

# Get Ready for Chapter 8

**Diagnose Readiness** You have two options for checking Prerequisite Skills.

## Text Option

Take the Quick Check below. Refer to the Quick Review for help.

### QuickCheck

Rewrite each expression using the Distributive Property. Then simplify.

(Lesson 1-4)

1.  $a(a + 5)$
2.  $2(3 + x)$
3.  $n(n - 3n^2 + 2)$
4.  $-6(x^2 - 5x + 6)$
5. **FINANCIAL LITERACY** Five friends will pay \$9 per ticket, \$3 per drink, and \$6 per popcorn at the movies. Write an expression that could be used to determine the cost for them to go to the movies.

Find each product. (Lesson 7-7)

6.  $(x + 2)(x - 5)$
7.  $(x + 4)(x - 1)$
8.  $(2a - 3)(5a + 4)$
9.  $(3x - 4)(x + 5)$
10.  $(x + 4)(x + 7)$
11.  $(6a - 2b)(9a + b)$
12. **TABLECLOTH** The dimensions of a tablecloth are represented by a width of  $2x + 3$  and a length of  $x + 1$ . Find an expression for the area of the tablecloth.

Find each product. (Lesson 7-8)

13.  $(3 - a)^2$
14.  $(x + 5)^2$
15.  $(3x - 2y)^2$
16.  $(2x + 5y)(2x - 5y)$
17. **PHOTOGRAPHY** A photo is  $x + 6$  inches by  $x - 6$  inches. What is the area of the photo?

### QuickReview

#### EXAMPLE 1

Rewrite  $6x(-3x - 5x - 5x^2 + x^3)$  using the Distributive Property. Then simplify.

$$\begin{aligned} 6x(-3x - 5x - 5x^2 + x^3) &= 6x(-3x) + 6x(-5x) + 6x(-5x^2) + 6x(x^3) \\ &= -18x^2 - 30x^2 - 30x^3 + 6x^4 \\ &= -48x^2 - 30x^3 + 6x^4 \end{aligned}$$

#### EXAMPLE 2

Find  $(x + 3)(2x - 1)$ .

$(x + 3)(2x - 1)$	<b>Original expression</b>
$= x(2x) + x(-1) + 3(2x) + 3(-1)$	<b>FOIL method</b>
$= 2x^2 - x + 6x - 3$	<b>Multiply.</b>
$= 2x^2 + 5x - 3$	<b>Combine like terms.</b>

#### EXAMPLE 3

Find  $(y + 8)^2$ .

$(a + b)^2 = a^2 + 2ab + b^2$	<b>Square of a sum</b>
$(y + 8)^2 = (y)^2 + 2(y)(8) + 8^2$	<b><math>a = y, b = 8</math></b>
$= y^2 + 16y + 64$	<b>Simplify.</b>

## Online Option

### Math Online

Take a self-check Chapter Readiness Quiz at [glencoe.com](http://glencoe.com).



# Get Started on Chapter 8

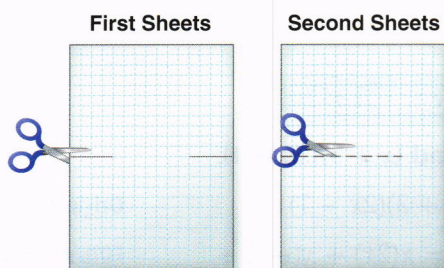
You will learn several new concepts, skills, and vocabulary terms as you study Chapter 8. To get ready, identify important terms and organize your resources. You may wish to refer to **Chapter 0** to review prerequisite skills.

## FOLDABLES®

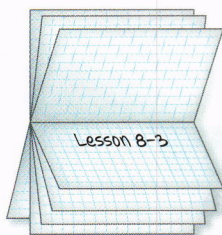
### Study Organizer

**Factoring and Quadratic Equations** Make this Foldable to help you organize your Chapter 8 notes about factoring and quadratic equations. Begin with four sheets of grid paper.

- 1 Fold** in half along the width. On the first two sheets, cut 5 centimeters along the fold at the ends. On the second two sheets cut in the center, stopping 5 centimeters from the ends.



- 2 Insert** the first sheets through the second sheets and align the folds. Label the front Chapter 8, Factoring and Quadratic Equations. Label the pages with lesson numbers and the last page with vocabulary.



## New Vocabulary

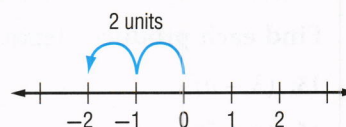
### English

### Español

- |  |                                       |
|--|---------------------------------------|
| <b>factored form</b> • p. 471                | <b>forma reducida</b>                 |
| <b>greatest common factor (GCF)</b> • p. 471 | <b>máximo común divisor (MCD)</b>     |
| <b>factoring</b> • p. 476                    | <b>factorización</b>                  |
| <b>factoring by grouping</b> • p. 477        | <b>factorización por agrupamiento</b> |
| <b>Zero Product Property</b> • p. 478        | <b>propiedad del producto de cero</b> |
| <b>quadratic equation</b> • p. 488           | <b>ecuación cuadrática</b>            |
| <b>prime polynomial</b> • p. 495             | <b>polinomio primo</b>                |
| <b>difference of two squares</b> • p. 499    | <b>diferencia de cuadrados</b>        |
| <b>perfect square trinomial</b> • p. 505     | <b>trinomio cuadrado perfecto</b>     |
| <b>Square Root Property</b> • p. 508         | <b>Propeidad de la raíz cuadrada</b>  |

## Review Vocabulary

**absolute value** • p. P11 • **valor absoluto** the absolute value of any number  $n$  is the distance the number is from zero on a number line and is written  $|n|$



The absolute value of  $-2$  is 2 because it is 2 units from 0.

**perfect square** • p. P7 • **cuadrado perfecto** a number with a square root that is a rational number

**prime number** • p. 861 • **numero primo** a whole number, greater than 1, with the only factor being 1 and itself

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- Get extra help from your own **Personal Tutor**
- Use **Extra Examples** for additional help
- Take a **Self-Check Quiz**
- Review Vocabulary** in fun ways

**Multilingual eGlossary** [glencoe.com](http://glencoe.com)



# 8-1

## Monomials and Factoring

### Then

You multiplied monomials and divided a polynomial by a monomial.

(Lesson 7-1 and 7-2)

### Now

- Factor monomials.
- Find the greatest common factors of monomials.

### New Vocabulary

factored form

greatest common factor (GCF)

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### Why?

Susie is making beaded bracelets for extra money. She has 60 gemstone beads and 15 glass beads. She wants each bracelet to have only one type of bead and all of the bracelets to have the same number of beads. Susie needs to determine the *greatest common factor* of 60 and 15.



**Factor Monomials** Factoring a monomial is similar to factoring a whole number. A monomial is in **factored form** when it is expressed as the product of prime numbers and variables, and no variable has an exponent greater than 1.

### EXAMPLE 1 Monomial in Factored Form

Factor  $20x^3y^2$  completely.

$$20x^3y^2 = 1 \cdot 20x^3y^2$$

$$= 1 \cdot 2 \cdot 10 \cdot x \cdot x \cdot x \cdot y \cdot y$$

$$= 1 \cdot 2 \cdot 2 \cdot 5 \cdot x \cdot x \cdot x \cdot y \cdot y$$

Express 20 as  $1 \cdot 20$ .

$$20 = 2 \cdot 10, x^3 = x \cdot x \cdot x, \text{ and } y^2 = y \cdot y$$

$$10 = 2 \cdot 5$$

Thus,  $20x^3y^2$  in factored form is  $1 \cdot 2 \cdot 2 \cdot 5 \cdot x \cdot x \cdot x \cdot y \cdot y$ .

### Check Your Progress

Factor each monomial completely.

1A.  $34x^4y^3$

1B.  $-52a^2b$

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**Greatest Common Factor** Two or more whole numbers may have some common prime factors. The product of the common prime factors is called their greatest common factor. The **greatest common factor (GCF)** is the greatest number that is a factor of both original numbers. The GCF of two or more monomials can be found in a similar way.

### EXAMPLE 2 GCF of a Set of Monomials

Find the GCF of  $12a^2b^2c$  and  $18ab^3$ .

$$12a^2b^2c = \overset{\circ}{2} \cdot \overset{\circ}{2} \cdot \overset{\circ}{3} \cdot \overset{\circ}{a} \cdot \overset{\circ}{a} \cdot \overset{\circ}{b} \cdot \overset{\circ}{b} \cdot c$$

$$18ab^3 = \overset{\circ}{2} \cdot \overset{\circ}{3} \cdot \overset{\circ}{3} \cdot \overset{\circ}{a} \cdot \overset{\circ}{b} \cdot \overset{\circ}{b} \cdot \overset{\circ}{b}$$

Factor each number, and write all powers of variables as products.

Circle the common prime factors.

The GCF of  $12a^2b^2c$  and  $18ab^3$  is  $2 \cdot 3 \cdot a \cdot b \cdot b$  or  $6ab^2$ .

### Check Your Progress

Find the GCF of each pair of monomials.

2A.  $6xy^3, 18yz$

2B.  $11a^2b, 21ab^2$

2C.  $30q^3r^2t, 50q^2rt$

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### Real-World EXAMPLE 3 Find a GCF

**FLOWERS** A florist has 20 roses and 30 tulips to make bouquets. What is the greatest number of identical bouquets she can make without having any flowers left over? How many of each kind of flower will be in each bouquet?

Find the GCF of 20 and 30.

$$20 = 2^2 \cdot 5 \quad \text{Write the prime factorization of each number.}$$

$$30 = 2 \cdot 3 \cdot 5 \quad \text{The common prime factors are 2 and 5 or 10.}$$

The GCF of 20 and 30 is 10. So, the florist can make 10 bouquets. Since  $2 \times 10 = 20$  and  $3 \times 10 = 30$ , each bouquet will have 2 roses and 3 tulips.

**Math in Motion,**  
Animation [glencoe.com](http://glencoe.com)

### Check Your Progress

3. What is the greatest possible value for the widths of two rectangles if their areas are 84 square inches and 70 square inches, respectively, and the length and width are whole numbers?

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### Check Your Understanding

#### Example 1 p. 471

Factor each monomial completely.

1.  $12g^2h^4$
2.  $-38rp^2t^2$
3.  $-17x^3y^2z$
4.  $23ab^3$

#### Examples 2 and 3 pp. 471–472

Find the GCF of each pair of monomials.

5.  $24cd^3, 48c^2d$
6.  $7gh, 11mp$
7.  $8x^2y^5, 31xy^3$
8.  $10ab, 25a$

9. **GEOMETRY** The areas of two rectangles are 15 square inches and 16 square inches. The length and width of both figures are whole numbers. If the rectangles have the same width, what is the greatest possible value for their widths?

### Practice and Problem Solving

**Step-by-Step Solutions** begin on page R12.  
**Extra Practice** begins on page 815.

#### Example 1 p. 471

Factor each monomial completely.

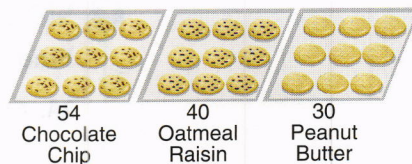
10.  $95xy^2$
11.  $-35a^3c^2$
12.  $42g^3h^3$
13.  $81n^5p$
14.  $-100q^4r$
15.  $121abc^3$

#### Examples 2 and 3 pp. 471–472

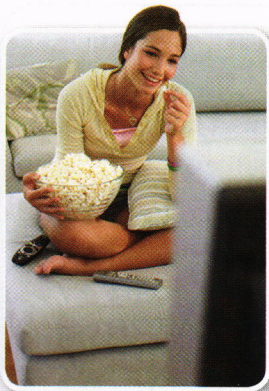
Find the GCF of each set of monomials.

16.  $25x^3, 45x^4, 65x^2$
17.  $26z^2, 32z, 44z^4$
18.  $30gh^2, 42g^2h, 66g$
19.  $12qr, 8r^2, 16rt$
20.  $42a^2b, 6a^2, 18a^3$
21.  $15r^2t, 35t^2, 70rt$

22. **BAKING** Delsin wants to package the same number of cookies in each bag, and each bag should have every type of cookie. If he puts the greatest possible number of cookies in each bag, how many bags can he make?





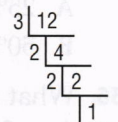


### Real-World Link

About 77% of 18- to 25-year-olds say their favorite way to watch a movie at home is watching a DVD or video, while only 17% say they watch movies that are on television.

Source: 2006 Gen Next Survey

23. **GEOMETRY** The area of a triangle is 28 square inches. What are possible whole-number dimensions for the base and height of the triangle?
24. **MUSIC** In what ways can Clara organize her 36 CDs so that she has the same number of CDs on each shelf, at least 4 per shelf, and at least 2 shelves of CDs?
25. **MOVIES** In what ways can Shannon arrange her 80 DVDs so that she has at least 4 shelves of DVDs, the same number on each shelf, and at least 5 on each shelf?
26. **VOLUNTEER** Denzell is donating packages of school supplies to an elementary school where he volunteers. He bought 200 pencils, 150 glue sticks, and 120 folders. How many packages can Denzell make using an equal number of each item? How many items of each type will each package contain?
27. **NUMBER THEORY** *Twin primes* are two consecutive odd numbers that are prime. The first two pairs of twin primes are 3 and 5 and 5 and 7. List the next five pairs.
28. **MULTIPLE REPRESENTATIONS** In this problem, you will investigate a method of factoring a number.
  - a. **ANALYTICAL** Copy the ladder diagram shown at the right six times and record six whole numbers, two of which are prime, in the top right portion of the diagrams.
  - b. **ANALYTICAL** Choose a prime factor of one of your numbers. Record the factor on the left of the number in the diagram. Divide the two numbers. Keep dividing by prime factors until the quotient is 1. Add to or subtract boxes from the diagram as necessary. Repeat this process with all of your numbers.
  - c. **VERBAL** What is the prime factorization of your six numbers?



So, the prime factorization of 12 is  $2^2 \cdot 3$ .

## H.O.T. Problems

Use **H**igher-**O**rders **T**hinking Skills

29. **CHALLENGE** Find the least pair of numbers that satisfies the following conditions. The GCF of the numbers is 11. One number is even and the other number is odd. One number is not a multiple of the other.
30. **REASONING** The *least common multiple* (LCM) of two or more numbers is the least number that is a multiple of each number. Compare and contrast the GCF and LCM of two or more numbers.
31. **REASONING** Determine whether the following statement is *true* or *false*. Provide an example or counterexample.  
*Two monomials always have a greatest common factor that is not equal to 1.*
32. **CHALLENGE** Two or more integers or monomials with a GCF of 1 are said to be *relatively prime*. Copy and complete the chart to determine which pairs of monomials are relatively prime.
33. **OPEN ENDED** Name three monomials with a GCF of  $6y^3$ . Explain your answer.
34. **WRITING IN MATH** Define *prime factorization* in your own words. Explain how to find the prime factorization of a monomial, and how a prime factorization helps you determine the GCF of two or more monomials.

Monomial	Prime Factorization
$15a^2bc^3$	
$6b^3c^3d$	
$12cd^2f$	
$22d^3fg^2$	
$30f^2gh^2$	



## Standardized Test Practice

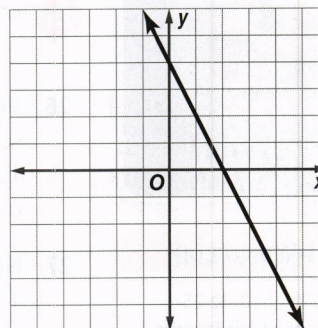
35. Abigail surveyed 320 of her classmates about what type of movie they prefer. The results of the survey are shown below. What percent of her classmates enjoyed action movies?

Type of Movie	Number of Responses
comedy	160
drama	25
science fiction	55
action	80

- A 25%                      C 75%  
 B 50%                      D 95%
36. What is the value of  $c$  in the equation  $4c - 27 = 19 + 2c$ ?
- F -4  
 G 4  
 H 23  
 J 46

37. Which equation best represents a line parallel to the line shown below?

- A  $y = 2x + 4$   
 B  $y = -2x - 5$   
 C  $y = \frac{1}{2}x - 6$   
 D  $y = -\frac{1}{2}x + 3$



38. **SHORT RESPONSE** The table shows a five-day forecast indicating high (H) and low (L) temperatures. Organize the temperatures in a matrix.

