$$
$$3x + \frac{1}{2}y = -3$$

F parallel

G perpendicular

H neither

J not enough information

Write an equation in slope-intercept form for the line that passes through the given point and is perpendicular to the graph of the equation. (Lesson 4-4)

22. $(3, -4)$; $y = -\frac{1}{3}x - 5$

23. $(0, -3)$; $y = -2x + 4$

24. $(-4, -5)$; $-4x + 5y = -6$

25. $(-1, -4)$; $-x - 2y = 0$

Scatter Plots and Lines of Fit

Why?

The graph shows the number of people from the United States who travel to other countries. The points do not all lie on the same line; however, you may be able to draw a line that is close to all of the points. That line would show a linear relationship between the year x and the number of travelers each year y. Generally, international travel has increased.

International Travel
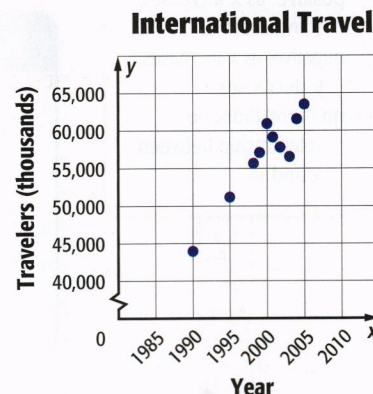
Source: *Statistical Abstract of the United States*

Investigate Relationships Using Scatter Plots Data with two variables are called **bivariate data**. A **scatter plot** shows the relationship between a set of data with two variables, graphed as ordered pairs on a coordinate plane. Scatter plots are used to investigate a relationship between two quantities.
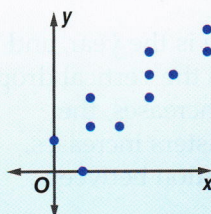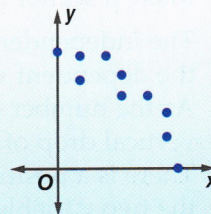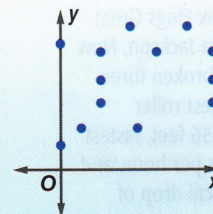
Concept Summary — Scatter Plots

For Your **FOLDABLE**

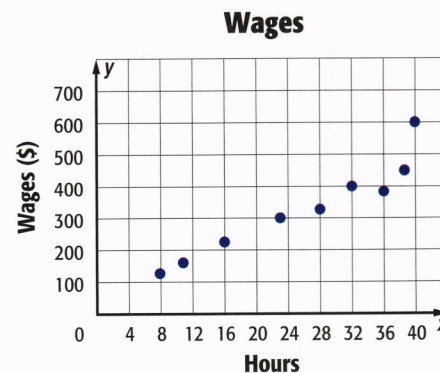**Positive Correlation**

Negative Correlation

No Correlation

🌎 Real-World EXAMPLE 1 — Evaluate a Correlation

WAGES Determine whether the graph shows a *positive*, *negative*, or *no* correlation. If there is a positive or negative correlation, describe its meaning in the situation.

Wages

The graph shows a positive correlation. As the number of hours worked increases, the wages usually increase.

✔ Check Your Progress

1. Refer to the graph on international travel. Determine whether the graph shows a *positive*, *negative*, or *no* correlation. If there is a positive or negative correlation, describe its meaning.

▶ **Personal Tutor** glencoe.com

Use Lines of Fit Scatter plots can show whether there is a trend in a set of data. When the data points all lie close to a line, a **line of fit** or *trend line* can model the trend.

Key Concept   Using a Linear Function to Model Data   For Your FOLDABLE

Step 1   Make a scatter plot. Determine whether any relationship exists in the data.

Step 2   Draw a line that seems to pass close to most of the data points.

Step 3   Use two points on the line of fit to write an equation for the line.

Step 4   Use the line of fit to make predictions.

▶ **Math *in Motion*, Interactive Lab** glencoe.com
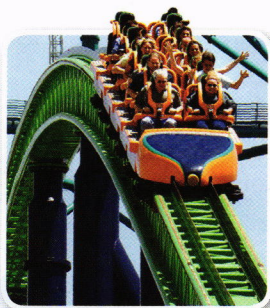
🌐 **Real-World EXAMPLE 2**   **Write a Line of Fit**

ROLLER COASTERS The table shows the largest vertical drops of nine roller coasters in the United States and the number of years after 1988 that they were opened. Identify the independent and the dependant variables. Is there a relationship in the data? If so, predict the vertical drop in a roller coaster built 25 years after 1988.

Years Since 1988	1	3	5	8	12	12	12	13	15
Vertical Drop (ft)	151	155	225	230	306	300	255	255	400

Source: Ultimate Roller Coaster

Step 1   Make a scatter plot.

The independent variable is the year, and the dependent variable is the vertical drop. As the number of years increases, the vertical drop of roller coasters increases. There is a positive correlation between the two variables.

Vertical Drops of Roller Coasters

Step 2   Draw a line of fit.

No one line will pass through all of the data points. Draw a line that passes close to the points. A line of fit is shown.

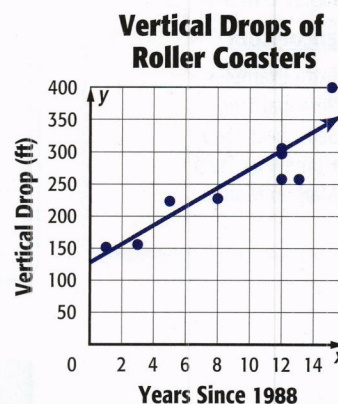**Step 3**   Write the slope-intercept form of an equation for the line of fit.

The line of fit passes close to (2, 150) and the data point (12, 300).

Find the slope.

$m = \dfrac{y_2 - y_1}{x_2 - x_1}$ $(x_1, y_1) = (2, 150),$
               $(x_2, y_2) = (12, 300)$

$= \dfrac{300 - 150}{12 - 2}$

$= \dfrac{150}{10}$ or 15

Use $m = 15$ and either the point-slope form or the slope-intercept form to write the equation of the line of fit.

$y - y_1 = m(x - x_1)$

$y - 150 = 15(x - 2)$

$y - 150 = 15x - 30$

$y = 15x + 120$

A slope of 15 means that the vertical drops increased an average of 15 feet per year. To predict the vertical drop of a roller coaster built 25 years after 1988, substitute 25 for x in the equation. The vertical drop is $15(25) + 120$ or 495 feet.

2. **MUSIC** The table shows the dollar value in millions for the sales of CDs for the year. Make a scatter plot and determine what relationship exists, if any.

Year	2000	2001	2002	2003	2004	2005
Sales	13,215	12,909	12,044	11,233	11,447	10,520

▶ **Personal Tutor glencoe.com**

In Lesson 4-2, you learned that linear extrapolation is used to predict values *outside* the range of the data. You can also use a linear equation to predict values *inside* the range of the data. This is called **linear interpolation**.

🌐 **Real-World EXAMPLE 3**   **Use Interpolation or Extrapolation**

TRAVEL Use the scatter plot to find the approximate number of United States travelers to international countries in 1996.

International Travel

Step 1 Draw a line of fit. The line should be as close to as many points as possible.

Step 2 Write the slope-intercept form of the equation. The line of fit passes through $(0, 44{,}623)$ and $(15, 63{,}866)$.

Source: Statistical Abstract of the United States

Find the slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad \text{Slope Formula}$$

$$= \frac{63{,}866 - 44{,}623}{15 - 0} \qquad \begin{array}{l}(x_1, y_1) = (0, 44{,}623), \\ (x_2, y_2) = (15, 63{,}866)\end{array}$$

$$= \frac{19{,}243}{15} \qquad \text{Simplify.}$$

Use $m = \dfrac{19{,}243}{15}$ and either the point-slope form or the slope-intercept form to write the equation of the line of fit.

$$y - y_1 = m(x - x_1)$$

$$y - 44{,}623 = \frac{19{,}243}{15}(x - 0)$$

$$y - 44{,}623 = \frac{19{,}243}{15}x$$

$$y = \frac{19{,}243}{15}x + 44{,}623$$

Step 3 Evaluate the function for $x = 1996 - 1990$ or 6.

$$y = \frac{19{,}243}{15}x + 44{,}623 \qquad \text{Equation of best-fit line}$$

$$= \frac{19{,}243}{15}(6) + 44{,}623 \qquad x = 6$$

$$= 7697\frac{1}{5} + 44{,}623 \text{ or } 52{,}320\frac{1}{5} \qquad \text{Add.}$$

In 1996, there were approximately 52,320 thousand or 52,320,000 people who traveled from the United States to international countries.

✔ **Check Your Progress**

3. **MUSIC** Use the equation for the line of fit in Check Your Progress 2 to estimate CD sales in 2015.
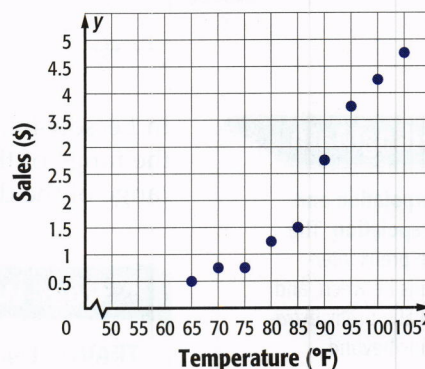
▶ **Personal Tutor glencoe.com**

Example 1
p. 245

Determine whether each graph shows a *positive*, *negative*, or *no* correlation. If there is a positive or negative correlation, describe its meaning in the situation.

1.   **Free Throws**

Free Throws Made (%) (y-axis)
Practice Minutes Per Day (x-axis)

2.   **Lemonade Sales**
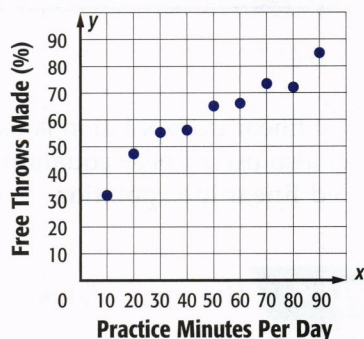
Sales ($) (y-axis)
Temperature (°F) (x-axis)

Example 2
p. 246

3. MARRIAGE The table shows the median age of females when they were first married.

a. Make a scatter plot and determine what relationship exists, if any, in the data. Identify the independent and the dependant variables.

b. Draw a line of fit for the scatter plot.

c. Write an equation in slope-intercept form for the line of fit.

Example 3
p. 247

d. Predict what the median age of females when they are first married will be in 2016.

e. Do you think the equation can give a reasonable estimate for the year 2056? Explain.

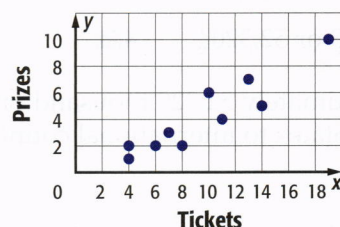| Year	Age
1996	24.8
1997	25.0
1998	25.0
1999	25.1
2000	25.1
2001	25.1
2002	25.3
2003	25.3
2005	25.5
2006	25.9

Source: U.S. Bureau of Census

Practice and Problem Solving

● = **Step-by-Step Solutions** begin on page R12.
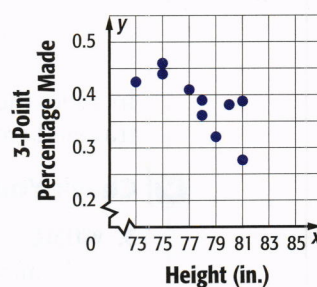**Extra Practice** begins on page 815.

