Get Ready for Chapter 7

Diagnose Readiness You have two options for checking Prerequisite Skills.

Text Option

Take the Quick Check below. Refer to the Quick Review for help.

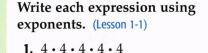
EXAMPLE 1

4 factors of 5 is 5^4 .

QuickCheck

QuickReview

Write $5 \cdot 5 \cdot 5 \cdot 5 + x \cdot x \cdot x$ using exponents.

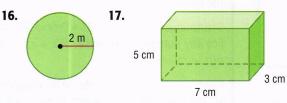


- **2.** $y \cdot y \cdot y$
- 3. 6 6
- **5.** *b b b b b b*
- 6. $m \cdot m \cdot m \cdot p \cdot p \cdot p \cdot p \cdot p \cdot p$
- 7. $\frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3}$ 8. $\frac{x}{y} \cdot \frac{x}{y} \cdot \frac{x}{y} \cdot \frac{x}{y} \cdot \frac{w}{z} \cdot \frac{w}{z}$
- Evaluate each expression. (Lesson 1-2)

9. 2 ³	10. $(-5)^2$	11. 3 ³
12. $(-4)^3$	13. $\left(\frac{2}{3}\right)^2$	14. $\left(\frac{1}{2}\right)^4$

15. SCHOOL The probability of guessing correctly on 5 true-false questions is $\left(\frac{1}{2}\right)$ Express this probability as a fraction without exponents.

Find the area or volume of each figure. (Lessons 0-8 and 0-9)

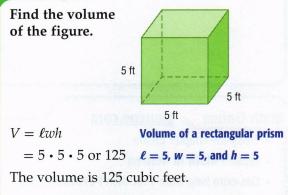


18. PHOTOGRAPHY A photo is 4 inches by 6 inches. What is the area of the photo?

Math Online

	3 factors of x is x^3 .
	So, $5 \cdot 5 \cdot 5 \cdot 5 + x \cdot x \cdot x = 5^4 + x^3$.
=	dia dan pinak al ap
	thing with the second sheet
	EXAMPLE 2
	Evaluate $\left(\frac{5}{7}\right)^2$.
	$\left(\frac{5}{7}\right)^2 = \frac{5^2}{7^2}$ Power of a Quotient
	$=\frac{25}{49}$ Simplify.

EXAMPLE 3

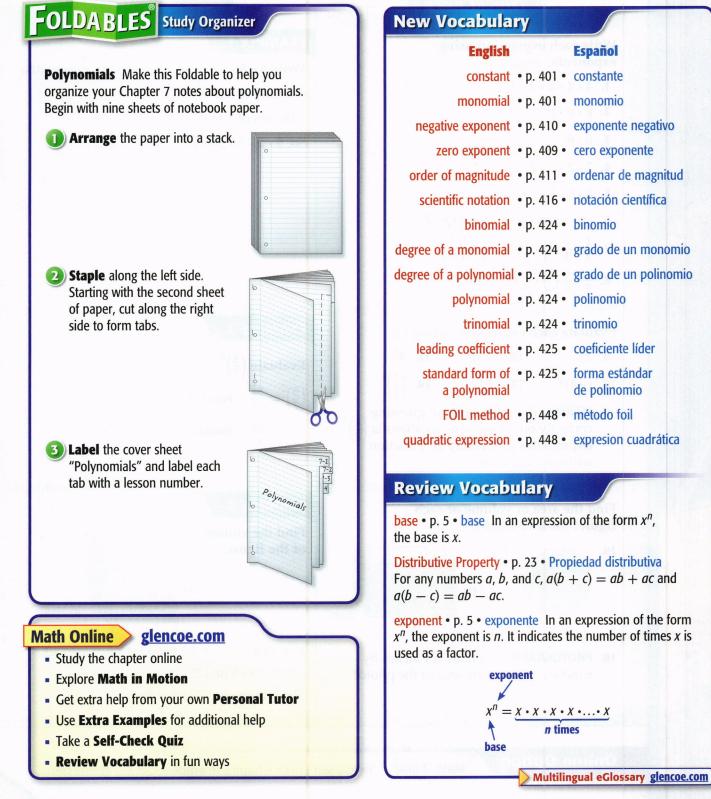


Online Option

Take a self-check Chapter Readiness Quiz at glencoe.com.

Get Started on Chapter 7

You will learn several new concepts, skills, and vocabulary terms as you study Chapter 7. To get ready, identify important terms and organize your resources. You may wish to refer to **Chapter 0** to review prerequisite skills.



Multiplying Monomials

Why?

Many formulas contain *monomials*. For example, the formula for the horsepower of a car is $H = w \left(\frac{v}{234}\right)^3$. *H* represents the horsepower produced by the engine, *w* equals the weight of the car with passengers, and *v* is the velocity of the car at the end of a quarter of a mile. As the velocity increases, the horsepower increases.



Monomials A **monomial** is a number, a variable, or the product of a number and one or more variables with nonnegative integer exponents. It has only one term. In the formula to calculate the horsepower of a car, the term $w\left(\frac{v}{234}\right)^3$ is a monomial. An expression that involves division by a variable, like $\frac{ab}{c}$, is not a monomial.

A **constant** is a monomial that is a real number. The monomial 3x is an example of a *linear expression* since the exponent of x is 1. The monomial $2x^2$ is a *nonlinear expression* since the exponent is a positive number other than 1.

EXAMPLE 1 Identify Monomials

Determine whether each expression is a monomial. Write *yes* or *no*. Explain your reasoning.

- **a.** 10 Yes; this is a constant, so it is a monomial.
- **b.** f + 24 No; this expression has addition, so it has more than one term.
- **c.** h^2 Yes; this expression is a product of variables.
- **d.** *j* Yes; single variables are monomials.

Check Your Progress





1B. $23abcd^2$ **1D.** $\frac{mp}{n}$ **Personal Tutor glencoe.com**

Recall that an expression of the form x^n is called a *power* and represents the result of multiplying x by itself n times. x is the *base*, and n is the *exponent*. The word *power* is also used sometimes to refer to the exponent.

exponent

$$3^4 = 3 \cdot 3 \cdot 3 \cdot 3 = 81$$

base

You performed operations on expressions with exponents. (Lesson 1-1)

Now

Then

- Multiply monomials.
- Simplify expressions involving monomials.

New/ Vocabulary/ monomial constant

Math Online

- Extra Examples
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- Self-Check Quiz
- Homework Help

By applying the definition of a power, you can find the product of powers. Look for a pattern in the exponents.

These examples demonstrate the property for the product of powers.

Key (Concept Product of Powers	For Your
Words	To multiply two powers that have the same base, add their exponents.	tonon vollar
Symbols	For any real number <i>a</i> and any integers <i>m</i> and <i>p</i> , $a^m \cdot a^p = a^{m+p}$.	
Examples	$b^3 \cdot b^5 = b^{3+5} \text{ or } b^8$ $g^4 \cdot g^6 = g^{4+6} \text{ or } g^{10}$	han Paraki

EXAMPLE 2 Product of Powers

Simplify each expression.

Check Your Progress

2A. $(3y^4)(7y^5)$

 \checkmark

a. $(6n^3)(2n^7)$ $(6n^3)(2n^7) = (6 \cdot 2)(n^3 \cdot n^7)$ $= (6 \cdot 2)(n^3 + 7)$ $= 12n^{10}$

b.
$$(3pt^3)(p^3t^4)$$

 $(3pt^3)(p^3t^4) = (3 \cdot 1)(p \cdot p^3)(t^3 \cdot t^4)$
 $= (3 \cdot 1)(p^{1+3})(t^{3+4})$
 $= 3p^4t^7$

Group the coefficients and the variables. Product of Powers Simplify.

Group the coefficients and the variables. Product of Powers Simplify.

2B.
$$(-4rx^2t^3)(-6r^5x^2t)$$

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We can use the Product of Powers Property to find the power of a power. In the following examples, look for a pattern in the exponents.

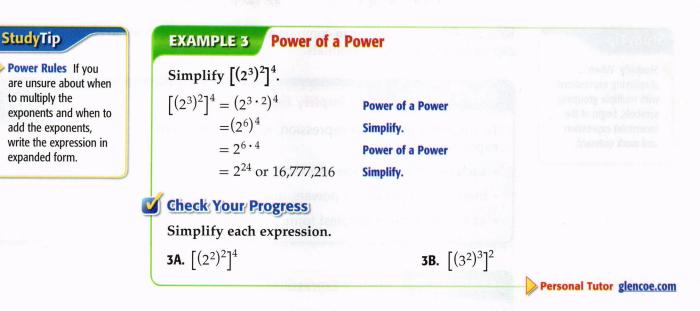
$$(3^{2})^{4} = \overbrace{(3^{2})(3^{2})(3^{2})(3^{2})}^{4 \text{ factors}} \qquad (r^{4})^{3} = \overbrace{(r^{4})(r^{4})(r^{4})}^{3 \text{ factors}} = 3^{8} = r^{12}$$

These examples demonstrate the property for the power of a power.

Key Cor	For You For You FOLDAB
Words	To find the power of a power, multiply the exponents.
Symbols	For any real number <i>a</i> and any integers <i>m</i> and <i>p</i> , $(a^m)^p = a^m \cdot p$.
Examples	$(b^3)^5 = b^{3+5} \text{ or } b^{15} \qquad (g^6)^7 = g^{6+7} \text{ or } g^{42}$

StudyTip

Coefficients and Powers of 1 A variable with no exponent or coefficient shown can be assumed to have an exponent and coefficient of 1. For example, $x = 1x^{1}$.



We can use the Product of Powers Property and the Power of a Power Property to find the power of a product. In the following examples, look for a pattern in the exponents.

$$(tw)^{3} = \underbrace{(tw)(tw)(tw)}_{= (t \cdot t \cdot t)(w \cdot w \cdot w)}_{= t^{3}w^{3}} = \underbrace{(2yz^{2})(2yz^{2})(2yz^{2})}_{= (2 \cdot 2 \cdot 2)(y \cdot y \cdot y)(z^{2} \cdot z^{2} \cdot z^{2})}_{= 2^{3}y^{3}z^{6} \text{ or } 8y^{3}z^{6}}$$

These examples demonstrate the property for the power of a product.

Key Co	ncept Power of a Product For Your
Words	To find the power of a product, find the power of each factor and multiply.
Symbols	For any real numbers a and b and any integer m , $(ab)^m = a^m b^m$.
Example	$(-2xy^3)^5 = (-2)^5 x^5 y^{15}$ or $-32x^5 y^{15}$

EXAMPLE 4 Power of a Product

GEOMETRY Express the area of the circle as a monomial.

Area = πr^2 = $\pi (2xy^2)^2$ = $\pi (2^2x^2y^4)$ = $4x^2y^4\pi$ Formula Replace Power of Simplify.

Formula for the area of a circle Replace *r* with 2*xy*². Power of a Product

1	
($2xy^2$
	-)

The area of the circle is $4x^2y^4\pi$ square units.

Check Your Progress

4A. Express the area of a square with sides of length $3xy^2$ as a monomial.

4B. Express the area of a triangle with a height of 4a and a base of $5ab^2$ as a monomial.

StudyTip

Simplify When simplifying expressions with multiple grouping symbols, begin at the innermost expression and work outward. **Simplify Expressions** We can combine and use these properties to simplify expressions involving monomials.

Power of a Power

Power of a Product

Power of a Power

Product of Powers

Commutative

Key Concept Simplify Expressions

To simplify a monomial expression, write an equivalent expression in which:

- each variable base appears exactly once,
- there are no powers of powers, and
- all fractions are in simplest form.

EXAMPLE 5 Simplify Expressions

Simplify $(3xy^4)^2[(-2y)^2]^3$. $(3xy^4)^2[(-2y)^2]^3 = (3xy^4)^2(-2y)^6$ $= (3)^2x^2(y^4)^2(-2)^6y^6$ $= 9x^2y^8(64)y^6$ $= 9(64)x^2 \cdot y^8 \cdot y^6$ $= 576x^2y^{14}$

Check Your Progress

5. Simplify
$$\left(\frac{1}{2}a^2b^{-2}\right)^3 [(-4b)^2]^2$$
.

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For Your

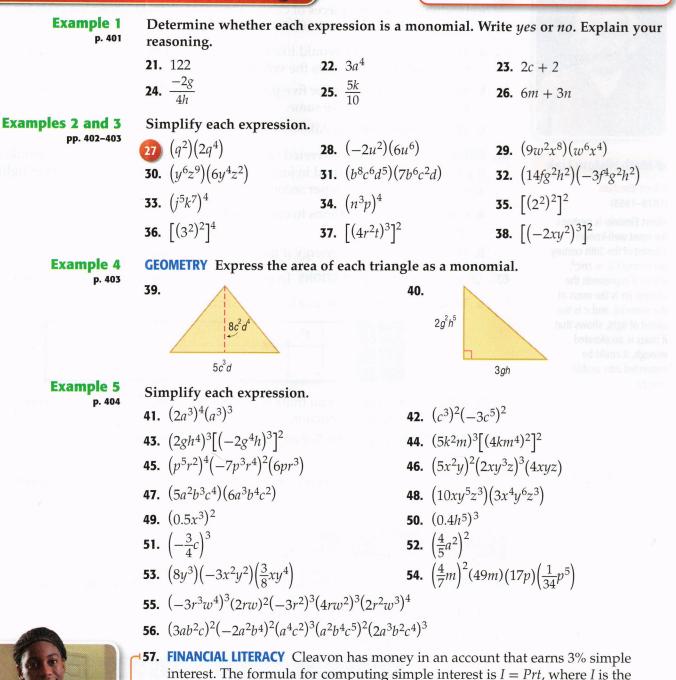
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🗹 Check Your Understanding

Example 1 p. 401	Determine whether or reasoning.	each expression is a monom	ial. Write <i>yes</i> or <i>no</i> . Explain your	
	1. 15	2. 2 – 3 <i>a</i>	3. $\frac{5c}{d}$	
	4. $-15g^2$	5. $\frac{r}{2}$	6. 7b + 9	
Examples 2 and 3	Simplify each expres	sion.		
рр. 402–403	7. $k(k^3)$	8. $m^4(m^2)$	9 $2q^2(9q^4)$	
	10. $(5u^4v)(7u^4v^3)$	11. $[(3^2)^2]^2$	12. $(xy^4)^6$	
	13. $(4a^4b^9c)^2$	14. $(-2f^2g^3h^2)^3$	15. $(-3p^5t^6)^4$	
Example 4 p. 403	16. GEOMETRY The formula for the surface area of a cube is $SA = 6s^2$, where <i>SA</i> is the surface area and <i>s</i> is the length of any side.			
	a. Express the sur	face area of the cube as a mo	nomial.	
	b. What is the sur	face area of the cube if $a = 3$	and $b = 4$?	
Example 5	Simplify each expres	ssion.		
p. 404	17. $(5x^2y)^2(2xy^3z)^3(4x^3y^3)^2(2xy^3z)^3(4x^3y^3)^3)^3(4x^$	<i>xyz</i>) 18. (-3	$(3d^2f^{-3}g)^2[(-3d^2f)^3]^2$	
	19. $(-2g^{3}h)(-3gj^{4})^{2}($	$-ghj)^2$ 20. (-7	$(2a^2c)^3[(2a^2c)^2]^3$	

Practice and Problem Solving

Step-by-Step Solutions begin on page R12. Extra Practice begins on page 815.



Real-World Link

84% of teens have some money saved. The average teen has saved \$1044.

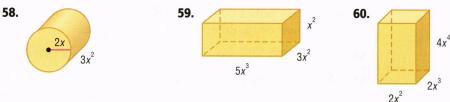
Source: Charles Schwab Teens & Money Survey

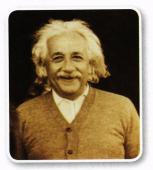
interest rate (in decimal form), and *t* represents time in years.a. Cleavon makes a deposit of \$2*c* and leaves it for 2 years. Write a monomial that represents the interest earned.

interest earned, P represents the principal that he put into the account, r is the

b. If *c* represents a birthday gift of \$250, how much will Cleavon have in this account after 2 years?

GEOMETRY Express the volume of each solid as a monomial.





Math History Link

Albert Einstein (1879–1955)

Albert Einstein is perhaps the most well-known scientist of the 20th century. His formula $E = mc^2$, where *E* represents the energy, *m* is the mass of the material, and *c* is the speed of light, shows that if mass is accelerated enough, it could be converted into usable energy.

- **PACKAGING** For a commercial art class, Aiko must design a new container for individually wrapped pieces of candy. The shape that she chose is a cylinder. The formula for the volume of a cylinder is $V = \pi r^2 h$.
 - **a.** The radius that Aiko would like to use is $2p^3$, and the height is $4p^3$. Write a monomial that represents the volume of her container.
 - **b.** Make a table of values for five possible radius widths and heights if the volume is to remain the same.
 - **c.** What is the volume of Aiko's container if the height is doubled?
- **62. ENERGY** Matter can be converted completely into energy by using the formula at the left. Energy is measured in joules, mass in kilograms, and the speed of light is about 300 million meters per second.
 - **a.** Complete the calculations to convert 3 kilograms of gasoline completely into energy.
 - **b.** What happens to the energy if the amount of gasoline is doubled?
- **63. WULTIPLE REPRESENTATIONS** In this problem, you will explore exponents.
 - **a. TABULAR** Copy and use a calculator to complete the table.

Power	3 ⁴	3 ³	3 ²	3 ¹	3 ⁰	3 ⁻¹	3-2	3-3	3 ⁻⁴
Value						$\frac{1}{3}$	<u>1</u> 9	$\frac{1}{27}$	<u>1</u> 81

- **b. ANALYTICAL** What do you think the values of 5^0 and 5^{-1} are? Verify your conjecture using a calculator.
- **c. ANALYTICAL** Complete: For any nonzero number *a* and any integer *n*, $a^{-n} =$ _____.
- d. VERBAL Describe the value of a nonzero number raised to the zero power.

H.O.T. Problems Use Higher-Ord

Use Higher-Order Thinking Skills

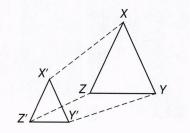
- **64. CHALLENGE** For any nonzero real numbers *a* and *b* and any integers *m* and *t*, simplify the expression $\left(-\frac{a^m}{b^t}\right)^{2t}$ and describe each step.
- **65. REASONING** Copy the table below.

Equation	Related Expression	Power of <i>x</i>	Linear or Nonlinear
y = x	ied. ^D represents	nter et van	
$y = x^2$	un decimal Nam	star berriat	
$y = x^3$	nteless à deposit e	no de la	

- **a.** For each equation, write the related expression and record the power of *x*.
- **b**. Graph each equation using a graphing calculator.
- **c.** Classify each graph as *linear* or *nonlinear*.
- **d.** Explain how to determine whether an equation, or its related expression, is linear or nonlinear without graphing.
- **66. OPEN ENDED** Write three different expressions that can be simplified to x^6 .
- **67.** WRITING IN MATH Write two formulas that have monomial expressions in them. Explain how each is used in a real-world situation.

Standardized Test Practice

- 68. Which of the following is not a monomial?
 - **A** -6xy **C** $-\frac{1}{2b^3}$
 - **B** $\frac{1}{2}a^2$ **D** $5gh^4$
- **69. GEOMETRY** The accompanying diagram shows the transformation of $\triangle XYZ$ to $\triangle X'Y'Z'$.



This transformation is an example of a

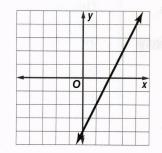
- F dilation
- **G** line reflection
- H rotation
- J translation

70. CARS In 1994, the average price of a new domestic car was \$16,930. In 2002, the average price was \$19,126. Based on a linear model, what is the predicted average price for 2010?

.0,773
.(

B \$21,322 **D** \$18,577

71. SHORT RESPONSE If a line has a positive slope and a negative *y*-intercept, what happens to the *x*-intercept if the slope and the *y*-intercept are both doubled?



Spiral Review

Solve each system of inequalities by graphing. (Lesson 6-8)

72. $y < 4x$	73. $y \ge 2$	74. $y > -2x - 1$	75. $3x + 2y < 10$
$2x + 3y \ge -21$	$2y + 2x \le 4$	$2y \le 3x + 2$	2x + 12y < -6

Perform the indicated matrix operations. If an operation cannot be performed, write *impossible*. (Lesson 6-7)

76.	$\begin{bmatrix} 2 & 5 & 3 \\ -5 & -1 & 10 \\ 4 & -4 & 0 \end{bmatrix} +$	$\begin{bmatrix} -8 & 2 & -3 \\ 3 & 6 & -4 \\ -6 & -10 \end{bmatrix}$	$ \begin{bmatrix} -6 \\ -1 \\ 6 \end{bmatrix} $ 77. $ \begin{bmatrix} 11 & 0 & 7 \\ 8 & 11 & -10 \end{bmatrix} - \begin{bmatrix} -3 & 0 & 4 \end{bmatrix} $
78.	$\begin{bmatrix} -5 & 2 & -11 \\ 2 & -2 & 1 \end{bmatrix}$	$+\begin{bmatrix}2&5\\3&-9\end{bmatrix}$	79. $\begin{bmatrix} 2 & -5 & -7 \\ -1 & 11 & 1 \\ 6 & -3 & 4 \end{bmatrix} + \begin{bmatrix} -4 & 0 & -9 \\ 12 & -12 & 8 \\ 12 & 0 & 8 \end{bmatrix}$

80. BABYSITTING Alexis charges \$10 plus \$4 per hour to babysit. Alexis needs at least \$40 more to buy a television for which she is saving. Write an inequality for this situation. Will she be able to get her television if she babysits for 5 hours? (Lesson 5-6)

Skills Review

Find each quotient. (Lesson 0–3)

81.	$-64 \div (-8)$
84.	$-23.94 \div 10.5$

82.	$-78 \div 1.3$
85.	$-32.5 \div (-2.5)$

83. 42.3 ÷ (−6)
86. −98.44 ÷ 4.6

Then

You multiplied monomials. (Lesson 7-1)

Now

- Find the quotient of two monomials.
- Simplify expressions containing negative and zero exponents.