	Mon	Tue	Wed	Thurs	Fri
H	92	87	85	88	90
L	68	64	62	65	66

## Spiral Review

Find each product. (Lesson 7-8)

39.  $(a - 4)^2$                       40.  $(c + 6)^2$                       41.  $(z - 5)^2$   
 42.  $(n - 3)(n + 3)$                       43.  $(y + 2)^2$                       44.  $(d - 7)(d + 7)$

Find each product. (Lesson 7-7)

45.  $(2m - 3)(m + 4)$                       46.  $(h - 2)(3h - 5)$                       47.  $(t + 2)(t + 9)$   
 48.  $(8r - 1)(r - 6)$                       49.  $(p + 3q)(p + 3q)$                       50.  $(n - 4)(n + 2)(n + 1)$

Write an augmented matrix to solve each system of equations. (Lesson 6-7)

51.  $y = 2x + 3$                       52.  $8x + 2y = 13$                       53.  $-x + \frac{1}{3}y = 5$   
 $y = 4x - 1$                        $4x + y = 11$                        $2x + 3y = 1$

54. **FINANCIAL LITERACY** Suppose you have already saved \$50 toward the cost of a new television. You plan to save \$5 more each week. Write and graph an equation for the total amount  $T$  that you will have  $w$  weeks from now. (Lesson 4-1)

## Skills Review

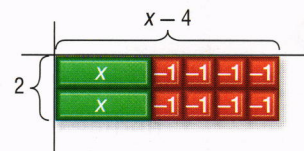
Use the Distributive Property to rewrite each expression. (Lesson 1-4)

55.  $2(4x - 7)$                       56.  $\frac{1}{2}d(2d + 6)$                       57.  $-h(6h - 1)$   
 58.  $9m - 9p$                       59.  $5y - 10$                       60.  $3z - 6x$

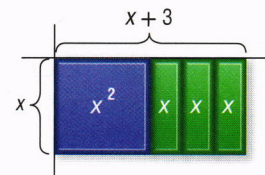


Factoring Using  
the Distributive Property

When two or more numbers are multiplied, these numbers are *factors* of the product. Sometimes you know the product of binomials and are asked to find the factors. This is called factoring. You can use algebra tiles and a product mat to factor binomials.

**ACTIVITY 1** Use algebra tiles to factor  $2x - 8$ .**Step 1** Model  $2x - 8$ .**Step 2** Arrange the tiles into a rectangle. The total area of the rectangle represents the product, and its length and width represent the factors.

The rectangle has a width of 2 and a length of  $x - 4$ . Therefore,  $2x - 8 = 2(x - 4)$ .

**ACTIVITY 2** Use algebra tiles to factor  $x^2 + 3x$ .**Step 1** Model  $x^2 + 3x$ .**Step 2** Arrange the tiles into a rectangle.

The rectangle has a width of  $x$  and a length of  $x + 3$ . Therefore,  $x^2 + 3x = x(x + 3)$ .

**Model and Analyze**

Use algebra tiles to factor each binomial.

1.  $4x + 12$

2.  $4x - 6$

3.  $3x^2 + 4x$

4.  $10 - 2x$

Determine whether each binomial can be factored. Justify your answer with a drawing.

5.  $6x - 9$

6.  $5x - 4$

7.  $4x^2 + 7$

8.  $x^2 + 3x$

9. **WRITING IN MATH** Write a paragraph that explains how you can use algebra tiles to determine whether a binomial can be factored. Include an example of one binomial that can be factored and one that cannot.



## Using the Distributive Property

**Then**

You found the GCF of a set of monomials. (Lesson 8-1)

**Now**

- Use the Distributive Property to factor polynomials.
- Solve equations of the form  $ax^2 + bx = 0$ .

**New Vocabulary**

factoring

factoring by grouping

Zero Product Property

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**Why?**

The cost of rent for Ms. Cole's store is determined by the square footage of the space. The area of the store can be modeled by the equation  $A = 1.6w^2 + 6w$ , where  $w$  is the width of the store in feet. We can use factoring and the Zero Product Property to find possible dimensions of the store.

**Use the Distributive Property to Factor** In Chapter 7, the Distributive Property was used to multiply a monomial by a polynomial.

$$5z(4z + 7) = 5z(4z) + 5z(7) \\ = 20z^2 + 35z$$

You can work backward to express a polynomial as a product of a monomial factor and a polynomial factor.

$$1.6w^2 + 6w = 1.6w(w) + 6(w) \\ = w(1.6w + 6)$$

So,  $5z(4z + 7)$  is the *factored form* of  $20z^2 + 35z$ . **Factoring** a polynomial involves finding the *completely factored form*.

**EXAMPLE 1 Use the Distributive Property**

Use the Distributive Property to factor each polynomial.

a.  $27y^2 + 18y$

Find the GCF of each term.

$$27y^2 = 3 \cdot 3 \cdot 3 \cdot y \cdot y$$

$$18y = 2 \cdot 3 \cdot 3 \cdot y$$

$$\text{GCF} = 3 \cdot 3 \cdot y \text{ or } 9y$$

Write each term as the product of the GCF and the remaining factors. Use the Distributive Property to *factor out* the GCF.

$$27y^2 + 18y = 9y(3y) + 9y(2) \\ = 9y(3y + 2)$$

Factor each term.

Circle common factors.

Rewrite each term using the GCF. Distributive Property

b.  $-4a^2b - 8ab^2 + 2ab$

$$-4a^2b = -1 \cdot 2 \cdot 2 \cdot a \cdot a \cdot b$$

$$-8ab^2 = -1 \cdot 2 \cdot 2 \cdot 2 \cdot a \cdot b \cdot b$$

$$2ab = 2 \cdot a \cdot b$$

$$\text{GCF} = 2 \cdot a \cdot b \text{ or } 2ab$$

$$-4a^2b - 8ab^2 + 2ab = 2ab(-2a) - 2ab(4b) + 2ab(1) \\ = 2ab(-2a - 4b + 1)$$

Factor each term.

Circle common factors.

Rewrite each term using the GCF. Distributive Property

**Check Your Progress**

1A.  $15w - 3v$

1B.  $7u^2t^2 + 21ut^2 - ut$



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Using the Distributive Property to factor polynomials with four or more terms is called **factoring by grouping** because terms are put into groups and then factored. The Distributive Property is then applied to a common binomial factor.



## Key Concept

## Factoring by Grouping

For Your  
FOLDABLE

- Words** A polynomial can be factored by grouping only if all of the following conditions exist.
- There are four or more terms.
  - Terms have common factors that can be grouped together.
  - There are two common factors that are identical or additive inverses of each other.

**Symbols**  $ax + bx + ay + by = (ax + bx) + (ay + by)$   
 $= x(a + b) + y(a + b)$   
 $= (x + y)(a + b)$

### EXAMPLE 2 Factor by Grouping

Factor  $4qr + 8r + 3q + 6$ .

$$\begin{aligned} 4qr + 8r + 3q + 6 \\ &= (4qr + 8r) + (3q + 6) \\ &= 4r(q + 2) + 3(q + 2) \end{aligned}$$

Original expression

Group terms with common factors.

Factor the GCF from each group.

Notice that  $(q + 2)$  is common in both groups, so it becomes the GCF.

$$= (4r + 3)(q + 2)$$

Distributive Property



### Check Your Progress

Factor each polynomial.

2A.  $rn + 5n - r - 5$

2B.  $3np + 15p - 4n - 20$

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It can be helpful to recognize when binomials are additive inverses of each other. For example  $6 - a = -1(a - 6)$ .

### StudyTip

**Check** To check your factored answers, multiply your factors out. You should get your original expression as a result.

### EXAMPLE 3 Factor by Grouping with Additive Inverses

Factor  $2mk - 12m + 42 - 7k$ .

$$\begin{aligned} 2mk - 12m + 42 - 7k \\ &= (2mk - 12m) + (42 - 7k) \\ &= 2m(k - 6) + 7(6 - k) \\ &= 2m(k - 6) + 7[(-1)(k - 6)] \\ &= 2m(k - 6) - 7(k - 6) \\ &= (2m - 7)(k - 6) \end{aligned}$$

Group terms with common factors.

Factor the GCF from each group.

$$6 - k = -1(k - 6)$$

Associative Property

Distributive Property



### Check Your Progress

Factor each polynomial.

3A.  $c - 2cd + 8d - 4$

3B.  $3p - 2p^2 - 18p + 27$

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**Solve Equations by Factoring** Some equations can be solved by factoring. Consider the following.

$$3(0) = 0$$

$$0(2 - 2) = 0$$

$$-312(0) = 0$$

$$0(0.25) = 0$$

Notice that in each case, at least one of the factors is 0. These examples are demonstrations of the **Zero Product Property**.



### Key Concept

### Zero Product Property

For Your  
**FOLDABLE**

#### Words

If the product of two factors is 0, then at least one of the factors must be 0.

#### Symbols

For any real numbers  $a$  and  $b$ , if  $ab = 0$ , then  $a = 0$ ,  $b = 0$ , or both  $a$  and  $b$  equal zero.

Recall from Lesson 3-2 that a solution or root of an equation is any value that makes the equation true.

### Watch Out!

#### Unknown Value

It may be tempting to solve an equation by dividing each side by the variable. However, the variable has an unknown value, so you may be dividing by 0, which is undefined.

### EXAMPLE 4 Solve Equations

Solve each equation. Check your solutions.

a.  $(2d + 6)(3d - 15) = 0$

$$(2d + 6)(3d - 15) = 0$$

Original equation

$$2d + 6 = 0 \quad \text{or} \quad 3d - 15 = 0$$

Zero Product Property

$$2d = -6$$

$$3d = 15$$

Solve each equation.

$$d = -3$$

$$d = 5$$

Divide.

The roots are  $-3$  and  $5$ .

**CHECK** Substitute  $-3$  and  $5$  for  $d$  in the original equation.

$$(2d + 6)(3d - 15) = 0$$

$$(2d + 6)(3d - 15) = 0$$

$$[2(-3) + 6][3(-3) - 15] \stackrel{?}{=} 0$$

$$[2(5) + 6][3(5) - 15] \stackrel{?}{=} 0$$

$$(-6 + 6)(-9 - 15) \stackrel{?}{=} 0$$

$$(10 + 6)(15 - 15) \stackrel{?}{=} 0$$

$$(0)(-24) \stackrel{?}{=} 0$$

$$16(0) \stackrel{?}{=} 0$$

$$0 = 0 \quad \checkmark$$

$$0 = 0 \quad \checkmark$$

b.  $c^2 = 3c$

$$c^2 = 3c$$

Original equation

$$c^2 - 3c = 0$$

Subtract  $3c$  from each side to get 0 on one side of the equation.

$$c(c - 3) = 0$$

Factor by using the GCF to get the form  $ab = 0$ .

$$c = 0 \quad \text{or} \quad c - 3 = 0$$

Zero Product Property

$$c = 3$$

Solve each equation.

The roots are 0 and 3.

Check by substituting 0 and 3 for  $c$ .



### Check Your Progress

4A.  $3n(n + 2) = 0$

4B.  $8b^2 - 40b = 0$

4C.  $x^2 = -10x$

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### Real-World Link

Dog agility tests a person's skills as a trainer and handler. Competitors race through an obstacle course that includes hurdles, tunnels, a see-saw, and line poles.

Source: United States Dog Agility Association

## Real-World EXAMPLE 5 Use Factoring

**AGILITY** Penny is a Labrador Retriever who competes with her trainer in the agility course. Within the course, Penny must leap over a hurdle. Penny's jump can be modeled by the equation  $h = -16t^2 + 20t$ , where  $h$  is the height of the leap in inches at  $t$  seconds. Find the values of  $t$  when  $h = 0$ .

$$h = -16t^2 + 20t$$

Original equation

$$0 = -16t^2 + 20t$$

Substitution,  $h = 0$

$$0 = 4t(-4t + 5)$$

Factor by using the GCF.

$$4t = 0 \quad \text{or} \quad -4t + 5 = 0$$

Zero Product Property

$$t = 0$$

$$-4t = -5$$

Solve each equation.

$$t = \frac{5}{4} \text{ or } 1.25$$

Divide each side by  $-4$ .

Penny's height is 0 inches at 0 seconds and 1.25 seconds into the jump.

### Check Your Progress

5. **KANGAROOS** The hop of a kangaroo can be modeled by  $h = 24t - 16t^2$  where  $h$  represents the height of the hop in meters and  $t$  is the time in seconds. Find the values of  $t$  when  $h = 0$ .

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## Check Your Understanding

### Example 1 p. 476

Use the Distributive Property to factor each polynomial.

1.  $21b - 15a$

2.  $14c^2 + 2c$

3.  $10g^2h^2 + 9gh^2 - g^2h$

4.  $12jk^2 + 6j^2k + 2j^2k^2$

### Examples 2 and 3 p. 477

Factor each polynomial.

5.  $np + 2n + 8p + 16$

6.  $xy - 7x + 7y - 49$

7.  $3bc - 2b - 10 + 15c$

8.  $9fg - 45f - 7g + 35$

### Example 4 p. 478

Solve each equation. Check your solutions.

9.  $3k(k + 10) = 0$

10.  $(4m + 2)(3m - 9) = 0$

11.  $20p^2 - 15p = 0$

12.  $r^2 = 14r$

### Example 5 p. 479

13. **SPIDERS** Jumping spiders can commonly be found in homes and barns throughout the United States. A jumping spider's jump can be modeled by the equation  $h = 33.3t - 16t^2$ , where  $t$  represents the time in seconds and  $h$  is the height in feet.

- When is the spider's height at 0 feet?
- What is the spider's height after 1 second? after 2 seconds?

14. **ROCKETS** At a Fourth of July celebration, a rocket is launched straight up with an initial velocity of 125 feet per second. The height  $h$  of the rocket in feet above sea level is modeled by the formula  $h = 125t - 16t^2$ , where  $t$  is the time in seconds after the rocket is launched.

- What is the height of the rocket when it returns to the ground?
- Let  $h = 0$  in the equation and solve for  $t$ .
- How many seconds will it take for the rocket to return to the ground?



# Practice and Problem Solving

= **Step-by-Step Solutions** begin on page R12.  
**Extra Practice** begins on page 815.

## Example 1 p. 476

Use the Distributive Property to factor each polynomial.

15.  $16t - 40y$
16.  $30v + 50x$
17.  $2k^2 + 4k$
18.  $5z^2 + 10z$
19.  $4a^2b^2 + 2a^2b - 10ab^2$
20.  $5c^2v - 15c^2v^2 + 5c^2v^3$

## Examples 2 and 3 p. 477

Factor each polynomial.

21.  $fg - 5g + 4f - 20$
22.  $a^2 - 4a - 24 + 6a$
23.  $hj - 2h + 5j - 10$
24.  $xy - 2x - 2 + y$
25.  $45pq - 27q - 50p + 30$
26.  $24ty - 18t + 4y - 3$
27.  $3dt - 21d + 35 - 5t$
28.  $8r^2 + 12r$
29.  $21th - 3t - 35h + 5$
30.  $vp + 12v + 8p + 96$
31.  $5br - 25b + 2r - 10$
32.  $2nu - 8u + 3n - 12$
33.  $5gf^2 + g^2f + 15gf$
34.  $rp - 9r + 9p - 81$
35.  $27cd^2 - 18c^2d^2 + 3cd$
36.  $18r^3t^2 + 12r^2t^2 - 6r^2t$
37.  $48tu - 90t + 32u - 60$
38.  $16gh + 24g - 2h - 3$

## Example 4 p. 478

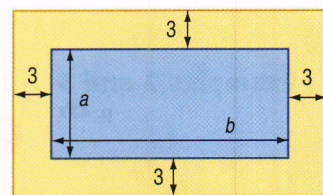
Solve each equation. Check your solutions.

39.  $3b(9b - 27) = 0$
40.  $2n(3n + 3) = 0$
41.  $(8z + 4)(5z + 10) = 0$
42.  $(7x + 3)(2x - 6) = 0$
43.  $b^2 = -3b$
44.  $a^2 = 4a$

## Example 5 p. 479

45. **GEOMETRY** Use the drawing at the right.

- a. Write an expression in factored form to represent the area of the blue section.
- b. Write an expression in factored form to represent the area of the region formed by the outer edge.
- c. Write an expression in factored form to represent the orange region.



46. **FIREWORKS** A ten-inch fireworks shell is fired from ground level. The height of the shell in feet is given by the formula  $h = 263t - 16t^2$ , where  $t$  is the time in seconds after launch.
  - a. Write the expression that represents the height in factored form.
  - b. At what time will the height be 0? Is this answer practical? Explain.
  - c. What is the height of the shell 8 seconds and 10 seconds after being fired?
  - d. At 10 seconds, is the shell rising or falling?
47. **ARCHITECTURE** The frame of a doorway is an arch that can be modeled by the graph of the equation  $y = -3x^2 + 12x$ , where  $x$  and  $y$  are measured in feet. On a coordinate plane, the floor is represented by the  $x$ -axis.
  - a. Make a table of values for the height of the arch if  $x = 0, 1, 2, 3$ , and 4 feet.
  - b. Plot the points from the table on a coordinate plane and connect the points to form a smooth curve to represent the arch.
  - c. How high is the doorway?





### Real-World Link

The Xtreme Skyflyer at Virginia's Kings Dominion uses a free-fall/cable swing to hoist people about 160 feet in the air. Then the riders experience a free-fall followed by a period of swinging back and forth.

Source: Theme Park Insider

48. **RIDES** Suppose the height of a rider after being dropped can be modeled by  $h = -16t^2 - 96t + 160$ , where  $h$  is the height in feet and  $t$  is time in seconds.
- Write an expression to represent the height in factored form.
  - From what height is the rider initially dropped?
  - At what height will the rider be after 3 seconds of falling? Is this possible? Explain.

49. **ARCHERY** The height  $h$  in feet of an arrow can be modeled by the equation  $h = 64t - 16t^2$ , where  $t$  is time in seconds. Ignoring the height of the archer, how long after the arrow is released does it hit the ground?

50. **TENNIS** A tennis player hits a tennis ball upward with an initial velocity of 80 feet per second. The height  $h$  in feet of the tennis ball can be modeled by the equation  $h = 80t - 16t^2$ , where  $t$  is time in seconds. Ignoring the height of the tennis player, how long does it take the ball to hit the ground?