Example 1
p. 245

Determine whether each graph shows a *positive*, *negative*, or *no* correlation. If there is a positive or negative correlation, describe its meaning in the situation.
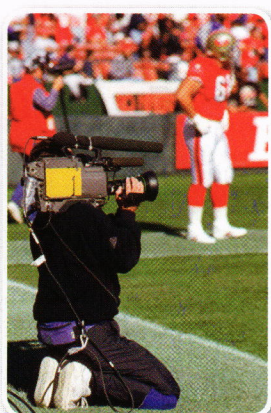
4.   **Game Tickets at the Fair**

Prizes (y-axis)
Tickets (x-axis)

5   **NBA 3-Point Percentage**

3-Point Percentage Made (y-axis)
Height (in.) (x-axis)

6.

Salaries

Years of Formal Education

7

Gas Mileage of Various Vehicles

Speed of Vehicle (mph)

Examples 2 and 3
pp. 246-247
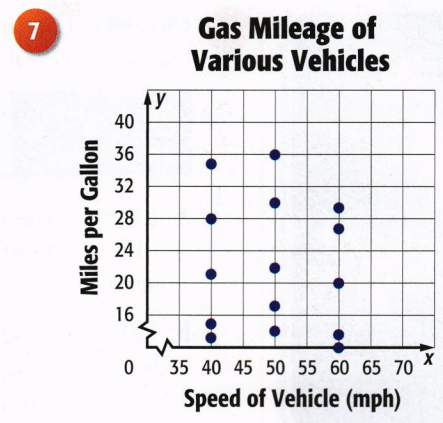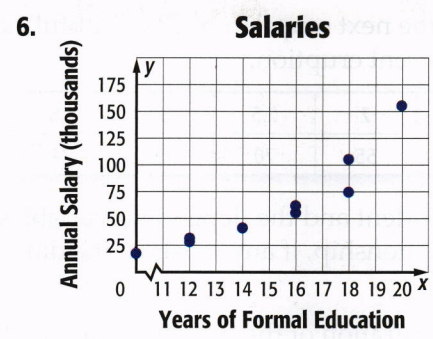
8. MILK Refer to the scatter plot of gallons of milk consumption per person for selected years.

a. Use the points (2, 21.75) and (4, 21) to write the slope-intercept form of an equation for the line of fit.

b. Predict the milk consumption in 2015.

c. Predict in what year milk consumption will be 10 gallons.

d. Is it reasonable to use the equation to estimate the consumption of milk for any year? Explain.

Consumption of Milk in Gallons

Years Since 2000
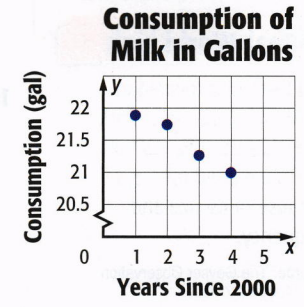
9. FOOTBALL Use the scatter plot.

a. Use the points (3, 73,000) and (6, 67,650) to write the slope-intercept form of an equation for the line of fit shown in the scatter plot.

b. Predict the average attendance at a game in 2012.

c. Can you use the equation to make a decision about the average attendance in any given year in the future? Explain.

Buffalo Bills Attendance

Years Since 2000
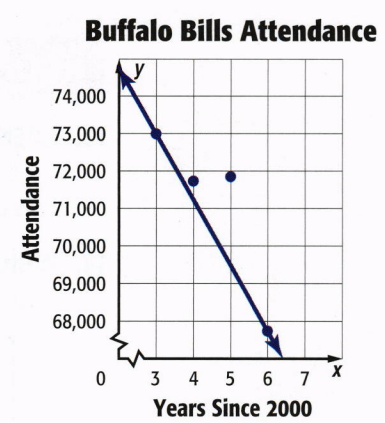
10. WEIGHT The Body Mass Index (BMI) is a measure of body fat using height and weight. The heights and weights of twelve men with normal BMI are given in the table at the right.

a. Make a scatter plot comparing the height in inches to the weight in pounds.

b. Draw a line of fit for the data.

c. Write the slope-intercept form of an equation for the line of fit.

d. Predict the normal weight for a man who is 84 inches tall.

e. A man's weight is 188 pounds. Use the equation of the line of fit to predict the height of the man.

Height (in.)	Weight (lb)
62	115
63	124
65	120
67	134
67	140
68	138
68	144
68	152
69	147
72	155
73	168
73	166

11 **GEYSERS** The time to the next eruption of Old Faithful can be predicted by using the duration of the current eruption.

Duration (min)	1.5	2	2.5	3	3.5	4	4.5	5
Interval (min)	48	55	70	72	74	82	93	100

a. Identify the independent and the dependent variables. Make a scatter plot and determine what relationship, if any, exists in the data. Draw a line of fit for the scatter plot.

b. Let x represent the duration of the previous interval. Let y represent the time between eruptions. Write the slope-intercept form of the equation for the line of fit. Predict the interval after a 7.5-minute eruption.

c. Make a critical judgment about using the equation to predict the duration of the next eruption. Would the equation be a useful model?

12. **COLLECT DATA** Use a tape measure to measure both the foot size and the height in inches of ten individuals.

a. Record your data in a table.

b. Make a scatter plot and draw a line of fit for the data.

c. Write an equation for the line of fit.

d. Make a conjecture about the relationship between foot size and height.

H.O.T. Problems   Use **H**igher-**O**rder **T**hinking Skills

13. **OPEN ENDED** Describe a real-life situation that can be modeled using a scatter plot. Decide whether there is a *positive*, *negative*, or *no* correlation. Explain what this correlation means.

14. **WHICH ONE DOESN'T BELONG?** Analyze the following situations and determine which one does not belong.

hours worked and amount of money earned	height of an athlete and favorite color
seedlings that grow an average of 2 centimeters each week	number of photos stored on a camera and capacity of camera

15. **CHALLENGE** Determine which line of fit is better for the scatter plot. Explain your reasoning.

16. **REASONING** What can make a scatter plot and line of fit more useful for accurate predictions? Does an accurate line of fit always predict what will happen in the future? Explain.

17. **WRITING IN MATH** Make a scatter plot that shows the height of a person and age. Explain how you could use the scatter plot to predict the age of a person given his or her height. How can the information from a scatter plot be used to identify trends and make decisions?
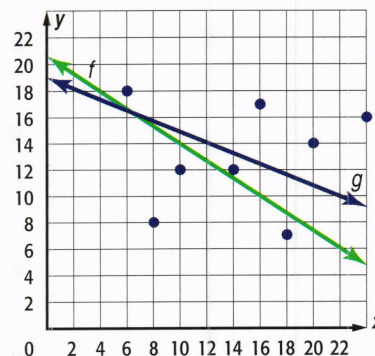
18. Which equation best describes the relationship between the values of x and y in the table?

A $y = x - 5$
B $y = 2x - 5$
C $y = 3x - 7$
D $y = 4x - 7$

x	y
−1	−7
0	−5
2	−1
4	3

19. STATISTICS Mr. Hernandez collected data on the heights and average stride lengths of a random sample of high school students. He then made a scatter plot. What kind of correlation did he most likely see?

F positive
G constant
H negative
J no

20. GEOMETRY Mrs. Aguilar's rectangular bedroom measures 13 feet by 11 feet. She wants to purchase carpet for the bedroom that costs $2.95 per square foot, including tax. How much will the carpet cost?

A $70.80
B $141.60
C $145.95
D $421.85

21. SHORT RESPONSE Nikia bought a one-month membership to a fitness center for $35. Each time she goes, she rents a locker for $0.25. If she spent $40.50 at the fitness center last month, how many days did she go?

Spiral Review

Determine whether the graphs of each pair of equations are *parallel*, *perpendicular*, or *neither*. (Lesson 4-4)

22. $y = -2x + 11$
$y + 2x = 23$

23. $3y = 2x + 14$
$2x + 3y = 2$

24. $y = -5x$
$y = 5x - 18$

25. $y = 3x + 2$
$y = -\frac{1}{3}x - 2$

Write each equation in standard form. (Lesson 4-3)

26. $y - 13 = 4(x - 2)$

27. $y - 5 = -2(x + 2)$

28. $y + 3 = -5(x + 1)$

29. $y + 7 = \frac{1}{2}(x + 2)$

30. $y - 1 = \frac{5}{6}(x - 4)$

31. $y - 2 = -\frac{2}{5}(x - 8)$

Graph each equation. (Lesson 4-1)

32. $y = 2x + 3$

33. $4x + y = -1$

34. $3x + 4y = 7$

Find the slope of the line that passes through each pair of points. (Lesson 3-3)

35. $(3, 4), (10, 8)$

36. $(-4, 7), (3, 5)$

37. $(3, 7), (-2, 4)$

38. $(-3, 2), (-3, 4)$

39. $(-2, -6), (-1, 10)$

40. $(1, -5), (-3, -5)$

41. DRIVING Latisha drove 248 miles in 4 hours. At that rate, how long will it take her to drive an additional 93 miles? (Lesson 2-6)

Skills Review

Express each relation as a graph. Then determine the domain and range. (Lesson 1-6)

**42.** $\{(4, 5), (5, 4), (-2, -2), (4, -5), (-5, 4)\}$

43. $\{(7, 6), (3, 4), (4, 5), (-2, 6), (-3, 2)\}$

You may be considering attending a college or technical school in the future. What factors cause tuition to rise—increased building costs, higher employee salaries, or the amount of bottled water consumed?

Let's see how bottled water and college tuition are related. The table shows the average college tuition and fees for public colleges and the per person consumption of bottled water per year for 2001 through 2005.

Year	2001	2002	2003	2004	2005
Water Consumed (gallons)	18.8	20.9	22.4	24.0	26.1
Tuition ($)	3725	4081	4694	5132	5491

Source: *The New York Times Almanac*
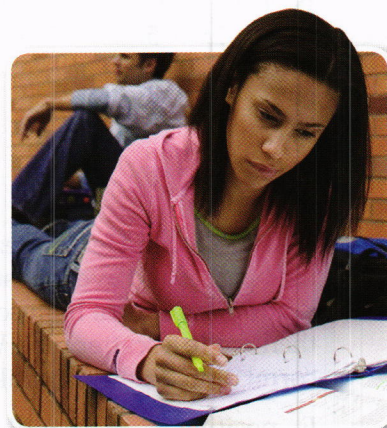
ACTIVITY

Follow the steps to learn about correlation and causation.

Step 1 Graph the ordered pairs (gallons, tuition) to create a scatter plot. For example, one ordered pair is (18.8, 2562). Describe the graph.

Step 2 Is the correlation *positive* or *negative*? Explain.

Step 3 Do you think drinking more bottled water *causes* college tuition costs to rise? Explain.

Step 4 **Causation** occurs when a change in one variable produces a change in another variable. Correlation can be observed between many variables, but causation can only be determined from data collected from a controlled experiment. Describe an experiment that could illustrate causation.

Exercises

For each exercise, determine whether each situation illustrates *correlation* or *causation*. Explain your reasoning, including other factors that might be involved.

1. A survey showed that sleeping with the light on was positively correlated to nearsightedness.

2. A controlled experiment showed a positive correlation between the number of cigarettes smoked and the probability of developing lung cancer.

3. A random sample of students found that owning a cell phone had a negative correlation with riding the bus to school.

4. A controlled experiment showed a positive correlation between the number of hours using headphones when listening to music and the level of hearing loss.

5. DeQuan read in the newspaper that shark attacks are positively correlated with monthly ice cream sales.

Regression and Median-Fit Lines

Why?

Spider-Man comics have been around since 1962. Since then Spider-Man has made appearances in magazines, books, and movies.

The table shows the number of comic books and appearances that he has made. We can estimate how many appearances he will make in 2012.

Spider-Man Comics and Appearances

Year	Number
1962	1
1972	39
1982	110
1992	164
2002	278
2012	?

Best-Fit Lines You have learned how to find and write equations for lines of fit by hand. Many calculators use complex algorithms that find a more precise line of fit called the **best-fit line**. One algorithm is called **linear regression**.

Your calculator may also compute a number called the **correlation coefficient**. This number will tell you if your correlation is positive or negative and how closely the equation is modeling the data. The closer the correlation coefficient is to 1 or −1, the more closely the equation models the data.