New Vocabulary

zero exponents negative exponent order of magnitude

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- Self-Check Quiz
- Homework Help
- Math in Motion

Dividing Monomials

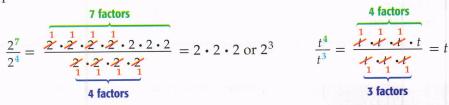
Why?

The tallest redwood tree is 112 meters or about 10^2 meters tall. The average height of a redwood tree is 15 meters. The closest power of ten to 15 is 10^1 , so an average redwood is about 10^1 meters tall. The ratio of the tallest tree's height to the average tree's height is $\frac{10^2}{10^1}$ or 10^1 .

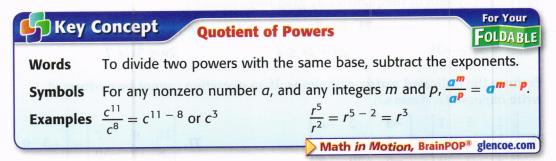
This means the tallest redwood tree is approximately 10 times as tall as the average redwood tree.



Quotients of Monomials We can use the principles for reducing fractions to find quotients of monomials like $\frac{10^2}{10^0}$. In the following examples, look for a pattern in the exponents.



These examples demonstrate the Quotient of Powers Rule.



EXAMPLE 1Quotient of PowersSimplify $\frac{g^3h^5}{gh^2}$. Assume that no denominator equals zero. $\frac{g^3h^5}{gh^2} = \left(\frac{g^3}{g}\right) \left(\frac{h^5}{h^2}\right)$ Group powers with the same base. $= (g^{3-1})(h^{5-2})$ Quotient of Powers $= g^2h^3$ Simplify.Check Your ProgressSimplify each expression. Assume that no denominator equals zero.1A. $\frac{x^3y^4}{x^2y}$ 1B. $\frac{k^7m^{10}p}{k^5m^3p}$

We can use the Product of Powers Rule to find the powers of quotients for monomials. In the following example, look for a pattern in the exponents.

$$\left(\frac{3}{4}\right)^3 = \left(\frac{3}{4}\right)\left(\frac{3}{4}\right)\left(\frac{3}{4}\right) = \frac{3}{4} + \frac{3}{4} + \frac{3}{4} = \frac{3}{4^3}$$

$$\left(\frac{3}{4}\right)^3 = \left(\frac{3}{4}\right)\left(\frac{3}{4}\right)\left(\frac{3}{4}\right) = \frac{3}{4} + \frac{3}{4} + \frac{3}{4} = \frac{3}{4^3}$$

$$3 \text{ factors}$$

$$\left(\frac{c}{4}\right)^2 = \left(\frac{c}{6}\right)\left(\frac{c}{6}\right) = \frac{2}{6} + \frac{c}{6} + \frac{c}{6}$$

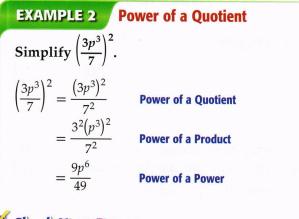
$$\left(\frac{\overline{d}}{\overline{d}}\right)^{-} = \left(\frac{\overline{d}}{\overline{d}}\right)\left(\frac{\overline{d}}{\overline{d}}\right) = \frac{\overline{d}}{\overline{d} \cdot \overline{d}} = \frac{\overline{d}^2}{\overline{d}^2}$$
2 factors

Power Rules with Variables

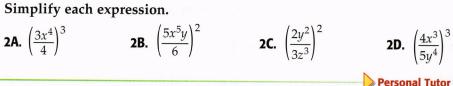
StudyTip

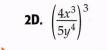
The power rules apply to variables as well as numbers. For example, $\left(\frac{3a}{4b}\right)^3 = \frac{(3a)^3}{(4b)^3}$ or $\frac{27a^3}{64b^3}$.

Key (Concept Power of a Quotient For Your
Words	To find the power of a quotient, find the power of the numerator and the power of the denominator.
Symbols	For any real numbers a and $b \neq 0$, and any integer m , $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$.
Examples	$\left(\frac{3}{5}\right)^4 = \frac{3^4}{5^4} \qquad \left(\frac{r}{t}\right)^5 = \frac{r^5}{t^5}$



Check Your Progress





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A calculator can be used to explore expressions with 0 as the exponent. There are two methods to explain why a calculator gives a value of 1 for 3⁰.

Method 1

Method 2

$\frac{3^5}{3^5} = 3^{5-5}$	Quotient of Powers	$\frac{3^5}{3^5} = \frac{\cancel{3} \cdot \cancel{3} \cdot \cancel{3} \cdot \cancel{3} \cdot \cancel{3} \cdot \cancel{3}}{\cancel{3} \cdot \cancel{3} \cdot \cancel{3} \cdot \cancel{3} \cdot \cancel{3} \cdot \cancel{3}}$	Definition of powers
$= 3^{0}$	Simplify.	= 1	Simplify.

Since $\frac{3^5}{3^5}$ can only have one value, we can conclude that $3^0 = 1$. A **zero exponent** is any nonzero number raised to the zero power.

Key Concept

Zero Exponent Property

Any nonzero number raised to the zero power is equal to 1. Words

For any nonzero number a, $a^0 = 1$. **Symbols Examples** $15^0 = 1$

$$\left(\frac{b}{c}\right)^0 = 1 \qquad \qquad \left(\frac{2}{7}\right)^0 = 1$$

EXAMPLE 3 Zero Exponent

Simplify each expression. Assume that no denominator equals zero.

a. $\left(-\frac{4n^2q^5r^2}{9n^3q^2r}\right)^0$ $\left(-\frac{4n^2q^5r^2}{9n^3q^2r}\right)^0 = 1 \qquad a^0 = 1$ **b.** $\frac{x^5y^0}{x^3}$ $\frac{x^5 y^0}{x^3} = \frac{x^5(1)}{x^3} \qquad a^0 = 1$ $= x^{2}$ **Quotient of Powers**

Check Your Progress

Zero Exponent Be careful of parentheses. The expression $(5x)^0$ is 1

StudyTip

but $5x^0 = 5$.

Negative Exponents Any nonzero real number raised to a negative power is a negative exponent. To investigate the meaning of a negative exponent, we can simplify expressions like $\frac{c^2}{c^5}$ using two methods.

Method 1

3A. $\frac{b^4c^2d^0}{b^2c}$

Method 2

 $\frac{c^2}{c^5} = c^{2-5}$ Quotient of Powers $\frac{c^2}{c^5} = \frac{e^{-5}}{e^{-5}}$ $= c^{-3}$ Simplify.

$$\frac{c^2}{c^5} = \frac{\cancel{e} \cdot \cancel{e}}{\cancel{e} \cdot \cancel{e} \cdot \cancel{c} \cdot \cancel{c} \cdot \cancel{c}}$$
$$= \frac{1}{\sqrt{3}}$$

3B. $\left(\frac{2f^4g^7h^3}{15f^3g^9h^6}\right)^0$

Definition of powers

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For Your

FOLDABLE

Simplify.

Since $\frac{c^2}{c^5}$ can only have one value, we can conclude that $c^{-3} = \frac{1}{c^3}$.

Key 🤇	Concept Negative Exponent Property	For Your FOLDABLE
Words	For any nonzero number a and any integer n , a^{-n} is the reciprocal of a^n . Also, the reciprocal of a^{-n} is a^n .	
Symbols	For any nonzero number <i>a</i> and any integer <i>n</i> , $a^{-n} = \frac{1}{a^n}$ and $\frac{1}{a^{-n}} = a^n$.	
Examples	$2^{-4} = \frac{1}{2^4} = \frac{1}{16} \qquad \qquad \frac{1}{j^{-4}} = j^4$	

StudyTip **Negative Signs** Be aware of where a negative sign is placed. $5^{-1} = \frac{1}{5}$, while $-5^1 \neq \frac{1}{5}$.

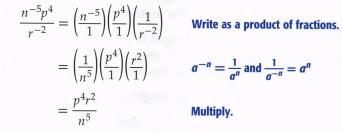
An expression is considered simplified when it contains only positive exponents, each base appears exactly once, there are no powers of powers, and all fractions are in simplest form.

Simplify each expression. Assume that no denominator equals zero.

Negative Exponents EXAMPLE 4

a. $\frac{n^{-5}p^4}{r^{-2}}$

C.



b.
$$\frac{5r^{-3}t^{4}}{-20r^{2}t^{7}u^{-5}}$$
$$\frac{5r^{-3}t^{4}}{-20r^{2}t^{7}u^{-5}} = \left(\frac{5}{-20}\right)\left(\frac{r^{-3}}{r^{2}}\right)\left(\frac{t^{4}}{t^{7}}\right)\left(\frac{1}{u^{-5}}\right)$$
$$= \left(-\frac{1}{4}\right)(r^{-3}-2)(t^{4}-7)(u^{5})$$
$$= -\frac{1}{4}r^{-5}t^{-3}u^{5}$$
$$= -\frac{1}{4}\left(\frac{1}{r^{5}}\right)\left(\frac{1}{t^{3}}\right)(u^{5})$$
$$= -\frac{u^{5}}{4r^{5}t^{3}}$$

Group powers with the same base.

Quotient of Powers and Negative Exponents Property

Simplify.

Negative Exponent Property

Multiply.

$$\frac{2a^{2}b^{3}c^{-5}}{10a^{-3}b^{-1}c^{-4}}$$

$$\frac{2a^{2}b^{3}c^{-5}}{10a^{-3}b^{-1}c^{-4}} = \left(\frac{2}{10}\right)\left(\frac{a^{2}}{a^{-3}}\right)\left(\frac{b^{3}}{b^{-1}}\right)\left(\frac{c^{-5}}{c^{-4}}\right)$$

$$= \left(\frac{1}{5}\right)\left(a^{2}-(-3)\right)\left(b^{3}-(-1)\right)\left(c^{-5}-(-4)\right)$$

$$= \frac{1}{5}a^{5}b^{4}c^{-1}$$

$$= \frac{1}{5}(a^{5})\left(b^{4}\right)\left(\frac{1}{c}\right)$$

$$= \frac{a^{5}b^{4}}{5c}$$

Group powers with the same base.

Quotient of Powers and Negative Exponents Property

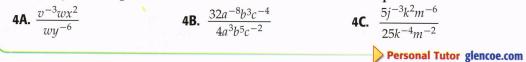
Simplify.

Negative Exponent Property

Multiply.

Check Your Progress

Simplify each expression. Assume that no denominator equals zero.



Order of magnitude is used to compare measures and to estimate and perform rough calculations. The order of magnitude of a quantity is the number rounded to the nearest power of 10. For example, the power of 10 closest to 95,000,000,000 is 10¹¹, or 100,000,000,000. So the order of magnitude of 95,000,000,000 is 10¹¹.



Real-World Link

An adult human weighs about 70 kilograms and an adult dairy cow weighs about 700 kilograms. Their weights differ by 1 order of magnitude.



Real-World Link

There are over 14,000 species of ants living all over the world. Some ants can carry objects that are 50 times their own weight.

Source: Maine Animal Coalition

Real-World EXAMPLE 5 Apply Properties of Exponents

HEIGHT Suppose the average height of a man is about 1.7 meters, and the average height of an ant is 0.0008 meter. How many orders of magnitude as tall as an ant is a man?

Understand We must find the order of magnitude of the heights of the man and ant. Then find the ratio of the orders of magnitude of the man's height to that of the ant's height.

- **Plan** Round each height to the nearest power of ten. Then find the ratio of the height of the man to the height of the ant.
- **Solve** The average height of a man is close to 1 meter. So, the order of magnitude is 10^0 meter. The average height of an ant is about 0.001 meter. So, the order of magnitude is 10^{-3} meters.

The ratio of the height of a man to the height of an ant is about $\frac{10^0}{10^{-3}}$.

 $\frac{10^{0}}{10^{-3}} = 10^{0 - (-3)}$ Quotient of Powers = 10^{3} 0 - (-3) = 0 + 3 or 3 = 1000 Simplify.

So, a man is approximately 1000 times as tall as an ant, or a man is 3 orders of magnitude as tall as an ant.

Check The ratio of the man's height to the ant's height is

 $\frac{1.7}{0.0008}$ = 2125. The order of magnitude of 2125 is 10³.

Check Your Progress

5. ASTRONOMY The order of magnitude of the mass of Earth is about 10²⁷. The order of magnitude of the Milky Way galaxy is about 10⁴⁴. How many orders of magnitude as big is the Milky Way galaxy as Earth?

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Check Your Understanding

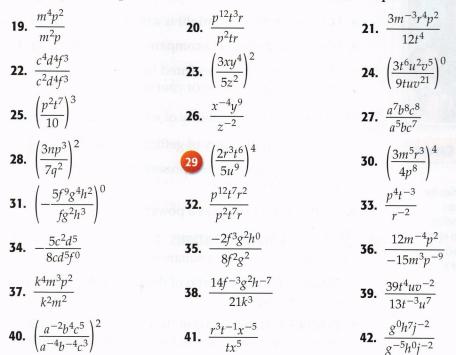
Examples 1-4 pp. 408-411

Simplify each expression. Assume that no denominator equals zero.

Example 5 p. 412 18. FINANCIAL LITERACY The gross domestic product (GDP) for the United States in 2006 was \$13.06 trillion, and the GDP per person was \$43,800. Use order of magnitude to approximate the population of the United States in 2006.

Practice and Problem Solving

Examples 1-4 pp. 408-411 Simplify each expression. Assume that no denominator equals zero.



Example 5 p. 412 **43. INTERNET** In a recent year, there were approximately 3.95 million Internet hosts. Suppose there were 208 million Internet users. Determine the order of magnitude for the Internet hosts and Internet users. Using the orders of magnitude, how many Internet users were there compared to Internet hosts?

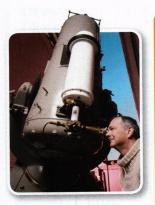
44. PROBABILITY The probability of rolling a die and getting an even number is $\frac{1}{2}$. If you roll the die twice, the probability of getting an even number both times is $\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$ or $\left(\frac{1}{2}\right)^2$. Write an expression to represent the probability of rolling a die *d* times and getting an even number every time. Write the expression as a power of 2.

Simplify each expression. Assume that no denominator equals zero.

- $45. \ \frac{-4w^{12}}{12w^3} \qquad 46. \ \frac{13r^7}{39r^4} \qquad 47. \ \frac{(4k^3m^2)^3}{(5k^2m^{-3})^{-2}} \\
 48. \ \frac{3wy^{-2}}{(w^{-1}y)^3} \qquad 49. \ \frac{20qr^{-2}t^{-5}}{4q^0r^4t^{-2}} \qquad 50. \ \frac{-12c^3d^0f^{-2}}{6c^5d^{-3}f^4} \\
 51. \ \frac{(2g^{3}h^{-2})^2}{(g^2h^0)^{-3}} \qquad 52. \ \frac{(5pr^{-2})^{-2}}{(3p^{-1}r)^3} \qquad 53. \ \left(\frac{-3x^{-6}y^{-1}z^{-2}}{6x^{-2}yz^{-5}}\right)^{-2} \\
 54. \ \left(\frac{2a^{-2}b^4c^2}{-4a^{-2}b^{-5}c^{-7}}\right)^{-1} \qquad 55. \ \frac{(16x^2y^{-1})^0}{(4x^0y^{-4}z)^{-2}} \qquad 56. \ \left(\frac{4^0c^2d^3f}{2c^{-4}d^{-5}}\right)^{-3} \\$
- **57. COMPUTERS** In 1993, the processing speed of a desktop computer was about 10⁸ instructions per second. By 2004, it had increased to 10¹⁰ instructions per second. The newer computer is how many times as fast as the older one?

= Step-by-Step Solutions begin on page R12.

Extra Practice begins on page 815.



Real-World Career

Astronomer

An astronomer studies the universe and analyzes space travel and satellite communications. To be a technician or research assistant, a bachelor's degree is required.

- **58. ASTRONOMY** The brightness of a star is measured in magnitudes. The lower the magnitude, the brighter the star. A magnitude 9 star is 2.51 times as bright as a magnitude 10 star. A magnitude 8 star is 2.51 2.51 or 2.51² times as bright as a magnitude 10 star.
 - **a.** How many times as bright is a magnitude 3 star as a magnitude 10 star?
 - **b**. Write an expression to compare a magnitude *m* star to a magnitude 10 star.
 - **c.** Magnitudes can be measured in negative numbers. Does your expression hold true? Give an example or counterexample.

PROBABILITY The probability of rolling a die and getting a 3 is $\frac{1}{6}$. If you roll the

die twice, the probability of getting a 3 both times is $\frac{1}{6} \cdot \frac{1}{6}$ or $\left(\frac{1}{6}\right)^2$.

- **a.** Write an expression to represent the probability of rolling a die *d* times and getting a 3 each time.
- **b.** Write the expression as a power of 6.
- **60. Solution MULTIPLE REPRESENTATIONS** To find the area of a circle, use $A = \pi r^2$. The formula for the area of a square is $A = s^2$.
 - **a. ALGEBRAIC** Find the ratio of the area of the circle to the area of the square.
 - **b. ALGEBRAIC** If the radius of the circle and the length of each side of the square are doubled, find the ratio of the area of the circle to the square.



c. TABULAR Copy and complete the table.

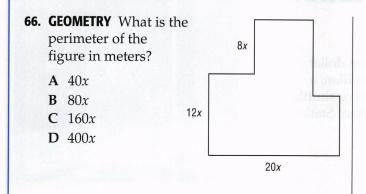
Radius	Area of Circle	Area of Square	Ratio
r	eave arounds	elegia Anoldi	
2r	di dina 10	ereren e cocia	
3r	e alvi i vice i	a declarador.	
4 <i>r</i>	e ne stuive in e		
5 <i>r</i>	1973 RE (1184)	g ur chroann) C C C Th	
6 <i>r</i>			0

d. ANALYTICAL What conclusion can be drawn from this?

H.O.T. Problems Use Higher-Order Thinking Skills

- **61. REASONING** Is $x^y \cdot x^z = x^{yz}$ sometimes, always, or never true? Explain.
- **62. OPEN ENDED** Name two monomials with a quotient of $24a^2b^3$.
- **63.** CHALLENGE Use the Quotient of Powers Property to explain why $x^{-n} = \frac{1}{x^n}$.
- **64. REASONING** Write a convincing argument to show why $3^0 = 1$ using the following pattern: $3^5 = 243$, $3^4 = 81$, $3^3 = 27$, $3^2 = 9$.
- **65.** WRITING IN MATH Explain how to use the Quotient of Powers property and the Power of a Quotient property.

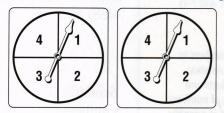
Standardized Test Practice



67. In researching her science project, Leigh learned that light travels at a constant rate and that it takes 500 seconds for light to travel the 93 million miles from the Sun to Earth. Mars is 142 million miles from the Sun. About how many seconds will it take for light to travel from the Sun to Mars?

- F 235 seconds
- G 327 seconds
- H 642 seconds
- J 763 seconds

68. EXTENDED RESPONSE Jessie and Jonas are playing a game using the spinners below. Each spinner is equally likely to stop on any of the four numbers. In the game, a player spins both spinners and calculates the product of the two numbers on which the spinners have stopped.



a. What product has the greatest probability of occurring?

77. $5(u-8) \le 3(u+10)$ **80.** -6(b+5) > 3(b-5)

85. 10⁴
89. 4⁶

b. What is the probability of that product occurring?

69. Simplify $(4^{-2} \cdot 5^0 \cdot 64)^3$.

A	$\frac{1}{64}$	С	320
	64	D	1024

Spiral Review

70. GEOLOGY The seismic waves of a magnitude 6 earthquake are 10^2 times as great as a magnitude 4 earthquake. The seismic waves of a magnitude 4 earthquake are 10 times as great as a magnitude 3 earthquake. How many times as great are the seismic waves of a magnitude 6 earthquake as those of a magnitude 3 earthquake? (Lesson 7-1)

Solve each system of inequalities by graphing. (Lesson 6-8)

71. <i>y</i> ≥ 1	72. $y \ge -3$	73. $y < 3x + 2$	74. $y - 2x < 2$
x < -1	y - x < 1	$y \ge -2x + 4$	y - 2x > 4

Solve each inequality. Check your solution. (Lesson 5-3)

75.	5(2h-6) > 4h	76.	$22 \ge 4(b - 8) + 10$
78.	$8 + t \le 3(t + 4) + 2$	79.	$9n + 3(1 - 6n) \le 21$

81. GRADES In a high school science class, a test is worth three times as much as a quiz. What is the student's average grade? (Lesson 2-9)

Science	Grades
Tests	Quizzes
85	82
92	75
	95

Skills Review

Evaluate each expression.	(Lesson 1-1)		
82. 9 ²	83. 11 ²	84. 10 ⁶	
86. 3 ⁵	87. 5 ³	88. 12 ³	

1-3

W

Then

You found products and quotients of monomials. (Lessons 7-1 and 7-2)

Now/

- Express numbers in scientific notation.
- Find products and quotients of numbers expressed in scientific notation.