51. **MULTIPLE REPRESENTATIONS** In this problem, you will explore the *box method* of factoring. To factor  $x^2 + x - 6$ , write the first term in the top left-hand corner of the box, and then write the last term in the lower right-hand corner.

	?	?
?	$x^2$	?
?	?	-6

- ANALYTICAL** Determine which two factors have a product of  $-6$  and a sum of 1.
- SYMBOLIC** Write each factor in an empty square in the box. Include the positive or negative sign and variable.
- ANALYTICAL** Find the factor for each row and column of the box. What are the factors of  $x^2 + x - 6$ ?
- VERBAL** Describe how you would use the box method to factor  $x^2 - 3x - 40$ .

### H.O.T. Problems

Use **H**igher-**O**rders **T**hinking Skills

52. **FIND THE ERROR** Hernando and Rachel are solving  $2m^2 = 4m$ . Is either of them correct? Explain your reasoning.

**Hernando**

$$2m^2 = 4m$$

$$\frac{2m^2}{m} = \frac{4m}{2m}$$

$$2m = 2$$

$$m = 1$$

**Rachel**

$$2m^2 = 4m$$

$$2m^2 - 4m = 0$$

$$2m(m - 2) = 0$$

$$2m = 0 \text{ or } m - 2 = 0$$

$$m = 0 \text{ or } 2$$

53. **CHALLENGE** Given the equation  $(ax + b)(ax - b) = 0$ , solve for  $x$ . What do we know about the values of  $a$  and  $b$ ?
54. **OPEN ENDED** Write a four-term polynomial that can be factored by grouping. Then factor the polynomial.
55. **REASONING** Given the equation  $c = a^2 - ab$ , for what values of  $a$  and  $b$  does  $c = 0$ ?
56. **WRITING IN MATH** Explain how to solve a quadratic equation by using the Zero Product Property.



## Standardized Test Practice

57. Which is a factor of  $6z^2 - 3z - 2 + 4z$ ?

A  $2z + 1$

C  $z + 2$

B  $3z - 2$

D  $2z - 1$

58. **PROBABILITY** Hailey has 10 blocks: 2 red, 4 blue, 3 yellow, and 1 green. What is the probability that it will be either a red or a yellow block?

F  $\frac{3}{10}$

H  $\frac{1}{2}$

G  $\frac{1}{5}$

J  $\frac{7}{10}$

59. **GRIDDED RESPONSE** Cho is making a 140-inch by 160-inch quilt with quilt squares that measure 8 inches on each side. How many will be needed to make the quilt?

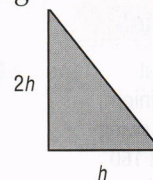
60. **GEOMETRY** The area of the right triangle shown below is  $5h$  square centimeters. What is the height of the triangle?

A 2 cm

B 5 cm

C 8 cm

D 10 cm



## Spiral Review

Find the GCF of each set of monomials. (Lesson 8-1)

61. 15, 25

62. 40, 100

63.  $16x, 24x^2$

64.  $30a^2, 50ab^2$

65.  $8c^2d^3, 16c^3d$

66.  $4y, 18y^2, 6y^3$

67. **GENETICS** Brown genes  $B$  are dominant over blue genes  $b$ . A person with genes  $BB$  or  $Bb$  has brown eyes. Someone with genes  $bb$  has blue eyes. Elisa has brown eyes with  $Bb$  genes, and Bob has blue eyes. Write an expression for the possible eye coloring of Elisa and Bob's children. Determine the probability that their child would have blue eyes. (Lesson 7-8)

Simplify. (Lesson 7-1)

68.  $(ab^4)(ab^2)$

69.  $(p^5r^4)(p^2r)$

70.  $(-7c^3d^4)(4cd^3)$

71.  $(9xy^7)^2$

72.  $[(3^2)^4]^2$

73.  $[(4^2)^3]^2$

74. **BASKETBALL** In basketball, a free throw is 1 point and a field goal is either 2 or 3 points. In a season, Tim Duncan of the San Antonio Spurs scored a total of 1342 points. The total number of 2-point field goals and 3-point field goals was 517, and he made 305 of the 455 free throws that he attempted. Find the number of 2-point field goals and 3-point field goals Duncan made that season. (Lesson 6-4)

Solve each inequality. Check your solution. (Lesson 5-3)

75.  $3y - 4 > -37$

76.  $-5q + 9 > 24$

77.  $-2k + 12 < 30$

78.  $5q + 7 \leq 3(q + 1)$

79.  $\frac{z}{4} + 7 \geq -5$

80.  $8c - (c - 5) > c + 17$

## Skills Review

Find each product. (Lesson 7-7)

81.  $(a + 2)(a + 5)$

82.  $(d + 4)(d + 10)$

83.  $(z - 1)(z - 8)$

84.  $(c + 9)(c - 3)$

85.  $(x - 7)(x - 6)$

86.  $(g - 2)(g + 11)$



## 8-3

## Factoring Trinomials

You can use algebra tiles to factor trinomials. If a polynomial represents the area of a rectangle formed by algebra tiles, then the rectangle's length and width are *factors* of the area. If a rectangle cannot be formed to represent the trinomial, then the trinomial is not factorable.

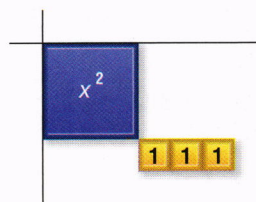
**ACTIVITY 1** Factor  $x^2 + bx + c$ 

Use algebra tiles to factor  $x^2 + 4x + 3$ .

**Step 1** Model  $x^2 + 4x + 3$ .



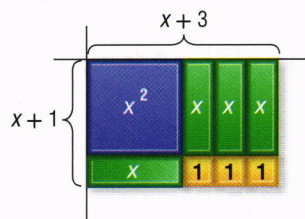
**Step 2** Place the  $x^2$ -tile at the corner of the product mat. Arrange the 1-tiles into a rectangular array. Because 3 is prime, the 3 tiles can be arranged in a rectangle in one way, a 1-by-3 rectangle.



**Step 3** Complete the rectangle with the  $x$ -tiles.

The rectangle has a width of  $x + 1$  and a length of  $x + 3$ .

Therefore,  $x^2 + 4x + 3 = (x + 1)(x + 3)$ .

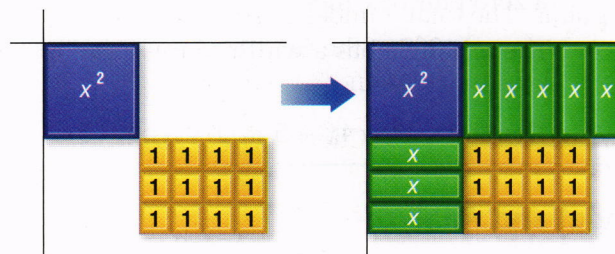
**ACTIVITY 2** Factor  $x^2 + bx + c$ 

Use algebra tiles to factor  $x^2 + 8x + 12$ .

**Step 1** Model  $x^2 + 8x + 12$ .



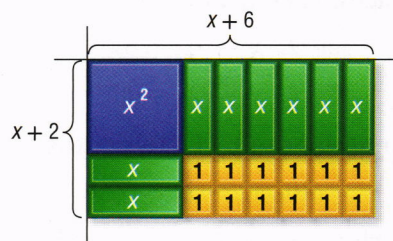
**Step 2** Place the  $x^2$ -tile at the corner of the product mat. Arrange the 1-tiles into a rectangular array. Since  $12 = 3 \times 4$ , try a 3-by-4 rectangle. Try to complete the rectangle. Notice that there is an extra  $x$ -tile.



**Step 3** Arrange the 1-tiles into a 2-by-6 rectangular array. This time you can complete the rectangle with the  $x$ -tiles.

The rectangle has a width of  $x + 2$  and a length of  $x + 6$ .

Therefore,  $x^2 + 8x + 12 = (x + 2)(x + 6)$ .



(continued on the next page)



### ACTIVITY 3 Factor $x^2 - bx + c$

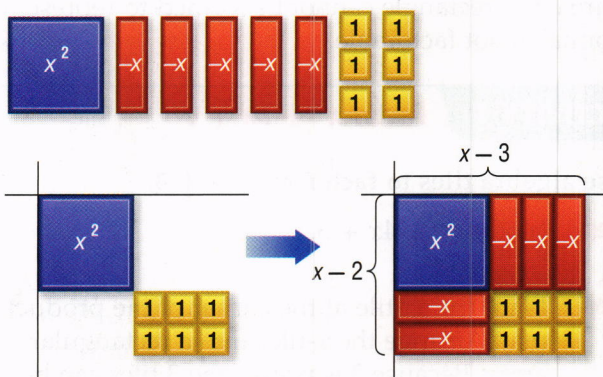
Use algebra tiles to factor  $x^2 - 5x + 6$ .

**Step 1** Model  $x^2 - 5x + 6$ .

**Step 2** Place the  $x^2$ -tile at the corner of the product mat. Arrange the 1-tiles into a 2-by-3 rectangular array as shown.

**Step 3** Complete the rectangle with the  $x$ -tiles. The rectangle has a width of  $x - 2$  and a length of  $x - 3$ .

Therefore,  $x^2 - 5x + 6 = (x - 2)(x - 3)$ .



### ACTIVITY 4 Factor $x^2 - bx - c$

Use algebra tiles to factor  $x^2 - 4x - 5$ .

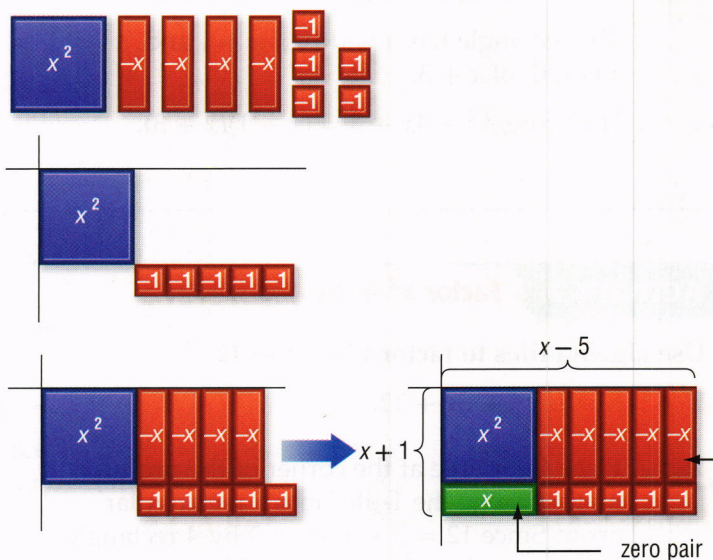
**Step 1** Model  $x^2 - 4x - 5$ .

**Step 2** Place the  $x^2$ -tile at the corner of the product mat. Arrange the 1-tiles into a 1-by-5 rectangular array as shown.

**Step 3** Place the  $x$ -tile as shown. Recall that you can add zero pairs without changing the value of the polynomial. In this case, add a zero pair of  $x$ -tiles.

The rectangle has a width of  $x + 1$  and a length of  $x - 5$ .

Therefore,  $x^2 - 4x - 5 = (x + 1)(x - 5)$ .



### Model and Analyze

Use algebra tiles to factor each trinomial.

- |                    |                   |                   |                    |
|--------------------|-------------------|-------------------|--------------------|
| 1. $x^2 + 3x + 2$  | 2. $x^2 + 6x + 8$ | 3. $x^2 + 3x - 4$ | 4. $x^2 - 7x + 12$ |
| 5. $x^2 + 7x + 10$ | 6. $x^2 - 2x + 1$ | 7. $x^2 + x - 12$ | 8. $x^2 - 8x + 15$ |

Tell whether each trinomial can be factored. Justify your answer with a drawing.

- |                   |                    |                   |               |
|-------------------|--------------------|-------------------|---------------|
| 9. $x^2 + 3x + 6$ | 10. $x^2 - 5x - 6$ | 11. $x^2 - x - 4$ | 12. $x^2 - 4$ |
|-------------------|--------------------|-------------------|---------------|

13. **WRITING IN MATH** How can you use algebra tiles to determine whether a trinomial can be factored?



# Quadratic Equations: $x^2 + bx + c = 0$

## Then

You multiplied binomials by using the FOIL method. (Lesson 7-7)

## Now

- Factor trinomials of the form  $x^2 + bx + c$ .
- Solve equations of the form  $x^2 + bx + c = 0$ .

## New Vocabulary

quadratic equation

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## Why?

Diana is having a rectangular in-ground swimming pool installed and she wants to include a 24-foot fence around the pool. The pool requires a space of 36 square feet. What dimensions should the pool have?

To solve this problem, the landscape architect needs to find two numbers that have a product of 36 and a sum of 12, half the perimeter of the pool.



**Factor  $x^2 + bx + c$**  In Lesson 7-7, you learned how to multiply two binomials by using the FOIL method. Each of the binomials was a factor of the product. The pattern for multiplying two binomials can be used to factor certain types of trinomials.

$$\begin{aligned}(x + 3)(x + 4) &= x^2 + 4x + 3x + 3 \cdot 4 \\ &= x^2 + (4 + 3)x + 3 \cdot 4 \\ &= x^2 + 7x + 12\end{aligned}$$

Use the FOIL method.

Distributive Property

Simplify.

Notice that the coefficient of the middle term,  $7x$ , is the sum of 3 and 4, and the last term, 12, is the product of 3 and 4.

Observe the following pattern in this multiplication.

$$\begin{aligned}(x + 3)(x + 4) &= x^2 + (4 + 3)x + (3 \cdot 4) \\ (x + m)(x + p) &= x^2 + (p + m)x + mp \\ &= x^2 + \underbrace{(m + p)}_{b}x + \underbrace{mp}_{c} \\ x^2 + bx + c &\end{aligned}$$

Let  $3 = m$  and  $4 = p$ .

Commutative (+)

$b = m + p$  and  $c = mp$

Notice that the coefficient of the middle term is the sum of  $m$  and  $p$ , and the last term is the product of  $m$  and  $p$ . This pattern can be used to factor trinomials of the form  $x^2 + bx + c$ .



## Key Concept

## Factoring $x^2 + bx + c$

For Your

FOLDABLE

**Words** To factor trinomials in the form  $x^2 + bx + c$ , find two integers,  $m$  and  $p$ , with a sum of  $b$  and a product of  $c$ . Then write  $x^2 + bx + c$  as  $(x + m)(x + p)$ .

**Symbols**  $x^2 + bx + c = (x + m)(x + p)$  when  $m + p = b$  and  $mp = c$

**Example**  $x^2 + 6x + 8 = (x + 2)(x + 4)$ , because  $2 + 4 = 6$  and  $2 \cdot 4 = 8$ .

When  $c$  is positive, its factors have the same signs. Both of the factors are positive or negative based upon the sign of  $b$ . If  $b$  is positive, the factors are positive. If  $b$  is negative, the factors are negative.



**Problem-SolvingTip****Guess and Check**

When factoring a trinomial, make an educated guess, check for reasonableness, and then adjust the guess until the correct answer is found.

**EXAMPLE 1** *b and c Are Positive*

Factor  $x^2 + 9x + 20$ .

In this trinomial,  $b = 9$  and  $c = 20$ . Since  $c$  is positive and  $b$  is positive, you need to find two positive factors with a sum of 9 and a product of 20. Make an organized list of the factors of 20, and look for the pair of factors with a sum of 9.

Factors of 20	Sum of Factors
1, 20	21
2, 10	12
4, 5	9

The correct factors are 4 and 5.

$$\begin{aligned} x^2 + 9x + 20 &= (x + m)(x + p) \\ &= (x + 4)(x + 5) \end{aligned}$$

Write the pattern.

$$m = 4 \text{ and } p = 5$$

**CHECK** You can check this result by multiplying the two factors. The product should be equal to the original expression.

$$\begin{aligned} (x + 4)(x + 5) &= x^2 + 5x + 4x + 20 \\ &= x^2 + 9x + 20 \quad \checkmark \end{aligned}$$

**FOIL Method**  
Simplify.



**Check Your Progress** Factor each polynomial.

1A.  $d^2 + 11d + 24$

1B.  $9 + 10t + t^2$

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When factoring a trinomial in which  $b$  is negative and  $c$  is positive, use what you know about the product of binomials to narrow the list of possible factors.

**StudyTip**

**Finding Factors** Once the correct factors are found, it is not necessary to test any other factors. In Example 2,  $-2$  and  $-6$  are the correct factors, so  $-3$  and  $-4$  do not need to be tested.

**EXAMPLE 2** *b Is Negative and c Is Positive*

Factor  $x^2 - 8x + 12$ .

In this trinomial,  $b = -8$  and  $c = 12$ . Since  $c$  is positive and  $b$  is negative, you need to find two negative factors with a sum of  $-8$  and a product of 12.