🌐 Real-World EXAMPLE 1   Best-Fit Line

MOVIES The table shows the amount of money made by movies in the United States. Use a graphing calculator to write an equation for the best-fit line for that data.

Year	2000	2001	2002	2003	2004	2005	2006
Income ($ billion)	7.66	8.41	9.52	9.49	9.54	8.99	9.49

Before you begin, make sure that your Diagnostic setting is on. You can find this under the **CATALOG** menu. Press **D** and then scroll down and click **DiagnosticOn**. Then press ENTER.

Step 1 Enter the data by pressing STAT and selecting the Edit option. Let the year 2000 be represented by 0. Enter the years since 2000 into List 1 (L1). These will represent the x-values. Enter the income ($ billion) into List 2 (L2). These will represent the y-values.

Step 2 Perform the regression by pressing STAT and selecting the **CALC** option. Scroll down to **LinReg (ax+b)** and press ENTER twice.

```
LinReg
  y=ax+b
  a=.2382142857        ← slope
  b=8.299642857        ← y-intercept
  r²=.4989334739
  r=.7063522308        ← correlation coefficient
```

Step 3 Write the equation of the regression line by rounding the a and b values. The form that we chose was $ax + b$, so the equation is $y = 0.24x + 8.30$. The correlation coefficient is about 0.7064, which means that the equation models the data fairly well.

✓ **Check Your Progress**

Write an equation of the best-fit line for the data in each table. Name the correlation coefficient. Round to the nearest ten-thousandth. Let x be the number of years since 2003.

1A. HOCKEY The table shows the number of goals of leading scorers.

Mustang Girls Hockey Leading Scorers								
Year	2003	2004	2005	2006	2007	2008	2009	2010
Goals	30	23	41	35	31	43	33	45

1B. HOCKEY The table gives the number of goals scored by the team each season.

Mustang Girls Hockey Team Goals								
Year	2003	2004	2005	2006	2007	2008	2009	2010
Goals	63	44	55	63	81	85	93	84

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We can use points on the best-fit line to estimate values that are not in the data. Recall that when we estimate values that are between known values, this is called *linear interpolation*. When we estimate a number outside of the range of the data, it is called *linear extrapolation*.

🌐 **Real-World EXAMPLE 2** **Use Interpolation and Extrapolation**

PAINTBALL The table shows the points received by the top ten paintball teams at a tournament. How many points did the 20th-ranked team receive?

Top Ten Teams										
Rank	1	2	3	4	5	6	7	8	9	10
Score	100	89	96	99	97	98	78	70	64	80

Write an equation of the best-fit line for the data. Then extrapolate to find the missing value.
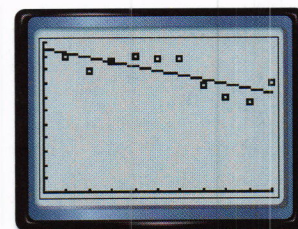
Step 1 Enter the data from the table into the lists as you did before. Let the ranks be the x-values and the scores be the y-values. Then graph the scatter plot.
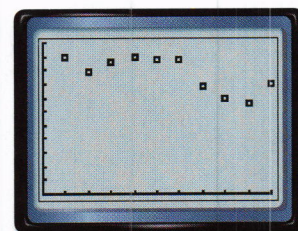
[0, 10] scl: 1 by [0, 110] scl: 10

Step 2 Perform the linear regression using the data in the lists. Find the equation of the best-fit line.

The equation is $y = -3.32x + 105.3$.
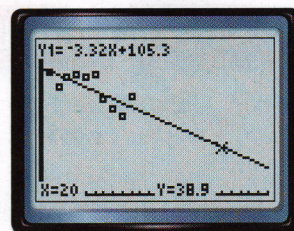
[0, 10] scl: 1 by [0, 110] scl: 10

Step 3 Graph the best-fit line. Press 2nd [CALC] ENTER to find that when $x = 20$, $y \approx 39$.

It is estimated that the 20th ranked team received 39 points.

[0, 25] scl: 1 by [0, 125] scl: 10

✔ **Check Your Progress**

ONLINE GAMES Use linear interpolation to find the percent of Americans that play online games for the following ages.

Percent of Americans Who Play Online Games					
Age	15	20	30	40	50
Percent	81	54	37	29	25

Source: Pew Internet & American Life Survey

2A. 35 years   **2B.** 18 years

Median-Fit Lines A second type of fit line that can be found using a graphing calculator is a **median-fit line**. The equation of a median-fit line is calculated using the medians of the coordinates of the data points.
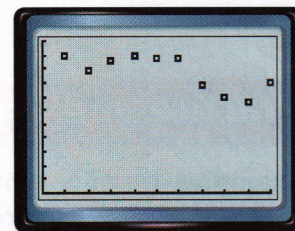
EXAMPLE 3   **Median-Fit Line**

PAINTBALL Find and graph the equation of a median-fit line for the data in Example 2. Then predict the score of the 15th ranked team.

Step 1 The data should be in the lists. Graph the scatter plot.

[0, 10] scl: 1 by [0, 110] scl: 10

Step 2 To find the median-fit equation, press the STAT key and select the **CALC** option. Scroll down to the **Med-Med** option and press ENTER. The value of a is the slope, and the value of b is the y-intercept.

The equation for the median-fit line is $y = -3.71x + 108.26$.

Step 3 Copy the equation to the Y= list and graph. Use the value option to find the value of y when $x = 15$.

The 15th place team scored about 53 points.
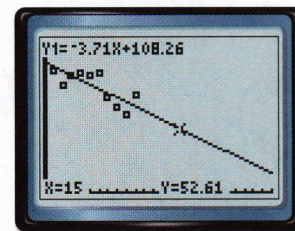
Notice that the equations for the regression line and the median-fit line are very similar.

[0, 25] scl: 1 by [0, 125] scl: 10

✔ **Check Your Progress**

3. Use the data from Check Your Progress 2 and a median-fit line to estimate the numbers of 18- and 35-year-olds who play online games. Compare these values with the answers from the regression line.

Example 1
p. 253

1. **POTTERY** A local university is keeping track of the number of art students who use the pottery studio each day. Write an equation of the regression line and find the correlation coefficient.

Students Throwing Pottery							
Day	1	2	3	4	5	6	7
Students	10	15	18	15	13	19	20

Example 2
p. 254

2. **COMPUTERS** The table below shows the percent of Americans with a broadband connection at home in a recent year. Use linear extrapolation and a regression equation to estimate the percentage of 60-year-olds with broadband at home.

Percentage of Americans with Broadband At Home					
Age	25	30	35	40	45	50
Percent	40	42	36	35	36	32

Example 3
p. 255

3. **VACATION** The Smiths want to rent a house on the lake that sleeps eight people. The cost of the house per night is based on how close it is to the water.

Rental Properties							
Distance from Lake (mi)	0.0 (houseboat)	0.3	0.5	1.0	1.25	1.5	2.0
Price/Night ($)	785	325	250	200	150	140	100

a. Find and graph an equation for the median-fit line.

b. What would you estimate is the cost of a rental 1.75 miles from the lake?

Practice and Problem Solving

Examples 1 and 2
pp. 253–254

Write an equation of the regression line for the data in each table. Then find the correlation coefficient.

4. **SKYSCRAPERS** The table ranks the ten tallest buildings in the world.

Tallest Buildings and Stories										
Rank	1	2	3	4	5	6	7	8	9	10
Stories	101	88	110	88	88	80	69	102	78	70

5. **MUSIC** The table gives the number of annual violin auditions held by a youth symphony each year since 2000. Let x be the number of years since 2000.

Youth Symphony Violin Auditions							
Year	2000	2001	2002	2003	2004	2005	2006
Auditions	22	19	25	'37	32	35	42

6. **RETAIL** The table gives the sales of jeans at a clothing chain since 2004. Let x be the number of years since 2004.

Jeans Sales By Year						
Year	2004	2005	2006	2007	2008	2009
Sales (Millions of Dollars)	6.84	7.6	10.9	15.4	17.6	21.2

Example 3
p. 255

7 MARATHON The number of entrants in the Boston Marathon every five years since 1975 is shown. Let x be the number of years since 1975.

Year	1975	1980	1985	1990	1995	2000	2005
Entrants	2395	5417	5594	9412	9416	17,813	20,453

Source: Boston Athletic Association

a. Find an equation for the median-fit line.

b. According to the equation, how many entrants were there in 2003?

8. CAMPING A campground keeps a record of the number of campsites rented the week of July 4 for several years. Let x be the number of years since 2000.

Campsites Rented July 4th Week						
Year	2002	2003	2004	2005	2006	2007
Sites Rented	34	45	42	53	58	47

a. Find an equation for the regression line.

b. Predict the number of campsites that will be rented in 2010.

c. Predict the number of campsites that will be rented in 2020.

9. ICE CREAM An ice cream company keeps a count of the tubs of cookie dough ice cream delivered to each of their stores in a particular area.

Store Delivery of Cookie Dough Ice Cream					
Store Size (ft^2)	2100	2225	3135	3569	4587
Tubs (hundreds)	110	102	215	312	265

a. Find an equation for the median-fit line.

b. Graph the points and the median-fit line.

c. How many tubs would be delivered to a 1500-square-foot store? a 5000-square-foot store?

10. COLLEGE TESTING The ACT is an exam that evaluates students' readiness to perform college-level work. The table below shows the number of participants who took the test in the given years. Let x be the number of years since 1990.

Years	1990	1995	2000	2001	2002	2003	2004	2005	2006
Participants (thousands)	817	945	1065	1070	1116	1175	1171	1186	1206

Source: ACT

a. Find an equation for the regression line.

b. According to the equation, how many participants were there in 1998?

c. How many students would you predict will participate in 2011?

11. FINANCIAL LITERACY The prices of the eight top-selling brands of jeans at Jeanie's Jeans are given in the table below.

Sales Rank	1	2	3	4	5	6	7	8
Price ($)	43	44	50	61	64	135	108	78

a. Find the equation for the regression line.

b. According to the equation, what would be the price of a pair of the 12th best-selling brand?

c. Is this a reasonable prediction? Explain.

12. **STATE FAIRS** Opening day attendance at the North Carolina State Fair for 2000 was 53,331, for 2001 it was 47,940, for 2002 it was 54,036, for 2003 it was 61,364, for 2004 it was 61,289, and for 2005 it was 52,201.

    a. Construct a table for the given data.
    b. Graph the points and the regression line.
    c. Predict the attendance on opening day in 2012.

13 **MUSIC** For the following data, find the equation of the regression line and give the correlation coefficient to the nearest ten-thousandth place.

Battle of the Bands Concessions									
Year	1998	1999	2000	2001	2002	2003	2004	2005	2006
Amount Raised ($)	1236	1560	1423	1740	2230	2563	3215	4517	4219

14. **FIREFIGHTERS** The table shows statistics from the U.S. Fire Administration.

    a. Find an equation for the median-fit line.
    b. Graph the points and the median-fit line.
    c. Does the median-fit line give you an accurate picture of the number of firefighters? Explain.

Age	Number of Firefighters
18	40,919
25	245,516
35	330,516
45	296,665
55	167,087
65	54,559

15. **ATHLETICS** The table shows the number of participants in high school athletics.

Year Since 1970	1	10	20	30	35
Athletes	3,960,932	5,356,913	5,298,671	6,705,223	7,159,904

    a. Find an equation for the regression line.
    b. According to the equation, how many participated in 1988?

16. **ART** A count was kept on the number of paintings sold at an auction by the year in which they were painted. Let *x* be the number of years since 1950.

Paintings Sorted by Year of Execution						
Year Painted	1950	1955	1960	1965	1970	1975
Paintings Solds	8	5	25	21	9	22

    a. Find the equation for the linear regression line.
    b. How many paintings were sold that were painted in 1961?
    c. Is the linear regression equation an accurate model of the data? Explain why or why not.