New/ Vocabulary/

Math Online

- glencoe.comExtra Examples
- Personal Tutor
- Self-Check Quiz
- Homework Help

Scientific Notation

Why?

Space tourism is a multibillion dollar industry. For a price of \$20 million, a civilian can travel on a rocket or shuttle and visit the International Space Station (ISS) for a week.



Scientific Notation Very large and very small numbers such as \$20 million can be cumbersome to use in calculations. For this reason, numbers are often expressed in scientific notation. A number written in **scientific notation** is of the form $a \times 10^n$, where $1 \le a < 10$ and n is an integer.

ney c	oncept Standard Form to Scientific Notation For You
Step 1	Move the decimal point until it is to the right of the first nonzero digit. The result is a real number <i>a</i> .
Step 2	Note the number of places <i>n</i> and the direction that you moved the decimal point.
Step 3	If the decimal point is moved left, write the number as $a \times 10^{n}$. If the decimal point is moved right, write the number as $a \times 10^{-n}$.
	P
Step 4	Remove the unnecessary zeros.
EXAMI Express	PLE 1 Standard Form to Scientific Notation
EXAMI	PLE 1 Standard Form to Scientific Notation

- **Step 2** The decimal point moved 8 places to the left, so n = 8.
- **Step 3** $201,000,000 = 2.01000000 \times 10^8$

Step 4 2.01×10^8

b. 0.000051

Step 1 0.000051 → 00005.1

a = 00005.1

Step 2 The decimal point moved 5 places to the right, so n = 5.

Step 3 $0.000051 = 00005.1 \times 10^{-5}$

Step 4 5.1×10^{-5}

Check Your Progress

1A. 68,700,000,000

1B. 0.0000725

You can also rewrite numbers in scientific notation in standard form.

Negative Signs Be careful about the placement of negative signs. A negative sign in the exponent means that the number is between 0 and 1. A negative sign before the number means that it is less than 0.

Watch Out!

For Your **Key Concept Scientific Notation to Standard Form** FOLDABLE **Step 1** In $a \times 10^n$, note whether n > 0 or n < 0. **Step 2** If n > 0, move the decimal point *n* places right. If n < 0, move the decimal point -n places left. Step 3 Insert zeros, decimal point, and commas as needed for place value. EXAMPLE 2 **Scientific Notation to Standard Form** Express each number in standard form. a. 6.32×10^9 **Step 1** The exponent is 9, so n = 9. **Step 2** Since n > 0, move the decimal point 9 places to the right. $6.32 \times 10^9 \longrightarrow 6320000000$ **Step 3** $6.32 \times 10^9 = 6,320,000,000$ **Rewrite: insert commas. b.** 4×10^{-7} **Step 1** The exponent is -7, so n = -7. **Step 2** Since n < 0, move the decimal point 7 places to the left. $4 \times 10^{-7} \longrightarrow 0000004$

Step 3 $4 \times 10^{-7} = 0.0000004$

Rewrite; insert a 0 before the decimal point.

Check Your Progress

2A. 3.201×10^6

2B. 9.03×10^{-5}

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Product and Quotients in Scientific Notation You can use scientific notation to simplify multiplying and dividing very large and very small numbers.

Problem-SolvingTip

Estimate Reasonable Answers Estimating an answer before computing the solution can help you determine if your answer is reasonable.

EXAMPLE 3 Multiply with Scientific Notation

Evaluate $(3.5 \times 10^{-3})(7 \times 10^5)$. Express the result in both scientific notation and standard form.

 $(3.5 \times 10^{-3})(7 \times 10^{5}) = (3.5 \times 7)(10^{-3} \times 10^{5}) = 24.5 \times 10^{2} = (2.45 \times 10^{1}) \times 10^{2} = 2.45 \times 10^{3} = 2450$

Original expression Commutative and Associative Properties Product of Powers $24.5 = 2.45 \times 10$ Product of Powers Standard form

Check Your Progress

Evaluate each product. Express the results in both scientific notation and standard form.

3A. $(6.5 \times 10^{12})(8.7 \times 10^{-15})$

3B. $(1.95 \times 10^{-8})(7.8 \times 10^{-2})$

StudyTip

Quotient of Powers Recall that the Quotient of Powers Property is only valid for powers that have the same base. Since 10^8 and 10^3 have the same base, the property applies.

EXAMPLE 4 Divide with Scientific Notation

Evaluate $\frac{3.066 \times 10^8}{7.3 \times 10^3}$. Express the result in both scientific notation and standard form.

 $\frac{3.066 \times 10^8}{7.3 \times 10^3} = \left(\frac{3.066}{7.3}\right) \left(\frac{10^8}{10^3}\right) \\ = 0.42 \times 10^5 \\ = 4.2 \times 10^{-1} \times 10^5 \\ = 4.2 \times 10^4 \\ = 42,000$

Quotient of Powers $0.42 = 4.2 \times 10^{-1}$ Product of Powers Standard form

Product rule for fractions

Check Your Progress

Evaluate each quotient. Express the results in both scientific notation and standard form.

4A. $\frac{2.3958 \times 10^3}{1.98 \times 10^8}$

4B. $\frac{1.305 \times 10^3}{1.45 \times 10^{-4}}$

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Real-World Link

The platinum award was created in 1976. In 2004, the criteria for the award was extended to digital sales. The top-selling artist of all time is the Beatles with 170 million units sold.

Source: Recording Industry Association of America

Real-World EXAMPLE 5 Use Scientific Notation

MUSIC In the United States, a CD reaches gold status once 500 thousand copies are sold. A CD reaches platinum status once 1 million or more copies are sold.

a. Express the number of copies of CDs that need to be sold to reach each status in standard notation.

gold status: 500 thousand = 500,000; platinum status: 1 million = 1,000,000

b. Write each number in scientific notation.

gold status: $500,000 = 5 \times 10^5$; platinum status: $1,000,000 = 1 \times 10^6$

c. How many copies of a CD have sold if it has gone platinum 13 times? Write your answer in scientific notation and standard form.

A CD reaches platinum status once it sells 1 million records. Since the CD has gone platinum 13 times, we need to multiply by 13.

$(13)(1 \times 10^6)$	Original expression
$=(13 \times 1)(10^{6})$	Associative Property
$= 13 \times 10^{6}$	$13 \times 1 = 13$
$=(1.3 \times 10^{1}) \times 10^{6}$	$13 = 1.3 \times 10$
$= 1.3 \times 10^{7}$	Product of Powers
= 13,000,000	Standard form

Check Your Progress

- **5. SATELLITE RADIO** Suppose a satellite radio company earned \$125.4 million in one year.
 - A. Write this number in standard form.
 - **B.** Write this number in scientific notation.
 - **C.** If the following year the company earned 2.5 times the amount earned the previous year, determine the amount earned. Write your answer in scientific notation and standard form.

Check Your Understanding

E	- · · · ·			
Example 1 p. 416	Express each number in scientific notation.			
	1. 185,000,000		1,902,500,000	
	3. 0.000564	4.	0.00000804	
	MONEY Express each n	umber in scientific no	otation.	
	5. Teenagers spend \$1	3 billion annually on c	lothing.	
	6. Teenagers have an i \$1.5 billion of discre		lies' spending h	abit. They control about
Example 2	Express each number i	n standard form.		
р. 417	7. 1.98×10^7	8.	4.052×10^{6}	
	9. 3.405×10^{-8}	10.	6.8×10^{-5}	
Example 3 p. 417	Evaluate each product. standard form.	Express the results in	n both scientific	notation and
	11. $(1.2 \times 10^3)(1.45 \times 10^$	0 ¹²) 12.	$(7.08 \times 10^{14})(5$	$\times 10^{-9}$)
	13. $(5.18 \times 10^2)(9.1 \times 10$	0 ⁻⁵) 14 .	$(2.9 \times 10^{-2})(5.2)$	2×10^{-9})
Example 4 p. 418	Evaluate each quotient standard form.	. Express the results i	n both scientifi	c notation and
	15. $\frac{1.035 \times 10^8}{2.3 \times 10^4}$	16.	$\frac{2.542 \times 10^5}{4.1 \times 10^{-10}}$	
			111 /1 10	
	17. $\frac{1.445 \times 10^{-7}}{1.7 \times 10^5}$	18.	$\frac{2.05 \times 10^{-8}}{4 \times 10^{-2}}$	
Example 5 p. 418	19. AIR FILTERS Salvador bought an air purifier to help him deal with his allergies. The filter in the purifier will stop particles as small as one hundredth of a micror A micron is one millionth of a millimeter.			
	a. Write one hundre	edth and one micron in	n standard form.	
	b. Write one hundre	edth and one micron ir	n scientific notati	on.
		lest size particle in me ndard form and scient		r will stop? Write the
Practice and I	Problem Solving			Solutions begin on page R12. a Practice begins on page 815.
Example 1	Express each number i	n scientific notation.		
р. 416	20. 1,220,000	21) 58,600,000	22. 1	,405,000,000,000
	23. 0.0000013	24. 0.000056	25. 0	0.000000000709
	E-MAIL Express each nu	umber in scientific no	tation.	
	26. Approximately 100 Archives.			re put into the National
	27. By 2010, the e-mail	security market will g	enerate \$5.5 billi	on.
Example 2	Express each number i	n standard form.		
p. 417	28. 1×10^{12}	29. 9.4×10^7	30. 8	3.1×10^{-3}
	31. 5×10^{-4}	32. 8.73×10^{11}		0.22×10^{-6}
			Lesson 7-3	Scientific Notation 419

Lesson 7-3 Scientific Notation 419

Example 2 p. 417

INTERNET Express each number in standard form.

34. About 2.1×10^7 people, aged 12 to 17, use the Internet.

35. Approximately 1.1×10^7 teens go online daily.

Examples 3 and 4 pp. 417-418

Evaluate each product or quotient. Express the results in both scientific notationand standard form.

36. $(3.807 \times 10^3)(5 \times 10^2)$	37. $\frac{9.6 \times 10^3}{1.2 \times 10^{-4}}$
38. $\frac{2.88 \times 10^3}{1.2 \times 10^{-5}}$	$(6.5 \times 10^7) (7.2 \times 10^{-2})$
40. $(9.5 \times 10^{-18})(9 \times 10^9)$	41. $\frac{8.8 \times 10^3}{4 \times 10^{-4}}$
42. $\frac{9.15 \times 10^{-3}}{6.1 \times 10}$	43. $(2.01 \times 10^{-4})(8.9 \times 10^{-3})$
44. $(2.58 \times 10^2)(3.6 \times 10^6)$	45. $\frac{5.6498 \times 10^{10}}{8.2 \times 10^4}$
46. $\frac{1.363 \times 10^{16}}{2.9 \times 10^6}$	47. $(9.04 \times 10^6)(5.2 \times 10^{-4})$
48. $(1.6 \times 10^{-5})(2.3 \times 10^{-3})$	49. $\frac{6.25 \times 10^{-4}}{1.25 \times 10^2}$
50. $\frac{3.75 \times 10^{-9}}{1.5 \times 10^{-4}}$	51. $(3.4 \times 10^4)(7.2 \times 10^{-15})$
52. $\frac{8.6 \times 10^4}{2 \times 10^{-6}}$	53. $(6.3 \times 10^{-2})(3.5 \times 10^{-4})$

Example 5 p. 418 **54. ASTRONOMY** The distance between Earth and the Sun varies throughout the year. Earth is closest to the Sun in January when the distance is 91.4 million miles. In July, the distance is greatest at 94.4 million miles.

a. Write 91.4 million in both standard form and in scientific notation.

- **b.** Write 94.4 million in both standard form and in scientific notation.
- **c.** What is the percent increase in distance from January to July? Round to the nearest tenth of a percent.

Evaluate each product or quotient. Express the results in both scientific notation and standard form.

55. $(4.65 \times 10^{-2})(5 \times 10^{6})$	56. $\frac{2.548 \times 10^5}{2.8 \times 10^{-2}}$
57. $\frac{2.135 \times 10^5}{3.5 \times 10^{12}}$	58. $(4.8 \times 10^5)(3.16 \times 10^{-5})$
59. $(4.3 \times 10^{-3})(4.5 \times 10^4)$	60. $\frac{5.184 \times 10^{-5}}{7.2 \times 10^3}$
61. $(5 \times 10^3)(1.8 \times 10^{-7})$	62. $\frac{1.032 \times 10^{-4}}{8.6 \times 10^{-5}}$

LIGHT The speed of light is approximately 3×10^8 meters per second.

- **63.** Write an expression to represent the speed of light in kilometers per second.
- 64. Write an expression to represent the speed of light in kilometers per hour.
- **65.** Make a table to show how many kilometers light travels in a day, a week, a' 30-day month, and a 365-day year. Express your results in scientific notation.
- **66.** The distance from Earth to the Moon is approximately 3.844×10^5 kilometers. How long would it take light to travel from Earth to the Moon?



Real-World Link

The distance from Earth to the Sun does not determine the seasons. The seasons are determined by the tilt of the Earth's axis and the elliptical orbit around the Sun.

Source: University of British Columbia Okanagan



Real-World Link

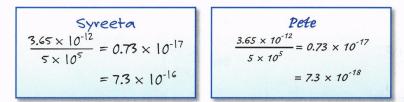
The longest river on Earth is the Nile River in Africa. It is 4,160 miles long, beginning in Burundi to its mouth at the Mediterranean Sea. The Nile River basin covers an area of 1.293×10^6 square miles.

Source: Encarta Encyclopedia

- **EARTH** The population of Earth is about 6.623×10^9 . The land surface of Earth is 1.483×10^8 square kilometers. What is the population density for the land surface area of Earth?
- **68. RIVERS** A drainage basin separated from adjacent basins by a ridge, hill, or mountain is known as a watershed. The watershed of the Amazon River is 2,300,000 square miles. The watershed of the Mississippi River is 1,200,000 square miles.
 - **a.** Write each of these numbers in scientific notation.
 - **b.** How many times as large is the Amazon River watershed as the Mississippi River watershed?
- **69. AGRICULTURE** In a recent year, farmers planted approximately 92.9 million acres of corn. They also planted 64.1 million acres of soybeans and 11.1 million acres of cotton.
 - a. Write each of these numbers in scientific notation and in standard form.
 - **b.** How many times as much corn was planted as soybeans? Write your results in standard form and in scientific notation. Round your answer to four decimal places.
 - **c.** How many times as much corn was planted as cotton? Write your results in standard form and in scientific notation. Round your answer to four decimal places.

H.O.T. Problems Use Higher-Order Thinking Skills

- **70. REASONING** Which is greater, 100^{10} or 10^{100} ? Explain your reasoning.
- **71. FIND THE ERROR** Syreeta and Pete are solving a division problem with scientific notation. Is either of them correct? Explain your reasoning.



72. CHALLENGE Order these numbers from least to greatest without converting them to standard form.

 5.46×10^{-3} , 6.54×10^{3} , 4.56×10^{-4} , -5.64×10^{4} , -4.65×10^{5}

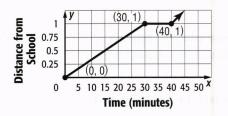
73. REASONING Determine whether the statement is *always, sometimes,* or *never* true. Give examples or a counterexample to verify your reasoning.

When multiplying two numbers written in scientific notation, the resulting number can have no more than two digits to the left of the decimal point.

- **74. OPEN ENDED** Write two numbers in scientific notation with a product of 1.3×10^{-3} . Then name two numbers in scientific notation with a quotient of 1.3×10^{-3} .
- **75.** WRITING IN MATH Write the steps that you would use to divide two numbers written in scientific notation. Then describe how you would write the results in standard form.

Standardized Test Practice

- **76.** Which number represents 0.05604×10^8 written in standard form?
 - A0.000000005604C5,604,000B560,400D50,604,000
- **77.** Toni left school and rode her bike home. The graph below shows the relationship between her distance from the school and time.



Which explanation could account for the section of the graph from x = 30 to x = 40?

- **F** Toni rode her bike down a hill.
- **G** Toni ran all the way home.
- H Toni stopped at a friend's house on her way home.
- J Toni returned to school to get her mathematics book.

- **78. SHORT RESPONSE** In his first four years of coaching football, Coach Delgato's team won 5 games the first year, 10 games the second year, 8 games the third year, and 7 games the fourth year. How many games does the team need to win during the fifth year to have an average of 8 wins per year?
- **79.** The table shows the relationship between Calories and grams of fat contained in an order of fried chicken from various restaurants.

Calories	305	410	320	500	510	440
Fat (g)	28	34	28	41	42	38

Assuming that the data can best be described by a linear model, about how many grams of fat would you expect to be in a 275-Calorie order of fried chicken?

- A 22
- **B** 25
- **C** 27
- **D** 28

Spiral Review

Simplify. Assume that no denominator is equ	al to zero.	(Lesson 7-2)
---	-------------	--------------

80. $\frac{8^5}{8^6}$	81. $\frac{6^{\circ}}{6^{3}}$
83. $\left(\frac{3a^4b^4}{8c^2}\right)^4$	84. $\left(\frac{5d^3g^2}{3h^4}\right)^2$

82. $\frac{r^8 t^{12}}{r^2 t^7}$ 85. $\left(\frac{4n^2 p^4}{8p^3}\right)^3$

86. CHEMISTRY Lemon juice is 10² times as acidic as tomato juice. Tomato juice is 10³ times as acidic as egg whites. How many times as acidic is lemon juice as egg whites? (Lesson 7-1)

Write each equation in slope-intercept form. (Lesson 4-2)

87. $y - 2 = 3(x - 1)$	88. $y - 5 = 6(x + 1)$	89. $y + 2 = -2(x + 5)$
90. $y + 3 = \frac{1}{2}(x + 4)$	91. $y - 1 = \frac{2}{3}(x + 9)$	92. $y + 3 = -\frac{1}{4}(x + 2)$

Skills Review

Simplify each expression. If not possible, write simplified. (Lesson 1-4)

93. 3 <i>u</i> + 10 <i>u</i>	94. 5 <i>a</i> - 2 + 6 <i>a</i>	95. $6m^2 - 8m$
96. $4w^2 + w + 15w^2$	97. 13(5 + 4 <i>a</i>)	98. (4 <i>t</i> - 6)16

422 Chapter 7 Polynomials

Algebra tiles can be used to model polynomials. A polynomial is a monomial or the sum of monomials. The diagram at the right shows the models.

Algebra Lab

Polynomials

Polynomi	al Models
Polynomials are modeled using three types of tiles.	1 x x ²
Each tile has an opposite.	-1 -x -x ²

ACTIVITY

• 5x

EXPLORE

-1

To model this polynomial, you will need 5 green *x*-tiles.

Use algebra tiles to model each polynomial.

• $3x^2 - 1$

To model this polynomial, you will need 3 blue x^2 -tiles and 1 red -1-tile.

• $-2x^2 + x + 3$

To model this polynomial, you will need 2 red $-x^2$ -tiles, 1 green *x*-tile, and 3 yellow 1-tiles.



Use algebra tiles to model each polynomial. Then draw a diagram of your model.

1.	$-4x^2$	2		
_	a 2	•		

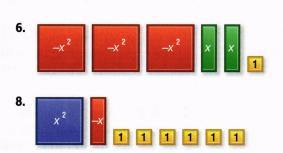
3. $2x^2 - 3x$

Write an algebraic expression for each model.



9. MAKE A CONJECTURE Write a sentence or two explaining why algebra tiles are sometimes called area tiles.

		-
2	. 3 <i>x</i> –	- 5
4	$x^{2} +$	-2x + 1

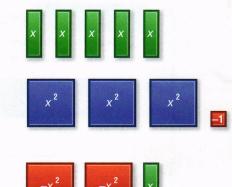


x	x	×	
x ²	x ²	x ²	-1
-× ²	-x ²		

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1

Then

You identified monomials and their characteristics. (Lesson 7-1)

Now/

- Find the degree of a polynomial.
- Write polynomials in standard form.

New Vocabulary

polynomial binomial trinomial degree of a monomial degree of a polynomial standard form of a polynomial leading coefficient

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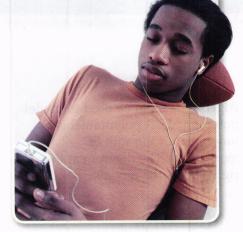
- Extra Examples
- Personal Tutor
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Polynomials

Why?

In 2011, sales of digital audio players are expected to reach record numbers. The sales data can be modeled by the equation $U = -2.7t^2 + 49.4t + 128.7$, where *U* is the number of units shipped in millions and *t* is the number of years since 2005.

The expression $-2.7t^2 + 49.4t + 128.7$ is an example of a polynomial. Polynomials can be used to model situations.



Degree of a Polynomial A **polynomial** is a monomial or the sum of monomials, each called a *term* of the polynomial. Some polynomials have special names. A **binomial** is the sum of *two* monomials, and a **trinomial** is the sum of *three* monomials.