Factors of 12	Sum of Factors
-1, -12	-13
-2, -6	-8
-3, -4	-7

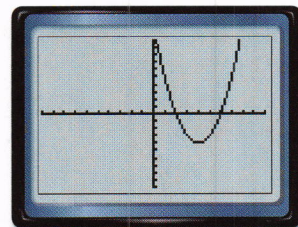
The correct factors are  $-2$  and  $-6$ .

$$\begin{aligned} x^2 - 8x + 12 &= (x + m)(x + p) \\ &= (x - 2)(x - 6) \end{aligned}$$

Write the pattern.

$$m = -2 \text{ and } p = -6$$

**CHECK** Graph  $y = x^2 - 8x + 12$  and  $y = (x - 2)(x - 6)$  on the same screen. Since only one graph appears, the two graphs must coincide. Therefore, the trinomial has been factored correctly.  $\checkmark$



$[-10, 10]$  scl: 1 by  $[-10, 10]$  scl: 1



**Check Your Progress** Factor each polynomial.

2A.  $21 - 22m + m^2$

2B.  $w^2 - 11w + 28$

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## Review Vocabulary

**absolute value** the distance a number is from zero on a number line, written  $|n|$   
(Lesson 0-2)

When  $c$  is negative, its factors have opposite signs. To determine which factor is positive and which is negative, look at the sign of  $b$ . The factor with the greater absolute value has the same sign as  $b$ .

### EXAMPLE 3 $c$ is Negative

Factor each polynomial.

a.  $x^2 + 2x - 15$

In this trinomial,  $b = 2$  and  $c = -15$ . Since  $c$  is negative, the factors  $m$  and  $p$  have opposite signs. So either  $m$  or  $p$  is negative, but not both. Since  $b$  is positive, the factor with the greater absolute value is also positive.

List the factors of  $-15$ , where one factor of each pair is negative. Look for the pair of factors with a sum of 2.

Factors of $-15$	Sum of Factors
$-1, 15$	14
$-3, 5$	2

$$\begin{aligned} x^2 + 2x - 15 &= (x + m)(x + p) \\ &= (x - 3)(x + 5) \end{aligned}$$

**CHECK**  $(x - 3)(x + 5) = x^2 + 5x - 3x - 15$   
 $= x^2 + 2x - 15$  ✓

The correct factors are  $-3$  and  $5$ .

Write the pattern.

$m = -3$  and  $p = 5$

FOIL Method

Simplify.

b.  $x^2 - 7x - 18$

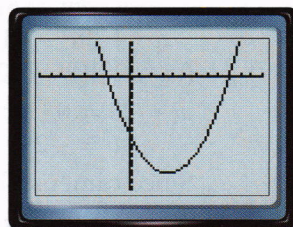
In this trinomial,  $b = -7$  and  $c = -18$ . Either  $m$  or  $p$  is negative, but not both. Since  $b$  is negative, the factor with the greater absolute value is also negative.

List the factors of  $-18$ , where one factor of each pair is negative. Look for the pair of factors with a sum of  $-7$ .

Factors of $-18$	Sum of Factors
$1, -18$	$-17$
$2, -9$	$-7$
$3, -6$	$-3$

$$\begin{aligned} x^2 - 7x - 18 &= (x + m)(x + p) \\ &= (x + 2)(x - 9) \end{aligned}$$

**CHECK** Graph  $y = x^2 - 7x - 18$  and  $y = (x + 2)(x - 9)$  on the same screen.



$[-10, 15]$  scl: 1 by  $[-40, 20]$  scl: 1

The graphs coincide. Therefore, the trinomial has been factored correctly. ✓

### Check Your Progress

3A.  $y^2 + 13y - 48$

3B.  $r^2 - 2r - 24$



**Solve Equations by Factoring** A **quadratic equation** can be written in the standard form  $ax^2 + bx + c = 0$ , where  $a \neq 0$ . Some equations of the form  $x^2 + bx + c = 0$  can be solved by factoring and then using the Zero Product Property.

#### EXAMPLE 4 Solve an Equation by Factoring

Solve  $x^2 + 6x = 27$ . Check your solutions.


$$\begin{array}{ll} x^2 + 6x = 27 & \text{Original equation} \\ x^2 + 6x - 27 = 0 & \text{Subtract 27 from each side.} \\ (x - 3)(x + 9) = 0 & \text{Factor.} \\ x - 3 = 0 \quad \text{or} \quad x + 9 = 0 & \text{Zero Product Property} \\ x = 3 \quad \quad \quad x = -9 & \text{Solve each equation.} \end{array}$$

The roots are 3 and  $-9$ .

**CHECK** Substitute 3 and  $-9$  for  $x$  in the original equation.

$$\begin{array}{l} x^2 + 6x = 27 \\ (3)^2 + 6(3) \stackrel{?}{=} 27 \\ 9 + 18 \stackrel{?}{=} 27 \\ 27 = 27 \quad \checkmark \end{array}$$

$$\begin{array}{l} x^2 + 6x = 27 \\ (-9)^2 + 6(-9) \stackrel{?}{=} 27 \\ 81 - 54 \stackrel{?}{=} 27 \\ 27 = 27 \quad \checkmark \end{array}$$

 **Check Your Progress** Solve each equation. Check your solutions.

4A.  $z^2 - 3z = 70$

4B.  $x^2 + 3x - 18 = 0$

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Factoring can be useful when solving real-world problems.



#### Real-World Link

A company that produces event signs recommends foamcore boards for event signs that will be used only once. For signs used more than once, use a stronger type of foamcore board.

Source: MegaPrint Inc.

#### Real-World EXAMPLE 5 Solve a Problem by Factoring

**DESIGN** Ling is designing a poster. The top of the poster is 4 inches long and the rest of the poster is 2 inches longer than the width. If the poster requires 616 square inches of poster board, find the width  $w$  of the poster.

**Understand** You want to find the width of the poster.

**Plan** Since the poster is a rectangle, width  $\cdot$  length = area.

**Solve** Let  $w$  = the width of the poster.  
The length is  $w + 2 + 4$  or  $w + 6$ .

$$\begin{array}{l} w(w + 6) = 616 \\ w^2 + 6w = 616 \\ w^2 + 6w - 616 = 0 \\ (w + 28)(w - 22) = 0 \\ w + 28 = 0 \quad \text{or} \quad w - 22 = 0 \\ w = -28 \quad \quad \quad w = 22 \end{array}$$

Since dimensions cannot be negative, the width is 22 inches.

**Check** If the width is 22 inches, then the area of the poster is  $22 \cdot (22 + 6)$  or 616 square inches, which is the amount the poster requires.  $\checkmark$



**Write the equation.**


**Multiply.**

**Subtract 616 from each side.**

**Factor.**

**Zero Product Property**

**Solve each equation.**

 **Check Your Progress**

5. **GEOMETRY** The height of a parallelogram is 18 centimeters less than its base. If the area is 175 square centimeters, what is its height?

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## Check Your Understanding

### Examples 1–3 pp. 486–487

Factor each polynomial.

1.  $x^2 + 14x + 24$

3.  $n^2 + 4n - 21$

2.  $y^2 - 7y - 30$

4.  $m^2 - 15m + 50$

### Example 4 p. 488

Solve each equation. Check your solutions.

5.  $x^2 - 4x - 21 = 0$

7.  $x^2 - 15x + 54 = 0$

9.  $x^2 - x - 72 = 0$

6.  $n^2 - 3n + 2 = 0$

8.  $x^2 + 12x = -32$

10.  $x^2 - 10x = -24$

### Example 5 p. 488

11. **FRAMING** Tina bought a frame for a photo, but the photo is too big for the frame. Tina needs to reduce the width and length of the photo by the same amount. The area of the photo should be reduced to half the original area. If the original photo is 12 inches by 16 inches, what will be the dimensions of the smaller photo?

## Practice and Problem Solving

 = **Step-by-Step Solutions** begin on page R12.  
**Extra Practice** begins on page 815.

### Examples 1–3 pp. 486–487

Factor each polynomial.

12.  $x^2 + 17x + 42$

14.  $a^2 + 8a - 48$

16.  $44 + 15h + h^2$

18.  $-24 - 10x + x^2$

13.  $y^2 - 17y + 72$

15.  $n^2 - 2n - 35$

17.  $40 - 22x + x^2$

19.  $-42 - m + m^2$

### Example 4 p. 488

Solve each equation. Check your solutions.

20.  $x^2 - 7x + 12 = 0$

22.  $x^2 - 6x = 27$

24.  $c^2 + 10c + 9 = 0$

26.  $n^2 - 120 = 7n$

28.  $y^2 - 90 = 13y$

21.  $y^2 + y = 20$

23.  $a^2 + 11a = -18$

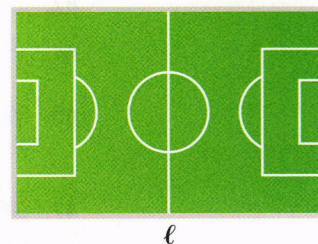
25.  $x^2 - 18x = -32$

27.  $d^2 + 56 = -18d$

29.  $h^2 + 48 = 16h$

### Example 5 p. 488

30. **GEOMETRY** A triangle has an area of 36 square feet. If the height of the triangle is 6 feet more than its base, what are its height and base?
31. **GEOMETRY** A rectangle has an area represented by  $x^2 - 4x - 12$  square feet. If the length is  $x + 2$  feet, what is the width of the rectangle?
32. **SOCCER** The width of a high school soccer field is 45 yards shorter than its length.
- Define a variable, and write an expression for the area of the field.
  - The area of the field is 9000 square yards. Find the dimensions.



$l - 45$

Factor each polynomial.

33.  $q^2 + 11qr + 18r^2$

35.  $x^2 - 6xy + 5y^2$

34.  $x^2 - 14xy - 51y^2$

36.  $a^2 + 10ab - 39b^2$





### Real-World Link

Australian Libby Trickett has set the world record three times for the 100 meter freestyle event. In the 2008 Beijing Olympics, she won the silver medal for the event.

Source: Fox Sports

- 37. SWIMMING** The length of a rectangular swimming pool is 20 feet greater than its width. The area of the pool is 525 square feet.

- Define a variable and write an equation for the area of the pool.
- Solve the equation.
- Interpret the solutions. Do both solutions make sense? Explain.

**GEOMETRY** Find an expression for the perimeter of a rectangle with the given area.

**38.**  $A = x^2 + 24x - 81$

**39.**  $A = x^2 + 13x - 90$

- 40. MULTIPLE REPRESENTATIONS** In this problem, you will explore factoring when the leading coefficient is not 1.

- a. TABULAR** Copy and complete the table below.

Product of Two Binomials	$ax^2 + mx + px + c$	$ax^2 + bx + c$	$m \times p$	$a \times c$
$(2x + 3)(x + 4)$	$2x^2 + 8x + 3x + 12$	$2x^2 + 11x + 12$	24	24
$(x + 1)(3x + 5)$				
$(2x - 1)(4x + 1)$				
$(3x + 5)(4x - 2)$				

- b. ANALYTICAL** How are  $m$  and  $p$  related to  $a$  and  $c$ ?

- c. ANALYTICAL** How are  $m$  and  $p$  related to  $b$ ?

- d. VERBAL** Describe a process you can use for factoring a polynomial of the form  $ax^2 + bx + c$ .

### H.O.T. Problems

Use **H**igher-**O**rders **T**hinking Skills

- 41. FIND THE ERROR** Jerome and Charles have factored  $x^2 + 6x - 16$ . Is either of them correct? Explain your reasoning.

Jerome

$$x^2 + 6x - 16 = (x + 2)(x - 8)$$

Charles

$$x^2 + 6x - 16 = (x - 2)(x + 8)$$

**CHALLENGE** Find all values of  $k$  so that each polynomial can be factored using integers.

**42.**  $x^2 + kx - 19$

**43.**  $x^2 + kx + 14$

**44.**  $x^2 - 8x + k, k > 0$

**45.**  $x^2 - 5x + k, k > 0$

- 46. REASONING** For any factorable trinomial,  $x^2 + bx + c$ , will the absolute value of  $b$  sometimes, always, or never be less than the absolute value of  $c$ ? Explain.

- 47. OPEN ENDED** Give an example of a trinomial that can be factored using the factoring techniques presented in this lesson. Then factor the trinomial.

- 48. CHALLENGE** Factor  $(4y - 5)^2 + 3(4y - 5) - 70$ .

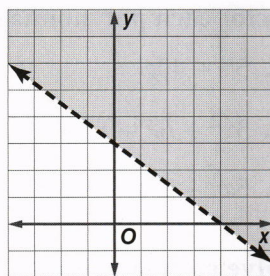
- 49. WRITING IN MATH** Explain how to factor trinomials of the form  $x^2 + bx + c$  and how to determine the signs of the factors of  $c$ .



## Standardized Test Practice

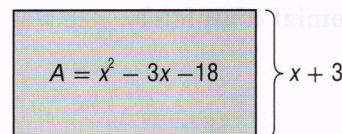
50. Which inequality is shown in the graph below?

- A  $y \leq -\frac{3}{4}x + 3$   
 B  $y < -\frac{3}{4}x + 3$   
 C  $y > -\frac{3}{4}x + 3$   
 D  $y \geq -\frac{3}{4}x + 3$



51. **SHORT RESPONSE** Olivia must earn more than \$254 from selling candy bars in order to go on a trip with the National Honor Society. If each candy bar is sold for \$1.25, what is the fewest candy bars she must sell?

52. **GEOMETRY** Which expression represents the length of the rectangle?



- F  $x + 5$   
 G  $x + 6$   
 H  $x - 6$   
 J  $x - 5$

53. The difference of 21 and a number  $n$  is 6. Which equation shows the relationship?

- A  $21 - n = 6$       C  $21n = 6$   
 B  $21 + n = 6$       D  $6n = -21$

## Spiral Review

Factor each polynomial. (Lesson 8-2)

54.  $10a^2 + 40a$

55.  $11x + 44x^2y$

56.  $2m^3p^2 - 16mp^2 + 8mp$

57.  $2ax + 6xc + ba + 3bc$

58.  $8ac - 2ad + 4bc - bd$

59.  $x^2 - xy - xy + y^2$

60. **FLOORING** Emma is replacing her dining room floor, which is 10 feet by 12 feet. The flooring comes in pieces 1 foot by 1 foot, 2 foot by 2 foot, 3 foot by 3 foot, and 2 foot by 3 foot. Without cutting the pieces, which of the four sizes of flooring can Emma use? Explain. (Lesson 8-1)

Perform the indicated matrix operations. If an operation cannot be performed, write *impossible*. (Lesson 6-6)

61.  $\begin{bmatrix} 10 & 0 & 8 \\ 2 & -5 & -7 \end{bmatrix} + \begin{bmatrix} 2 & -9 & 4 \\ -1 & 2 & -3 \end{bmatrix}$

62.  $\begin{bmatrix} 7 & 3 & 1 \\ 9 & -2 & -4 \\ 4 & -8 & 7 \end{bmatrix} - \begin{bmatrix} -7 & 4 & 2 \\ 6 & 3 & -3 \\ 11 & -16 & 5 \end{bmatrix}$

63.  $\begin{bmatrix} 25 \\ -8 \\ -23 \end{bmatrix} + \begin{bmatrix} -9 & 21 & -2 & 7 \end{bmatrix}$

64.  $3 \begin{bmatrix} -5 & 3 \\ 4 & 0 \\ -1 & 8 \end{bmatrix}$

65. **LANDSCAPING** Kendrick is planning a circular flower garden with a low fence around the border. He has 38 feet of fence. What is the radius of the largest garden he can make? (Hint:  $C = 2\pi r$ ) (Lesson 5-2)

## Skills Review

Factor each polynomial. (Lesson 8-2)

66.  $6mx - 4m + 3rx - 2r$

67.  $3ax - 6bx + 8b - 4a$

68.  $2d^2g + 2fg + 4d^2h + 4fh$



Factor each monomial completely. (Lesson 8-1)

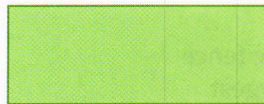
1.  $16x^3y^2$
2.  $35ab^4$
3.  $-20m^5n^2$
4.  $-13xy^3$
5. **ROOM DESIGN** The area of a rectangular room is 120 square feet. What are the possible whole-number dimensions for the length and width of the room? (Lesson 8-1)

Find the GCF of each set of monomials. (Lesson 8-1)

6.  $10a, 20a^2, 25a$
7.  $13c, 25d$
8.  $21ab, 35a, 56ab^3$
9. **FASHION** A sales clerk is organizing 24 pairs of shoes for a sales display. In what ways can she organize the shoes so that she has the same number of shoes on each shelf, at least 4 pairs of shoes per shelf, and at least 2 shelves of shoes?

Use the Distributive Property to factor each polynomial. (Lesson 8-2)

10.  $3xy - 9x$
11.  $6ab + 12ab^2 + 18b$
12. **MULTIPLE CHOICE** The area of the rectangle is  $3x^2 + 6x - 12$  square units. What is the width of the rectangle? (Lesson 8-2)



$$x^2 + 2x - 4$$

- A 2 units
- B 3 units
- C 4 units
- D 6 units

Factor each polynomial. (Lesson 8-2)

13.  $5h + 40g$
14.  $3x^2 + 6x + x + 2$
15.  $5a^2 - 25a - a + 5$

Solve each equation. Check your solutions. (Lesson 8-2)

16.  $2x(x - 5) = 0$
17.  $6p^2 - 3p = 0$
18.  $a^2 = 15a$

19. **ARCHITECTURE** The curve of the archway under a bridge can be modeled by the equation  $y = -\frac{1}{5}x^2 + 6x$ , where  $x$  and  $y$  are measured in feet. Copy and complete the table for each value of  $x$ . (Lesson 8-2)

$x$	$y$
0	
10	
15	
20	
30	

Factor each polynomial. (Lesson 8-3)

20.  $x^2 - 4x - 21$
21.  $x^2 - 10x + 24$
22.  $x^2 + 4x - 21$

Solve each equation. Check your solutions. (Lesson 8-3)

23.  $x^2 - 5x = 14$
24.  $x^2 - 3x - 18 = 0$
25.  $24 + x^2 = 10x$

26. **MULTIPLE CHOICE** A rectangle has a length that is 2 inches longer than its width. The area of the rectangle is 48 square inches. What is the length of the rectangle? (Lesson 8-3)

- F 48 in.
- G 8 in.
- H 6 in.
- J 2 in.