17. **SCHOOL** The table shows the average cost of a technical school between 2000 and 2006. Let *x* be 0 for the school year 2000–2001.

Average Cost of Public 2-Year Institution					
Year	2000–2001	2001–2002	2002–2003	2003–2004	2004–2005	2005–2006
Cost ($)	4839	5137	5601	6020	6375	6492

Source: U.S. Department of Education

    a. Find the equation of the median-fit line.
    b. Graph the points and the median-fit line.
    c. What would you estimate the cost to be in 2020–2021?

18. SPACE EXPLORATION As of 2006, the names of the space shuttles launched each year since 1993 are given below. Let x be the year.

Space Shuttle Launches	
Year	**Launches**
1993	STS-61, STS-58, STS-51, STS-57, STS-55, STS-56, STS-54
1994	STS-66, STS-68, STS-64, STS-65, STS-59, STS-62, STS-60
1995	STS-74, STS-73, STS-69, STS-70, STS-71, STS-67, STS-63
1996	STS-80, STS-79, STS-78, STS-77, STS-76, STS-75, TSTS-72
1997	STS-87, STS-86, STS-85, STS-94, STS-84, STS-83, STS-82, STS-81
1998	STS-88, STS-95, STS-91, STS-90, STS-89
1999	STS-103, STS-93, STS-96
2000	STS-97, STS-92, STS-106, STS-101, STS-99
2001	STS-108, STS-105, STS-104, STS-100, STS-102, STS-98
2002	STS-113, STS-112, STS-111, STS-110, STS-109
2003	STS-107
2004	
2005	STS-114
2006	STS-116, STS-115, STS-121

a. Construct a table of values for the data that could be used for graphing.

b. Find an equation for the regression line.

c. Predict the number of launches there will be in 2015. Is your prediction reasonable? What influences might cause the actual number of flights to be different from your prediction? Explain.

H.O.T. Problems   Use **H**igher-**O**rder **T**hinking Skills

19. CHALLENGE Below are the results of the World Superpipe Championships in 2008.

Men	Score	Rank	Women	Score
Shaun White	93.00	1	Torah Bright	96.67
Mason Aguirre	90.33	2	Kelly Clark	93.00
Janne Korpi	85.33	3	Soko Yamaoka	85.00
Luke Mitrani	85.00	4	Ellery Hollingsworth	79.33
Keir Dillion	81.33	5	Sophie Rodriguez	71.00

Find an equation of the regression line for each, and graph them on the same coordinate plane. Compare and contrast the men's and women's graphs.

20. REASONING For a class project, the scores that 10 randomly selected students earned on the first 8 tests of the school year are given. Explain how to find a line of best fit. Could it be used to predict the scores of other students? Explain your reasoning.

21. OPEN ENDED For 10 different people, measure their heights and the lengths of their heads from chin to top. Use these data to generate a linear regression equation and a median-fit equation. Make a prediction using both of the equations.

22. WRITING IN MATH Using the data at the beginning of the lesson, describe the steps you would take to determine the number of appearances Spiderman will make in 2012.

23. GEOMETRY Sam is putting a border around a poster. x represents the poster's width, and y represents the poster's length. Which equation represents how much border Sam will use if he doubles the length and the width?

A $4xy$ **C** $4(x + y)$
B $(x + y)^4$ **D** $16(x + y)$

24. SHORT RESPONSE Tatiana wants to run 5 miles at an average pace of 9 minutes per mile. After 4 miles, her average pace is 9 minutes 10 seconds. In how many minutes must she complete the final mile to reach her goal?

25. What is the slope of the line that passes through $(1, 3)$ and $(-3, 1)$?

F -2 **H** $\frac{1}{2}$
G $-\frac{1}{2}$ **J** 2

26. What is an equation of the line that passes through $(0, 1)$ and has a slope of 3?

A $y = 3x - 1$
B $y = 3x - 2$
C $y = 3x + 4$
D $y = 3x + 1$

Spiral Review

27. USED CARS Gianna wants to buy a specific make and model of a used car. She researched prices from dealers and private sellers and made the graph shown. (Lesson 4-5)

a. Describe the relationship in the data.

b. Use the line of fit to predict the price of a car that is 7 years old.

c. Is it reasonable to use this line of fit to predict the price of a 10-year-old car? Explain.

Used Cars

28. GEOMETRY A quadrilateral has sides with equations $y = -2x$, $2x + y = 6$, $y = \frac{1}{2}x + 6$, and $x - 2y = 9$. Is the figure a rectangle? Explain your reasoning. (Lesson 4-4)
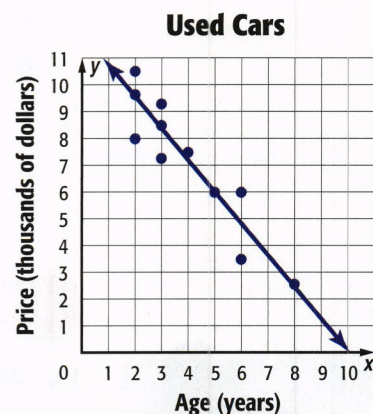
Write each equation in standard form. (Lesson 4-3)

29. $y - 2 = 3(x - 1)$

30. $y - 5 = 6(x + 1)$

31. $y + 2 = -2(x - 5)$

32. $y + 3 = \frac{1}{2}(x + 4)$

33. $y - 1 = \frac{2}{3}(x + 9)$

34. $y + 3 = -\frac{1}{4}(x + 2)$

Find the slope of the line that passes through each pair of points. (Lesson 3-3)

35. $(3, 4), (10, 8)$ **36.** $(-4, 7), (3, 5)$ **37.** $(3, 7), (-2, 4)$ **38.** $(-3, 2), (-3, 4)$

Skills Review

If $f(x) = x^2 - x + 1$, find each value. (Lesson 1-7)

39. $f(-1)$ **40.** $f(5) - 3$ **41.** $f(a)$ **42.** $f(b + 2)$

Graph each equation. (Lesson 3-1)

43. $y = x + 2$ **44.** $x + 5y = 4$ **45.** $2x - 3y = 6$ **46.** $5x + 2y = 6$

Now

- Identify and graph step functions.
- Identify and graph absolute value and piecewise-defined functions.

New Vocabulary

step function
piecewise-linear function
greatest integer function
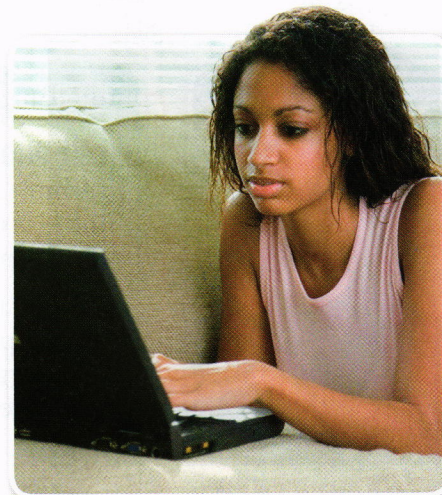absolute value function
piecewise-defined
 function

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Special Functions

Why?

Kim is ordering books online. The site charges for shipping based on the amount of the order. If the order is less than $10, shipping costs $3. If the order is more than $10 but less than $20, it will cost $5 to ship it.

Step Functions The graph of a **step function** is a series of line segments. Because each part of a step function is linear, this type of function is called a **piecewise-linear function**.

One example of a step function is the **greatest integer function**, written as $f(x) = [\![x]\!]$, where $f(x)$ is the greatest integer not greater than x. For example, $[\![6.8]\!] = 6$ because 6 is the greatest integer that is not greater than 6.8.
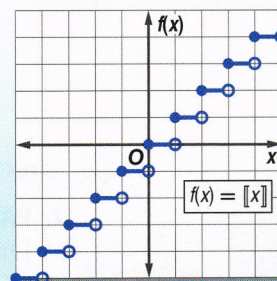
Key Concept   Greatest Integer Function

For Your
FOLDABLE

|	
Parent function:	$f(x) = [\![x]\!]$
Type of graph:	disjointed line segments
Domain:	all real numbers
Range:	all integers

$f(x) = [\![x]\!]$

EXAMPLE 1   Greatest Integer Function

Graph $f(x) = [\![x + 2]\!]$. **State the domain and range.**

First, make a table. Select a few values between integers. On the graph, dots represent included points. Circles represent points not included.

x	$x + 2$	$[\![x + 2]\!]$
0	2	2
0.25	2.25	2
0.5	2.5	2
1	3	3
1.25	3.25	3
1.5	3.5	3
2	4	4
2.25	4.25	4

$f(x) = [\![x + 2]\!]$
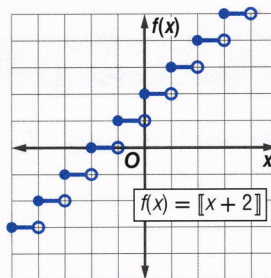
Note that this is the graph of $f(x) = [\![x]\!]$ shifted 2 units to the left.

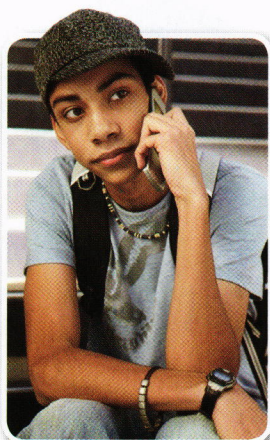Because the dots and circles overlap, the domain is all real numbers. The range is all integers.

✓ Check Your Progress

1. Graph $g(x) = 2[\![x]\!]$. State the domain and range.

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Step functions can be used to represent many real-world situations involving money.

🌐 Real-World EXAMPLE 2   Step Function

CELL PHONE PLANS Cell phone companies charge by the minute, not by the second. A cell phone company charges $0.45 per minute or any fraction thereof for exceeding the number of minutes allotted on each plan. Draw a graph that represents this situation.

The total cost for the extra minutes will be a multiple of $0.45, and the graph will be a step function. If the time is greater than 0 but less than or equal to 1 minute, the charge will be $0.45. If the time is greater than 2 but is less than or equal to 3 minutes, you will be charged for 3 minutes or $1.35.

x	$f(x)$
$0 < x \le 1$	0.45
$1 < x \le 2$	0.90
$2 < x \le 3$	1.35
$3 < x \le 4$	1.80
$4 < x \le 5$	2.25
$5 < x \le 6$	2.70
$6 < x \le 7$	3.15

Cell Phone Overage
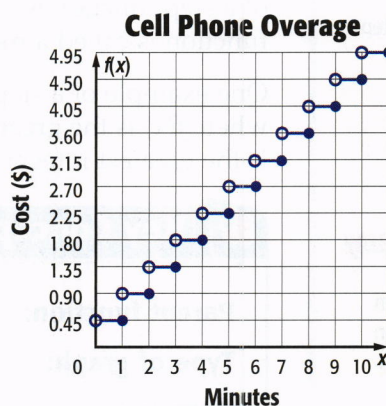
✓ Check Your Progress

2. PARKING A garage charges $4 for the first hour and $1 for each additional hour. Draw a graph that represents this situation.

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Absolute Value Functions Another type of piecewise-linear function is the **absolute value function**. Recall that the absolute value of a number is always nonnegative. So in the absolute value parent function, written as $f(x) = |x|$, all of the values of the range are nonnegative.

📋 Key Concept   Absolute Value Function   For Your FOLDABLE
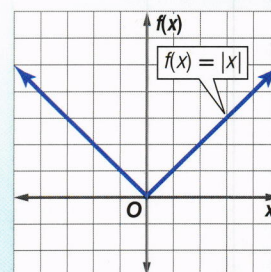
Parent function: $f(x) = |x|$, defined as

$$f(x) = \begin{cases} x & \text{if } x > 0 \\ 0 & \text{if } x = 0 \\ -x & \text{if } x < 0 \end{cases}$$

Type of graph: V-shaped

Domain: all real numbers

Range: all nonnegative real numbers

The absolute value function is called a **piecewise-defined function** because it is defined using two or more expressions.