EXAMPLE 1 Identify Polynomials

Determine whether each expression is a polynomial. If so, identify the polynomial as a *monomial*, *binomial*, or *trinomial*.

Expression	Is it a polynomial?	Monomial, binomial, or trinomial?
a. $4y - 5xz$	Yes; $4y - 5xz$ is the sum of the two monomials $4y$ and $-5xz$.	binomial
b 6.5	Yes; -6.5 is a real number.	monomial
c. $7a^{-3} + 9b$	No; $7a^{-3} = \frac{7}{a^3}$, which is not a monomial.	none of these
d. $6x^3 + 4x + x + 3$	Yes; $6x^3 + 4x + x + 3 = 6x^3 + 5x + 3$, the sum of three monomials.	trinomial

Check Your Progress

- 1**A.** *x*
- **1C.** 5rx + 7tuv

1B.
$$-3y^2 - 2y + 4y - 1$$

1D. $10x^{-4} - 8x^a$

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The **degree of a monomial** is the sum of the exponents of all its variables. A nonzero constant has degree 0. Zero has no degree.

The **degree of a polynomial** is the greatest degree of any term in the polynomial. To find the degree of a polynomial, you must find the degree of each term. Some polynomials have special names based on their degree.

ReadingMath

Prefixes The prefixes mono, bi, and tri mean one, two, and three, respectively. Hence, a monomial has one term, a binomial has two terms, and a trinomial has three terms.

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Math in Motion, **Interactive Lab**

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Degree of a Polynomial EXAMPLE 2

Find the degree of each polynomial.

a. $3a^2b^3 + 6$

- **Step 1** Find the degree of each term.
 - $3a^2b^3$: degree = 2 + 3 or 5 6: degree 0

Step 2 The degree of the polynomial is the greatest degree, 5.

```
b. 2d^3 - 5c^5d - 7
```

 $-5c^{5}d$: degree = 5 + 1 or 6 The degree of the polynomial is 6.

Check Your Progress

 $2d^3$: degree = 3

-7: degree 0

2A. $7xy^5z$

2B. $2rt - 3rt^2 - 7r^2t^2 - 13$

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Polynomials in Standard Form The terms of a polynomial may be written in any order. Polynomials written in only one variable are usually written in standard form.

The **standard form of a polynomial** is written with the terms in order from greatest degree to least degree. When a polynomial is written in standard form, the coefficient of the first term is called the leading coefficient.



EXAMPLE 3 Standard Form of a Polynomial

Write each polynomial in standard form. Identify the leading coefficient.

a.
$$3x^2 + 4x^5 - 7x$$

Step 1 Find the degree of each term.

Degree:
$$2$$
 5 1
Polynomial: $3x^2 + 4x^5 - 7x$

Step 2 Write the terms in descending order: $4x^5 + 3x^2 - 7x$. The leading coefficient is 4.

b. $5y - 9 - 2y^4 - 6y^3$

Step 1 Degree:

Degree: 1 0 4 3 \uparrow \uparrow \uparrow \uparrow Polynomial: $5y - 9 - 2y^4 - 6y^3$

Step 2 $-2y^4 - 6y^3 + 5y - 9$ The leading coefficient is -2.

Check Your Progress

3A. $8 - 2x^2 + 4x^4 - 3x$

3B.
$$y + 5y^3 - 2y^2 - 7y^6 + 10$$



Real-World Link

The world's biggest skateboard was built in 1996. The skateboard is 10 feet long, 4 feet wide, 3 feet tall and is fully functional. It is on display in San Diego, California.

Source: Foundation Skateboard Company

We can use polynomials to estimate values between two points. We can also use them to predict values of events before they occur.

Real-World EXAMPLE 4 Use a Polynomial

BUSINESS From 2000 through 2006, the number *U* of skateboards (in thousands) produced at a manufacturing plant can be modeled by the equation $U = 3t^2 - 2t + 10$, where *t* is the number of years since 2000. How many skateboards were produced in 2002?

Find the value of *t*, and substitute the value of *t* to find the number of skateboards produced.

Since *t* is the number of years since 2000, *t* equals 2002 – 2000 or 2.

 $U = 3t^2 - 2t + 10$ Original equation $= 3(2)^2 - 2(2) + 10$ t = 2= 3(4) - 4 + 10Simplify.= 12 - 4 + 10Multiply.= 18Simplify.

Since *U* is in thousands, the number of skateboards produced was 18 thousand or 18,000.

Check Your Progress

- **4A.** How many skateboards were produced in 2005?
- 4B. If this trend continues, how many skateboards will be produced in 2015?

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🗹 Check Your Understanding

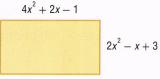
Example 1 Determine whether each expression is a polynomial. If so, identify p. 424 the polynomial as a monomial, binomial, or trinomial. **2.** $2y - 5 + 3y^2$ 1. $7ab + 6b^2 - 2a^3$ **4.** $\frac{4m}{3p}$ **6.** $5q^{-4} + 6q$ **3.** $3x^2$ 5. $5m^2p^3 + 6$ Example 2 Find the degree of each polynomial. p. 425 8. $6p^3 - p^4$ 7. -3 10. $\frac{3}{4}$ 9. -7z12. $2a^2b^5 + 5 - ab$ $12 - 7q^2t + 8r$ 14. $9hjk - 4h^2j^3 + 5j^2k^2 - h^3k^3$ **13.** $6df^3 + 3d^2f^2 + 2d + 1$ **Example 3** Write each polynomial in standard form. Identify the leading coefficient. p. 425 16. $-y^3 + 3y - 3y^2 + 2$ 15. $2x^5 - 12 + 3x$ **18.** $2a + 4a^3 - 5a^2 - 1$ 17. $4z - 2z^2 - 5z^4$ **19. ENROLLMENT** Suppose the number N (in hundreds) of students projected **Example 4** p. 426 to attend a high school from 1998 to 2007 can be modeled by the equation $N = t^2 + 1.5t + 0.5$, where t is the number of years since 1998. a. How many students were enrolled in the high school in 2003?

b. How many students were enrolled in the high school in 2005?

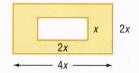
Practice and Problem Solving

= Step-by-Step Solutions begin on page R12. Extra Practice begins on page 815.

Example 1 p. 424		etermine whether e polynomial as a					
	20.	$\frac{5y^3}{x^2} + 4x$	21.	21		22. $c^4 - 2c^2 + 1$	
		$d^{x^2} + 3d^{-c}$	24.	$a - a^2$		25. $5n^3 + nq^3$	
Example 2	Fir	nd the degree of e	ach polyno	omial.			
p. 425	26.	$13 - 4ab + 5a^3b$	27.	3x - 8		28. -4	
	29.	$17g^{2}h$	30.	$10 + 2cd^4 -$	$-6d^2g$	31. $2z^2y^2 - 7 + 5y^3w$,4
Example 3	Wr	ite each polynom	ial in stan	dard form. I	Identify the	leading coefficient.	
p. 425	32.	$5x^2 - 2 + 3x$		3	33. $8y + 7y^3$		
	34.	$4 - 3c - 5c^2$		3	$-4d^4 + 1$	$-d^2$ over being state	
	36.	$11t + 2t^2 - 3 + t^3$	5		37. $2 + r - r^3$		
	38.	$\frac{1}{2}x - 3x^4 + 7$		3	39. $-9b^2 + 10^{-10}$	$0b - b^6$	
Example 4 p. 426	40.	FIREWORKS A fire of 150 feet per se equation $H = -1$	cond. The l	neight H of t	the firework s	om the ground at a spe shell is modeled by th econds.	ed e
		a. How high will	l the firewo	ork be after 3	seconds?		•
		b. How high will	l the firewo	ork be after 5	seconds?	J	
	Cla	assify each polyne	omial acco	rding to its	degree and 1	number of terms.	
	41.	$4x - 3x^2 + 5$	42.	$11z^{3}$		43. $9 + y^4$	
	44.	$3x^3 - 7$	45.	$-2z^5 - x^2 - x^2$	+5x - 8	46. $10t - 4t^2 + 6t^3$	
	47.	ICE CREAM An ice	e cream sho	p is changi	ng the size of	f their cone.	
						square of the radius <i>r,</i> nts the volume.	
		b. How much wi 4 inches?	ll the cone	hold if the r	adius is 1.5 iı	nches and the height is	S
		c. If the volume of cone is 3 inche				and the radius of the	
	48.	GEOMETRY Write	two expres	ssions for th	e perimeter a	and area of the rectang	gle.
				+ 2 <i>x</i> – 1	-		



49. GEOMETRY Write a polynomial for the area of the shaded region shown.





Real-World Link

A space shuttle has three parts: the orbiter, an external tank, and two solid rocket boosters. Including fuel, the shuttle weighs a total of 4.4 million pounds at launch.

Source: How Stuff Works

50. PROJECT Rocky and Arturo are designing a rocket for a competition. The top must be cone-shaped and the body of the rocket must be cylindrical. The volume of a cone is the product of $\frac{1}{3}$, π , the height *h*, and the square of the radius *r*. The volume of a cylinder is the product of π , the height *t*, and the square of the radius *r*.



- a. Write a polynomial that represents the volume of the rocket.
- **b.** If the height of the body of the rocket is 8 inches, the height of the top is 6 inches, and the radius is 3 inches, find the volume of the rocket.
- **c.** If the height of the body of the rocket is 9 inches, the height of the top is 5 inches, and the radius is 4 inches, find the volume of the rocket.
- **MULTIPLE REPRESENTATIONS** In this problem, you will explore perimeter and area.
 - a. GEOMETRIC Draw three rectangles that each have a perimeter of 400 feet.
 - **b. TABULAR** Record the width and length of each rectangle in a table like the one shown below. Find the area of each rectangle.

Rectangle	Length	Width	Area
1	100 ft	C HINNE - AG	82
2	50 ft	2010/02/02/02/02/02	as s ale
3	75 ft	an tool in the	p. 426
4	x ft	a Tél mont-unios	

- **c. GRAPHICAL** On a coordinate system, graph the area of rectangle 4 in terms of the length, *x*. Use the graph to determine the largest area possible.
- **d. ANALYTICAL** Determine the length and width that produce the largest area.

H.O.T. Problems

Use Higher-Order Thinking Skills

52. FIND THE ERROR Chuck and Claudio are writing $2x^2 - 3 + 5x$ in standard form. Is either of them correct? Explain your reasoning.

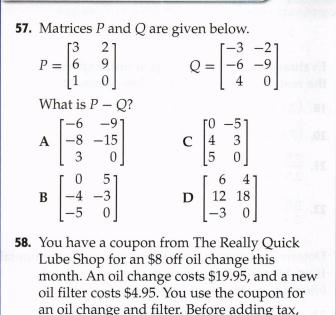
Chuck	Claudio
2x ² : degree 2	2x ² : degree 2
-3: degree 0	-3 : degree 0
5x: degree	5x : degree 1
$2x^2 - 5x + 3$	$2x^2+5x-3$

- **53. CHALLENGE** Write a polynomial that represents any odd integer if *x* is an integer. Explain.
- 54. REASONING Is the following statement sometimes, always, or never true? Explain.

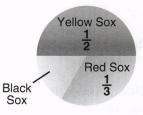
A binomial can have a degree of zero.

- **55. OPEN ENDED** Write an example of a cubic trinomial.
- **56.** WRITING IN MATH Explain how to write a polynomial in standard form and how to identify the leading coefficient.

Standardized Test Practice



59. SHORT RESPONSE In a recent poll, 3000 people were asked to pick their favorite baseball team. The accompanying circle graph shows the results of that poll. How many people polled picked the Black Sox as their favorite team?



60. What value for *y* satisfies the system of equations below?

$$2x + y = 19$$
$$4x - 6y = -2$$

w much should you pay?				
\$11.95			A	5
\$16.90			B	7
\$24.90			С	8
\$27.95			D	10
	w much should you pay? \$11.95 \$16.90 \$24.90 \$27.95	\$11.95 \$16.90 \$24.90	\$11.95 \$16.90 \$24.90	\$11.95 A \$16.90 B \$24.90 C

Spiral Review

Express each number in standard r	iotation. (Lesson 7-3)			
61. 6×10^{-7}	62. 7.2×10^{-10}	63.	8.1×10^5	
64. 7×10^6	65. 0.132×10^{-6}	66.	1.88×10^0	
Simplify. Assume that no denomin	ator is equal to zero. (Lesson 7-2)			
67. $a^0(a^4)(a^{-8})$	68. $\frac{(4m^{-3}c^6)^0}{mc}$	69.	$\frac{(3f^2g^6)^0}{(18f^6g^2)^0}$	
70. 12 ⁻¹	71. $\frac{k^{-4}}{m^2 p^{-8}}$	72.	$\frac{(nq^{-1})^3}{(n^4q^8)^{-1}}$	

73. FINANCIAL LITERACY The owners of a new restaurant have hired enough servers to handle 17 tables of customers. The fire marshal has approved the restaurant for a limit of 56 customers. How many two-seat tables and how many four-seat tables should the owners buy? (Lesson 6-4)

Skills Review

Simplify each expression. If not possible, write simplified. (Lesson 1-5)

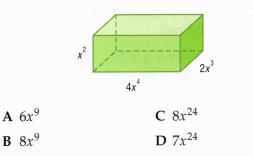
- **74.** $7b^2 + 14b 10b$ **77.** $7h^5 - 7j^5 + 8k^5$
- **75.** $5t + 12t^2 8t$ **78.** $n + \frac{n}{3} + \frac{2}{3}n$

76. $3y^4 + 2y^4 + 2y^5$ **79.** $2u + \frac{u}{2} + u^2$

Mid-Chapter Quiz Lessons 7-1 through 7-4

Simplify each expression. (Lesson 7-1)

- 1. $(x^3)(4x^5)$
- **2.** $(m^2p^5)^3$
- **3.** $[(2xy^3)^2]^3$
- **4.** $(6ab^3c^4)(-3a^2b^3c)$
- **5. MULTIPLE CHOICE** Express the volume of the solid as a monomial. (Lesson 7-1)



Simplify each expression. Assume that no denominator equals 0. (Lesson 7-2)



10. ASTRONOMY Physicists estimate that the number of stars in the universe has an order of magnitude of 10^{21} . The number of stars in the Milky Way galaxy is around 100 billion. Using orders of magnitude, how many times as many stars are there in the universe as the Milky Way? (Lesson 7-2)

Express each number in scientific notation. (Lesson 7-3)

11.	0.00000054	12.	0.0042
13.	234,000	14.	418,000,000

Express each number in standard form. (Lesson 7-3)

- **15.** 4.1×10^{-3}
- **16.** 2.74×10^5
- 17. 3×10^9
- **18.** 9.1×10^{-5}

430 Chapter 7 Polynomials

Evaluate each product or quotient. Express the results in scientific notation. (Lesson 7-3)

- **19.** $(2.13 \times 10^2)(3 \times 10^5)$
- **20.** $(7.5 \times 10^6)(2.5 \times 10^{-2})$

21.
$$\frac{7.5 \times 10^8}{2.5 \times 10^4}$$

22. $\frac{6.6 \times 10^5}{2 \times 10^{-3}}$

Determine whether each expression is a polynomial. If so, identify the polynomial as a *monomial*, *binomial*, or *trinomial*. (Lesson 7-4)

23.	$3y^2 - 2$
24.	$4t^5 + 3t^2 + t$
25.	$\frac{3x}{5y}$
26.	ax^{-3}
27.	3 <i>b</i> ²
28.	$2x^{-3} - 4x + 1$

29. POPULATION The table shows the population density for Nevada for various years. (Lesson 7-4)

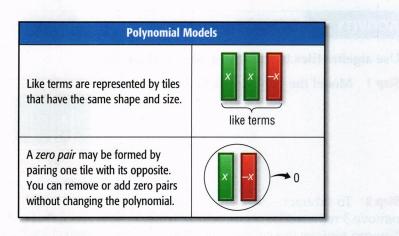
Year	Years Since 1930	People/ Square Mile
1930	0	0.8
1960	30	2.6
1980	50	7.3
1990	60	10.9
2000	70	18.2

- **a.** The population density *d* of Nevada from 1930 to 2000 can be modeled by $d = 0.005y^2 0.127y + 1$, where *y* represents the number of years since 1930. Identify the type of polynomial for $0.005y^2 0.127y + 1$.
- **b.** What is the degree of the polynomial?
- **c.** Predict the population density of Nevada for 2020. Explain your method.
- **d.** Predict the population density of Nevada for 2030. Explain your method.

EXPLORE

Algebra Lab **Adding and Subtracting** Polynomials

Monomials such as 3x and -2x are called *like terms* because they have the same variable to the same power. When you use algebra tiles, you can recognize like terms because the individual tiles have the same size and shape.



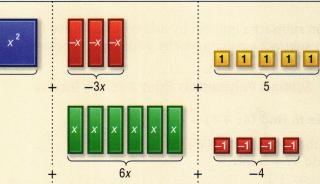
Add Polynomials ACTIVITY 1

Use algebra tiles to find $(2x^2 - 3x + 5) + (x^2 + 6x - 4)$.

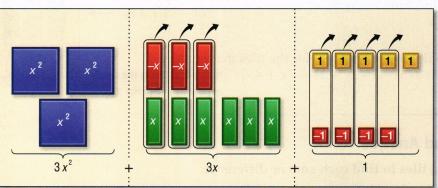
Step 1 Model each polynomial.

$$2x^2 - 3x + 5 \longrightarrow$$
$$x^2 + 6x - 4 \longrightarrow$$

 $2x^2$



Step 2 Combine like terms and remove zero pairs.



Step 3 Write the polynomial for the tiles that remain.

 $(2x^2 - 3x + 5) + (x^2 + 6x - 4) = 3x^2 + 3x + 1$

(continued on the next page)

ACTIVITY 2 Subtract Polynomials

Use algebra tiles to find (4x + 5) - (-3x + 1).

Step 1 Model the polynomial 4x + 5.

 $\begin{array}{c} x \\ x \\ 4x \\ 4x \end{array} + \begin{array}{c} 1 \\ 5 \\ 5 \end{array}$

7x

Step 2 To subtract -3x + 1, you must remove 3 red -x-tiles and 1 yellow 1-tile. You can remove the yellow 1-tile, but there are no red -x-tiles. Add 3 zero pairs of *x*-tiles. Then remove the 3 red -x-tiles.

Step 3 Write the polynomial for the tiles that remain. (4x + 5) - (-3x + 1) = 7x + 4

Recall that you can subtract a number by adding its additive inverse or opposite. Similarly, you can subtract a polynomial by adding its opposite.

ACTIVITY 3 **Subtract Polynomials Using Additive Inverse** Use algebra tiles to find (4x + 5) - (-3x + 1). 5 **Step 1** To find the difference of 4x + 5and -3x + 1, add 4x + 5 and the opposite of -3x + 1. 4x + 51 1 The opposite of -3x + 1 is 3x - 1. **Step 2** Write the polynomial for the tiles that remain. 3x(4x + 5) - (-3x + 1) = 7x + 4.Notice that this is the same answer as in Activity 2.

Model and Analyze

Use algebra tiles to find each sum or difference.

- **1.** $(x^2 + 5x 2) + (3x^2 2x + 6)$ **2.** $(2x^2 + 8x + 1) (x^2 4x 2)$ **3.** $(-4x^2 + x) (x^2 + 5x)$
- **4.** WRITING IN MATH Find $(4x^2 x + 3) (2x + 1)$ using each method from Activity 2 and Activity 3. Illustrate with drawings, and explain in writing how zero pairs are used in each case.

7-5

You wrote polynomials in

Then

standard form.

(Lesson 7-4)

Now/

Add polynomials.

Math Online

Extra Examples

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Subtract polynomials.

Adding and Subtracting Polynomials

Why?

From 2000 to 2003, sales (in millions of dollars) of rap/hip-hop music R and country music C in the United States can be modeled by the following equations, where t is the number of years since 2000.

 $R = -132.3t^{3} + 624.7t^{2} - 773.6t + 1847.7$ $C = -3.4t^{3} + 8.6t^{2} - 95t + 1532.6$

The total music sales *T* of rap/hip-hop music and country music is R + C.



Add Polynomials Adding polynomials involves adding like terms. You can group like terms by using a horizontal or vertical format.

EXAMPLE 1 Add Polynomials

Find each sum.

a. $(2x^2 + 5x - 7) + (3 - 4x^2 + 6x)$

Horizontal Method

$$(2x^{2} + 5x - 7) + (3 - 4x^{2} + 6x)$$

= $[2x^{2} + (-4x^{2})] + [5x + 6x] + [-7 + 3]$
= $-2x^{2} + 11x - 4$

Group like terms. Combine like terms.

Group like terms. Combine like terms.

Vertical Method

$2x^2 + 5x - 7$	
$(+) -4x^2 + 6x + 3$	
$-2x^2 + 11x - 4$	

Align like terms in columns and combine.

b. $(3y + y^3 - 5) + (4y^2 - 4y + 2y^3 + 8)$

Horizontal Method

$$(3y + y^3 - 5) + (4y^2 - 4y + 2y^3 + 8)$$

= $[y^3 + 2y^3] + 4y^2 + [3y + (-4y)] + [(-5) + 8]$
= $3y^3 + 4y^2 - y + 3$

Vertical Method

 $\frac{y^3 + 0y^2 + 3y - 5}{(+) 2y^3 + 4y^2 - 4y + 8} \\ \frac{3y^3 + 4y^2 - 4y + 8}{3y^3 + 4y^2 - y + 3}$

Insert a placeholder to help align the terms. Align and combine like terms.