## Quadratic Equations: $ax^2 + bx + c = 0$

### Why?

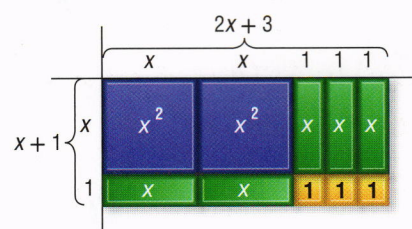
At amusement parks around the country, the paths of riders can be modeled by the expression  $16t^2 - 5t + 120$ .

Factoring this expression can help the ride operators determine how long a rider rides on the initial swing.

**Factor  $ax^2 + bx + c$**  In the last lesson, you factored quadratic expressions of the form  $ax^2 + bx + c$ , where  $a = 1$ . In this lesson, you will apply the factoring methods to quadratic expressions in which  $a$  is not 1.

The dimensions of the rectangle formed by the algebra tiles are the factors of  $2x^2 + 5x + 3$ . The factors of  $2x^2 + 5x + 3$  are  $x + 1$  and  $2x + 3$ .

You can also use the method of factoring by grouping to solve this expression.



### Then

You factored trinomials of the form  $x^2 + bx + c$ . (Lesson 8-3)

### Now

- Factor trinomials of the form  $ax^2 + bx + c$ .
- Solve equations of the form  $ax^2 + bx + c = 0$ .

### New Vocabulary

prime polynomial

### Math Online

[glencoe.com](http://glencoe.com)

- Extra Examples
- Personal Tutor
- Self-Check Quiz
- Homework Help

**Step 1** Apply the pattern:  $2x^2 + 5x + 3 = 2x^2 + mx + px + 3$ .

**Step 2** Find two numbers that have a product of  $2 \cdot 3$  or 6 and a sum of 5.

Factors of 6	Sum of Factors
1, 6	7
2, 3	5

**Step 3** Use grouping to find the factors.

$$\begin{aligned}
 2x^2 + 5x + 3 &= 2x^2 + mx + px + 3 \\
 &= 2x^2 + 2x + 3x + 3 \\
 &= (2x^2 + 2x) + (3x + 3) \\
 &= 2x(x + 1) + 3(x + 1) \\
 &= (2x + 3)(x + 1)
 \end{aligned}$$

Write the pattern.

$m = 2$  and  $p = 3$

Group terms with common factors.

Factor the GCF.

$x + 1$  is the common factor.

Therefore,  $2x^2 + 5x + 3 = (2x + 3)(x + 1)$ .



### Key Concept

### Factoring $ax^2 + bx + c$

For Your

**FOLDABLE**

#### Words

To factor trinomials of the form  $ax^2 + bx + c$ , find two integers,  $m$  and  $p$ , with a sum of  $b$  and a product of  $ac$ . Then write  $ax^2 + bx + c$  as  $ax^2 + mx + px + c$ , and factor by grouping.

#### Example

$$\begin{aligned}
 5x^2 - 13x + 6 &= 5x^2 - 10x - 3x + 6 & m = -10 \text{ and } p = -3 \\
 &= 5x(x - 2) + (-3)(x - 2) \\
 &= (5x - 3)(x - 2)
 \end{aligned}$$



**StudyTip****Greatest Common**

**Factor** Always look for a GCF of the terms of a polynomial before you factor.

**EXAMPLE 1** Factor  $ax^2 + bx + c$ 

Factor each trinomial.

a.  $7x^2 + 29x + 4$

In this trinomial,  $a = 7$ ,  $b = 29$ , and  $c = 4$ . You need to find two numbers with a sum of 29 and a product of  $7 \cdot 4$  or 28. Make a list of the factors of 28 and look for the pair of factors with the sum of 29.

Factors of 28	Sum of Factors
1, 28	29

The correct factors are 1 and 28.

$$\begin{aligned} 7x^2 + 29x + 4 &= 7x^2 + mx + px + 4 \\ &= 7x^2 + 1x + 28x + 4 \\ &= (7x^2 + 1x) + (28x + 4) \\ &= x(7x + 1) + 4(7x + 1) \\ &= (x + 4)(7x + 1) \end{aligned}$$

Write the pattern.

$m = 1$  and  $p = 28$

Group terms with common factors.

Factor the GCF.

$7x + 1$  is the common factor.

b.  $3x^2 + 15x + 18$

The GCF of the terms  $3x^2$ ,  $15x$ , and 18 is 3. Factor this first.

$$\begin{aligned} 3x^2 + 15x + 18 &= 3(x^2 + 5x + 6) \\ &= 3(x + 3)(x + 2) \end{aligned}$$

Distributive Property

Find two factors of 6 with a sum of 5.

**Check Your Progress**

1A.  $5x^2 + 13x + 6$

1B.  $6x^2 + 22x - 8$

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Sometimes the coefficient of the  $x$ -term is negative.

**EXAMPLE 2** Factor  $ax^2 - bx + c$ 

Factor  $3x^2 - 17x + 20$ .

In this trinomial,  $a = 3$ ,  $b = -17$ , and  $c = 20$ . Since  $b$  is negative,  $m + p$  will be negative. Since  $c$  is positive,  $mp$  will be positive.

To determine  $m$  and  $p$ , list the negative factors of  $ac$  or 60. The sum of  $m$  and  $p$  should be  $-17$ .

Factors of 60	Sum of Factors
-2, -30	-32
-3, -20	-23
-4, -15	-19
-5, -12	-17

The correct factors are  $-5$  and  $-12$ .

$$\begin{aligned} 3x^2 - 17x + 20 &= 3x^2 - 12x - 5x + 20 \\ &= (3x^2 - 12x) + (-5x + 20) \\ &= 3x(x - 4) + (-5)(x - 4) \\ &= (3x - 5)(x - 4) \end{aligned}$$

$m = -12$  and  $p = -5$

Group terms with common factors.

Factor the GCF.

Distributive Property

**Check Your Progress**

2A.  $2n^2 - n - 1$

2B.  $10y^2 - 35y + 30$

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A polynomial that cannot be written as a product of two polynomials with integral coefficients is called a **prime polynomial**.

### EXAMPLE 3 Determine Whether a Polynomial is Prime

Factor  $4x^2 - 3x + 5$ , if possible. If the polynomial cannot be factored using integers, write *prime*.

In this trinomial,  $a = 4$ ,  $b = -3$ , and  $c = 5$ . Since  $b$  is negative,  $m + p$  is negative. Since  $c$  is positive,  $mp$  is positive. So,  $m$  and  $p$  are both negative. Next, list the factors of 20. Look for the pair with a sum of  $-3$ .

Factors of 20	Sum of Factors
-20, -1	-21
-4, -5	-9
-2, -10	-12

There are no factors with a sum of  $-3$ . So the quadratic expression cannot be factored using integers. Therefore,  $4x^2 - 3x + 5$  is prime.

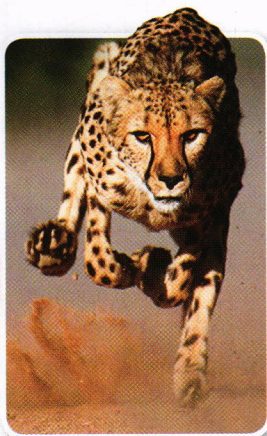
#### Check Your Progress

Factor each polynomial, if possible. If the polynomial cannot be factored using integers, write *prime*.

3A.  $4r^2 - r + 7$

3B.  $2x^2 + 3x - 5$

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#### Real-World Link

Cheetahs are the fastest land animals in the world, reaching speeds of up to 70 mph. It can accelerate from 0 to 40 mph in 3 strides. It takes just seconds for the cheetah to reach the full speed of 70 mph.

Source: Cheetah Conservation Fund

### Real-World EXAMPLE 4 Solve Equations by Factoring

**WILDLIFE** Suppose a cheetah pouncing on an antelope leaps with an initial velocity of 19 feet per second. How long is the cheetah in the air if it lands on the antelope's hind quarter, 3 feet from the ground?

$$h = -16t^2 + vt + h_0$$

$$3 = -16t^2 + 19t + 0$$

$$0 = -16t^2 + 19t - 3$$

$$0 = 16t^2 - 19t + 3$$

$$0 = (16t - 3)(t - 1)$$

$$16t - 3 = 0 \quad \text{or} \quad t - 1 = 0$$

$$16t = 3 \quad t = 1$$

$$t = \frac{3}{16}$$

Equation for height

$h = 3$ ,  $v = 19$ , and  $h_0 = 0$

Subtract 3 from each side.

Multiply each side by  $-1$ .

Factor  $16t^2 - 19t + 3$ .

Zero Product Property

Solve each equation.

The solutions are  $\frac{3}{16}$  and 1 second. It takes the cheetah  $\frac{3}{16}$  second to reach a height of 3 feet on his way up. It takes the cheetah 1 second to reach a height of 3 feet on his way down. So, the cheetah is in the air 1 second before he catches the antelope.

#### Check Your Progress

4. **PHYSICAL SCIENCE** A person throws a ball upward from a 506-foot tall building. The ball's height  $h$  in feet after  $t$  seconds is given by the equation  $h = -16t^2 + 48t + 506$ . The ball lands on a balcony that is 218 feet above the ground. How many seconds was it in the air?

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#### Watch Out!

**Keep the  $-1$**  Do not forget to carry the  $-1$  that was factored out through the rest of the steps or multiply both sides by  $-1$ .



## Check Your Understanding

### Examples 1–3 pp. 494–495

Factor each polynomial, if possible. If the polynomial cannot be factored using integers, write *prime*.

1.  $3x^2 + 17x + 10$
2.  $2x^2 + 22x + 56$
3.  $5x^2 - 3x + 4$
4.  $3x^2 - 11x - 20$

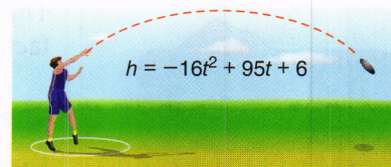
### Example 4 p. 495

Solve each equation. Check your solutions.

5.  $2x^2 + 9x + 9 = 0$
6.  $3x^2 + 17x + 20 = 0$
7.  $3x^2 - 10x + 8 = 0$
8.  $2x^2 - 17x + 30 = 0$

9. **DISC** Ken throws the discus at a school meet.

- a. What is the initial height of the discus?
- b. After how many seconds does the discus hit the ground?



## Practice and Problem Solving

= **Step-by-Step Solutions** begin on page R12.  
**Extra Practice** begins on page 815.

### Examples 1–3 pp. 494–495

Factor each polynomial, if possible. If the polynomial cannot be factored using integers, write *prime*.

10.  $5x^2 + 34x + 24$
11.  $2x^2 + 19x + 24$
12.  $4x^2 + 22x + 10$
13.  $4x^2 + 38x + 70$
14.  $2x^2 - 3x - 9$
15.  $4x^2 - 13x + 10$
16.  $2x^2 + 3x + 6$
17.  $5x^2 + 3x + 4$
18.  $12x^2 + 69x + 45$
19.  $4x^2 - 5x + 7$
20.  $5x^2 + 23x + 24$
21.  $3x^2 - 8x + 15$

### Example 4 p. 495

22. **SHOT PUT** An athlete throws a shot put with an initial velocity of 29 feet per second and from an initial height of 6 feet.

- a. Write an equation that models the height of the shot put in feet with respect to time in seconds.
- b. After how many seconds will the shot put hit the ground?

Solve each equation. Check your solutions.

23.  $2x^2 + 9x - 18 = 0$
24.  $4x^2 + 17x + 15 = 0$
25.  $-3x^2 + 26x = 16$
26.  $-2x^2 + 13x = 15$
27.  $-3x^2 + 5x = -2$
28.  $-4x^2 + 19x = -30$

29. **BASKETBALL** When Jerald shoots a free throw, the ball is 6 feet from the floor and has an initial velocity of 20 feet per second. The hoop is 10 feet from the floor.

- a. Use the vertical motion model to determine an equation that models Jerald's free throw.
- b. How long is the basketball in the air before it reaches the hoop?
- c. Raymond shoots a free throw that is 5 foot 9 inches from the floor with the same initial velocity. Will the ball be in the air more or less time? Explain.

30. **DIVING** Ben dives from a 10-foot platform. The equation  $h = -16t^2 + 27t + 10$  models the dive. How long will it take Ben to reach the water?





### Real-World Career

#### Urban Planner

Urban planners design the layout of an area. They take into consideration the available land and geographical and environmental factors to design an area that benefits the community the most. City planners have a bachelor's degree in planning and almost half have a master's degree.

- 31. NUMBER THEORY** Six times the square of a number  $x$  plus 11 times the number equals 2. What are possible values of  $x$ ?

Factor each polynomial, if possible. If the polynomial cannot be factored using integers, write *prime*.

**32.**  $-6x^2 - 23x - 20$

**33.**  $-4x^2 - 15x - 14$

**34.**  $-5x^2 + 18x + 8$

**35.**  $-6x^2 + 31x - 35$

**36.**  $-4x^2 + 5x - 12$

**37.**  $-12x^2 + x + 20$

- 38. URBAN PLANNING** The city has commissioned the building of a new park. The area of the park can be expressed as  $660x^2 + 524x + 85$ . Factor this expression to find binomials with integer coefficients that represent possible dimensions of the park. If  $x = 8$ , what is the perimeter of the park?

- 39. MULTIPLE REPRESENTATIONS** In this problem, you will explore factoring a special type of polynomial.

- GEOMETRIC** Draw a square and label the sides  $a$ . Within this square, draw a smaller square that shares a vertex with the first square. Label the sides  $b$ . What are the areas of the two squares?
- GEOMETRIC** Cut and remove the small square. What is the area of the remaining region?
- ANALYTICAL** Draw a diagonal line between the inside corner and outside corner of the figure, and cut along this line to make two congruent pieces. Then rearrange the two pieces to form a rectangle. What are the dimensions?
- ANALYTICAL** Write the area of the rectangle as the product of two binomials.
- VERBAL** Complete this statement:  $a^2 - b^2 = \dots$  Why is this statement true?

### H.O.T. Problems

Use **H**igher-**O**rders **T**hinking Skills

- 40. FIND THE ERROR** Zachary and Samantha are solving  $6x^2 - x = 12$ . Is either of them correct? Explain your reasoning.

Zachary

$$\begin{aligned} 6x^2 - x &= 12 \\ x(6x - 1) &= 12 \\ x = 12 \text{ or } 6x - 1 &= 12 \\ 6x &= 13 \\ x &= \frac{13}{6} \end{aligned}$$

Samantha

$$\begin{aligned} 6x^2 - x &= 12 \\ 6x^2 - x - 12 &= 0 \\ (2x - 3)(3x + 4) &= 0 \\ 2x - 3 = 0 \text{ or } 3x + 4 &= 0 \\ x = \frac{3}{2} \quad x &= -\frac{4}{3} \end{aligned}$$

- 41. REASONING** A square has an area of  $9x^2 + 30xy + 25y^2$  square inches. The dimensions are binomials with positive integer coefficients. What is the perimeter of the square? Explain.
- 42. CHALLENGE** Find all values of  $k$  so that  $2x^2 + kx + 12$  can be factored as two binomials using integers.
- 43. OPEN ENDED** Write a quadratic equation with integer coefficients that has  $\frac{1}{2}$  and  $-\frac{3}{5}$  as solutions. Explain your reasoning.
- 44. WRITING IN MATH** Explain how to determine which values should be chosen for  $m$  and  $n$  when factoring a polynomial of the form  $ax^2 + bx + c$ .



## Standardized Test Practice

- 45. GRIDDED RESPONSE** Savannah has two sisters. One sister is 8 years older than her and the other sister is 2 years younger than her. The product of Savannah's sisters' ages is 56. How old is Savannah?

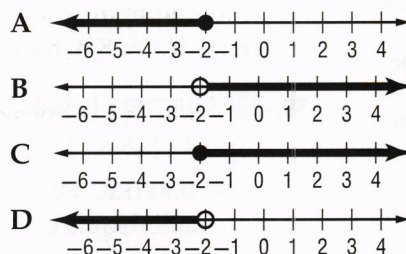
- 46.** What is the product of  $\frac{2}{3}a^3b^5$  and  $\frac{3}{5}a^5b^2$ ?

A  $\frac{2}{5}a^8b^7$   
 B  $\frac{2}{5}a^2b^3$   
 C  $\frac{2}{5}a^8b^3$   
 D  $\frac{2}{5}a^2b^7$

- 47.** What is the solution set of  $x^2 + 2x - 24 = 0$ ?