EXAMPLE 3   **Absolute Value Function**

Graph $f(x) = |x - 4|$. **State the domain and range.**

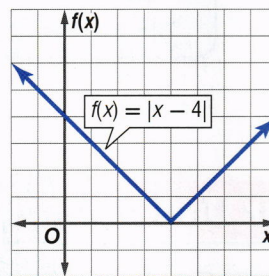Since $f(x)$ cannot be negative, the minimum point of the graph is where $f(x) = 0$.

$$f(x) = |x - 4| \quad \text{Original function}$$
$$0 = x - 4 \quad \text{Replace } f(x) \text{ with 0 and } |x - 4| \text{ with } x - 4.$$
$$4 = x \quad \text{Add 4 to each side.}$$

Next make a table of values. Include values for $x > 4$ and $x < 4$.

$f(x) = \|x - 4\|$	
x	$f(x)$
-2	6
0	4
2	2
4	0
5	1
6	2
7	3
8	4

The domain is all real numbers. The range is all real numbers greater than or equal to 0. Note that this is the graph of $f(x) = |x|$ shifted 4 units to the right.

✔ **Check Your Progress**

3. Graph $f(x) = |2x + 1|$. State the domain and range.

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Not all piecewise-defined functions are absolute value functions. Step functions are also piecewise-defined functions. In fact, all piecewise-linear functions are piecewise-defined.

StudyTip

Piecewise Functions
To graph a piecewise-defined function, graph each "piece" separately. There should be a dot or line that contains each member of the domain.
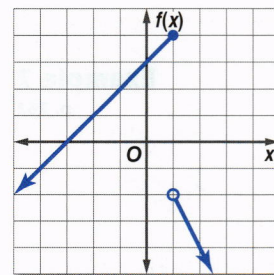
EXAMPLE 4   **Piecewise-Defined Function**

Graph $f(x) = \begin{cases} -2x \text{ if } x > 1 \\ x + 3 \text{ if } x \le 1 \end{cases}$. **State the domain and range.**

Graph the first expression. Create a table of values for when $x > 1$, $f(x) = -2x$ and draw the graph. Since x is not equal to 1, place a circle at $(1, -2)$.

Next, graph the second expression. Create a table of values for when $x \le 1$, $f(x) = x + 3$ and draw the graph. If $x = 1$, then $f(x) = 4$; place a dot at $(1, 4)$.

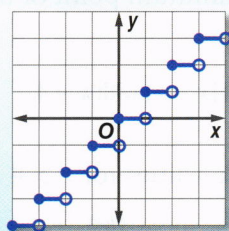

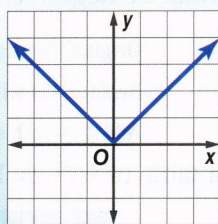The domain is all real numbers. The range is $y \le 4$.

✔ **Check Your Progress**

4. Graph $f(x) = \begin{cases} 2x + 1 \text{ if } x > 0 \\ 3 \text{ if } x \le 0 \end{cases}$. State the domain and range.
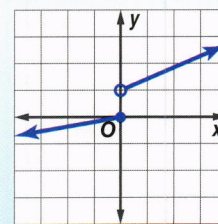
▶ **Personal Tutor glencoe.com**

Step Function   **Absolute Value Function**   **Piecewise-Defined Function**

✓ Check Your Understanding

Example 1
p. 261

Graph each function. State the domain and range.

1. $f(x) = \frac{1}{2}[\![x]\!]$

2. $g(x) = -[\![x]\!]$

3. $h(x) = [\![2x]\!]$

Order Total ($)	Shipping Cost ($)
0–15	3.99
15.01–30	5.99
30.01–50	6.99
50.01–75	7.99
75.01–100	8.99
Over $100	9.99

Example 2
p. 262

4. SHIPPING Elan is ordering a gift for his dad online. The table shows the shipping rates. Graph the step function.

Examples 3 and 4
p. 263

Graph each function. State the domain and range.

5. $f(x) = |x - 3|$

6. $g(x) = |2x + 4|$

7. $f(x) = \begin{cases} 2x - 1 \text{ if } x > -1 \\ -x \text{ if } x \leq -1 \end{cases}$

8. $g(x) = \begin{cases} -3x - 2 \text{ if } x > -2 \\ -x + 1 \text{ if } x \leq -2 \end{cases}$

Practice and Problem Solving

● = **Step-by-Step Solutions** begin on page R12.
Extra Practice begins on page 815.

Example 1
p. 261

Graph each function. State the domain and range.

9 $f(x) = 3[\![x]\!]$

10. $f(x) = [\![-x]\!]$

11. $g(x) = -2[\![x]\!]$

12. $g(x) = [\![x]\!] + 3$

13. $h(x) = [\![x]\!] - 1$

14. $h(x) = \frac{1}{2}[\![x]\!] + 1$

Example 2
p. 262

15. CAB FARES Lauren wants to take a taxi from a hotel to a friend's house. The rate is $3 plus $1.50 per mile after the first mile. Every fraction of a mile is rounded up to the next mile.

a. Draw a graph to represent the cost of using a taxi cab.

b. What is the cost if the trip is 8.5 miles long?

16. POSTAGE The United States Postal Service increases the rate of postage periodically. The table shows the cost to mail a letter weighing 1 ounce or less from 1988 through 2007. Draw a step graph to represent the data.

Year	1988	1991	1995	1999	2001	2002	2006	2007
Cost ($)	0.25	0.29	0.32	0.33	0.34	0.37	0.39	0.41

Graph each function. State the domain and range.

17. $f(x) = |2x - 1|$

18. $f(x) = |x + 5|$

19. $g(x) = |-3x - 5|$

20. $g(x) = |-x - 3|$

21. $f(x) = \left|\frac{1}{2}x - 2\right|$

22. $f(x) = \left|\frac{1}{3}x + 2\right|$

23. $g(x) = |x + 2| + 3$

24. $g(x) = |2x - 3| + 1$

25. $f(x) = \begin{cases} \frac{1}{2}x - 1 \text{ if } x > 3 \\ -2x + 3 \text{ if } x \le 3 \end{cases}$

26. $f(x) = \begin{cases} 2x - 5 \text{ if } x > 1 \\ 4x - 3 \text{ if } x \le 1 \end{cases}$

27. $f(x) = \begin{cases} 2x + 3 \text{ if } x \ge -3 \\ -\frac{1}{3}x + 1 \text{ if } x < -3 \end{cases}$

28. $f(x) = \begin{cases} 3x + 4 \text{ if } x \ge 1 \\ x + 3 \text{ if } x < 1 \end{cases}$

29. $f(x) = \begin{cases} 3x + 2 \text{ if } x > -1 \\ -\frac{1}{2}x - 3 \text{ if } x \le -1 \end{cases}$

30. $f(x) = \begin{cases} 2x + 1 \text{ if } x < -2 \\ -3x - 1 \text{ if } x \ge -2 \end{cases}$
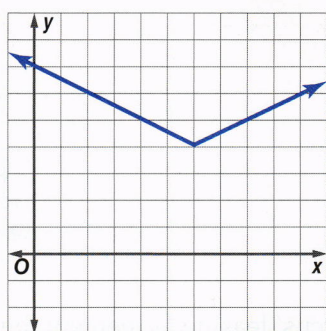
Determine the domain and range of each function.

StudyTip

Nonlinear Functions
The greatest integer function, absolute value function, and piecewise defined functions are examples of nonlinear functions.

31

32.
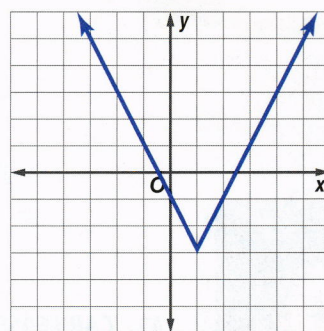
33.
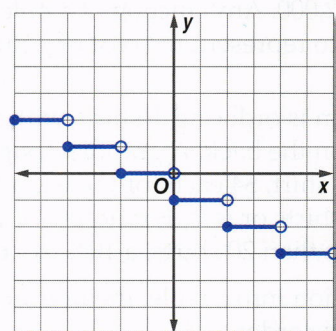
34.

35.

36.

37. **BOATING** According to Boat Minnesota, the maximum number of people that can safely ride in a boat is determined by the boat's length and width. The table shows some guidelines for the length of a boat that is 6 feet wide. Graph this relation.

Length of Boat (ft)	18–19	20–22	23–24
Number of People	7	8	9

For Exercises 38–41, match each graph to one of the following equations.

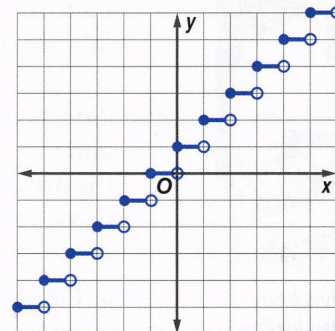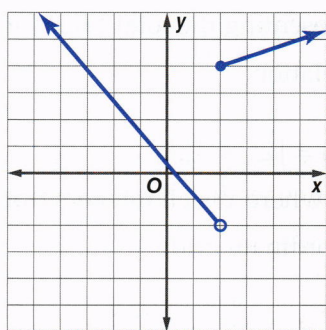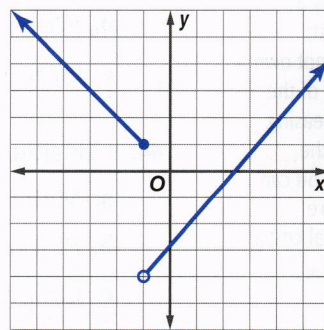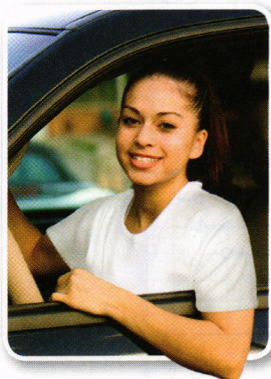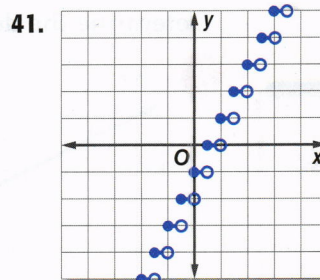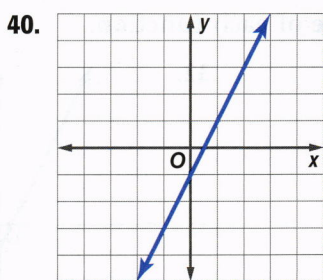| A	B	C	D
$y = 2x - 1$	$y = [\![2x]\!] - 1$	$y = |2x| - 1$	$y = \begin{cases} 2x + 1 & \text{if } x > 0 \\ -2x + 1 & \text{if } x \leq 0 \end{cases}$
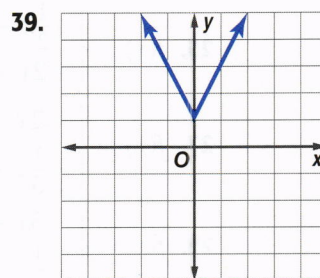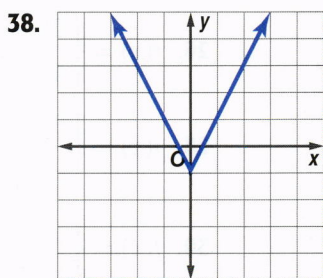
38.

39.

40.

41.

42. CAR LEASE As part of Marcus' leasing agreement, he will be charged $0.20 per mile for each mile over 12,000. Any fraction of a mile is rounded up to the next mile. Make a step graph to represent the cost of going over the mileage.