Check Your Progress

1A. Find $(5x^2 - 3x + 4) + (6x - 3x^2 - 3)$. **1B.** Find $(y^4 - 3y + 7) + (2y^3 + 2y - 2y^4 - 11)$.

StudyTip

Additive Inverse When finding the additive inverse of a polynomial, you are multiplying every term by -1.

StudyTip

Vertical Method Notice that the polynomials are written in standard form with like terms aligned. **Subtract Polynomials** Recall that you can subtract a real number by adding its opposite or additive inverse. Similarly, you can subtract a polynomial by adding its additive inverse.

To find the additive inverse of a polynomial, write the opposite of each term in the polynomial.

 $-(3x^2 + 2x - 6) = -3x^2 - 2x + 6$ Additive Inverse

EXAMPLE 2 Subtract Polynomials

Find each difference.

a. $(3 - 2x + 2x^2) - (4x - 5 + 3x^2)$

Horizontal Method

Subtract $4x - 5 + 3x^2$ by adding its additive inverse.

$$3 - 2x + 2x^{2}) - (4x - 5 + 3x^{2})$$

= $(3 - 2x + 2x^{2}) + (-4x + 5 - 3x^{2})$
= $[2x^{2} + (-3x^{2})] + [(-2x) + (-4x)] + [3 + 5]$
= $-x^{2} - 6x + 8$
The additive inverse of $4x - 5 + 3x^{2}$.
is $-4x + 5 - 3x^{2}$.
Group like terms.
Combine like terms.

Vertical Method

Align like terms in columns and subtract by adding the additive inverse.

$$2x^{2} - 2x + 3$$

$$(-) 3x^{2} + 4x - 5$$
Add the opposite.
$$2x^{2} - 2x + 3$$

$$(+) -3x^{2} - 4x + 5$$

$$(+) -3x^{2} - 4x + 5$$

$$-x^{2} - 6x + 8$$
Thus $(3 - 2x + 2x^{2}) - (4x - 5 + 3x^{2}) = -x^{2} - 6x + 8$

b.
$$(7p + 4p^3 - 8) - (3p^2 + 2 - 9p)$$

Horizontal Method

Subtract $3p^2 + 2 - 9p$ by adding its additive inverse.

$$\begin{array}{l} \left(7p + 4p^3 - 8\right) - \left(3p^2 + 2 - 9p\right) \\ = \left(7p + 4p^3 - 8\right) + \left(-3p^2 - 2 + 9p\right) & \text{The additive inverse of } 3p^2 + 2 - 9p \\ = \left[7p + 9p\right] + 4p^3 + \left(-3p^2\right) + \left[(-8) + (-2)\right] & \text{Group like terms.} \\ = 4p^3 - 3p^2 + 16p - 10 & \text{Combine like terms.} \end{array}$$

Vertical Method

Align like terms in columns and subtract by adding the additive inverse.

$$4p^{3} + 0p^{2} + 7p - 8$$

$$(-) \quad 3p^{2} - 9p + 2$$
Add the opposite.
$$4p^{3} + 0p^{2} + 7p - 8$$

$$(+) \quad -3p^{2} + 9p - 2$$

$$4p^{3} - 3p^{2} + 16p - 10$$

$$(-) \quad -3p^{2} + 9p - 2$$

$$4p^{3} - 3p^{2} + 16p - 10$$

Thus, $(7p + 4p^3 - 8) - (3p^2 + 2 - 9p) = 4p^3 - 3p^2 + 16p - 10.$

Check Your Progress

2A. Find $(4x^3 - 3x^2 + 6x - 4) - (-2x^3 + x^2 - 2)$. **2B.** Find $(8y - 10 + 5y^2) - (7 - y^3 + 12y)$.



Real-World Link

Sales of digital cameras recently increased by 42% in one year. Sales are expected to increase by at least 15% each year as consumers upgrade their cameras.

Source: Big Planet Marketing Company

Real-World EXAMPLE 3 Add and Subtract Polynomials

CONSUMER ELECTRONICS An electronics store is starting to track sales of cell phones and digital cameras. The equations below represent the number of cell phones *P* and the number of digital cameras *C* sold in *m* months.

> P = 7m + 137C = 4m + 78

a. Write an equation for the monthly sales *T* of phones and cameras.

Add the polynomial for *P* with the polynomial for *C*.

total sales = cell phone sales + digital camera sales

T = 7m + 137 + 4m + 78Substitution

= 11m + 215

Combine like terms.

An equation is T = 11m + 215.

b. Use the equation to predict the number of cell phones and digital cameras sold in 10 months.

T = 11(10) + 215	Substitute 10 for m.
= 110 + 215	Simplify.
= 325	

Thus, a total of 325 cell phones and digital cameras will be sold in 10 months.

Check Your Progress

3. Use the information above to write an equation that represents the difference in the monthly sales of cell phones and the monthly sales of digital cameras. Use the equation to predict the difference in monthly sales in 24 months.

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🗹 Check Your Understanding

Examples 1 and 2 pp. 433-434

Find each sum or difference.

5.
$$(-4z^3 - 2z + 8) - (4z^3 + 3z^2 - 5)$$

$$(2c^2 + 6c + 4) + (5c^2 - 7)$$

1. $(6x^3 - 4) + (-2x^3 + 9)$ 2. $(g^3 - 2g^2 + 5g + 6) - (g^2 + 2g)$ 3. $(4 + 2a^2 - 2a) - (3a^2 - 8a + 7)$ 4. $(8y - 4y^2) + (3y - 9y^2)$ 5. $(-4z^3 - 2z + 8) - (4z^3 + 3z^2 - 5)$ 6. $(-3d^2 - 8 + 2d) + (4d - 12 + d^2)$ **8.** $(3n^3 - 5n + n^2) - (-8n^2 + 3n^3)$

Example 3 p. 435 **9.** VACATION The total number of students T who traveled for spring break consists of two groups: students who flew to their destinations F and students who drove to their destination D. The number (in thousands) of students who flew and the total number of students who flew or drove can be modeled by the following equations, where *n* is the number of years since 1995.

$$T = 14n + 21$$
 $F = 8n + 7$

- **a.** Write an equation that models the number of students who drove to their destination for this time period.
- **b.** Predict the number of students who will drive to their destination in 2012.
- c. How many students will drive or fly to their destination in 2015?

Practice and Problem Solving

Examples 1 and 2 pp. 433-434 Find each sum or difference.

- **10.** $(y + 5) + (2y + 4y^2 2)$
 - **12.** $(3c^3 c + 11) (c^2 + 2c + 8)$
 - 14. (2x 2y + 1) (3y + 4x)
 - **16.** $(x^2y 3x^2 + y) + (3y 2x^2y)$
 - **18.** $(5n 2p^2 + 2np) (4p^2 + 4n)$
 - **20.** $(6ab^2 + 2ab) + (3a^2b 4ab + ab^2)$

Example 3 p. 435

- 22. PETS From 1997 through 2007, the number of dogs D and the number of cats C (in hundreds) adopted from animal shelters in the United States are modeled by the following equations, where *n* is the number of years since 1997.
 - C = n + 4D = 2n + 3
 - a. Write an equation that models the total number T of dogs and cats adopted in hundreds for this time period.
 - **b.** If this trend continues, how many dogs and cats will be adopted in 2011?

Find each sum or difference.

- **23.** (4x + 2y 6z) + (5y 2z + 7x) + (-9z 2x 3y)
- **24.** $(5a^2 4) + (a^2 2a + 12) + (4a^2 6a + 8)$
- **25.** $(3c^2 7) + (4c + 7) (c^2 + 5c 8)$
- **26.** $(3n^3 + 3n 10) (4n^2 5n) + (4n^3 3n^2 9n + 4)$
- **27. GEOMETRY** Write a polynomial that represents the perimeter of the figure at the right.
- **28. PAINTING** Kin is painting two walls of her bedroom. The area of one wall can be modeled by $3x^2 + 14$, and the area of the other wall can be modeled by 2x - 3. What is the total area of the two walls?
- **29. GEOMETRY** The perimeter of the figure at the right is represented by the expression $3x^2 - 7x + 2$. Write a polynomial that represents the measure of the third side.
- **30.** FOOTBALL The National Football League is divided into two conferences, the American A and the National N. From 2002 through 2009, the total attendance T (in thousands) for both conferences and for the American Conference games can be modeled by the following equations, where *x* is the number of years since 2002.

$$T = -0.69x^3 + 55.83x^2 + 643.31x + 10,538$$

$$A = -3.78x^3 + 58.96x^2 + 265.96x + 5257$$

Estimate how many people attended a National Conference football game in 2009.

31. GEOMETRY The width of a rectangle is represented by 5x + 2y, and the length is represented by 6y - 2x. Write a polynomial that represents the perimeter.



Real-World Link

The Pro Football Hall of Fame located in Canton, Ohio, is approximately 83,000 square feet and has had more than eight million visitors.

Source: Pro Football Hall of Fame

= Step-by-Step Solutions begin on page R12. Extra Practice begins on page 815.

(1) $(2x + 3x^2) - (7 - 8x^2)$

15. $(4a - 5b^2 + 3) + (6 - 2a + 3b^2)$

17. $(-8xy + 3x^2 - 5y) + (4x^2 - 2y + 6xy)$

19. $(4rxt - 8r^2x + x^2) - (6rx^2 + 5rxt - 2x^2)$

2x + 1

 $5x + \frac{1}{4}$

 $2x^2 - 10x + 6$

 $2x + \frac{1}{2}$

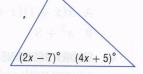
 $x^2 - x - 4$

 $3x - \frac{1}{2}$

21. $(cd^2 + 2cd - 4) + (-6 + 4cd - 2cd^2)$

13. $(z^2 + z) + (z^2 - 11)$

- 32. GARDENING Candida is planting flowers on the perimeter of a rectangular patio.
 - **a.** If the perimeter of the patio is 210x and one side measures 32x, find the length of the other side.
 - **b**. Write a polynomial that represents the area of the rectangular patio.
- **33. GEOMETRY** The sum of the measures of the angles in a triangle is 180°.
 - a. Write an expression to represent the measure of the third angle of the triangle.



- **b.** If x = 23, find the measures of the three angles.
- 34. SALES An electronics store estimates that the cost, in dollars, of selling t units of LCD televisions is given by the expression $0.002t^2 + 4t + 400$. The revenue from the sales of *t* LCD televisions is 8*t*.
 - **a.** Write a polynomial that represents the profit of selling *t* units.
 - **b.** If 750 LCD televisions are sold, how much did the store earn?
 - c. If 575 LCD televisions are sold, how much did the store earn?

CAR RENTAL The cost to rent a car for a day is \$15 plus \$0.15 for each mile driven. 35

- **a.** Write a polynomial that represents the cost of renting a car for *m* miles.
- **b**. If a car is driven 145 miles, how much would it cost to rent?
- **c.** If a car is driven 105 miles each day for four days, how much would it cost to rent a car?
- d. If a car is driven 220 miles each day for seven days, how much would it cost to rent a car?

H.O.T. Problems

Use Higher-Order Thinking Skills

36. FIND THE ERROR Cheyenne and Sebastian are finding $(2x^2 - x) - (3x + 3x^2 - 2)$. Is either of them correct? Explain your reasoning.

Cheyenne $(2x^2 - x) - (3x + 3x^2 - 2)$ $= (2x^2 - x) + (-3x + 3x^2 - 2)$ $=5x^{2}-4x-2$ $= -x^{2} - 4x - 2$

sebastian $(2x^2 - x) - (3x + 3x^2 - 2)$ $= (2x^{2} - x) + (-3x - 3x^{2} - 2)$

- **37. OPEN ENDED** Write two trinomials with a difference of $2x^3 7x + 8$.
- **38.** CHALLENGE Write a polynomial that represents the sum of an odd integer 2n + 1and the next two consecutive odd integers.
- **39. REASONING** Find a counterexample to the following statement.

The order in which polynomials are subtracted does not matter.

- **40. OPEN ENDED** Write three trinomials with a sum of $4x^4 + 3x^2$.
- 41. WRITING IN MATH Describe how to add and subtract polynomials using both the vertical and horizontal formats. Which one do you think is easier? Why?



Real-World Link

On average, an LCD television lasts about 60,000 hours. This means the lifespan of an LCD television can be 20 years or more if the television is used less than 8 hours per day.

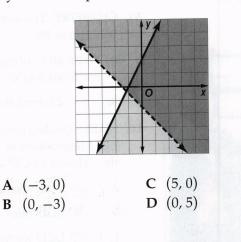
Source: LCD TV Buying Guide

Standardized Test Practice

42. Three consecutive integers can be represented by x, x + 1, and x + 2. What is the sum of these three integers?

- **A** x(x+1)(x+2) **C** 3x+3**B** x^3+3 **D** x+3
- **43. SHORT RESPONSE** What is the perimeter of a square with sides that measure 2x + 3 units?
- 44. Jim cuts a board in the shape of a regular hexagon and pounds in a nail at each vertex, as shown. How many rubber bands will he need to stretch a rubber band across every possible pair of nails?
 F 15 G 12 H 14 J 9

45. Which ordered pair is in the solution set of the system of inequalities shown in the graph?



Spiral Review

Find the degree of each polynomial. (Lesson 7-4)

46. 6b ⁴	47. 10 <i>t</i>	48. $5g^2h$
49. $7np^4$	50. 25	51. $t^3 + 6u$
52. $2 + 3ab^3 - a^2b + 4a^6$		53. $6 - v^4 + v^2 z^3 + 6v^3$

- **54. POPULATION** The 2005 population of North Carolina's Beaufort County was approximately 46,000. Express this number in scientific notation. (Lesson 7-3)
- **55. JOBS** Kimi received an offer for a new job. She wants to compare the offer with her current job. What is total amount of sales that Kimi must get each month to make the same income at either job? (Lesson 6-2)

Determine whether each sequence is an arithmetic sequence. If it is, state the common difference. (Lesson 3-5)

56. 24, 16, 8, 0,	57. $3\frac{1}{4}, 6\frac{1}{2}, 13, 26, \dots$	58. 7, 6, 5, 4,
59. 10, 12, 15, 18,	60. -15, -11, -7, -3,	61. -0.3, 0.2, 0.7, 1.2, .

Skills Review

Simplify. (Lesson 7-1)	
62. $t(t^5)(t^7)$	63. $n^3(n^2)(-2n^3)$
64. $(5t^5v^2)(10t^3v^4)$	65. $(-8u^4z^5)(5uz^4)$
66. [(3) ²] ³	67. [(2) ³] ²
68. $(2m^4k^3)^2(-3mk^2)^3$	69. $(6xy^2)^2(2x^2y^2z^2)^3$

61. -0.3, 0.2, 0.7, 1.2, ...

New Offer

CurrentJob

^^^^

\$600/mo 2% commission

\$1000/mo 1.5% commission

Multiplying a Polynomial by a Monomial

Then

You multiplied monomials. (Lesson 7-1)

Now/

- Multiply a polynomial by a monomial.
- Solve equations involving the products of monomials and polynomials.

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Why?

Charmaine Brooks is opening a fitness club. She tells the contractor that the length of the fitness room should be three times the width plus 8 feet.

To cover the floor with mats for exercise classes, Ms. Brooks needs to know the area of the floor. So she multiplies the width times the length, w(3w + 8).



Polynomial Multiplied by Monomial To find the product of a polynomial and a monomial, you can use the Distributive Property.

EXAMPLE 1 Multiply a Polynomial b	y a Monomial
Find $-3x^2(7x^2 - x + 4)$.	
Horizontal Method $-3x^{2}(7x^{2} - x + 4)$ $= -3x^{2}(7x^{2}) - (-3x^{2})(x) + (-3x^{2})(4)$ $= -21x^{4} - (-3x^{3}) + (-12x^{2})$ $= -21x^{4} + 3x^{3} - 12x^{2}$	Original expression Distributive Property Multiply. Simplify.
Vertical Method $7x^2 - x + 4$ (\times) $-3x^2$ $-21x^4 + 3x^3 - 12x^2$ Distributive PropertyMultiply.	
Check Your Progress Find each product.	
-	1B. $-6d^3(3d^4 - 2d^3 - d + 9)$ Personal Tutor glencoe.com

We can use this same method more than once to simplify large expressions.

EXAMPLE 2 Simplify Expressions

Simplify $2p(-4p^2 + 5p) - 5(2p^2 + 20)$. $2p(-4p^2 + 5p) - 5(2p^2 + 20)$ $= (2p)(-4p^2) + (2p)(5p) + (-5)(2p^2) + (-5)(20)$ $= -8p^3 + 10p^2 - 10p^2 - 100$ $= -8p^3 + (10p^2 - 10p^2) - 100$ $= -8p^3 - 100$

Original expression Distributive Property Multiply. Commutative and Associative Properties Combine like terms.

Check Your Progress

Simplify each expression.

2A. $3(5x^2 + 2x - 4) - x(7x^2 + 2x - 3)$ **2B.** $15t(10y^3t^5 + 5y^2t) - 2y(yt^2 + 4y^2)$

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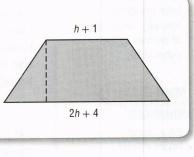
We can use the Distributive Property to multiply monomials by polynomials and solve real world problems.

Test-TakingTip

Formulas Many standardized tests provide formula sheets with commonly used formulas. If you are unsure of the correct formula, check the sheet before beginning to solve the problem.



GRIDDED RESPONSE The theme for a school dance is "Solid Gold." For one decoration, Kana is covering a trapezoid-shaped piece of poster board with metallic gold paper to look like a bar of gold. If the height of the poster board is 18 inches, how much metallic paper will Kana need in square inches?



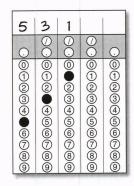
Read the Test Item

The question is asking you to find the area of the trapezoid with a height of *h* and bases of h + 1 and 2h + 4.

Solve the Test Item

Write an equation to represent the area of the trapezoid. Let $b_1 = h + 1$, let $b_2 = 2h + 4$ and let h = height of the trapezoid.

 $A = \frac{1}{2}h(b_1 + b_2)$ Area of a trapezoid $= \frac{1}{2}h[(h+1) + (2h+4)] \qquad b_1 = h + 1 \text{ and } b_2 = 2h + 4$ $=\frac{1}{2}h(3h+5)$ Add and simplify. $=\frac{3}{2}h^2+\frac{5}{2}h$ **Distributive Property** $=\frac{3}{2}(18)^2+\frac{5}{2}(18)$ h = 18= 531Simplify.



Kana will need 531 square inches of metallic paper. Grid in your response of 531.

Check Your Progress

- 3. Kachima is making triangular bandanas for the dogs and cats in her pet club. The base of the bandana is the length of the collar with 4 inches added to each end to tie it on. The height is $\frac{1}{2}$ of the collar length.
 - A. If Kachima's dog has a collar length of 12 inches, how much fabric does she need in square inches?
 - B. If Kachima makes a bandana for her friend's cat with a 6-inch collar, how much fabric does Kachima need in square inches?

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Real-World Link

In a recent year, the pet supply business hit an estimated \$7.05 billion in sales. This business ranges from gourmet food to rhinestone tiaras, pearl collars, and cashmere coats.

Source: Entrepreneur Magazine

StudyTip

Combining Like Terms When simplifying a long expression, it may be helpful to put a circle around one set of like terms, a rectangle around another set, a triangle around another set, and so on. Solve Equations with Polynomial Expressions We can use the Distributive Property to solve equations that involve the products of monomials and polynomials.

Equations with Polynomials on Both Sides EXAMPLE 4

Solve 2a(5a - 2) + 3a(2a + 6) + 8 = a(4a + 1) + 2a(6a - 4) + 50.

2a(5a - 2) + 3a(2a + 6) + 8 = a(4a + 1) + 2a(6a - 4) + 50 $10a^2 - 4a + 6a^2 + 18a + 8 = 4a^2 + a + 12a^2 - 8a + 50$ $16a^2 + 14a + 8 = 16a^2 - 7a + 50$ 14a + 8 = -7a + 50-21a + 8 = 5021a = 42a = 2

Original equation Distributive Property Combine like terms. Subtract 16a² from each side. Add 7a to each side. Subtract 8 from each side. Divide each side by 21.

CHECK

2a(5a-2) + 3a(2a+6) + 8 = a(4a+1) + 2a(6a-4) + 50 $2(2)[5(2) - 2] + 3(2)[2(2) + 6] + 8 \stackrel{?}{=} 2[4(2) + 1] + 2(2)[6(2) - 4] + 50$ $4(8) + 6(10) + 8 \stackrel{?}{=} 2(9) + 4(8) + 50$ Simplify. $32 + 60 + 8 \stackrel{?}{=} 18 + 32 + 50$ **Multiply.** $100 = 100 \checkmark$ Add and subtract. Check Your Progress

Solve each equation.