F  $\{-4, 6\}$                       H  $\{-3, 8\}$   
 G  $\{3, -8\}$                       J  $\{4, -6\}$

- 48.** Which is the solution set of  $x \geq -2$ ?



## Spiral Review

Factor each polynomial. (Lesson 8-3)

**49.**  $x^2 - 9x + 14$

**50.**  $n^2 - 8n + 15$

**51.**  $x^2 - 5x - 24$

**52.**  $z^2 + 15z + 36$

**53.**  $r^2 + 3r - 40$

**54.**  $v^2 + 16v + 63$

Solve each equation. Check your solutions. (Lesson 8-2)

**55.**  $a(a - 9) = 0$

**56.**  $(2y + 6)(y - 1) = 0$

**57.**  $10x^2 - 20x = 0$

**58.**  $8b^2 - 12b = 0$

**59.**  $15a^2 = 60a$

**60.**  $33x^2 = -22x$

- 61. ART** A painter has 32 units of yellow dye and 54 units of blue dye to make two shades of green. The units needed to make a gallon of light green and a gallon of dark green are shown. Make a graph showing the numbers of gallons of the two greens she can make, and list three possible solutions. (Lesson 6-8)

Color	Units of Yellow Dye	Units of Blue Dye
light green	4	1
dark green	1	6

Solve each compound inequality. Then graph the solution set. (Lesson 5-4)

**62.**  $k + 2 > 12$  and  $k + 2 \leq 18$

**63.**  $d - 4 > 3$  or  $d - 4 \leq 1$

**64.**  $3 < 2x - 3 < 15$

**65.**  $3t - 7 \geq 5$  and  $2t + 6 \leq 12$

**66.**  $h - 10 < -21$  or  $h + 3 < 2$

**67.**  $4 < 2y - 2 < 10$

- 68. FINANCIAL LITERACY** A home security company provides security systems for \$5 per week, plus an installation fee. The total cost for installation and 12 weeks of service is \$210. Write the point-slope form of an equation to find the total fee  $y$  for any number of weeks  $x$ . What is the installation fee? (Lesson 4-3)

## Skills Review

Find the principal square root of each number. (Lesson 0-2)

**69.** 16

**70.** 36

**71.** 64

**72.** 81

**73.** 121

**74.** 100



# Quadratic Equations: Differences of Squares

## Then

You factored trinomials into two binomials.  
(Lesson 8-3, 8-4)

## Now

- Factor binomials that are the difference of squares.
- Use the difference of squares to solve equations.

## New Vocabulary

difference of two squares

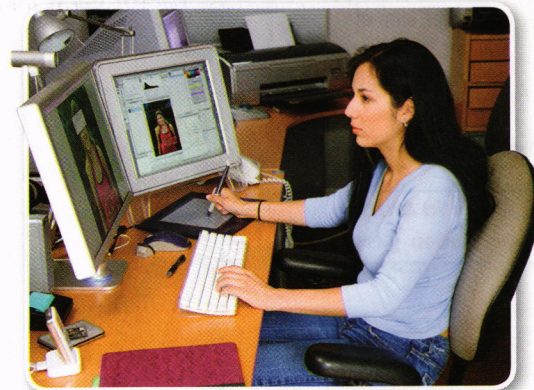
## Math Online

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- Personal Tutor
- Self-Check Quiz
- Homework Help
- Math in Motion

## Why?

Computer graphics designers use a combination of art and mathematics skills to design images and videos. They use equations to form shapes and lines on computers. Factoring can help to determine the dimensions and shapes of the figures.



**Factor Differences of Squares** Recall that in Lesson 7-8, you learned about the product of the sum and difference of two quantities. This resulting product is referred to as the **difference of two squares**. So, the factored form of the difference of squares is called the product of the sum and difference of the two quantities.



## Key Concept

## Difference of Squares

For Your

FOLDABLE

**Symbols**  $a^2 - b^2 = (a + b)(a - b)$  or  $(a - b)(a + b)$

**Examples**  $x^2 - 25 = (x + 5)(x - 5)$  or  $(x - 5)(x + 5)$

$t^2 - 64 = (t + 8)(t - 8)$  or  $(t - 8)(t + 8)$

## EXAMPLE 1 Factor Differences of Squares

Factor each polynomial.

a.  $16h^2 - 9a^2$

$$\begin{aligned} 16h^2 - 9a^2 &= (4h)^2 - (3a)^2 \\ &= (4h + 3a)(4h - 3a) \end{aligned}$$

Write in the form of  $a^2 - b^2$ .  
Factor the difference of squares.

b.  $121 - 4b^2$

$$\begin{aligned} 121 - 4b^2 &= (11)^2 - (2b)^2 \\ &= (11 - 2b)(11 + 2b) \end{aligned}$$

Write in the form of  $a^2 - b^2$ .  
Factor the difference of squares.

c.  $27g^3 - 3g$

Because the terms have a common factor, factor out the GCF first. Then proceed with other factoring techniques.

$$\begin{aligned} 27g^3 - 3g &= 3g(9g^2 - 1) \\ &= 3g[(3g)^2 - (1)^2] \\ &= 3g(3g - 1)(3g + 1) \end{aligned}$$

Factor out the GCF of  $3g$ .  
Write in the form  $a^2 - b^2$ .  
Factor the difference of squares.



## Check Your Progress

1A.  $81 - c^2$

1C.  $9x^3 - 4x$

1B.  $64g^2 - h^2$

1D.  $-4y^3 + 9y$

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**Watch Out!**

**Sum of Squares** The sum of squares,  $a^2 + b^2$ , does not factor into  $(a + b)(a + b)$ . The sum of squares is a prime polynomial and cannot be factored.

**Math in Motion,**  
Animation [glencoe.com](http://glencoe.com)

To factor a polynomial completely, a technique may need to be applied more than once. This also applies to the difference of squares pattern.

**EXAMPLE 2 Apply a Technique More than Once**

Factor each polynomial.

a.  $b^4 - 16$

$$\begin{aligned} b^4 - 16 &= (b^2)^2 - (4)^2 \\ &= (b^2 + 4)(b^2 - 4) \end{aligned}$$

Write  $b^4 - 16$  in  $a^2 - b^2$  form.

Factor the difference of squares.

Notice that the factor  $b^2 - 4$  is also the difference of squares.

$$\begin{aligned} &= (b^2 + 4)(b^2 - 2^2) \\ &= (b^2 + 4)(b + 2)(b - 2) \end{aligned}$$

Write  $b^2 - 4$  in  $a^2 - b^2$  form.

Factor the difference of squares.

b.  $625 - x^4$

$$\begin{aligned} 625 - x^4 &= (25)^2 - (x^2)^2 \\ &= (25 + x^2)(25 - x^2) \\ &= (25 + x^2)(5^2 - x^2) \\ &= (25 + x^2)(5 - x)(5 + x) \end{aligned}$$

Write  $625 - x^4$  in  $a^2 - b^2$  form.

Factor the difference of squares.

Write  $25 - x^2$  in  $a^2 - b^2$  form.

Factor the difference of squares.

**Check Your Progress**

Factor each polynomial.

2A.  $y^4 - 1$

2B.  $4a^4 - b^4$

2C.  $81 - x^4$

2D.  $16y^4 - 1$

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Sometimes more than one factoring technique needs to be applied to ensure that a polynomial is factored completely.

**EXAMPLE 3 Apply Different Techniques**

Factor each polynomial.

a.  $5x^5 - 45x$

$$\begin{aligned} 5x^5 - 45x &= 5x(x^4 - 9) \\ &= 5x[(x^2)^2 - (3)^2] \\ &= 5x(x^2 - 3)(x^2 + 3) \end{aligned}$$

Factor out GCF.

Write  $x^4 - 9$  in the form  $a^2 - b^2$ .

Factor the difference of squares.

Notice that the factor  $x^2 - 3$  is not the difference of squares because 3 is not a perfect square.

b.  $7x^3 + 21x^2 - 7x - 21$

$$\begin{aligned} 7x^3 + 21x^2 - 7x - 21 &= 7(x^3 + 3x^2 - x - 3) \\ &= 7[(x^3 + 3x^2) - (x + 3)] \\ &= 7[x^2(x + 3) - 1(x + 3)] \\ &= 7(x + 3)(x^2 - 1) \\ &= 7(x + 3)(x + 1)(x - 1) \end{aligned}$$

Original expression

Factor out GCF.

Group terms with common factors.

Factor each grouping.

$x + 3$  is the common factor.

Factor the difference of squares.

**Check Your Progress**

Factor each polynomial.

3A.  $2y^4 - 50$

3B.  $6x^4 - 96$

3C.  $2m^3 + m^2 - 50m - 25$

3D.  $r^3 + 6r^2 + 11r + 66$

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**Solve Equations by Factoring** After factoring, you can apply the Zero Product Property to an equation that is written as the product of factors set equal to 0.

### STANDARDIZED TEST EXAMPLE 4

In the equation  $y = x^2 - \frac{9}{16}$ , which is a value of  $x$  when  $y = 0$ ?

A  $-\frac{9}{4}$

B 0

C  $\frac{3}{4}$

D  $\frac{9}{4}$

#### Test-Taking Tip

##### Use Another Method

Another method that can be used to solve this equation is to substitute each answer choice into the equation.

#### Read the Test Item

Replace  $y$  with 0 and then solve.

#### Solve the Test Item

$$y = x^2 - \frac{9}{16}$$

Original equation

$$0 = x^2 - \frac{9}{16}$$

Replace  $y$  with 0.

$$0 = x^2 - \left(\frac{3}{4}\right)^2$$

Write in the form  $a^2 - b^2$ .

$$0 = \left(x + \frac{3}{4}\right)\left(x - \frac{3}{4}\right)$$

Factor the difference of squares.

$$0 = x + \frac{3}{4} \quad \text{or} \quad 0 = x - \frac{3}{4}$$

Zero Product Property

$$x = -\frac{3}{4} \quad x = \frac{3}{4}$$

The correct answer is C.



#### Check Your Progress

4. Which are the solutions of  $18x^3 = 50x$ ?

F  $0, \frac{5}{3}$

G  $-\frac{5}{3}, \frac{5}{3}$

H  $-\frac{5}{3}, \frac{5}{3}, 0$

J  $-\frac{5}{3}, \frac{5}{3}, 1$

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### Check Your Understanding

#### Examples 1–3

pp. 499–500

Factor each polynomial.

1.  $x^2 - 9$

2.  $4a^2 - 25$

3.  $9m^2 - 144$

4.  $2p^3 - 162p$

5.  $u^4 - 81$

6.  $2d^4 - 32f^4$

7.  $20r^4 - 45n^4$

8.  $256n^4 - c^4$

9.  $2c^3 + 3c^2 - 2c - 3$

10.  $f^3 - 4f^2 - 9f + 36$

11.  $3t^3 + 2t^2 - 48t - 32$

12.  $w^3 - 3w^2 - 9w + 27$

#### Example 4

p. 501

**EXTENDED RESPONSE** After an accident, skid marks may result from sudden breaking. The formula  $\frac{1}{24}s^2 = d$  approximates a vehicle's speed  $s$  in miles per hour given the length  $d$  in feet of the skid marks on dry concrete.

13. If skid marks on dry concrete are 54 feet long, how fast was the car traveling when the brakes were applied?
14. If the skid marks on dry concrete are 150 feet long, how fast was the car traveling when the brakes were applied?



# Practice and Problem Solving

= **Step-by-Step Solutions** begin on page R12.  
**Extra Practice** begins on page 815.

## Examples 1–3

pp. 499–500

Factor each polynomial.

15.  $q^2 - 121$

16.  $r^4 - k^4$

17.  $6n^4 - 6$

18.  $w^4 - 625$

19.  $r^2 - 9t^2$

20.  $2c^2 - 32d^2$

21.  $h^3 - 100h$

22.  $h^4 - 256$

23.  $2x^3 - x^2 - 162x + 81$

24.  $x^2 - 4y^2$

25.  $7h^4 - 7p^4$

26.  $3c^3 + 2c^2 - 147c - 98$

27.  $6k^2h^4 - 54k^4$

28.  $5a^3 - 20a$

29.  $f^3 + 2f^2 - 64f - 128$

30.  $3r^3 - 192r$

31.  $10q^3 - 1210q$

32.  $3xn^4 - 27x^3$

33.  $p^3r^5 - p^3r$

34.  $8c^3 - 8c$

35.  $r^3 - 5r^2 - 100r + 500$

36.  $3t^3 - 7t^2 - 3t + 7$

37.  $a^2 - 49$

38.  $4m^3 + 9m^2 - 36m - 81$

39.  $3m^4 + 243$

40.  $3x^3 + x^2 - 75x - 25$

41.  $12a^3 + 2a^2 - 192a - 32$

42.  $x^4 + 6x^3 - 36x^2 - 216x$

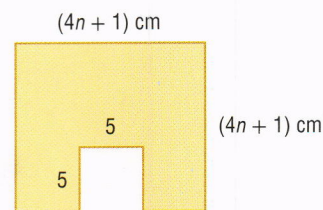
43.  $15m^3 + 12m^2 - 375m - 300$

## Example 4

p. 501

44. **GEOMETRY** The drawing at the right is a square with a square cut out of it.

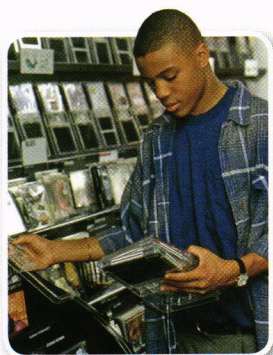
- Write an expression that represents the area of the shaded region.
- Find the dimensions of a rectangle with the same area as the shaded region in the drawing. Assume that the dimensions of the rectangle must be represented by binomials with integral coefficients.



45. **DECORATIONS** An arch decorated with balloons was used to decorate the gym for the spring dance. The shape of the arch can be modeled by the equation  $y = -0.5x^2 + 4.5x$ , where  $x$  and  $y$  are measured in feet and the  $x$ -axis represents the floor.

- Write the expression that represents the height of the arch in factored form.
- How far apart are the two points where the arch touches the floor?
- Graph this equation on your calculator. What is the highest point of the arch?

46. **DECKS** Zelda is building a deck in her backyard. The plans for the deck show that it is to be 24 feet by 24 feet. Zelda wants to reduce one dimension by a number of feet and increase the other dimension by the same number of feet. If the area of the reduced deck is 512 square feet, what are the dimensions of the deck?



## Real-World Link

Teens were asked "If you were given \$20 today, what would you most likely spend it on?" Of the teens surveyed, 32% would spend it on a music CD.

Source: USA TODAY

47. **SALES** The sales of a particular CD can be modeled by the equation  $S = -25m^2 + 125m$ , where  $S$  is the number of CDs sold in thousands, and  $m$  is the number of months that it is on the market.

- In what month should the music store expect the CD to stop selling?
- In what month will CD sales peak?
- How many copies will the CD sell at its peak?



**StudyTip****Solving an Equation By Factoring**

Remember to get 0 on one side of the equation before factoring.

Solve each equation by factoring. Check your solutions.

48.  $36w^2 = 121$

50.  $64x^2 - 1 = 0$

52.  $\frac{1}{4}b^2 = 16$


54.  $9d^2 - 81 = 0$

49.  $100 = 25x^2$

51.  $4y^2 - \frac{9}{16} = 0$

53.  $81 - \frac{1}{25}x^2 = 0$

55.  $4a^2 = \frac{9}{64}$

56.  **MULTIPLE REPRESENTATIONS** In this problem, you will investigate perfect square trinomials.

- a. **TABULAR** Copy and complete the table below by factoring each polynomial. Then write the first and last terms of the given polynomials as perfect squares.

Polynomial	Factored Polynomial	First Term	Last Term	Middle Term
$4x^2 + 12x + 9$	$(2x + 3)(2x + 3)$	$4x^2 = (2x)^2$	$9 = 3^2$	
$9x^2 - 24x + 16$				
$4x^2 - 20x + 25$				
$16x^2 + 24x + 9$				
$25x^2 + 20x + 4$				

- b. **ANALYTICAL** Write the middle term of each polynomial using the square roots of the perfect squares of the first and last terms.

- c. **ALGEBRAIC** Write the pattern for a perfect square trinomial.

- d. **VERBAL** What conditions must be met for a trinomial to be classified as a perfect square trinomial?

**H.O.T. Problems**

Use **H**igher-**O**rders **T**hinking Skills

57. **FIND THE ERROR** Elizabeth and Lorenzo are factoring an expression. Is either of them correct? Explain your reasoning.

Elizabeth

$$16x^4 - 25y^2 = (4x - 5y)(4x + 5y)$$

Lorenzo

$$16x^4 - 25y^2 = (4x^2 - 5y)(4x^2 + 5y)$$

58. **CHALLENGE** Factor and simplify  $9 - (k + 3)^2$ , a difference of squares.
59. **CHALLENGE** Factor  $x^{16} - 81$ .
60. **REASONING** Write and factor a binomial that is the difference of two perfect squares and that has a greatest common factor of  $5mk$ .
61. **REASONING** Determine whether the following statement is *true* or *false*. Give an example or counterexample to justify your answer.  
*All binomials that have a perfect square in each of the two terms can be factored.*
62. **OPEN ENDED** Write a binomial in which the difference of squares pattern must be repeated to factor it completely. Then factor the binomial.
63. **WRITING IN MATH** Describe why the difference of squares pattern has no middle term with a variable.