43 BASEBALL A baseball team is ordering T-shirts with the team logo on the front and the players' names on the back. A graphic design store charges $10 to set up the artwork plus $10 per shirt, $4 each for the team logo, and $2 to print the last name for an order of 10 shirts or less. For orders of 11–20 shirts, a 5% discount is given. For orders of more than 20 shirts, a 10% discount is given.

a. Organize the information into a table. Include a column showing the total order price for each size order.

b. Write an equation representing the total price for an order of x shirts.

c. Graph the piecewise relation.

44. Consider the function $f(x) = |2x + 3|$.

a. Make a table of values where x is all integers from -5 to 5, inclusive.

b. Plot the points on a coordinate grid.

c. Graph the function.

45. Consider the function $f(x) = |2x| + 3$.

a. Make a table of values where x is all integers from -5 to 5, inclusive.

b. Plot the points on a coordinate grid.

c. Graph the function.

d. Describe how this graph is different from the graph in Exercise 44.
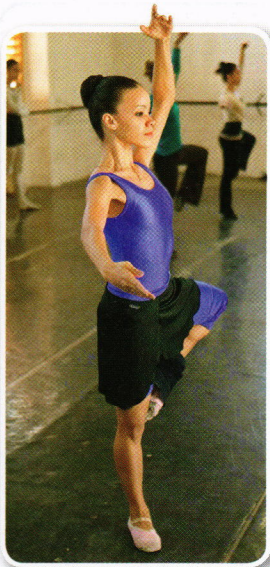
Real-World Link

In addition to traditional dance classes like ballet and tap, more studios are offering classes in hip-hop, jazz/hip-hip, and cardio dance.

Source: Ariel Dance Studio

46. DANCE A local studio must have at least 5 students enrolled in a class, or else the class will be canceled. Once 10 students are enrolled, a second class is started. Draw a graph for this situation.

47. THEATERS A certain theater will not have a show unless it has sold 50 tickets for that show. Once the capacity of 250 seats are sold, the theater begins selling tickets for the next show. Draw a graph that describes this situation.

Graph each function.

48. $f(x) = \frac{1}{2}|x| + 2$ **49** $g(x) = \frac{1}{3}|x| + 4$ **50.** $h(x) = -2|x - 3| + 2$

51. $f(x) = -4|x + 2| - 3$ **52.** $g(x) = -\frac{2}{3}|x + 6| - 1$ **53.** $h(x) = -\frac{3}{4}|x - 8| + 1$

54. MULTIPLE REPRESENTATIONS In this problem, you will explore piecewise-defined functions.

a. TABULAR Copy and complete the table of values for $f(x) = |[\![x]\!]|$ and $g(x) = [\![|x|]\!]$.

x	$[\![x]\!]$	$f(x) = |[\![x]\!]|$	$|x|$	$g(x) = [\![|x|]\!]$
−3	−3	3	3	3
−2.5				
−2				
0				
0.5				
1				
1.5				

b. GRAPHICAL Graph each function on a coordinate plane.

c. ANALYTICAL Compare and contrast the graphs of $f(x)$ and $g(x)$.

H.O.T. Problems Use Higher-Order Thinking Skills

55. REASONING Does the piecewise relation below represent a function? Why or why not?

$$y = \begin{cases} -2x + 4 & \text{if } x \geq 2 \\ -\frac{1}{2}x - 1 & \text{if } x \leq 4 \end{cases}$$

CHALLENGE Refer to the graph.

56. Write an absolute value function that represents the graph.

57. Write a piecewise function to represent the graph.

58. What are the domain and range?

59. WRITING IN MATH Refer to the information on cell phone plans in Example 2. Explain why the graph of this description is called a *step graph*.
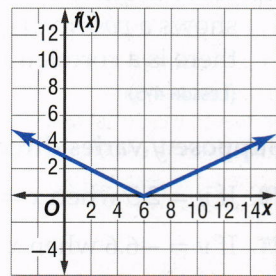
60. CHALLENGE A bicyclist travels up and down a hill. The hill has a vertical cross section that can be modeled by the equation $y = -\frac{1}{4}|x - 400| + 100$ where x and y are measured in feet.

a. If $0 \leq x \leq 800$, find the slope for the uphill portion of the trip and then the downhill portion of the trip.

b. Graph this function.

c. What are the domain and range of the graph?

61. Which equation represents a line that is perpendicular to the graph and passes through the point at (2, 0)?
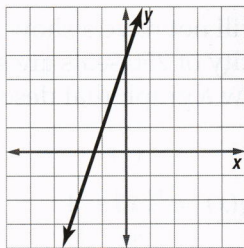
A $y = 3x - 6$

B $y = -3x + 6$

C $y = -\frac{1}{3}x + \frac{2}{3}$

D $y = \frac{1}{3}x - \frac{2}{3}$

62. A giant tortoise travels at a rate of 0.17 mile per hour. Which equation models the time t it would take the giant tortoise to travel 0.8 mile?

F $t = \frac{0.8}{0.17}$ H $t = \frac{0.17}{0.8}$

G $t = (0.17)(0.8)$ J $0.8 = \frac{0.17}{t}$

63. GEOMETRY If $\triangle JKL$ is similar to $\triangle JNM$ what is the value a?
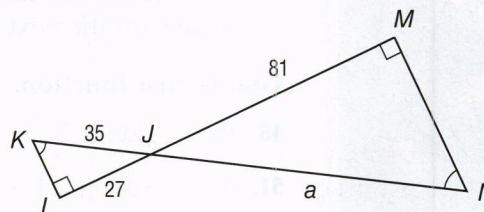
A 62.5

B 105

C 125

D 155.5

64. GRIDDED RESPONSE What is the difference in the value of $2.1(x + 3.2)$, when $x = 5$ and when $x = 3$?

Write an equation of the regression line for the data in each table. (Lesson 4-6)

65.

x	1	3	5	7	9
y	3	8	15	18	21

66.

x	3	5	7	9	11
y	7.2	23.5	41.2	56.4	73.1

67.

x	1	2	3	4	5
y	21	33	39	54	64

68.

x	2	4	6	8	10
y	1.4	2.4	2.9	3.3	4.2

69. TESTS Determine whether the graph at the right shows a *positive*, *negative*, or *no* correlation. If there is a correlation, describe its meaning. (Lesson 4-5)

Test Scores

Suppose y varies directly as x. (Lesson 3-4)

70. If $y = 2.5$ when $x = 0.5$, find y when $x = 20$.

71. If $y = -6.6$ when $x = 9.9$, find y when $x = 6.6$.

72. If $y = 2.6$ when $x = 0.25$, find y when $x = 1.125$.

73. If $y = 6$ when $x = 0.6$, find x when $y = 12$.
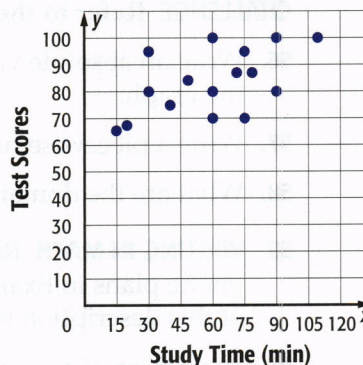
Solve each equation. (Lesson 2-2)

74. $104 = k - 67$

75. $-4 + x = -7$

76. $\frac{m}{7} = -11$

77. $\frac{2}{3}p = 14$

78. $-82 = n + 18$

79. $\frac{9}{t} = -27$

EXTEND
4-7

Graphing Technology Lab
Piecewise-Linear Functions

Math Online > glencoe.com
• Other Calculator Keystrokes
• Graphing Technology Personal Tutor

You can use a graphing calculator to graph and analyze various piecewise functions, including greatest integer functions and absolute value functions.

ACTIVITY 1 Greatest Integer Functions

Graph $f(x) = [\![x]\!]$ in the standard viewing window.

The calculator may need to be changed to dot mode for the function to graph correctly. Press **MODE** then use the arrow and **ENTER** keys to select **DOT**.

Enter the equation in the Y= list. Then graph the equation.

KEYSTROKES: Y= MATH ▶ 5 X,T,θ,n) Zoom 6

1A. How does the graph of $f(x) = [\![x]\!]$ compare to the graph of $f(x) = x$?

1B. What are the domain and range of the function $f(x) = [\![x]\!]$? Explain.

[−10, 10] scl: 1 by [−10, 10] scl: 1

The graphs of piecewise functions are affected by changes in parameters.

ACTIVITY 2 Absolute Value Functions

Graph $y = |x| − 3$ and $y = |x| + 1$ in the standard viewing window.

Enter the equations in the Y= list. Then graph.

KEYSTROKES: Y= MATH ▶ 1 X,T,θ,n) − 3 ENTER MATH ▶ 1 X,T,θ,n) + 1 Zoom 6

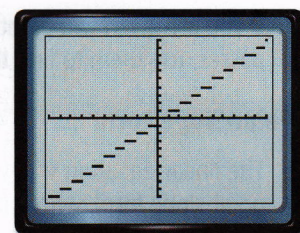**2A.** Compare and contrast the graphs to the graph of $y = |x|$.

2B. How does the value of c affect the graph of $y = |x| + c$?
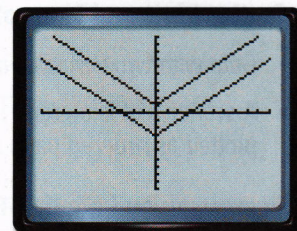
[−10, 10] scl: 1 by [−10, 10] scl: 1

Analyze The Results

1. A parking garage charges $4 for every hour or fraction of an hour. Is this situation modeled by a *linear* function or a *step* function? Explain your reasoning.

2. A maintenance technician is testing an elevator system. The technician starts the elevator at the fifth floor. It is sent to the ground floor, then back to the fifth floor. Assume the elevator travels at a constant rate. Should the height of the elevator be modeled by a step function or an absolute value function? Explain.

3. **MAKE A CONJECTURE** Explain why the greatest integer function is sometimes called the *floor function*.

4. Graph $y = −|x|$ in the standard viewing window. How is this graph to related to the graph of $y = |x|$?

5. **MAKE A CONJECTURE** Describe the transformation of the parent graph to $y = |x + c|$. Use a graphing calculator with different values of c to test your conjecture.

Chapter Summary

Key Concepts

Slope-Intercept Form (Lessons 4-1 and 4-2)

- The slope-intercept form of a linear equation is $y = mx + b$, where m is the slope and b is the y-intercept.

- If you are given two points through which a line passes, use them to find the slope first.

Point-Slope Form (Lesson 4-3)

- The linear equation $y - y_1 = m(x - x_1)$ is written in point-slope form, where (x_1, y_1) is a given point on a nonvertical line and m is the slope of the line.

Parallel and Perpendicular Lines (Lesson 4-4)

- Nonvertical parallel lines have the same slope.

- Lines that intersect at right angles are called perpendicular lines. The slopes of perpendicular lines are opposite reciprocals.

Scatter Plots and Lines of Fit (Lesson 4-5)

- Data with two variables are called bivariate data.

- A scatter plot is a graph in which two sets of data are plotted as ordered pairs in a coordinate plane.

Regression and Median-Fit Lines (Lesson 4-6)

- A graphing calculator can be used to find regression lines and median-fit lines.

Special Functions (Lesson 4-7)

- The greatest integer function is written as $f(x) = [\![x]\!]$, where $f(x)$ is the greatest integer not greater than x.

- The absolute value function is written as $f(x) = |x|$, where $f(x)$ is the distance between x and 0 on a number line.

FOLDABLES Study Organizer

Be sure the Key Concepts are noted in your Foldable.