4A.
$$2x(x + 4) + 7 = (x + 8) + 2x(x + 1) + 12$$

4B. $d(d + 3) - d(d - 4) - 9d - 16$

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🗹 Cł	neck Your	Understanding
	Example 1 p. 439	Find each product.1. $5w(-3w^2 + 2w - 4)$ 2. $6g^2(3g^3 + 4g^2 + 10g - 1)$ 3. $4km^2(8km^2 + 2k^2m + 5k)$ 4. $-3p^4r^3(2p^2r^4 - 6p^6r^3 - 5)$ 5. $2ab(7a^4b^2 + a^5b - 2a)$ 6. $c^2d^3(5cd^7 - 3c^3d^2 - 4d^3)$
	Example 2 p. 439	Simplify each expression. 7. $t(4t^2 + 15t + 4) - 4(3t - 1)$ 9. $-2d(d^3c^2 - 4dc^2 + 2d^2c) + c^2(dc^2 - 3d^4)$ 10. $-5w^2(8w^2x - 11wx^2) + 6x(9wx^4 - 4w - 3x^2)$
	Example 3 p. 440	11. GRIDDED RESPONSE Marlene is buying a new plasma television. The height of the screen of the television is one half the width plus 5 inches. The width is 30 inches. Find the height of the screen in inches.
	Example 4 p. 441	Solve each equation. 12. $-6(11 - 2c) = 7(-2 - 2c)$ 13. $t(2t + 3) + 20 = 2t(t - 3)$
		14. $-2(w + 1) + w = 7 - 4w$ 15. $3(y - 2) + 2y = 4y + 14$ 16. $a(a + 3) + a(a - 6) + 35 = a(a - 5) + a(a + 7)$ 17. $n(n - 4) + n(n + 8) = n(n - 13) + n(n + 1) + 16$

Practice and Problem Solving

Find each product. **Example 1** p. 439 19. $f(f^2 + 2f + 25)$ **18.** $b(b^2 - 12b + 1)$ **21.** $2j^2(5j^3 - 15j^2 + 2j + 2)$ **20.** $-3m^3(2m^3 - 12m^2 + 2m + 25)$ **23.** $4t^3u(2t^2u^2 - 10tu^4 + 2)$ **22.** $2pr^2(2pr + 5p^2r - 15p)$ Simplify each expression. Example 2 **25.** $a(-8a^2 + 2a + 4) + 3(6a^2 - 4)$ p. 439 **24.** $-3(5x^2 + 2x + 9) + x(2x - 3)$ **27.** $-9g(-2g+g^2) + 3(g^2+4)$ **26.** $-4d(5d^2 - 12) + 7(d + 5)$ **28.** $2i(7i^2k^2 + ik^2 + 5k) - 9k(-2i^2k^2 + 2k^2 + 3i)$ **29.** $4n(2n^3p^2 - 3np^2 + 5n) + 4p(6n^2p - 2np^2 + 3p)$ **30. DAMS** A new dam being built has the shape of a Example 3 trapezoid. The base at the bottom of the dam is p. 440 2 times the height. The base at the top of the dam is $\frac{1}{5}$ times the height minus 30 feet. a. Write an expression to find the area of the trapezoidal cross section of the dam. **b.** If the height of the dam is 180 feet, find the area of this cross section. Solve each equation. Example 4 p. 441 $31 7(t^2 + 5t - 9) + t = t(7t - 2) + 13$ **32.** $w(4w+6) + 2w = 2(2w^2 + 7w - 3)$ **33.** 5(4z+6) - 2(z-4) = 7z(z+4) - z(7z-2) - 48**34.** 9c(c-11) + 10(5c-3) = 3c(c+5) + c(6c-3) - 30**35.** $2f(5f-2) - 10(f^2 - 3f + 6) = -8f(f + 4) + 4(2f^2 - 7f)$ **36.** $2k(-3k + 4) + 6(k^2 + 10) = k(4k + 8) - 2k(2k + 5)$ Simplify each expression. **37.** $\frac{2}{3}np^2(30p^2 + 9n^2p - 12)$ **38.** $\frac{3}{5}r^2t(10r^3 + 5rt^3 + 15t^2)$ **39.** $-5q^2w^3(4q + 7w) + 4qw^2(7q^2w + 2q) - 3qw(3q^2w^2 + 9)$ **40.** $-x^2z(2z^2 + 4xz^3) + xz^2(xz + 5x^3z) + x^2z^3(3x^2z + 4xz)$ **41. PARKING** A parking garage charges \$30 per month plus \$0.50 per daytime hour

- and \$0.25 per hour during nights and weekends. Suppose Trent parks in the garage for 47 hours in January and h of those are night and weekend hours.
 - a. Find an expression for Trent's January bill.
 - **b.** Find the cost if Trent had 12 hours of night and weekend hours.
- **42. PETS** Che is building a dog house for his new puppy. The upper face of the dog house is a trapezoid. If the height of the trapezoid is 12 inches, find the area of the face of this piece of the dog house.



= Step-by-Step Solutions begin on page R12.

Extra Practice begins on page 815.



Real-World Link

Arthur Ashe is the only African-American male tennis player to win both Wimbledon and the U.S. Open and the first and only African-American player to be ranked the #1 tennis player in the world.

Source: CMG Worldwide

- **TENNIS** The tennis club is building a new tennis court with a path around it.
 - **a.** Write an expression for the area of the tennis court.
 - **b.** Write an expression for the area of the path.
 - c. Every three feet around the court, there will be a stepping stone placed in the path.If *x* = 36 feet, what is the perimeter of the outside of the path?
- **44. MULTIPLE REPRESENTATIONS** In this problem, you will investigate the degree of the product of a monomial and a polynomial.
 - **a. TABULAR** Write three monomials of different degrees and three polynomials of different degrees. Determine the degree of each monomial and polynomial. Multiply the monomials by the polynomials. Determine the degree of each product. Record your results in a table like the one shown below.

Monomial	Degree	Polynomial	Degree	Product of Monomial and Polynomial	Degree
					2.20 80%
	e esta	일하는 데 문화	124		
	Canie des	31-+	- 651.58	i i i i i i i i i i i i i i i i i i i	- (+ + 1)

b. VERBAL Make a conjecture about the degree of the product of a monomial and a polynomial. What is the degree of the product of a monomial of degree *a* and a polynomial of degree *b*?

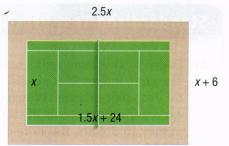
H.O.T. Problems

Use Higher-Order Thinking Skills

45. FIND THE ERROR Pearl and Ted both worked on this problem. Is either of them correct? Explain your reasoning.

PearlTed $2x^2(3x^2 + 4x + 2)$ $2x^2(3x^2 + 4x + 2)$ $6x^4 + 8x^2 + 4x^2$ $6x^4 + 8x^3 + 4x^2$ $6x^4 + 12x^2$ $6x^4 + 8x^3 + 4x^2$

- **46.** CHALLENGE Find *p* such that $3x^{p}(4x^{2p+3} + 2x^{3p-2}) = 12x^{12} + 6x^{10}$.
- **47.** CHALLENGE Simplify $4x^{-3}y^2(2x^5y^{-4} + 6x^{-7}y^6 4x^0y^{-2})$.
- **48. REASONING** Is there a value for *x* that makes the statement $(x + 2)^2 = x^2 + 2^2$ true? If so, find a value for *x*. Explain your reasoning.
- **49. OPEN ENDED** Write a monomial and a polynomial using *n* as the variable. Find their product.
- **50.** WRITING IN MATH Describe the steps to multiply a polynomial by a monomial.



Standardized Test Practice

- **51.** Every week a store sells *j* jeans and *t* T-shirts. The store makes \$8 for each T-shirt and \$12 for each pair of jeans. Which of the following expressions represents the total amount of money, in dollars, the store makes every week?
 - **A** 8j + 12t**C** 20(j+t)**B** 12j + 8t
 - **D** 96*jt*
- **52.** If a = 5x + 7y and b = 2y 3x, what is a + b?
 - **H** 2x + 9y $\mathbf{F} \quad 2x - 9y$ **G** 3y + 4xJ 2x - 5y

- 53. **GEOMETRY** A triangle has sides of length 5 inches and 8.5 inches. Which of the following cannot be the length of the third side?
 - A 3.5 inches
 - **B** 4 inches
 - C 5.5 inches
 - D 12 inches
- 54. SHORT RESPONSE Write an equation in which x varies directly as the cube of y and inversely as the square of *z*.

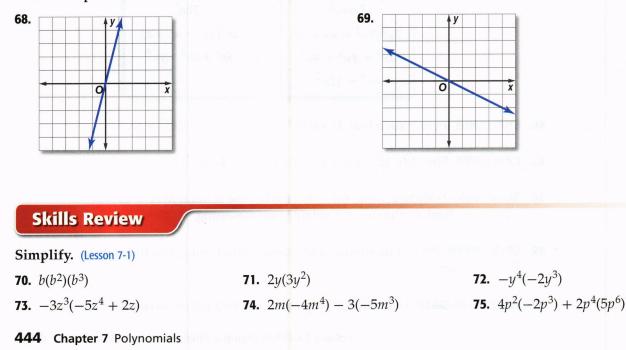
Spiral Review

Find each sum or difference. (Lesson 7-5)

55. $(2x^2 - 7) + (8 - 5x^2)$	56. $(3z^2 + 2z - 1) + (z^2 - 6)$	57. $(2a - 4a^2 + 1) - (5a^2 - 2a - 6)$
58. $(a^3 - 3a^2 + 4) - (4a^2 + 7)$	59. $(2ab - 3a + 4b) + (5a + 4ab)$	60. $(8c^3 - 3c^2 + c - 2) - (3c^3 + 9)$
Find the degree of each polynomia	al. (Lesson 7-4)	
61. 12 <i>y</i>	62. -10	63. $2x^2 - 5$
64. $9a - 8a^3 + 6$	65. $7b^2c^3$	66. $-3p^4r^5t^2$
67. TRAVEL In 1990, about 3.6 millio	on people took cruises. Between 1990	and 2000,

the number increased by about 300,000 each year. Write the point-slope form of an equation to find the total number of people *y* taking a cruise for any year *x*. Estimate the number of people who will take a cruise in 2010. (Lesson 4-3)

Write an equation in function notation for each relation. (Lesson 3-6)



You can use algebra tiles to find the product of two binomials.

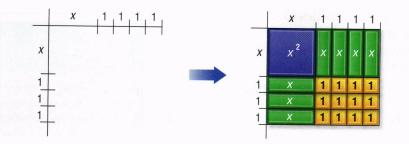
ACTIVITY 1 Multiply Binomials

EXPLORE

7-7/

Use algebra tiles to find (x + 3)(x + 4).

The rectangle will have a width of x + 3 and a length of x + 4. Use algebra tiles to mark off the dimensions on a product mat. Then complete the rectangle with algebra tiles.



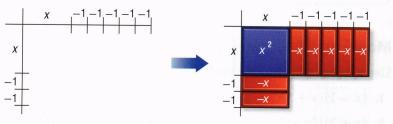
The rectangle consists of 1 blue x^2 -tile, 7 green x-tiles, and 12 yellow 1-tiles. The area of the rectangle is $x^2 + 7x + 12$. So, $(x + 3)(x + 4) = x^2 + 7x + 12$.

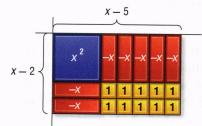
ACTIVITY 2 Multiply Binomials

Use algebra tiles to find (x - 2)(x - 5).

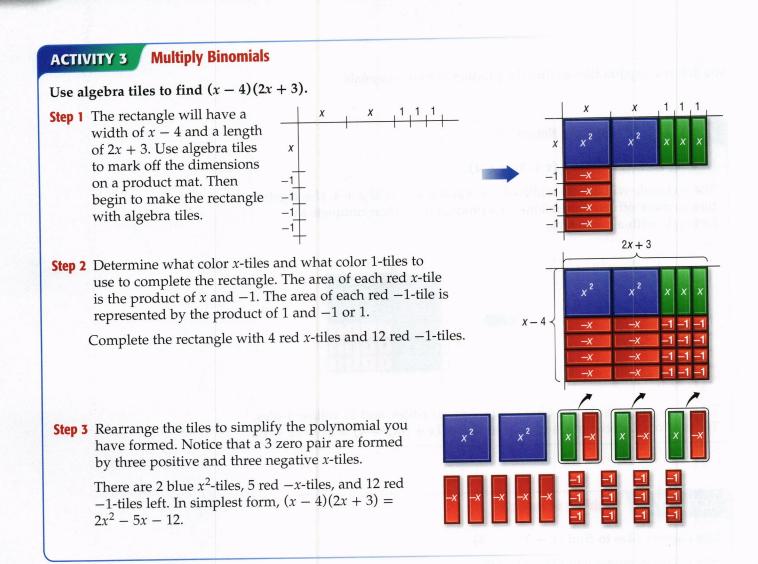
- **Step 1** The rectangle will have a width of x 2 and a length of x 5. Use algebra tiles to mark off the dimensions on a product mat. Then begin to make the rectangle with algebra tiles.
- **Step 2** Determine whether to use 10 yellow 1-tiles or 10 red -1-tiles to complete the rectangle. The area of each yellow tile is the product of -1 and -1. Fill in the space with 10 yellow 1-tiles to complete the rectangle.

The rectangle consists of 1 blue x^2 -tile, 7 red -x-tiles, and 10 yellow 1-tiles. The area of the rectangle is $x^2 - 7x + 10$. So, $(x - 2)(x - 5) = x^2 - 7x + 10$.





(continued on the next page)



Model and Analyze

Use algebra tiles to find each product.

- 1. (x+1)(x+4)
- 4. (x+2)(2x+3)

2. (x-3)(x-2)**3.** (x+5)(x-1)**5.** (x-1)(2x-1)**6.** (x+4)(2x-5)

Is each statement *true* or *false*? Justify your answer with a drawing of algebra tiles.

7. $(x-4)(x-2) = x^2 - 6x + 8$

8. $(x + 3)(x + 5) = x^2 + 15$

9. WRITING IN MATH You can also use the Distributive Property to find the product of two binomials. The figure at the right shows the model for (x + 4)(x + 5) separated into four parts. Write a sentence or two explaining how this model shows the use of the Distributive Property.



Multiplying Polynomials

Why?

Bodyboards, which are used to ride waves, are made of foam and are more rectangular than surfboards. A bodyboard's dimensions are determined by the height and skill level of the user.

The length of Ann's bodyboard should be Ann's height *h* minus 32 inches or h - 32. The board's width should be half of Ann's height plus 11 inches or $\frac{1}{2}h + 11$. To approximate the area of the bodyboard, you need to find $(h - 32)(\frac{1}{2}h + 11)$.



Multiply Binomials To multiply two binomials such as h - 32 and $\frac{1}{2}h + 11$, the Distributive Property is used. Binomials can be multiplied horizontally or vertically.

EXAMPLE 1 The Dis	tributive Property	
Find each product. a. $(2x + 3)(x + 5)$ Vertical Method		
Multiply by 5. 2x + 3 $(\times) x + 5$ 10x + 15 5(2x + 3) = 10x + 15	$\frac{2x^2+3x}{2x^2+3x}$	Combine like terms. $2x + 3$ $(x) x + 5$ $10x + 15$ $2x^{2} + 3x$ $2x^{2} + 13x + 15$
Horizontal Method $(2x + 3)(x + 5) = 2x(x + 3)(x + 5) = 2x^2$	(x + 5) + 3(x + 5) + 10x + 3x + 15 Rewrite Distribut	as the sum of two products. tive Property e like terms.
b. $(x - 2)(3x + 4)$ Vertical Method		
Multiply by 4.	Multiply by $3x$.	Combine like terms.
$\begin{array}{r} x-2\\ \underline{(\times)}\ 3x+4\\ 4x-8 \end{array}$	$ \begin{array}{r} x-2 \\ \underline{(\times) \ 3x+4} \\ 4x-8 \\ \underline{3x^2-6x} \end{array} $	$ \begin{array}{r} x-2 \\ \underline{(\times) \ 3x+4} \\ 4x-8 \\ \underline{3x^2-6x} \end{array} $
4(x-2) = 4x - 8	$3x(x-2) = 3x^2 - 6x$	$3x^2 - 2x - 8$
Horizontal Method		
	+4x-6x-8 Distribut	as the difference of two products. tive Property e like terms.

<u>Then</u>

You multiplied polynomials by monomials. (Lesson 7-6)

Now/

- Multiply polynomials by using the Distributive Property.
- Multiply binomials by using the FOIL method.

New/ Vocabulary/ FOIL method quadratic expression

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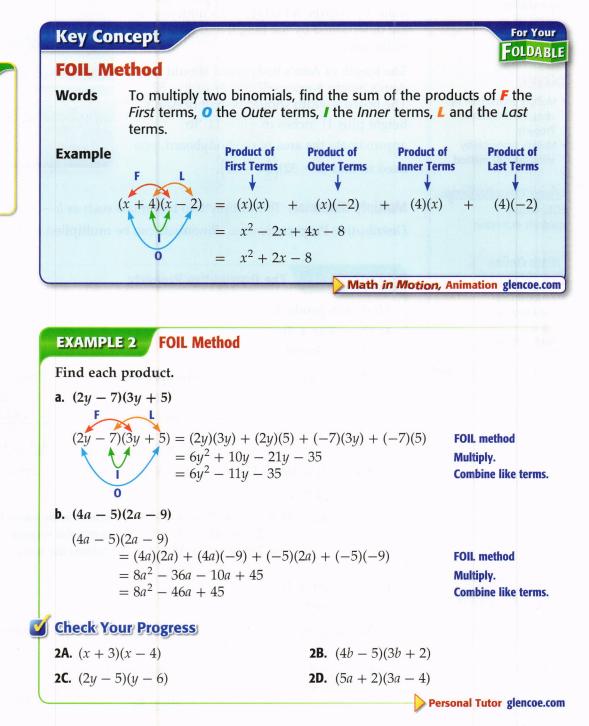
Lesson 7-7 Multiplying Polynomials 447

Check Your Progress

1A. (3m + 4)(m + 5)

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A shortcut version of the Distributive Property for multiplying binomials is called the **FOIL method**.



Notice that when two linear expressions are multiplied, the result is a quadratic expression. A **quadratic expression** is an expression in one variable with a degree of 2. When three linear expressions are multiplied, the result has a degree of 3.

The FOIL method can be used to find an expression that represents the area of a rectangular object when the lengths of the sides are given as binomials.

ReadingMath

Polynomials as Factors The expression (x + 4)(x - 2) is read the quantity x plus 4 times the quantity x minus 2.

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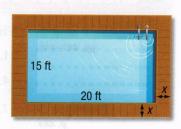
Real-World Link

The cost of a swimming pool depends on many factors, including the size of the pool, whether the pool is an above-ground or an in-ground pool, and the material used. Source: American Dream Homes

Real-World EXAMPLE 3 FOIL Method

SWIMMING POOL A contractor is building a deck around a rectangular swimming pool. The deck is *x* feet from every side of the pool. Write an expression for the total area of the pool and deck.

Understand We need to find an expression for the total area of the pool and deck.



- **Plan** Use the formula for the area of a rectangle and determine the length and width of the pool with the deck.
- **Solve** Since the deck is the same distance from every side of the pool, the length and width of the pool are 2x longer. So, the length can be represented by 2x + 20 and the width can be represented by 2x + 15.
 - Area = length \cdot width
 - = (2x + 20)(2x + 15)
 - = (2x)(2x) + (2x)(15) + (20)(2x) + (20)(15)
 - $= 4x^2 + 30x + 40x + 300$
 - $=4x^2+70x+300$

Area of a rectangle Substitution FOIL Method Multiply. Combine like terms.

So, the total area of the deck and pool is $4x^2 + 70x + 300$.

Check Choose a value for *x*. Substitute this value into (2x + 20)(2x + 15) and $4x^2 + 70x + 300$. The result should be the same for both expressions.

Check Your Progress

3. If the pool is 25 feet long and 20 feet wide, find the area of the pool and deck.

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Multiply Polynomials The Distributive Property can also be used to multiply any two polynomials.

EXAMPLE 4 The Distributive Property

Find each product.

a. $(6x + 5)(2x^2 - 3x - 5)$ $(6x + 5)(2x^2 - 3x - 5)$ $= 6x(2x^2 - 3x - 5) + 5(2x^2 - 3x - 5)$ $= 12x^3 - 18x^2 - 30x + 10x^2 - 15x - 25$ $= 12x^3 - 8x^2 - 45x - 25$

Distributive Property Multiply. Combine like terms.

b.
$$(2y^2 + 3y - 1)(3y^2 - 5y + 2)$$

 $(2y^2 + 3y - 1)(3y^2 - 5y + 2)$
 $= 2y^2(3y^2 - 5y + 2) + 3y(3y^2 - 5y + 2) - 1(3y^2 - 5y + 2)$

 $= 6y^4 - 10y^3 + 4y^2 + 9y^3 - 15y^2 + 6y - 3y^2 + 5y - 2$ = $6y^4 - y^3 - 14y^2 + 11y - 2$

Check Your Progress

4A. $(3x-5)(2x^2+7x-8)$

4B. $(m^2 + 2m - 3)(4m^2 - 7m + 5)$ **Personal Tutor glencoe.com**

Distributive Property

Combine like terms.