## Standardized Test Practice

64. One of the roots of  $2x^2 + 13x = 24$  is  $-8$ . What is the other root?

A  $-\frac{3}{2}$

C  $\frac{2}{3}$

B  $\frac{3}{2}$

D  $-\frac{2}{3}$

65. Which of the following is the sum of both solutions of the equation  $x^2 + 3x = 54$ ?

F  $-21$

H  $3$

G  $-3$

J  $21$

66. What are the  $x$ -intercepts of the graph of  $y = -3x^2 + 7x + 20$ ?

A  $\frac{5}{3}, -4$

C  $-\frac{5}{3}, 4$

B  $-\frac{5}{3}, -4$

D  $\frac{5}{3}, 4$

67. **EXTENDED RESPONSE** Two cars leave Cleveland at the same time from different parts of the city and both drive to Cincinnati. The distance in miles of the cars from the center of Cleveland can be represented by the two equations below, where  $t$  represents the time in hours.

Car A:  $65t + 15$

Car B:  $60t + 25$

- Which car is faster? Explain.
- Find an expression that models the distance between the two cars.
- How far apart are the cars after  $2\frac{1}{2}$  hours?

## Spiral Review

Factor each trinomial, if possible. If the trinomial cannot be factored using integers, write *prime*. (Lesson 8-4)

68.  $5x^2 - 17x + 14$

69.  $5a^2 - 3a + 15$

70.  $10x^2 - 20xy + 10y^2$

Solve each equation. Check your solutions. (Lesson 8-3)

71.  $n^2 - 9n = -18$

72.  $10 + a^2 = -7a$

73.  $22x - x^2 = 96$

74. **SAVINGS** Victoria and Trey each want to buy a scooter. In how many weeks will Victoria and Trey have saved the same amount of money, and how much will each of them have saved? (Lesson 6-1)

Solve each inequality. Graph the solution set on a number line. (Lesson 5-1)

75.  $t + 14 \geq 18$

76.  $d + 5 \leq 7$

77.  $-5 + k > -1$

78.  $5 < 3 + g$

79.  $2 \leq -1 + m$

80.  $2y > -8 + y$

81. **FITNESS** Silvia is beginning an exercise program that calls for 20 minutes of walking each day for the first week. Each week thereafter, she has to increase her daily walking for a week by 7 minutes. In which week will she first walk over an hour a day? (Lesson 3-5)



## Skills Review

Find each product. (Lesson 7-8)

82.  $(x - 6)^2$

83.  $(x - 2)(x - 2)$

84.  $(x + 3)(x + 3)$

85.  $(2x - 5)^2$

86.  $(6x - 1)^2$

87.  $(4x + 5)(4x + 5)$



## Quadratic Equations: Perfect Squares

### Why?

In a vacuum, a feather and a piano would fall at the same speed, or velocity. To find about how long it takes an object to hit the ground if it is dropped from an initial height of  $h_0$  feet above ground, you would need to solve the equation  $0 = -16t^2 + h_0$ , where  $t$  is time in seconds after the object is dropped.



### Then

You found the product of a sum and difference. (Lesson 7-8)

### Now

- Factor perfect square trinomials.
- Solve equations involving perfect squares.

### New Vocabulary

perfect square trinomial

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**Factor Perfect Square Trinomials** In Lesson 7-8, you learned the patterns for the products of the binomials  $(a + b)^2$  and  $(a - b)^2$ . Recall that these are special products that follow specific patterns.

$$(a + b)^2 = (a + b)(a + b)$$

$$= a^2 + ab + ab + b^2$$

$$= a^2 + 2ab + b^2$$

$$(a - b)^2 = (a - b)(a - b)$$

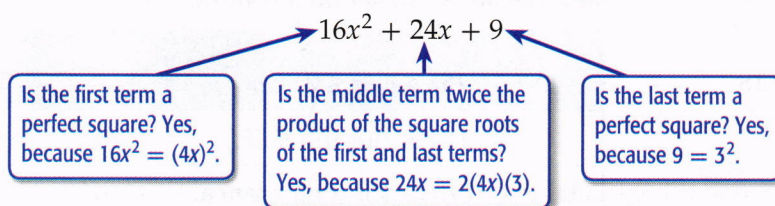
$$= a^2 - ab - ab + b^2$$

$$= a^2 - 2ab + b^2$$

These products are called **perfect square trinomials**, because they are the squares of binomials. The above patterns can help you factor perfect square trinomials.

For a trinomial to be factorable as a perfect square, the first and last terms must be perfect squares and the middle term must be two times the square roots of the first and last terms.

The trinomial  $16x^2 + 24x + 9$  is a perfect square trinomial, as illustrated below.



### Key Concept

### Factoring Perfect Square Trinomials

For Your

**FOLDABLE**

**Symbols**  $a^2 + 2ab + b^2 = (a + b)(a + b) = (a + b)^2$

$$a^2 - 2ab + b^2 = (a - b)(a - b) = (a - b)^2$$

**Examples**  $x^2 + 8x + 16 = (x + 4)(x + 4) \text{ or } (x + 4)^2$

$$x^2 - 6x + 9 = (x - 3)(x - 3) \text{ or } (x - 3)^2$$



**StudyTip****Recognizing Perfect Square Trinomials**

If the constant term of the trinomial is negative, the trinomial is not a perfect square trinomial, so it is not necessary to check the other conditions.

**EXAMPLE 1 Recognize and Factor Perfect Square Trinomials**

Determine whether each trinomial is a perfect square trinomial. Write *yes* or *no*. If so, factor it.

a.  $4y^2 + 12y + 9$

- ① Is the first term a perfect square? Yes,  $4y^2 = (2y)^2$ .
- ② Is the last term a perfect square? Yes,  $9 = 3^2$ .
- ③ Is the middle term equal to  $2(2y)(3)$ ? Yes,  $12y = 2(2y)(3)$

Since all three conditions are satisfied,  $4y^2 + 12y + 9$  is a perfect square trinomial.

$$4y^2 + 12y + 9 = (2y)^2 + 2(2y)(3) + 3^2 \\ = (2y + 3)^2$$

**Write as  $a^2 + 2ab + b^2$ .**  
**Factor using the pattern.**

b.  $9x^2 - 6x + 4$

- ① Is the first term a perfect square? Yes,  $9x^2 = (3x)^2$ .
- ② Is the last term a perfect square? Yes,  $4 = 2^2$ .
- ③ Is the middle term equal to  $-2(3x)(2)$ ? No,  $-6x \neq -2(3x)(2)$ .

Since the middle term does not satisfy the required condition,  $9x^2 - 6x + 4$  is not a perfect square trinomial.

**Check Your Progress**

1A.  $9y^2 + 24y + 16$

1B.  $2a^2 + 10a + 25$

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A polynomial is completely factored when it is written as a product of prime polynomials. More than one method might be needed to factor a polynomial completely. When completely factoring a polynomial, the Concept Summary can help you decide where to start.

Remember, if the polynomial does not fit any pattern or cannot be factored, the polynomial is prime.

Concept Summary		Factoring Methods	For Your FOLDABLE
Steps	Number of Terms	Examples	
<b>Step 1</b> Factor out the GCF.	any	$4x^3 + 2x^2 - 6x = 2x(2x^2 + x - 3)$	
<b>Step 2</b> Check for a difference of squares or a perfect square trinomial.	2 or 3	$9x^2 - 16 = (3x + 4)(3x - 4)$ $16x^2 + 24x + 9 = (4x + 3)^2$	
<b>Step 3</b> Apply the factoring patterns for $x^2 + bx + c$ or $ax^2 + bx + c$ (general trinomials), or factor by grouping.	3 or 4	$x^2 - 8x + 12 = (x - 2)(x - 6)$ $2x^2 + 13x + 6 = (2x + 1)(x + 6)$ $12y^2 + 9y + 8y + 6$ $= (12y^2 + 9y) + (8y + 6)$ $= 3y(4y + 3) + 2(4y + 3)$ $= (4y + 3)(3y + 2)$	



**EXAMPLE 2 Factor Completely**

Factor each polynomial, if possible. If the polynomial cannot be factored, write *prime*.

a.  $5x^2 - 80$

**Step 1** The GCF of  $5x^2$  and  $-80$  is 5, so factor it out.

**Step 2** Since there are two terms, check for a difference of squares.

$$\begin{aligned} 5x^2 - 80 &= 5(x^2 - 16) && \text{5 is the GCF of the terms.} \\ &= 5(x^2 - 4^2) && x^2 = x \cdot x \text{ and } 16 = 4 \cdot 4 \\ &= 5(x - 4)(x + 4) && \text{Factor the difference of squares.} \end{aligned}$$

b.  $9x^2 - 6x - 35$

**Step 1** The GCF of  $9x^2$ ,  $-6x$ , and  $-35$  is 1.

**Step 2** Since 35 is not a perfect square, this is not a perfect square trinomial.

**Step 3** Factor using the pattern  $ax^2 + bx + c$ . Are there two numbers with a product of  $9(-35)$  or  $-315$  and a sum of  $-6$ ? Yes, the product of 15 and  $-21$  is  $-315$ , and the sum is  $-6$ .

$$\begin{aligned} 9x^2 - 6x - 35 &= 9x^2 + mx + nx - 35 && \text{Write the pattern.} \\ &= 9x^2 + 15x - 21x - 35 && m = 15 \text{ and } n = -21 \\ &= (9x^2 + 15x) + (-21x - 35) && \text{Group terms with common factors.} \\ &= 3x(3x + 5) - 7(3x + 5) && \text{Factor out the GCF from each grouping.} \\ &= (3x + 5)(3x - 7) && 3x + 5 \text{ is the common factor.} \end{aligned}$$

**StudyTip****Check Your Answer**

You can check your answer by:

- Using the FOIL method.
- Using the Distributive Property.
- Graphing the original expression and factored expression and comparing the graphs.

If the product of the factors does not match the original expression exactly, the answer is incorrect.

**Check Your Progress**

2A.  $2x^2 - 32$

2B.  $12x^2 + 5x - 25$



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**Solve Equations with Perfect Squares** When solving equations involving repeated factors, it is only necessary to set one of the repeated factors equal to zero.

**EXAMPLE 3 Solve Equations with Repeated Factors**

Solve  $9x^2 - 48x = -64$ .

$$\begin{aligned} 9x^2 - 48x &= -64 && \text{Original equation} \\ 9x^2 - 48x + 64 &= 0 && \text{Add 64 to each side.} \\ (3x)^2 - 2(3x)(8) + (8)^2 &= 0 && \text{Recognize } 9x^2 - 48x + 64 \text{ as a perfect square trinomial.} \\ (3x - 8)^2 &= 0 && \text{Factor the perfect square trinomial.} \\ (3x - 8)(3x - 8) &= 0 && \text{Write } (3x - 8)^2 \text{ as two factors.} \\ 3x - 8 &= 0 && \text{Set the repeated factor equal to zero.} \\ 3x &= 8 && \text{Add 8 to each side.} \\ x &= \frac{8}{3} && \text{Divide each side by 3.} \end{aligned}$$

**Check Your Progress**

Solve each equation. Check your solutions.

3A.  $a^2 + 12a + 36 = 0$

3B.  $y^2 - \frac{4}{3}y + \frac{4}{9} = 0$



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You have solved equations like  $x^2 - 16 = 0$  by factoring. You can also use the definition of a square root to solve the equation.

### ReadingMath

#### Square Root

**Solutions**  $\pm\sqrt{16}$  is read as *plus or minus the square root of 16*.

$$x^2 - 16 = 0$$

Original equation

$$x^2 = 16$$

Add 16 to each side.

$$x = \pm\sqrt{16}$$

Take the square root of each side.

Remember that there are two square roots of 16, namely 4 and  $-4$ . Therefore, the solution set is  $\{-4, 4\}$ . You can express this as  $\{\pm 4\}$ .



### Key Concept

#### Square Root Property

For Your

FOLDABLE

**Words** To solve a quadratic equation in the form  $x^2 = n$ , take the square root of each side.

**Symbols** For any number  $n \geq 0$ , if  $x^2 = n$ , then  $x = \pm\sqrt{n}$ .

**Example**  $x^2 = 25$   
 $x = \pm\sqrt{25}$  or  $\pm 5$

In the equation  $x^2 = n$ , if  $n$  is not a perfect square, you need to approximate the square root. Use a calculator to find an approximation. If  $n$  is a perfect square, you will have an exact answer.

### EXAMPLE 4 Use the Square Root Property

Solve each equation. Check your solutions.

a.  $(y - 6)^2 = 81$

$$(y - 6)^2 = 81$$

Original equation

$$y - 6 = \pm\sqrt{81}$$

Square Root Property

$$y - 6 = \pm 9$$

$$81 = 9 \cdot 9$$

$$y = 6 \pm 9$$

Add 6 to each side.

$$y = 6 + 9 \quad \text{or} \quad y = 6 - 9$$

Separate into two equations.

$$= 15 \qquad = -3$$

Simplify.

The roots are 15 and  $-3$ .

Check in the original equation.

b.  $(x + 6)^2 = 12$

$$(x + 6)^2 = 12$$

Original equation

$$x + 6 = \pm\sqrt{12}$$

Square Root Property

$$x = -6 \pm \sqrt{12}$$

Subtract 6 from each side.

The roots are  $-6 \pm \sqrt{12}$  or  $-6 + \sqrt{12}$  and  $-6 - \sqrt{12}$ .

Using a calculator,  $-6 + \sqrt{12} \approx -2.54$  and  $-6 - \sqrt{12} \approx -9.46$ .



### Check Your Progress

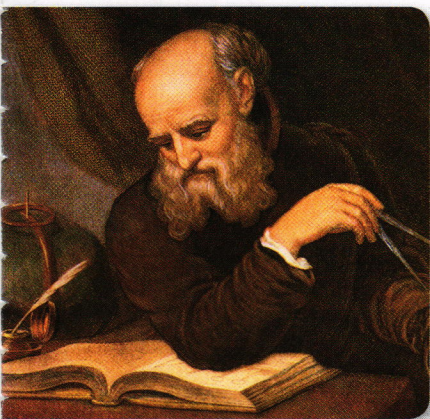
4A.  $(a - 10)^2 = 121$

4B.  $(z + 3)^2 = 26$

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### Math History Link

#### Galileo Galilei

(1564–1642)

Galileo was the first person to prove that objects of different weights fall at the same velocity by dropping two objects of different weights from the top of the Leaning Tower of Pisa in 1589.

### Real-World EXAMPLE 5 Solve an Equation

**PHYSICAL SCIENCE** During an experiment, a ball is dropped from a height of 205 feet. The formula  $h = -16t^2 + h_0$  can be used to approximate the number of seconds  $t$  it takes for the ball to reach height  $h$  from an initial height of  $h_0$  in feet. Find the time it takes the ball to reach the ground.

At ground level,  $h = 0$  and the initial height is 205, so  $h_0 = 205$ .

$$h = -16t^2 + h_0 \quad \text{Original Formula}$$

$$0 = -16t^2 + 205 \quad \text{Replace } h \text{ with } 0 \text{ and } h_0 \text{ with } 205.$$

$$-205 = -16t^2 \quad \text{Subtract 205 from each side.}$$

$$12.8125 = t^2 \quad \text{Divide each side by } -16.$$

$$\pm 3.6 \approx t \quad \text{Use the Square Root Property.}$$

Since a negative number does not make sense in this situation, the solution is 3.6. It takes about 3.6 seconds for the ball to reach the ground.

### Check Your Progress

- Find the time it takes a ball to reach the ground if it is dropped from a bridge that is half as high as the one described above.

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### Check Your Understanding

#### Example 1 p. 506

Determine whether each trinomial is a perfect square trinomial. Write *yes* or *no*. If so, factor it.

1.  $25x^2 + 60x + 36$

2.  $6x^2 + 30x + 36$

#### Example 2 p. 507

Factor each polynomial, if possible. If the polynomial cannot be factored, write *prime*.

3.  $2x^2 - x - 28$

4.  $6x^2 - 34x + 48$

5.  $4x^2 + 64$

6.  $4x^2 + 9x - 16$

#### Examples 3 and 4 pp. 507–508

Solve each equation. Check your solutions.

7.  $4x^2 = 36$

8.  $25a^2 - 40a = -16$

9.  $64y^2 - 48y + 18 = 9$

10.  $(z + 5)^2 = 47$

#### Example 5 p. 509

- PAINT** While painting his bedroom, Nick drops his paintbrush off his ladder from a height of 6 feet. Use the formula  $h = -16t^2 + h_0$  to approximate the number of seconds it takes for the paintbrush to hit the floor.

### Practice and Problem Solving

#### Example 1 p. 506

Determine whether each trinomial is a perfect square trinomial. Write *yes* or *no*. If so, factor it.

12.  $4x^2 - 42x + 110$

13.  $16x^2 - 56x + 49$

14.  $81x^2 - 90x + 25$

15.  $x^2 + 26x + 168$

 = **Step-by-Step Solutions** begin on page R12.  
**Extra Practice** begins on page 815.



**Example 2**  
p. 507

Factor each polynomial, if possible. If the polynomial cannot be factored, write *prime*.