Linear Functions and Relations

Key Vocabulary

absolute value function (p. 262)
best-fit line (p. 253)
bivariate data (p. 245)
correlation coefficient (p. 253)
greatest integer function (p. 261)
linear extrapolation (p. 226)
linear interpolation (p. 247)
linear regression (p. 253)
line of fit (p. 246)
median-fit line (p. 255)
parallel lines (p. 237)
perpendicular lines (p. 238)
piecewise-defined function (p. 262)
piecewise-linear function (p. 261)
point-slope form (p. 231)
scatter plot (p. 245)
slope-intercept form (p. 214)
step function (p. 261)

Vocabulary Check

State whether each sentence is *true* or *false*. If *false*, replace the underlined term to make a true sentence.

1. The <u>y-intercept</u> is the y-coordinate of the point where the graph crosses the y-axis.

2. The process of using a linear equation to make predictions about values that are beyond the range of the data is called <u>linear regression</u>.

3. A graph in which two sets of data are plotted as ordered pairs in a coordinate plane is called a <u>step function</u>.

4. The <u>correlation coefficient</u> describes whether the correlation between the variables is positive or negative and how closely the regression equation is modeling the data.

5. Lines in the same plane that do not intersect are called <u>parallel</u> lines.

6. Lines that intersect at <u>acute</u> angles are called perpendicular lines.

7. A function that is defined differently for different parts of its domain is called a <u>piecewise-defined function</u>.

8. The <u>range</u> of the greatest integer function is the set of all real numbers.

9. A <u>piecewise-linear function</u> is also called a step function.

Lesson-by-Lesson Review

Graphing Equations in Slope-Intercept Form (pp. 214–221)

Write an equation of a line in slope-intercept form with the given slope and y-intercept. Then graph the equation.

10. slope: 3, y-intercept: 5

11. slope: -2, y-intercept: -9

12. slope: $\frac{2}{3}$, y-intercept: 3

13. slope: $-\frac{5}{8}$, y-intercept: -2

Graph each equation.

14. $y = 4x - 2$

15. $y = -3x + 5$

16. $y = \frac{1}{2}x + 1$

17. $3x + 4y = 8$

18. **SKI RENTAL** Write an equation in slope-intercept form for the total cost of skiing for h hours with one lift ticket.
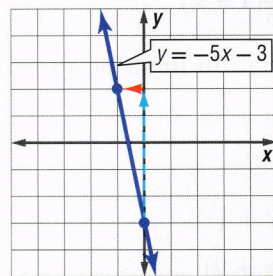
Slippery Slope
Ski Lodge

Lift Ticket $15/day
Ski Rental $5/hour

EXAMPLE 1

Write an equation of a line in slope-intercept form with slope -5 and y-intercept -3. Then graph the equation.

$y = mx + b$ **Slope-intercept form**

$y = -5x + (-3)$ $m = -5$ **and** $b = -3$

$y = -5x - 3$ **Simplify.**

To graph the equation, plot the y-intercept $(0, -3)$.

Then move up 5 units and left 1 unit. Plot the point. Draw a line through the two points.

$y = -5x - 3$

Writing Equations in Slope-Intercept Form (pp. 224–230)

Write an equation of the line that passes through the given point and has the given slope.

19. $(1, 2)$, slope 3

20. $(2, -6)$, slope -4

21. $(-3, -1)$, slope $\frac{2}{5}$

22. $(5, -2)$, slope $-\frac{1}{3}$

Write an equation of the line that passes through the given points.

23. $(2, -1), (5, 2)$

24. $(-4, 3), (1, 13)$

25. $(3, 5), (5, 6)$

26. $(2, 4), (7, 2)$

27. **CAMP** In 2000, a camp had 450 campers. Five years later, the number of campers rose to 750. Write a linear equation that represents the number of campers that attend camp.

EXAMPLE 2

Write an equation of the line that passes through $(3, 2)$ with a slope of 5.

Step 1   Find the y-intercept.

$y = mx + b$ **Slope-intercept form**

$2 = 5(3) + b$ $m = 5, y = 2,$ **and** $x = 3$

$2 = 15 + b$ **Simplify.**

$-13 = b$ **Subtract 15 from each side.**

Step 2   Write the equation in slope-intercept form.

$y = mx + b$ **Slope-intercept form**

$y = 5x - 13$ $m = 5$ **and** $b = -13$

4-3 Point-Slope Form (pp. 231–236)

Write an equation in point-slope form for the line that passes through the given point with the slope provided.

28. $(6, 3)$, slope 5

29. $(-2, 1)$, slope -3

30. $(-4, 2)$, slope 0

Write each equation in standard form.

31. $y - 3 = 5(x - 2)$

32. $y - 7 = -3(x + 1)$

33. $y + 4 = \frac{1}{2}(x - 3)$

34. $y - 9 = -\frac{4}{5}(x + 2)$

Write each equation in slope-intercept form.

35. $y - 2 = 3(x - 5)$

36. $y - 12 = -2(x - 3)$

37. $y + 3 = 5(x + 1)$

38. $y - 4 = \frac{1}{2}(x + 2)$

EXAMPLE 3

Write an equation in point-slope form for the line that passes through **(3, 4)** with a slope of **−2**.

$y - y_1 = m(x - x_1)$ **Point-slope form**

$y - 4 = -2(x - 3)$ **Replace m with −2 and (x_1, y_1) with (3, 4).**

EXAMPLE 4

Write $y + 6 = -4(x - 3)$ in standard form.

$y + 6 = -4(x - 3)$ **Original equation**

$y + 6 = -4x + 12$ **Simplify.**

$4x + y + 6 = 12$ **Add 4x to each side.**

$4x + y = 6$ **Subtract 6 from each side.**

4-4 Parallel and Perpendicular Lines (pp. 237–243)

Write an equation in slope-intercept form for the line that passes through the given point and is parallel to the graph of each equation.

39. $(2, 5)$, $y = x - 3$

40. $(0, 3)$, $y = 3x + 5$

41. $(-4, 1)$, $y = -2x - 6$

42. $(-5, -2)$, $y = -\frac{1}{2}x + 4$

Write an equation in slope-intercept form for the line that passes through the given point and is perpendicular to the graph of the given equation.

43. $(2, 4)$, $y = 3x + 1$

44. $(1, 3)$, $y = -2x - 4$

45. $(-5, 2)$, $y = \frac{1}{3}x + 4$

46. $(3, 0)$, $y = -\frac{1}{2}x$

EXAMPLE 5

Write an equation in slope-intercept form for the line that passes through **(−2, 4)** and is parallel to the graph of $y = 6x - 3$.

The slope of the line with equation $y = 6x - 3$ is 6. The line parallel to $y = 6x - 3$ has the same slope, 6.

$y - y_1 = m(x - x_1)$ **Point-slope form**
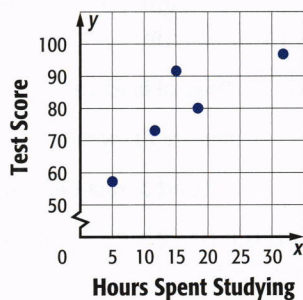
$y - 4 = 6(x - (-2))$ **Substitute.**

$y - 4 = 6(x + 2)$ **Simplify.**

$y - 4 = 6x + 12$ **Distributive Property**

$y = 6x + 16$ **Add 4 to each side.**

4-5 Scatter Plots and Lines of Fit (pp. 245–251)

47. Determine whether the graph shows a *positive*, *negative*, or *no* correlation. If there is a positive or negative correlation, describe its meaning.

48. **ATTENDANCE** A scatter plot of data compares the number of years since a business has opened and its annual number of sales. It contains the ordered pairs (2, 650) and (5, 1280). Write an equation in slope-intercept form for the line of fit for this situation.

EXAMPLE 6

The scatter plot displays the number of text messages and the number of phone calls made daily. Write an equation for the line of fit.
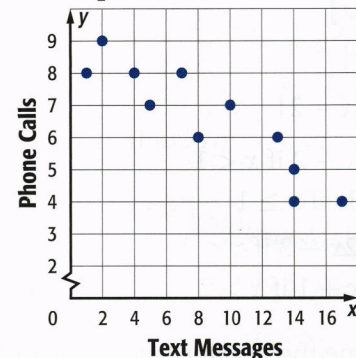
First, find the slope using (2, 9) and (17, 4).

$m = \dfrac{4 - 9}{17 - 2} = \dfrac{-5}{15}$ or $-\dfrac{1}{3}$ **Substitute and simplify.**

Then find the y-intercept.

$9 = -\dfrac{1}{3}(2) + b$ **Substitute.**

$9\dfrac{2}{3} = b$ **Add $\dfrac{2}{3}$ to each side.**

Write the equation. $y = -\dfrac{1}{3}x + 9\dfrac{2}{3}$

4-6 Regression and Median-Fit Lines (pp. 253–260)

49. **SALE** The table shows the number of sales made at an outerwear store during a sale. Write an equation of the regression line. Then estimate the daily sales on day 10 of the sale.

Days Since Sale Began	1	2	3	4	5	6	7
Daily Sales ($)	15	21	32	30	40	38	51

50. **MOVIES** The table shows ticket sales during the first week. Write an equation of the regression line. Then estimate the daily ticket sales on the 15th day after the movie opens.

Days Since Movie Opened	1	2	3	4	5	6	7
Daily Ticket Sales ($)	85	92	89	78	65	68	55

EXAMPLE 7

ATTENDANCE The table shows the annual attendance at an amusement park. Write an equation of the regression line for the data.

Year (since 2000)	0	1	2	3	4	5	6
Attendance (thousands)	75	80	72	68	65	60	53

Step 1 Enter the data by pressing [STAT] and selecting the **Edit** option.

Step 2 Perform the regression by pressing [STAT] and selecting the **CALC** option. Scroll down to **LinReg (ax + b)** and press [ENTER].

Step 3 Write the equation of the regression line by rounding the a- and b-values on the screen.
$y = -4.04x + 79.68$

4-7 Special Functions (pp. 261–268)

Graph each function. State the domain and range.

51. $f(x) = [\![x]\!]$

52. $f(x) = [\![2x]\!]$

53. $f(x) = |x|$

54. $f(x) = |2x - 2|$

55. $f(x) = \begin{cases} x - 2 \text{ if } x < 1 \\ 3x \text{ if } x \geq 1 \end{cases}$

56. $f(x) = \begin{cases} 2x - 3 \text{ if } x \leq 2 \\ x + 1 \text{ if } x > 2 \end{cases}$

57. Determine the domain and range of the function graphed below.
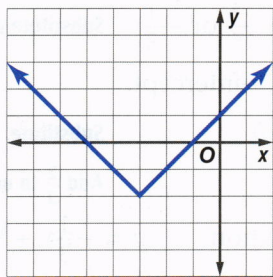
EXAMPLE 8

Graph $f(x) = |x + 3|$. State the domain and range.

Since $f(x)$ cannot be negative, the minimum point of the graph is where $f(x) = 0$.

$f(x) = |x + 3|$ **Original function**

$0 = x + 3$ **Replace $f(x)$ with 0.**

$-3 = x$ **Subtract 3 from each side.**

Next, make a table of values. Include values for $x > -3$ and $x < -3$.

x	−5	−4	−3	−2	−1
f(x)	2	1	0	1	2
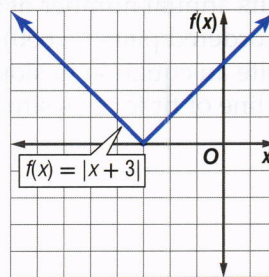
The domain is all real numbers, and the range is $f(x) \geq 0$.

1. Graph $y = 2x - 3$.

2. **MULTIPLE CHOICE** A popular pizza parlor charges $12 for a large cheese pizza plus $1.50 for each additional topping. Write an equation in slope-intercept form for the total cost C of a pizza with t toppings.

   A $C = 12t + 1.50$

   B $C = 13.50t$

   C $C = 12 + 1.50t$

   D $C = 1.50t - 12$

Write an equation of a line in slope-intercept form that passes through the given point and has the given slope.

3. $(-4, 2)$; slope -3 4. $(3, -5)$; slope $\frac{2}{3}$

Write an equation of the line in slope-intercept form that passes through the given points.