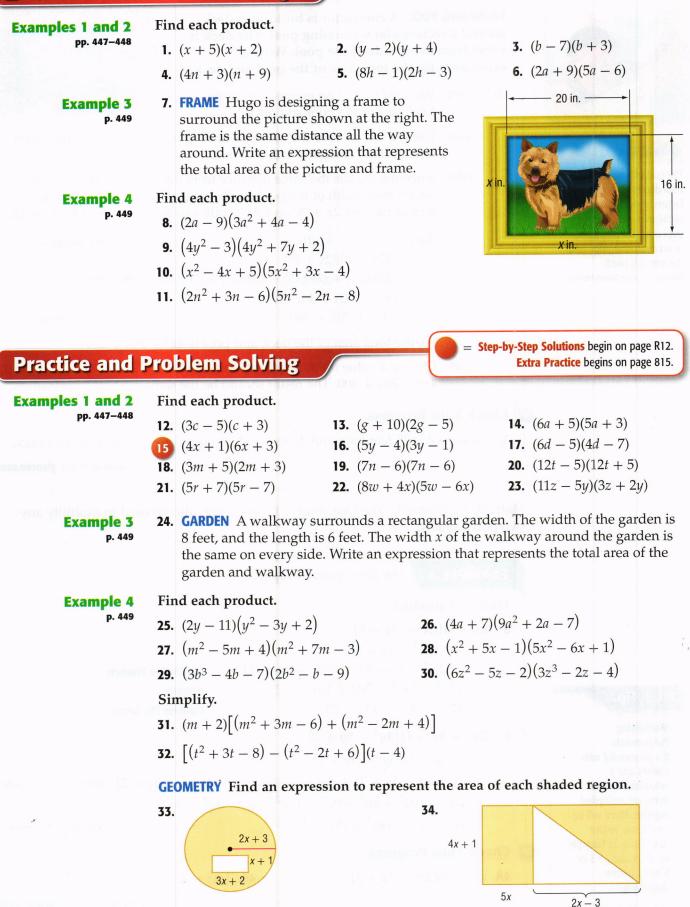
Multiply.

StudyTip

Multiplying Polynomials

If a polynomial with c terms and a polynomial with d terms are multiplied together, there will be $c \cdot d$ terms before simplifying. In Example 4a, there are $2 \cdot 3$ or 6 terms before simplifying.

🗹 Check Your Understanding





Real-World Link

On May 20, 2007, Misty May-Treanor won her 73rd professional beach volleyball title. May-Treanor has more wins than any other woman.

Source: Association of Volleyball Professionals

- **VOLLEYBALL** The dimensions of a sand volleyball court are represented by a width of 6y 5 feet and a length of 3y + 4 feet.
 - **a**. Write an expression that represents the area of the court.
 - **b**. The length of a sand volleyball court is 31 feet. Find the area of the court.
- **36. GEOMETRY** Write an expression for the area of a triangle with a base of 2x + 3 and a height of 3x 1.

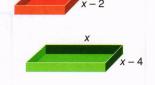
Find each product.

37. (a -

40. (2*r*

$(-2b)^2$	38.	$(3c+4d)^2$
$(-3t)^{3}$	41.	$(5g + 2h)^3$

- **39.** $(x 5y)^2$ **42.** $(4y + 3z)(4y - 3z)^2$
- **43. CONSTRUCTION** A sandbox kit allows you to build a square sandbox or a rectangular sandbox as shown.
 - **a.** What are the possible values of *x*? Explain.
 - **b.** Which shape has the greater area?



- **c.** What is the difference in areas between the two?
- **44. Solution MULTIPLE REPRESENTATIONS** In this problem, you will investigate the square of a sum.
 - a. TABULAR Copy and complete the table for each sum.

Expression	(Expression) ²
<i>x</i> + 5	
3y + 1	
z + q	

- **b. VERBAL** Make a conjecture about the terms of the square of a sum.
- **c. SYMBOLIC** For a sum of the form *a* + *b*, write an expression for the square of the sum.

H.O.T. Problems

Use Higher-Order Thinking Skills

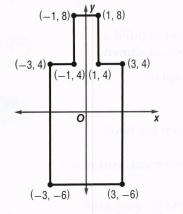
45. REASONING Determine if the following statement is *sometimes, always,* or *never* true. Explain your reasoning.

The FOIL method can be used to multiply a binomial and a trinomial.

- **46.** CHALLENGE Find $(x^m + x^p)(x^{m-1} x^{1-p} + x^p)$.
- **47. OPEN ENDED** Write a binomial and a trinomial involving a single variable. Then find their product.
- **48. REASONING** Compare and contrast the procedure used to multiply a trinomial by a binomial using the vertical method with the procedure used to multiply a three-digit number by a two-digit number.
- **49.** WRITING IN MATH Summarize the methods that can be used to multiply polynomials.

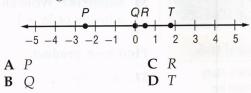
Standardized Test Practice

- **50.** What is the product of 2x 5 and 3x + 4?
 - **A** 5x 1 **B** $6x^2 - 7x - 20$ **C** $6x^2 - 20$
 - **D** $6x^2 + 7x 20$
- **51.** Which statement is correct about the symmetry of this design?

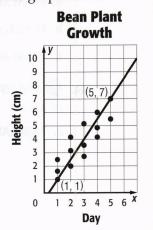


- **F** The design is symmetrical only about the *y*-axis.
- **G** The design is symmetrical only about the *x*-axis.
- **H** The design is symmetrical about both the *y* and the *x*-axes.
- J The design has no symmetry.

52. Which point on the number line represents a number that, when cubed, will result in a number greater than itself?



53. SHORT RESPONSE For a science project, Jodi selected three bean plants of equal height. Then, for five days, she measured their heights in centimeters and plotted the values on the graph below.



She drew a line of best fit on the graph. What is the slope of the line that she drew?

Spiral Review

54. SAVINGS Carrie has \$6000 to invest. She puts x dollars of this money into a savings account that earns 2% interest per year. She uses the rest of the money to purchase a certificate of deposit that earns 4% interest. Write an equation for the amount of money that Carrie will have in one year. (Lesson 7-6)

Find each sum or difference. (Lesson 7-5)

55. $(7a^2 - 5) + (-3a^2 + 10)$ **57.** $(4 + n^3 + 3n^2) + (2n^3 - 9n^2 + 6)$ **59.** (b + 4) + (c + 3b - 2)

61. $(-4m^3 - m + 10) - (3m^3 + 3m^2 - 7)$

56. $(8n - 2n^2) + (4n - 6n^2)$ **58.** $(-4u^2 - 9 + 2u) + (6u + 14 + 2u^2)$ **60.** $(3a^3 - 6a) - (3a^3 + 5a)$ **62.** (3a + 4ab + 3b) - (2b + 5a + 8ab)

Skills Review

Simplify. (Lesson 7-1) 63. $(-2t^4)^3 - 3(-2t^3)^4$ 64. $(-3h^2)^3 - 2(-h^3)^2$

65.
$$2(-5y^3)^2 + (-3y^3)^3$$
 66. $3(-6n^4)^2 + (-2n^2)^3$

Then

You multiplied binomials by using the FOIL method. (Lesson 7-7)

Now/

- Find squares of sums and differences.
- Find the product of a sum and a difference.

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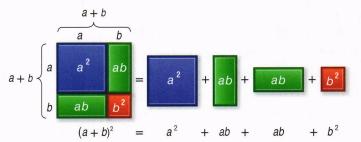
Why?

Colby wants to attach a dartboard to a square piece of corkboard. If the radius of the dartboard is r + 12, how large does the square corkboard need to be?

Colby knows that the diameter of the dartboard is 2(r + 12) or 2r + 24. Each side of the square also measures 2r + 24. To find how much corkboard is needed, Colby must find the area of the square: $A = (2r + 24)^2$.



Squares of Sums and Differences Some pairs of binomials, such as squares like $(2r + 24)^2$, have products that follow a specific pattern. Using the pattern can make multiplying easier. The square of a sum, $(a + b)^2$ or (a + b)(a + b), is one of those products.



For Your Key Concept Square of a Sum FOLDABLE The square of a + b is the square of a plus twice the Words product of *a* and *b* plus the square of *b*. **Example** $(x + 4)^2 = (x + 4)(x + 4)$ $(a + b)^2 = (a + b)(a + b)$ **Symbols** $= a^{2} + 2ab + b^{2}$ $= x^2 + 8x + 16$ Math in Motion, Animation glencoe.com EXAMPLE 1 Square of a Sum Find $(3x + 5)^2$. $(a + b)^2 = a^2 + 2ab + b^2$ Square of a sum

 $(3x + 5)^2 = (3x)^2 + 2(3x)(5) + 5^2$ $=9x^{2} + 30x + 25$

a = 3x, b = 5

Simplify. Use FOIL to check your solution.

Check Your Progress

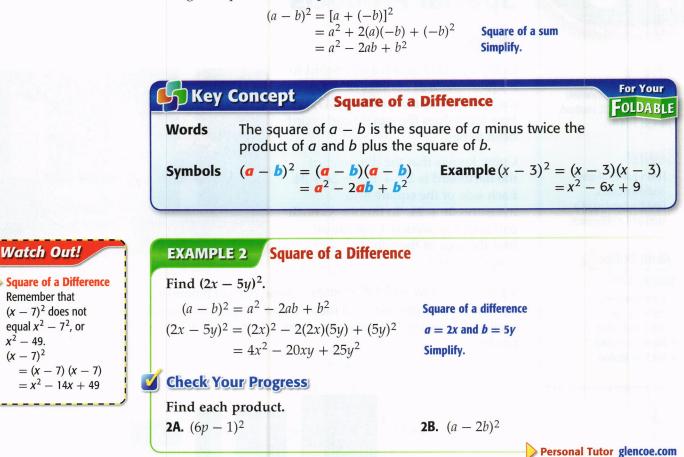
Find each product.

1A. $(8c + 3d)^2$

1B. $(3x + 4y)^2$

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There is also a pattern for the square of a difference. Write a - b as a + (-b) and square it using the square of a sum pattern.



The product of the square of a sum or the square of a difference is called a perfect square trinomial. We can use these to find patterns to solve real-world problems.

Square of a Difference Real-World EXAMPLE 3

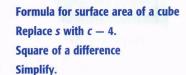
PHYSICAL SCIENCE Each edge of a cube of aluminum is 4 centimeters less than each edge of a cube of copper. Write an equation to model the surface area of the aluminum cube.



Let c = the length of each edge of the cube of copper. So, each edge of the cube of aluminum is c - 4.

- $SA = 6s^2$
- $SA = 6(c 4)^2$

 $SA = 6[c^2 - 2(4)(c) + 4^2]$ $SA = 6(c^2 - 8c + 16)$



/

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Check Your Progress

- **3. GARDENING** Alano has a garden that is g feet long and g feet wide. He wants to add 3 feet to the length and the width.
 - A. Show how the new area of the garden can be modeled by the square of a binomial.
 - **B.** Find the square of this binomial.

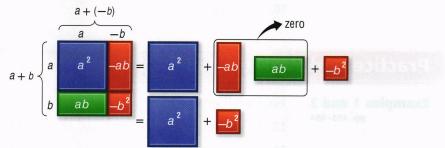
 $x^2 - 49$.

 $(x - 7)^2$

StudyTip

Patterns When using any of these patterns, a and b can be numbers, variables, or expressions with numbers and variables.

Product of a Sum and a Difference Now we will see what the result is when we multiply a sum and a difference, or (a + b)(a - b). Recall that a - b can be written as a + (-b).



Notice that the middle terms are opposites and add to a zero pair. So (a + b)(a - b) = $a^2 - ab + ab - b^2 = a^2 - b^2$.

	Concept Product of a Sum and a Difference For Your
Words	The product of $a + b$ and $a - b$ is the square of a minus the square of b .
Symbols	(a + b)(a - b) = (a - b)(a + b) = $a^2 - b^2$
	Math in Motion, Animation glencoe.com

EXAMPLE 4 Product of a Sum and a Difference

Find $(2x^2 + 3)(2x^2 - 3)$. $(a + b)(a - b) = a^2 - b^2$ $(2x^2 + 3)(2x^2 - 3) = (2x^2)^2 - (3)^2$ $=4x^{4}-9$

Product of a sum and difference $a = 2x^2$ and b = 3Simplify.

Check Your Progress

Find each product.

4A. (3n + 2)(3n - 2)

4B. (4c - 7d)(4c + 7d)

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D

DD

Dy

y

Dy

VV

Check Your Understanding

Find each product. Examples 1 and 2

- pp. 453-454 1. $(x + 5)^2$
- **2.** $(11 a)^2$ **4.** (3m-4)(3m-4)5. (g - 4h)(g - 4h)
- 3 $(2x + 7y)^2$ 6. $(3c + 6d)^2$

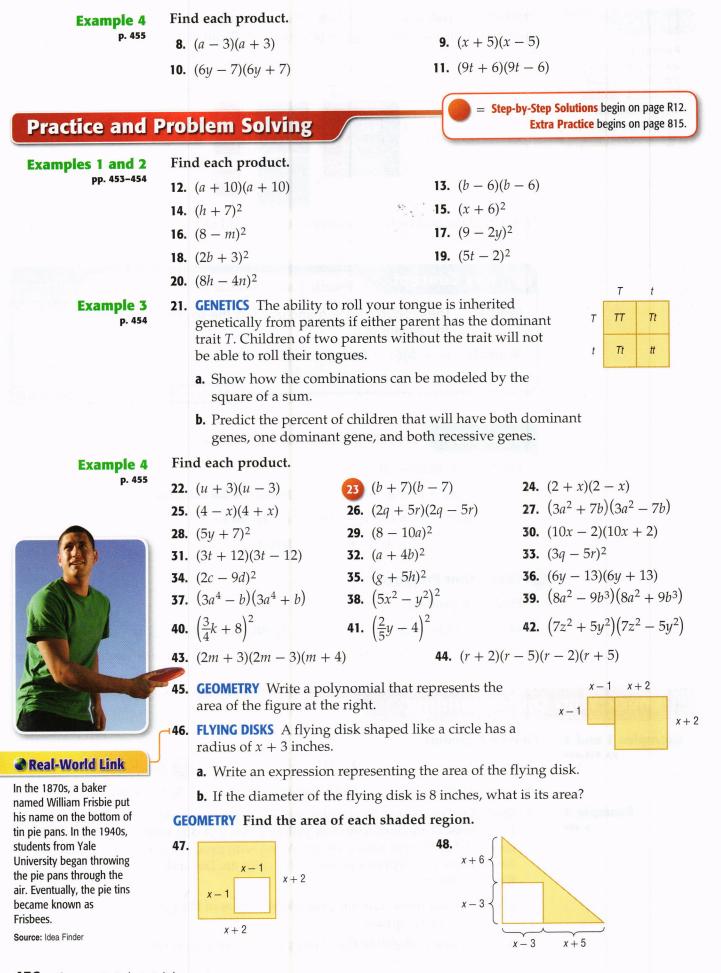
Example 3 p. 454

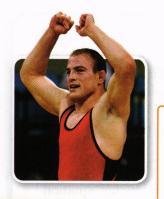
- 7. **GENETICS** The color of a Labrador retriever's fur is genetic. Dark genes *D* are dominant over yellow genes *y*. A dog with genes DD or Dy will have dark fur. A dog with genes yy will have yellow fur. Pepper's genes for fur color are Dy, and Ramiro's are yy.
 - **a**. Write an expression for the possible fur colors of Pepper's and Ramiro's puppies.

b. What is the probability that a puppy will have yellow fur?

D

V





Real-World Link

Cael Sanderson of Iowa State University is the only wrestler in NCAA Division I history to be undefeated for four years. He compiled a 159-0 record from 1999–2002.

Source: Team Sanderson

Find each product.

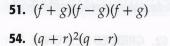
- **49.** (c + d)(c + d)(c + d) **50.** $(2a b)^3$
- **52.** (k m)(k + m)(k m) **53.** $(n p)^2(n + p)$
 - **WRESTLING** A high school wrestling mat must be a square with 38-foot sides and contain two circles as shown. Suppose the inner circle has a radius of *r* feet, and the radius of the outer circle is nine feet longer than the inner circle.
 - **a.** Write an expression for the area of the larger circle.
 - **b.** Write an expression for the area of the portion of the square outside the larger circle.

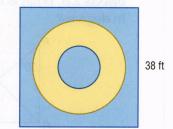
56. Solution Solu

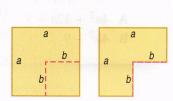
- a. NUMERICAL Find the area of each of the squares.
- **b. CONCRETE** Cut the smaller square out of the corner. What is the area of the shape?
- **c. ANALYTICAL** Remove the smaller rectangle on the bottom. Turn it and slide it next to the top rectangle. What is the length of the new arrangement? What is the width? What is the area?
- d. ANALYTICAL What pattern does this verify?
- H.O.T. Problems Use Higher-Order Thinking Skills
- 57. WHICH ONE DOESN'T BELONG? Which expression does not belong? Explain.

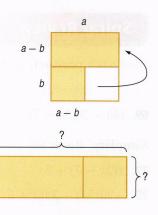
$$(2c-d)(2c-d)$$
 $(2c+d)(2c-d)$ $(2c+d)(2c+d)$ $(c+d)(c+d)$

- **58.** CHALLENGE Does a pattern exist for the cube of the sum $(a + b)^3$?
 - **a.** Investigate this question by finding the product (a + b)(a + b)(a + b).
 - **b.** Use the pattern you discovered in part *a* to find $(x + 2)^3$.
 - c. Draw a diagram of a geometric model for the cube of a sum.
 - **d**. What is the pattern for the cube of a difference, $(a b)^3$?
- **59. REASONING** Find *c* that makes $25x^2 90x + c$ a perfect square trinomial.
- **60. OPEN ENDED** Write two binomials with a product that is a binomial. Then write two binomials with a product that is not a binomial.
- **61.** WRITING IN MATH Describe how to square the sum of two quantities, square the difference of two quantities, and how to find the product of a sum of two quantities and a difference of two quantities.









Standardized Test Practice

62. GRIDDED RESPONSE In the right triangle, \overline{DB} bisects $\angle B$. What is the measure of $\angle ADB$ in degrees?

- **63.** What is the product of (2a 3) and (2a 3)?
 - **A** $4a^2 + 12a + 9$ C $4a^2 - 12a - 9$ **D** $4a^2 - 12a + 9$
 - **B** $4a^2 + 9$

64. Myron can drive 4 miles in *m* minutes. At this rate, how many minutes will it take him to drive 19 miles?

F	76m	H $\frac{4m}{19}$
G	$\frac{19m}{4}$	J $\frac{4}{19m}$

- 65. What property is illustrated by the equation 2x + 0 = 2x?
 - A Commutative Property of Addition
 - **B** Additive Inverse Property
 - C Additive Identity Property
 - D Associative Property of Addition

Spiral Review	Constant of the second states to any a set of the	
Find each product. (Lesson 7-	7)	
66. $(y-4)(y-2)$	67. $(2c-1)(c+3)$	68. $(d-9)(d+5)$
69. $(4h-3)(2h-7)$	70. $(3x + 5)(2x + 3)$	71. $(5m + 4)(8m + 3)$
Simplify. (Lesson 7-6)		
72. $x(2x-7) + 5x$	73. $c(c-8) + 2c(c+3)$	74. $8y(-3y+7) - 11y^2$
75. $-2d(5d) - 3d(d+6)$	76. $5m(2m^3 + m^2 + 8) + 4m$	77. $3p(6p-4) + 2\left(\frac{1}{2}p^2 - \frac{1}{2}p^2\right)$
Use substitution to solve e	ach system of equations. (Lesson 6-2)	
78. $4c = 3d + 3$	79. $c - 5d = 2$	80. $5r - t = 5$

81. BIOLOGY Each type of fish thrives in a specific range of temperatures. The best temperatures for sharks range from 18°C to 22°C, inclusive. Write a compound inequality to represent temperatures where sharks will not thrive. (Lesson 6-2)

2c + d = 4

Write an equation of the line that passes through each pair of points. (Lesson 4-2)

82. (1, 1), (7, 4)

c = d - 1

83. (5, 7), (0, 6)

84. (5, 1), (8, -2)

85. COFFEE A coffee store wants to create a mix using two coffees. How many pounds of coffee A should be mixed with 9 pounds of coffee B to get a mixture that can sell for \$6.95 per pound? (Lesson 2-9)



-4r + 5t = 17

3p

Skills Review

Find the prime factorization of each number. (Concepts and Skills Bank Lesson 3)

86. 40

87. 120

88. 900

89. 165

458 Chapter 7 Polynomials

Study Guide and Review

Chapter Summary

Key Concepts

For any nonzero real numbers *a* and *b* and any integers *m*, *n*, and *p*, the following are true.

Multiplying Monomials (Lesson 7-1)

- Product of Powers: $a^m \cdot a^n = a^{m+n}$
- Power of a Power: $(a^m)^n = a^m \cdot n$
- Power of a Product: $(ab)^m = a^m b^m$

Dividing Monomials (Lesson 7-2)

- Quotient of Powers: $\frac{a^m}{a^p} = a^{m-p}$
- Power of a Quotient: $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$
- Zero Exponent: $a^0 = 1$

• Negative Exponent:
$$a^{-n} = \frac{1}{a^n}$$
 and $\frac{1}{a^{-n}} = a^n$

Scientific Notation (Lesson 7-3)

- A number is in scientific notation if it is in the form $a \times 10^n$, where $1 \le a < 10$.
- To write in standard form:
 - If n > 0, move the decimal n places right.
 - If *n* < 0, move the decimal *n* places left.