5. $(1, 4)$, $(3, 10)$ 6. $(2, 5)$, $(-2, 8)$

7. $(0, 4)$, $(-3, 0)$ 8. $(7, -1)$, $(9, -4)$

9. **PAINTING** The data in the table show the size of a room in square feet and the time it takes to paint the room in minutes.

Room Size	100	150	200	400	500
Painting Time	160	220	270	500	680

   a. Use the points $(100, 160)$ and $(500, 680)$ to write an equation in slope-intercept form.

   b. Predict the amount of time required to paint a room measuring 750 square feet.

10. **SALARY** The table shows the relationship between years of experience and teacher salary.

Years Experience	1	5	10	15	20
Salary (thousands of dollars)	28	31	42	49	64

   a. Write an equation for the best-fit line.

   b. Find the correlation coefficient and explain what it tells us about the relationship between experience and salary.

Write an equation in slope-intercept form for the line that passes through the given point and is parallel to the graph of each equation.

11. $(2, -3)$, $y = 4x - 9$

12. $(-5, 1)$, $y = -3x + 2$

Write an equation in slope-intercept form for the line that passes through the given point and is perpendicular to the graph of the equation.

13. $(1, 4)$, $y = -2x + 5$ 14. $(-3, 6)$, $y = \frac{1}{4}x + 2$

15. **MULTIPLE CHOICE** The graph shows the relationship between outside temperature and daily ice cream cone sales. What type of correlation is shown?

   F positive correlation

   G negative correlation

   H no correlation

   J not enough information
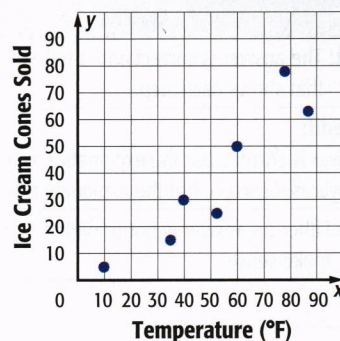
Graph each function.

16. $f(x) = |x - 1|$

17. $f(x) = -|2x|$

18. $f(x) = [\![x]\!]$

19. $f(x) = \begin{cases} 2x - 1 \text{ if } x < 2 \\ x - 3 \text{ if } x \geq 2 \end{cases}$

20. The table shows the number of children from Russia adopted by U.S. citizens.

Years Since 2000	0	1	2	3	4
Number of Children	4269	4279	4939	5209	6936

   a. Write the slope-intercept form of the equation for the regression line.

   b. Predict the number of children from Russia who will be adopted in 2025.

Short Answer Questions

Short answer questions require you to provide a solution to the problem, along with a method, explanation, and/or justification used to arrive at the solution.
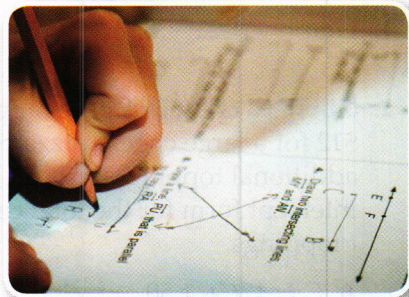
Strategies for Solving Short Answer Questions

Step 1

Short answer questions are typically graded using a **rubric**, or a scoring guide. The following is an example of a short answer question scoring rubric.

Scoring Rubric	
Criteria	**Score**
Full Credit: The answer is correct and a full explanation is provided that shows each step.	2
Partial Credit: • The answer is correct, but the explanation is incomplete. • The answer is incorrect, but the explanation is correct.	1
No Credit: Either an answer is not provided or the answer does not make sense.	0

Step 2

In solving short answer questions, remember to…

• explain your reasoning or state your approach to solving the problem.

• show all of your work or steps.

• check your answer if time permits.

EXAMPLE

Read the problem. Identify what you need to know. Then use the information in the problem to solve. Show your work.

The table shows production costs for building different numbers of skateboards. Determine the missing value, x, that will result in a linear model.
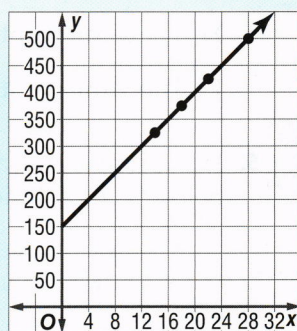
Skateboards Built	Production Costs
14	$325
28	$500
x	$375
22	$425

Read the problem carefully. You are given several data points and asked to find the missing value that results in a linear model.

Example of a 2-point response:

Set up a coordinate grid and plot the three given points: (14, 325), (28, 500), (22, 425).

Then draw a straight line through them and find the x-value that produces a y-value of 375.

So, building 18 skateboards would result in production costs of $375. These data form a linear model.

The steps, calculations, and reasoning are clearly stated. The student also arrives at the correct answer. So, this response is worth the full 2 points.

Exercises

Read each problem. Identify what you need to know. Then use the information in the problem to solve. Show your work.

1. Given points $M(-1, 7)$, $N(3, -5)$, $O(6, 1)$, and $P(-3, -2)$, determine two segments that are perpendicular to each other.

2. Write the equation of a line that is parallel to $4x + 2y = 8$ and has a y-intercept of 5.

3. Three vertices of a quadrilateral are shown on the coordinate grid. Determine a fourth vertex that would result in a trapezoid.
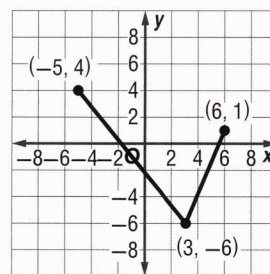
Multiple Choice

Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.

1. A shipping company charges $8.50 to ship packages that weigh up to 1 pound and $7.25 for each additional pound. Which of the following piecewise-defined functions represents the cost C of shipping a package that weighs p pounds?

   Shipping Rates
   - ◆ Up to 1 pound: **$8.50**
   - ◆ Each additional pound: **$7.25**

   A $C(p) = \begin{cases} 8.50 & \text{if } p \leq 1 \\ 8.50 + 7.25p & \text{if } p > 1 \end{cases}$

   B $C(p) = \begin{cases} 7.25 & \text{if } p \leq 1 \\ 7.25 + 8.50p & \text{if } p > 1 \end{cases}$

   C $C(p) = \begin{cases} 8.50 & \text{if } p \leq 1 \\ 8.50p + 7.25(p-1) & \text{if } p > 1 \end{cases}$

   D $C(p) = \begin{cases} 8.50 & \text{if } p \leq 1 \\ 8.50 + 7.25(p-1) & \text{if } p > 1 \end{cases}$

2. Refer to the information given in Exercise 1. How much would it cost a customer to ship a package that weighs 5 pounds 11 ounces? (Assume that partial pounds are rounded up to the nearest whole pound.)

   F $42.35   **H** $48.20

   G $44.75   **J** $52.00

3. Jaime bought a car in 2005 for $28,500. By 2008, the car was worth $23,700. Based on a linear model, what will the value of the car be in 2012?

   A $17,300   **C.** $18,100

   B $17,550   **D.** $18,475

4. If the graph of a line has a positive slope and a negative y-intercept, what happens to the x-intercept if the slope and the y-intercept are doubled?

   F The x-intercept becomes four times as great.

   G The x-intercept becomes twice as great.

   H The x-intercept becomes one-fourth as great.

   J The x-intercept remains the same.

5. Which absolute value equation has the graph below as its solution?
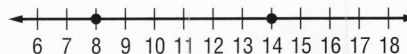
   A $|x - 3| = 11$

   B $|x - 4| = 12$

   C $|x - 11| = 3$

   D $|x - 12| = 4$

6. The table below shows the relationship between certain temperatures in degrees Fahrenheit and degrees Celsius. Which of the following linear equations correctly models this relationship?

   F $F = \frac{8}{5}C + 35$

   G $F = \frac{4}{5}C + 42$

   H $F = \frac{9}{5}C + 32$

   J $F = \frac{12}{5}C + 26$

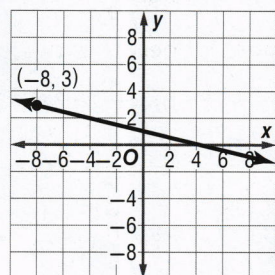 | Celsius (C) | Fahrenheit (F) |
   |---|---|
   | 10° | 50° |
   | 15° | 59° |
   | 20° | 68° |
   | 25° | 77° |
   | 30° | 86° |

   Test-TakingTip

   ▶ **Question 3** Find the average annual depreciation between 2005 and 2008. Then extend the pattern to find the car's value in 2012.

Short Response/Gridded Response

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

7. What is the equation of the line graphed below?

Express your answer in point slope form using the point $(-8, 3)$.

8. **GRIDDED RESPONSE** The linear equation below is a best fit model for the peak depth of the Mad River when x inches of rain fall. What would you expect the peak depth of the river to be after a storm that produces $1\frac{3}{4}$ inches of rain? Round your answer to the nearest tenth of a foot if necessary.

$$y = 2.5x + 14.8$$

9. Jacob formed an advertising company in 1992. Initially, the company only had 14 employees. In 2008, the company had grown to a total of 63 employees. Find the percent of change in the number of employees working at Jacob's company. Round to the nearest tenth of a percent if necessary.

10. The table shows the total amount of rain during a storm.

Hour	1	2	3	4
Inches	0.45	0.9	1.35	1.8

  a. Write an equation to fit the data in the table.

  b. Describe the relationship between the hour and the amount of rain received.

11. An electrician charges a $25 consultation fee plus $35 per hour for labor.

  a. Copy and complete the following table showing the charges for jobs that take 1, 2, 3, 4, or 5 hours.

Hours, h	Total Cost, C
1	
2	
3	
4	
5	

  b. Write an equation in slope-intercept form for the total cost of a job that takes h hours.

  c. If the electrician bills in quarter hours, how much would it cost for a job that takes 3 hours 15 minutes to complete?

Extended Response

Record your answer on a sheet of paper. Show your work.

12. Explain how you can determine whether two lines are parallel or perpendicular.

Need Extra Help?												
If you missed Question...	1	2	3	4	5	6	7	8	9	10	11	12
Go to Lesson or Page...	4-7	4-7	4-5	3-1	2-5	4-2	4-3	4-5	2-7	3-6	4-2	4-4

Then

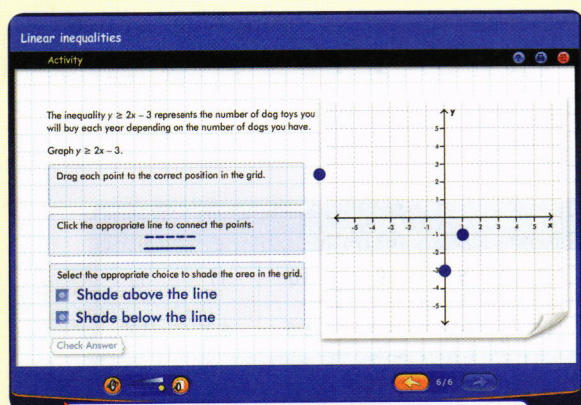In Chapter 2, you solved equations.

Now

In Chapter 5, you will:

- Solve one-step and multi-step inequalities.
- Solve compound inequalities and inequalities involving absolute value.
- Graph inequalities in two variables.

Why?

🌐 **PETS** In the United States, about 75 million dogs are kept as pets. Approximately 16% of these were adopted from animal shelters. About 14% of dog owners own more than 3 dogs.

Linear inequalities

Activity

The inequality $y \geq 2x - 3$ represents the number of dog toys you will buy each year depending on the number of dogs you have.

Graph $y \geq 2x - 3$.

Drag each point to the correct position in the grid.

Click the appropriate line to connect the points.

Select the appropriate choice to shade the area in the grid.
- Shade above the line
- Shade below the line

Check Answer

6/6

▶ **Math** *in Motion,* **Animation** **glencoe.com**