Operations with Polynomials (Lessons 7-5 through 7-8)

- To add or subtract polynomials, add or subtract like terms. To multiply polynomials, use the Distributive Property.
- Special products: $(a + b)^2 = a^2 + 2ab + b^2$

$$(a - b)^2 = a^2 - 2ab + b^2$$

 $(a + b)(a - b) = a^2 - b^2$

FOLDABLES Study Organizer

Be sure the Key Concepts

are noted in your Foldable.



Key Vocabulary

binomial (p. 424) constant (p. 401) degree of a monomial (p. 424) degree of a polynomial (p. 424) FOIL method (p. 448) leading coefficient (p. 425) monomial (p. 401) order of magnitude (p. 411) polynomial (p. 424) quadratic expression (p. 448) scientific notation (p. 416) standard form of a polynomial (p. 425) trinomial (p. 424)

Vocabulary Check

Choose a term from the Key Vocabulary list above that best describes each expression or equation.

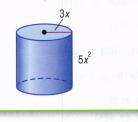
1. $x^{2} + 1$ 2. $5^{0} = 1$ 3. $x^{2} - 3x + 2$ 4. $(xy^{3})(x^{2}y^{4}) = x^{3}y^{7}$ 5. $(a^{7})^{3} = a^{21}$ 6. $5^{-2} = \frac{1}{5^{2}}$ 7. 6.2×10^{5} 8. $(x + 2)(x - 5) = x^{2} - 3x - 10$ 9. $x^{3} + 2x^{2} - 3x - 1$ 10. $7xy^{4}$

Lesson-by-Lesson Review



Simplify each expression. 11. $x \cdot x^3 \cdot x^5$ 12. $(2xy)(-3x^2y^5)$ 13. $(-4ab^4)(-5a^5b^2)$ 14. $(6x^3y^2)^2$

- **15.** $[(2r^3t)^3]^2$ **16.** $(-2u^3)(5u)$
- **17.** $(2x^2)^3(x^3)^3$ **18.** $\frac{1}{2}(2x^3)^3$
- **19. GEOMETRY** Use the formula $V = \pi r^2 h$ to find the volume of the cylinder.



EXAMPLE 1

Simplify $(5x^2y^3)(2x^4y)$. $(5x^2y^3)(2x^4y)$ $= (5 \cdot 2)(x^2 \cdot x^4)(y^3 \cdot y)$ $= 10x^6y^4$

Commutative Property Product of Powers

EXAMPLE 2

Simplify $(3a^2b^4)^3$. $(3a^2b^4)^3 = 3^3(a^2)^3(b^4)^3$ $= 27a^6b^{12}$

Power of a Product Simplify.

7-2 Dividing Monomials (pp. 408–415)

Simplify each expression. Assume that no denominator equals zero.

20.
$$\frac{(3x)^0}{2a}$$

21. $\left(\frac{3xy^3}{2z}\right)^3$
22. $\frac{12y^{-4}}{3y^{-5}}$
23. $a^{-3}b^0c^6$
24. $\frac{-15x^7y^8z^4}{-45x^3y^5z^3}$
25. $\frac{(3x^{-1})^{-2}}{(3x^2)^{-2}}$
26. $\left(\frac{6xy^{11}z^9}{48x^6yz^{-7}}\right)^0$
27. $\left(\frac{12}{2}\right)\left(\frac{x}{y^5}\right)\left(\frac{y^4}{x^4}\right)$

28. GEOMETRY The area of a rectangle is $25x^2y^4$ square feet. The width of the rectangle is 5xy feet. What is the length of the rectangle?



EXAMPLE 3

Simplify $\frac{2k^4m^3}{4k^2m}$. Assume that no denominator equals zero.

$$\frac{2k^4m^3}{4k^2m} = \left(\frac{2}{4}\right) \left(\frac{k^4}{k^2}\right) \left(\frac{m^3}{m}\right) \\ = \left(\frac{1}{2}\right) k^{4-2} m^{3-1} \\ = \frac{k^2m^2}{2}$$

Group powers with the same base.

Simplify.

EXAMPLE 4

Simplify $\frac{t^4uv^{-2}}{t^{-3}u^7}$. Assume that no denominator equals zero.

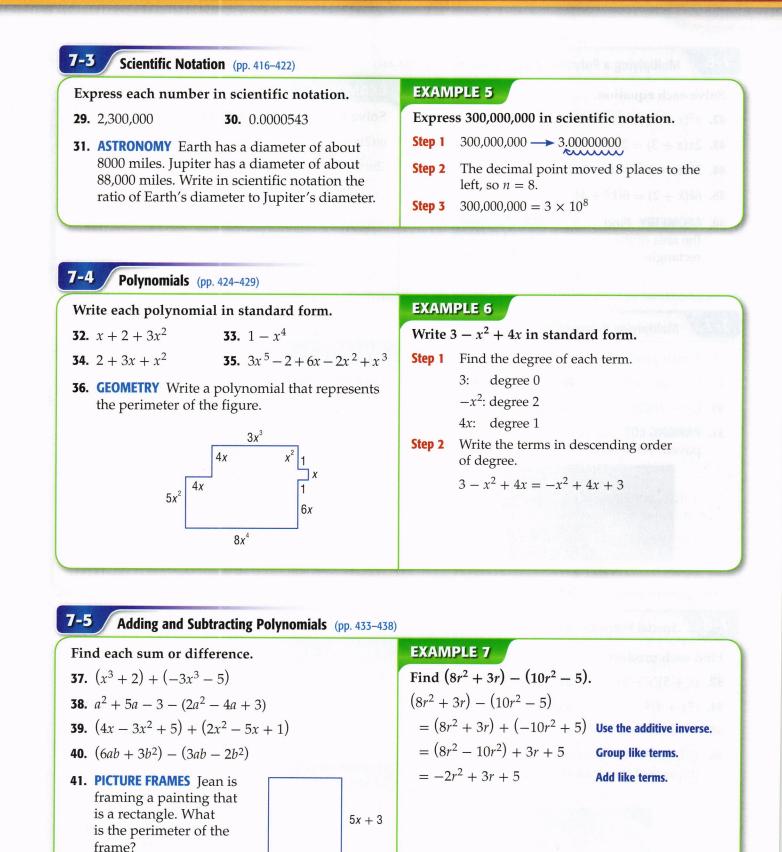
 $\frac{t^4 u v^{-2}}{t^{-3} u^7} = \left(\frac{t^4}{t^{-3}}\right) \left(\frac{u}{u^7}\right) (v^{-2})$ $= (t^4 + 3)(u^{1-7})(v^{-2})$ $= t^7 u^{-6} v^{-2}$ $= \frac{t^7}{u^6 v^2}$

Group the powers with the same base.

Quotient of Powers

Simplify.

Simplify.



 $2x^2 - 3x + 1$



Solve each equation.		EXAMPLE 8			
42. $x^2(x+2) = x(x^2+2x+1)$	Solve $m(2m - 5) + m = 2m(m - 6) + 16$.				
43. $2x(x+3) = 2(x^2+3)$	1.425	m(2m-5) + m = 2m(m-6) + 16			
44. $2(4w + w^2) - 6 = 2w(w - 4) + 10$	v 9352 -	$2m^2 - 5m + m = 2m^2 - 12m + 16$			
45. $6k(k+2) = 6(k^2+4)$		$2m^2 - 4m = 2m^2 - 12m + 16$			
46. GEOMETRY Find		-4m = -12m + 16			
the area of the	3 <i>x</i>	8m = 16			
rectangle. $x^2 + x - 7$		m = 2			
x + x - 7		Poistorials (p) starts			

7-7 Multiplying Polynomials (pp. 447–452)

Find	each	product.
TITTE	cucit	promotion

CHAPTER

47.	(x - 3)(x + 7)	48 .	(3a - 2)(6a + 5)

- **49.** (3r 7t)(2r + 5t) **50.** (2x + 5)(5x + 2)
- **51. PARKING LOT** The parking lot shown is to be paved. What is the area to be paved?



EXAMPLE 9 Find (6x - 5)(x + 4). (6x - 5)(x + 4)F 0 I L = (6x)(x) + (6x)(4) + (-5)(x) + (-5)(4) $= 6x^2 + 24x - 5x - 20$ Multiply. $= 6x^2 + 19x - 20$ Combine like terms.

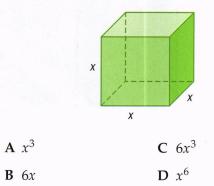
7-8 Special Products (pp. 453–458)

EXAMPLE 10
Find $(x - 7)^2$.
$(a - b)^2 = a^2 - 2ab + b^2$ Square of a Difference $(x - 7)^2 = x^2 - 2(x)(7) + (-7)^2$ $a = x$ and $b = 7$
$(x - 7)^{-2} = x^{-2} - 2(x)(7) + (-7)^{-3}$ is a range $= 1$ = $x^{2} - 14x + 49$ Simplify.
40 (bib + 369) - (3.16 - 267)
EXAMPLE 11
Find $(5a - 4)(5a + 4)$.
$(a + b)(a - b) = a^2 - b^2$ Product of a Sum and Difference
$(5a - 4)(5a + 4) = (5a)^2 - (4)^2$ $a = 5a$ and $b = 4$ = $25a^2 - 16$ Simplify.

Practice Test

Simplify each expression.

- 1. $(x^2)(7x^8)$
- **2.** $(5a^7bc^2)(-6a^2bc^5)$
- **3. MULTIPLE CHOICE** Express the volume of the solid as a monomial.



Simplify each expression. Assume that no denominator equals 0.

4.
$$\frac{x^{6}y^{8}}{x^{2}}$$

5. $\left(\frac{2a^{4}b^{3}}{c^{6}}\right)^{0}$
6. $\frac{2xy^{-7}}{8x}$

Express each number in scientific notation. (Lesson 7-3)

- **7.** 0.00021
- 8. 58,000

Express each number in standard form.

- **9.** 2.9×10^{-5}
- **10.** 9.1×10^6

Evaluate each product or quotient. Express the results in scientific notation.

11. $(2.5 \times 10^3)(3 \times 10^4)$

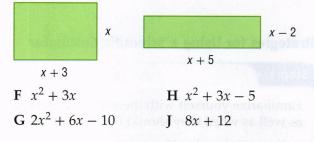
- 12. $\frac{8.8 \times 10^2}{4 \times 10^{-4}}$
- **13. ASTRONOMY** The average distance from Mercury to the Sun is 35,980,000 miles. Express this distance in scientific notation.

Find each sum or difference.

14.
$$(x + 5) + (x^2 - 3x + 7)$$

15. $(7m - 8n^2 + 3n) - (-2n^2 + 4m - 3n)$

16. MULTIPLE CHOICE Antonia is carpeting two of the rooms in her house. The dimensions are shown. What is the total area to be carpeted?



Find each product.

- **17.** $a(a^2 + 2a 10)$ **18.** (2a - 5)(3a + 5)
- **19.** $(x-3)(x^2+5x-6)$
- **20.** $(x + 3)^2$
- **21.** (2b-5)(2b+5)
- **22. GEOMETRY** A rectangular prism has dimensions x, x + 3, and 2x + 5.
 - **a**. Find the volume of the prism in terms of *x*.
 - **b.** Choose two values for *x*. How do the volumes compare?

Solve each equation.

- **23.** $5(t^2 3t + 2) = t(5t 2)$
- **24.** $3x(x+2) = 3(x^2-2)$
- **25. FINANCIAL LITERACY** Money invested in a certificate of deposit (CD) earns interest once per year. Suppose you invest \$4000 in a 2-year CD.
 - **a.** If the interest rate is 5% per year, the expression $4000(1 + 0.05)^2$ can be evaluated to find the total amount of money after two years. Explain the numbers in this expression.
 - **b**. Find the amount at the end of two years.
 - **c.** Suppose you invest \$10,000 in a CD for 4 years at an annual rate of 6.25%. What is the total amount of money you will have after 4 years?

CHAPTER

Preparing for Standardized Tests

Using a Scientific Calculator

Scientific calculators are powerful problem-solving tools. There are times when using a scientific calculator can be used to make computations faster and easier, such as computations with very large numbers. However, there are times when using a scientific calculator is necessary, like the estimation of irrational numbers.

Strategies for Using a Scientific Calculator

Step 1

Familiarize yourself with the various functions of a scientific calculator as well as when they should be used:

- Exponents scientific notation, calculating with large or small numbers
- Pi solving circle problems, like circumference and area
- Square roots distance on a coordinate plane, Pythagorean theorem
- **Graphs** analyzing paired data in a scatter plot, graphing functions, finding roots of equations

Step 2

Use your scientific or graphing calculator to solve the problem.

- Remember to work as efficiently as possible. Some steps may be done mentally or by hand, while others should be completed using your calculator.
- If time permits, check your answer.

EXAMPLE

Read the problem. Identify what you need to know. Then use the information in the problem to solve.

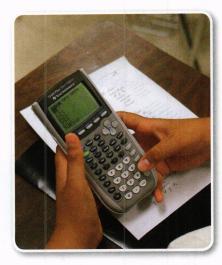
The distance from the Sun to Jupiter is approximately 7.786×10^{11} meters. If the speed of light is about 3×10^8 meters per second, how long does it take for light from the Sun to reach Jupiter? Round to the nearest minute.

A about 43 minutes

C about 1876 minutes

B about 51 minutes

D about 2595 minutes



Read the problem carefully. You are given the approximate distance from the Sun to Jupiter as well as the speed of light. Both quantities are given in scientific notation. You are asked to find how many minutes it takes for light from the Sun to reach Jupiter. Use the relationship distance = rate \times time to find the amount of time.

 $d = r \times t$

$$\frac{d}{r} = t$$

To find the amount of time, divide the distance by the rate. Notice, however, that the units for time will be seconds.

$$\frac{7.786 \times 10^{11} \text{ m}}{3 \times 10^8 \text{ m/s}} = t \text{ seconds}$$

Use a scientific calculator to quickly find the quotient. On most scientific calculators, the EE key is used to enter numbers in scientific notation.

KEYSTROKES: (7.786 2nd [EE] 11) / (3 2nd [EE] 8)

The result is 2595.333333333 seconds. To convert this number to minutes, use your calculator to divide the result by 60. This gives an answer of about 43.2555 minutes. The answer is A.

Exercises

Read each problem. Identify what you need to know. Then use the information in the problem to solve.

- 1. Since its creation 5 years ago, approximately 2.504×10^7 items have been sold or traded on a popular online website. What is the average daily number of items sold or traded over the 5-year period?
 - A about 9640 items per day
 - **B** about 13,720 items per day
 - C about 1,025,000 items per day
 - D about 5,008,000 items per day
- **2.** Evaluate \sqrt{ab} if a = 121 and b = 23.
 - F about 5.26
 - **G** about 9.90
 - H about 12
 - J about 52.75

- **3.** The population of the United States is about 3.034×10^8 people. The land area of the country is about 3.54×10^6 square miles. What is the average *population density* (number of people per square mile) of the United States?
 - A about 136.3 people per square mile
 - **B** about 112.5 people per square mile
 - **C** about 94.3 people per square mile
 - D about 85.7 people per square mile
- **4.** Eleece is making a cover for the marching band's bass drum. The drum has a diameter of 20 inches. Estimate the area of the face of the bass drum.
 - **F** 31.41 square inches
 - G 62.83 square inches
 - H 78.54 square inches
 - J 314.16 square inches

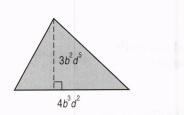
CHAPTER Standardized Test Practice

Cumulative, Chapters 1 through 7

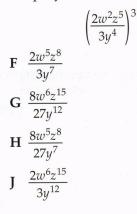
Multiple Choice

Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.

1. Express the area of the triangle below as a monomial.



- **A** $12b^{5}d^{7}$
- **B** 12b⁶d¹⁰
- **C** $6b^{6}d^{10}$
- $\mathbf{D} 6b^5d^7$
- **2.** Simplify the following expression.



3. Which equation of a line is perpendicular to $y = \frac{3}{5}x - 3?$

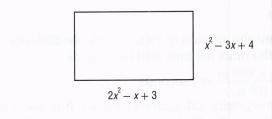
A $y = -\frac{5}{3}x + 2$ **B** $y = -\frac{3}{5}x + 2$ **C** $y = \frac{5}{3}x - 2$ **D** $y = \frac{3}{5}x - 2$

Test-TakingTip

Question 2 Use the laws of exponents to simplify the expression. Remember, to find the power of a power, multiply the exponents.

466 Chapter 7 Polynomials

4. Express the perimeter of the rectangle below as a polynomial.



F $3x^2 - 4x + 7$

G
$$3x^2 + x + 7$$

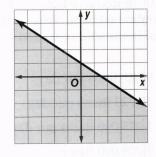
H
$$6x^2 - 8x + 14$$

- J $6x^2 4x + 7$
- **5.** Subtract the polynomials below.

$$(7a^2 + 6a - 2) - (-4a^3 + 3a^2 + 5)$$

A $4a^3 + 4a^2 + 6a - 7$

- **B** $11a^2 + 3a 7$
- **C** $4a^3 + 10a^2 + 6a + 3$
- **D** $4a^3 + 7a^3 3a$
- 6. Which inequality is shown in the graph?



$$F \quad y \le -\frac{2}{3}x - 1$$

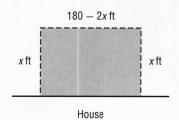
$$G \quad y \le -\frac{3}{4}x - 1$$

$$H \quad y \le -\frac{2}{3}x + 1$$

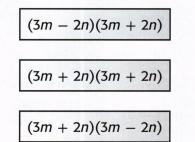
$$J \quad y \le -\frac{3}{4}x + 1$$

Short Response/Gridded Response

7. Mickey has 180 feet of fencing that she wants to use to enclose a play area for her puppy. She will use her house as one of the sides of the region.



- **a.** If she makes the play area *x* feet deep as shown in the figure, write a polynomial in standard form to represent the area of the region.
- **b.** How many square feet of area will the puppy have to play in if Mickey makes it 40 feet deep?
- **8.** Identify the expression below that does not belong with the other two. Explain.



9. What is the solution to the following system of equations? Show your work.

$$\begin{cases} y = 6x - 1 \\ y = 6x + 4 \end{cases}$$

10. GRIDDED RESPONSE At a family fun center, the Wilson and Sanchez families each bought video game tokens and batting cage tokens as shown in the table.

Family	Wilson	Sanchez		
Number of Video Game Tokens	25	30		
Number of Batting Cage Tokens	8	6		
Total Cost	\$26.50	\$25.50		

What is the cost in dollars of a batting cage token at the family fun center?

Extended Response

Record your answers on a sheet of paper. Show your work.

11. The table below shows the distances from the Sun to Mercury, Earth, Mars, and Saturn. Use the data to answer each question.

Planet	Distance from Sun (km)				
Mercury	5.79 × 10 ⁷				
Earth	1.50 × 10 ⁸				
Mars	2.28×10^{8}				
Saturn	1.43×10^{9}				

- **a.** Of the planets listed, which one is the closest to the Sun?
- **b.** About how many times as far from the Sun is Mars as Earth?

Need Extra Help?											
If you missed Question	1	2	3	4	5	6	7	8	9	10	11
Go to Lesson or Page	7-1	7-2	4-4	7-5	7-5	5-6	7-6	7-8	6-1	6-4	3-5

CHAPTER

Factoring and Quadratic Equations

Then

In Chapter 7, you multiplied monomials and polynomials.

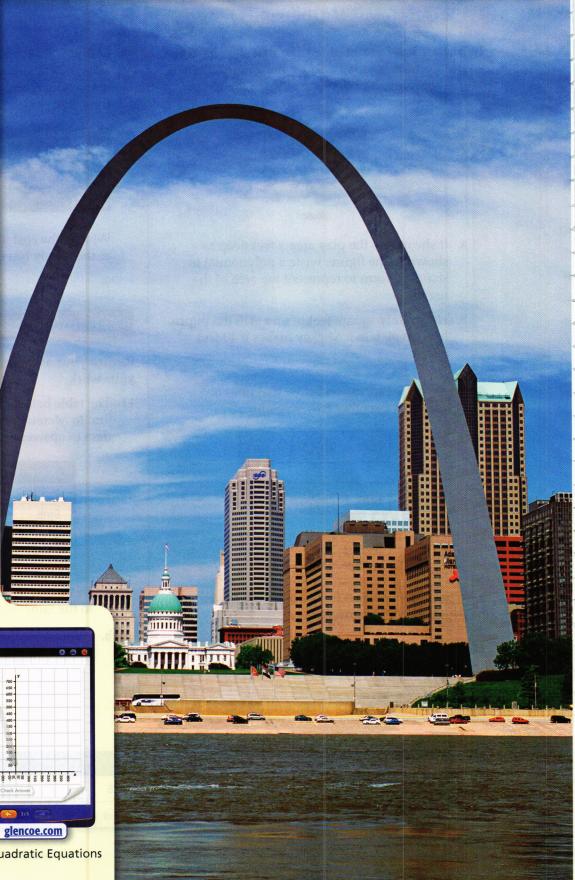
Now

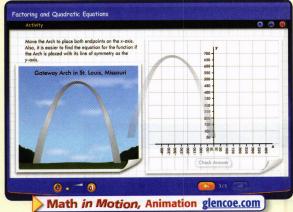
In Chapter 8, you will:

- Factor monomials.
- Factor trinomials.
- Factor differences of squares.
- Solve quadratic equations.

Why?

ARCHITECTURE Quadratic equations can be used to model the shape of architectural structures such as the tallest memorial in the United States, the Gateway Arch in St. Louis, Missouri.





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