16.  $24d^2 + 39d - 18$

18.  $2b^2 + 12b - 24$

20.  $16a^2 - 121b^2$

22.  $8c^2 - 88c + 242$

24.  $w^4 - w^2$

26.  $16q^3 - 48q^2 + 36q$

28.  $x^3 + 2x^2y - 4x - 8y$

30.  $2r^3 - r^2 - 72r + 36$

32.  $4c^4d - 10c^3d + 4c^2d^3 - 10cd^3$

17.  $8x^2 + 10x - 21$

19.  $8y^2 - 200z^2$

21.  $12m^3 - 22m^2 - 70m$

23.  $12x^2 - 84x + 147$

25.  $12p^3 - 3p$

27.  $4t^3 + 10t^2 - 84t$

29.  $2a^2b^2 - 2a^2 - 2ab^3 + 2ab$

31.  $3k^3 - 24k^2 + 48k$

33.  $g^2 + 2g - 3h^2 + 4h$

**Examples 3 and 4**  
pp. 507–508

Solve each equation. Check the solutions.

34.  $4m^2 - 24m + 36 = 0$

36.  $a^2 + \frac{10}{7}a + \frac{25}{49} = 0$

38.  $x^2 + 8x + 16 = 25$

40.  $4x^2 = 80x - 400$

42.  $4c^2 + 4c + 1 = 15$

35.  $(y - 4)^2 = 7$

37.  $x^2 - \frac{3}{2}x + \frac{9}{16} = 0$

39.  $5x^2 - 60x = -180$

41.  $9 - 54x = -81x^2$

43.  $x^2 - 16x + 64 = 6$

44. **PHYSICAL SCIENCE** For an experiment in physics class, a water balloon is dropped from the window of the school building. The window is 40 feet high. How long does it take until the balloon hits the ground? Round to the nearest hundredth.

45. **SCREENS** The area  $A$  in square feet of a projected picture on a movie screen can be modeled by the equation  $A = 0.25d^2$ , where  $d$  represents the distance from a projector to a movie screen. At what distance will the projected picture have an area of 100 square feet?

**Example 5**  
p. 509

46. **GEOMETRY** The area of a square is represented by  $9x^2 - 42x + 49$ . Find the length of each side.

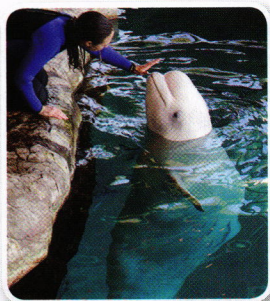
47. **GEOMETRY** The area of a square is represented by  $16x^2 + 40x + 25$ . Find the length of each side.

48. **ELECTION** For the student council elections, Franco is building the voting box shown with a volume of 672 cubic inches.



- a. Write a polynomial that represents the volume of the box.

- b. What are the dimensions of the voting box?



**Real-World Link**

The Shedd Aquarium in Chicago, Illinois, is home to over 1500 species of fish and mammals.

Source: Chicago Traveler

49. **AQUARIUM** Dexter has a fish tank shaped like a rectangular prism. It has a volume of 480 cubic inches. The height of the tank is 8 inches taller than the width, and the length of the tank is 6 inches longer than the width.

- a. Write a polynomial that represents the volume of the fish tank.

- b. What are the dimensions of the fish tank?





### Real-World Link

In 2005, 219,000 above-ground swimming pools were sold. This brought the total number of above-ground swimming pools in the United States to 3.6 million.

Source: Association of Pool & Spa Professionals

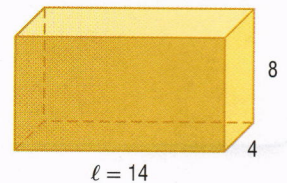
50. **GEOMETRY** The volume of a rectangular prism is represented by the expression  $8y^3 + 40y^2 + 50y$ . Find the possible dimensions of the prism if the dimensions are represented by polynomials with integer coefficients.

51. **POOLS** Ichiro wants to buy an above-ground swimming pool for his yard. Model A is 42 inches deep and holds 1750 cubic feet of water. The length of the rectangular pool is 5 feet more than the width.

- What is the surface area of the water?
- What are the dimensions of the pool?
- Model B pool holds twice as much water as Model A. What are some possible dimensions for this pool?
- Model C has length and width that are both twice as long as Model A, but the height is the same. What is the ratio of the volume of Model A to Model C?

52. **GEOMETRY** Use the rectangular prism at the right.

- Write an expression for the height and width of the prism in terms of the length,  $\ell$ .
- Write a polynomial for the volume of the prism in terms of the length.



### H.O.T. Problems

Use **H**igher-**O**rders **T**hinking Skills

53. **FIND THE ERROR** Debbie and Adriano are factoring the expression  $x^8 - x^4$  completely. Is either of them correct? Explain your reasoning.

Debbie

$$x^8 - x^4 = x^4(x^2 + 1)(x^2 - 1)$$

Adriano

$$x^8 - x^4 = x^4(x^2 + 1)(x - 1)(x + 1)$$

54. **CHALLENGE** Factor  $x^{n+6} + x^{n+2} + x^n$  completely.
55. **OPEN ENDED** Write a perfect square trinomial equation in which the coefficient of the middle term is negative and the last term is a fraction. Solve the equation.
56. **REASONING** Find a counterexample to the following statement.  
*A polynomial equation of degree three always has three real solutions.*
57. **WRITING IN MATH** Explain how to factor a polynomial completely.
58. **WHICH ONE DOESN'T BELONG?** Identify the trinomial that does not belong. Explain.

$$4x^2 - 36x + 81$$

$$25x^2 + 10x + 1$$

$$4x^2 + 10x + 4$$

$$9x^2 - 24x + 16$$

59. **OPEN ENDED** Write a binomial that can be factored using the difference of two squares twice. Set your binomial equal to zero and solve the equation.
60. **WRITING IN MATH** Explain how to determine whether a trinomial is a perfect square trinomial.

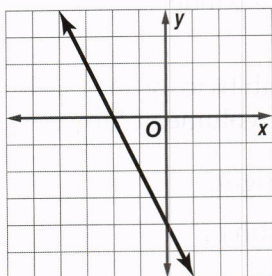


## Standardized Test Practice

61. What is the solution set for the equation  $(x - 3)^2 = 25$ ?

A  $\{-8, 2\}$                       C  $\{4, 14\}$   
B  $\{-2, 8\}$                       D  $\{-4, 14\}$

62. **SHORT RESPONSE** Write an equation in slope-intercept form for the graph shown below.



63. At an amphitheater, the price of 2 lawn seats and 2 pavilion seats is \$120. The price of 3 lawn seats and 4 pavilion seats is \$225. How much do lawn and pavilion seats cost?

F \$20 and \$41.25  
G \$10 and \$50  
H \$15 and \$45  
J \$30 and \$30

64. **GEOMETRY** The circumference of a circle is  $\frac{6\pi}{5}$  units. What is the area of the circle?

A  $\frac{3\pi}{5}$  units<sup>2</sup>                      C  $\frac{9\pi}{25}$  units<sup>2</sup>  
B  $\frac{12\pi}{5}$  units<sup>2</sup>                      D  $\frac{30\pi}{25}$  units<sup>2</sup>

## Spiral Review

Factor each polynomial, if possible. If the polynomial cannot be factored, write *prime*. (Lesson 8-5)

65.  $x^2 - 16$

66.  $4x^2 - 81y^2$

67.  $1 - 100p^2$

68.  $3a^2 - 20$

69.  $25n^2 - 1$

70.  $36 - 9c^2$

Solve each equation. Check your solutions. (Lesson 8-4)

71.  $4x^2 - 8x - 32 = 0$

72.  $6x^2 - 48x + 90 = 0$

73.  $14x^2 + 14x = 28$

74.  $2x^2 - 10x = 48$

75.  $5x^2 - 25x = -30$

76.  $8x^2 - 16x = 192$

**SOUND** The intensity of sound can be measured in watts per square meter. The table gives the watts per square meter for some common sounds. (Lesson 7-2)

77. How many times more intense is the sound from busy street traffic than sound from normal conversation?

78. Which sound is 10,000 times as loud as a busy street traffic?

79. How does the intensity of a whisper compare to that of normal conversation?

Watts Per Square Meter	Common Sounds
$10^{-11}$	rustling leaves
$10^{-10}$	whisper
$10^{-6}$	normal conversation
$10^{-5}$	busy street traffic
$10^{-4}$	vacuum cleaner
$10^{-1}$	front rows of rock concert
$10^1$	threshold of pain
$10^2$	military jet takeoff

## Skills Review

Find the slope of the line that passes through each pair of points. (Lesson 3-3)

80.  $(5, 7), (-2, -3)$

81.  $(2, -1), (5, -3)$

82.  $(-4, -1), (-3, -3)$

83.  $(-3, -4), (5, -1)$

84.  $(-2, 3), (8, 3)$

85.  $(-5, 4), (-5, -1)$



## Chapter Summary

### Key Concepts

#### Monomials and Factoring (Lesson 8-1)

- The greatest common factor (GCF) of two or more monomials is the product of their common prime factors.

#### Factoring Using the Distributive Property (Lesson 8-2)

- Using the Distributive Property to factor polynomials with four or more terms is called factoring by grouping.  

$$ax + bx + ay + by = x(a + b) + y(a + b)$$

$$= (a + b)(x + y)$$
- Factoring can be used to solve some equations. According to the Zero Product Property, for any real numbers  $a$  and  $b$ , if  $ab = 0$ , then either  $a = 0$ ,  $b = 0$ , or both  $a$  and  $b$  equal zero.

#### Factoring Trinomials and Differences of Squares (Lessons 8-3 through 8-5)

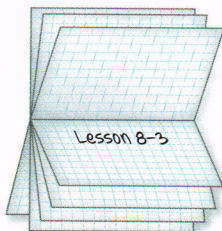
- To factor  $x^2 + bx + c$ , find  $m$  and  $p$  with a sum of  $b$  and a product of  $c$ . Then write  $x^2 + bx + c$  as  $(x + m)(x + p)$ .
- To factor  $ax^2 + bx + c$ , find  $m$  and  $p$  with a sum of  $b$  and a product of  $ac$ . Then write as  $ax^2 + mx + px + c$  and factor by grouping.
- $a^2 - b^2 = (a - b)(a + b)$

#### Perfect Squares and Factoring (Lesson 8-6)

- For a trinomial to be a perfect square, the first and last terms must be perfect squares, and the middle term must be twice the product of the square roots of the first and last terms.
- For any number  $n \geq 0$ , if  $x^2 = n$ , then  $x = \pm \sqrt{n}$ .

## FOLDABLES® Study Organizer

Be sure the Key Concepts are noted in your Foldable.



## Key Vocabulary

**difference of two squares** (p. 499)

**factored form** (p. 471)

**factoring** (p. 476)

**factoring by grouping** (p. 477)

**greatest common factor (GCF)** (p. 471)

**perfect square trinomial** (p. 505)

**prime polynomial** (p. 495)

**quadratic equation** (p. 488)

**Square Root Property** (p. 508)

**Zero Product Property** (p. 478)

## Vocabulary Check

State whether each sentence is *true* or *false*. If *false*, replace the underlined phrase or expression to make a true sentence.

- $x^2 + 5x + 6$  is an example of a prime polynomial.
- $(x + 5)(x - 5)$  is the factorization of a difference of squares.
- $5x$  is the greatest common factor of  $10x$  and  $15xy^2$ .
- $(x + 5)(x - 2)$  is the factored form of  $x^2 - 3x - 10$ .
- Expressions with four or more unlike terms can sometimes be factored by grouping.
- The Zero Product Property states that if  $ab = 1$ , then  $a$  or  $b$  is 1.
- $x^2 - 12x + 36$  is an example of a perfect square trinomial.
- $x - 2 = 0$  is a quadratic equation.
- $x^2 - 16$  is an example of a perfect square trinomial.
- The greatest common factor of  $8x$  and  $4x^2$  is  $4x$ .



## Lesson-by-Lesson Review

## 8-1 Monomials and Factoring (pp. 471–474)

Factor each monomial completely.

11.  $28x^3$

12.  $-33x^2y^3$

13.  $68cd^3$

14.  $120mq$

Find the greatest common factor of each set of monomials.

15.  $22b, 33c$

16.  $21xy, 28x^2y, 42xy^2$

17.  $6ab, 24ab^4$

18.  $10ab, 30a, 40a^2b$

19. **HOME IMPROVEMENT** A landscape architect is designing a stone path 36 inches wide and 120 inches long. What is the maximum size square stone that can be used so that none of the stones have to be cut?

## EXAMPLE 1

Factor  $24a^2b^3$  completely.

$$\begin{aligned} 24a^2b^3 &= 4 \cdot 6 \cdot a \cdot a \cdot b \cdot b \cdot b \\ &= 2 \cdot 2 \cdot 2 \cdot 3 \cdot a \cdot a \cdot b \cdot b \cdot b \end{aligned}$$

## EXAMPLE 2

Find the greatest common factor of  $12xy$  and  $8xy^2$ .

$$\begin{aligned} 12xy &= \textcircled{2} \cdot \textcircled{2} \cdot 3 \cdot \textcircled{x} \cdot \textcircled{y} \\ 8xy^2 &= \textcircled{2} \cdot \textcircled{2} \cdot 2 \cdot \textcircled{x} \cdot \textcircled{y} \cdot y \end{aligned}$$

Factor each monomial. Circle the common prime factors.

The greatest common factor is  $2 \cdot 2 \cdot x \cdot y$  or  $4xy$ .

## 8-2 Using the Distributive Property (pp. 476–482)

Use the Distributive Property to factor each polynomial.

20.  $12x + 24y$

21.  $14x^2y - 21xy + 35xy^2$

22.  $8xy - 16x^3y + 10y$

23.  $a^2 - 4ac + ab - 4bc$

24.  $2x^2 - 3xz - 2xy + 3yz$

25.  $24am - 9an + 40bm - 15bn$

Solve each equation. Check your solutions.

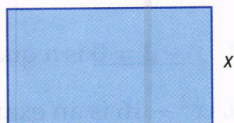
26.  $x(3x - 6) = 0$

27.  $6x^2 = 12x$

28.  $x^2 = 3x$

29.  $3x^2 = 5x$

30. **GEOMETRY** The area of the rectangle shown is  $x^3 - 2x^2 + 5x$  square units. What is the length?



## EXAMPLE 3

Factor  $12y^2 + 9y + 8y + 6$ .

$$\begin{aligned} 12y^2 + 9y + 8y + 6 &= (12y^2 + 9y) + (8y + 6) \\ &= 3y(4y + 3) + 2(4y + 3) \\ &= (4y + 3)(3y + 2) \end{aligned}$$

Group terms with common factors.

Factor the GCF from each group.

Distributive Property

## EXAMPLE 4

Solve  $x^2 - 6x = 0$ . Check your solutions.Write the equation so that it is of the form  $ab = 0$ .

$x^2 - 6x = 0$

Original equation

$x(x - 6) = 0$

Factor by using the GCF.

$x = 0 \quad \text{or} \quad x - 6 = 0$

Zero Product Property

$x = 6$

Solve.

The roots are 0 and 6. Check by substituting 0 and 6 for  $x$  in the original equation.



**8-3 Quadratic Equations:  $x^2 + bx + c = 0$**  (pp. 485–491)

Factor each trinomial.

31.  $x^2 - 8x + 15$       32.  $x^2 + 9x + 20$   
 33.  $x^2 - 5x - 6$       34.  $x^2 + 3x - 18$

Solve each equation. Check your solutions.

35.  $x^2 + 5x - 50 = 0$   
 36.  $x^2 - 6x + 8 = 0$   
 37.  $x^2 + 12x + 32 = 0$   
 38.  $x^2 - 2x - 48 = 0$   
 39.  $x^2 + 11x + 10 = 0$   
 40. **ART** An artist is working on a painting that is 3 inches longer than it is wide. The area of the painting is 154 square inches. What is the length of the painting?

**EXAMPLE 5**

 Factor  $x^2 + 10x + 21$ 

$b = 10$  and  $c = 21$ , so  $m + p$  is positive and  $mp$  is positive. Therefore,  $m$  and  $p$  must both be positive. List the positive factors of 21, and look for the pair of factors with a sum of 10.

Factors of 21	Sum of 10
1, 21	22
3, 7	10

The correct factors are 3 and 7.

$$\begin{aligned}
 x^2 + 10x + 21 &= (x + m)(x + p) && \text{Write the pattern.} \\
 &= (x + 3)(x + 7) && m = 3 \text{ and } p = 7
 \end{aligned}$$

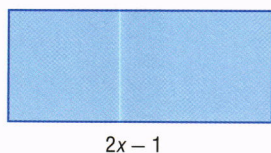
**8-4 Quadratic Equations:  $ax^2 + bx + c = 0$**  (pp. 493–498)

 Factor each trinomial, if possible. If the trinomial cannot be factored, write *prime*.

41.  $12x^2 + 22x - 14$   
 42.  $2y^2 - 9y + 3$   
 43.  $3x^2 - 6x - 45$   
 44.  $2a^2 + 13a - 24$

Solve each equation. Check your solutions.

45.  $40x^2 + 2x = 24$   
 46.  $2x^2 - 3x - 20 = 0$   
 47.  $-16t^2 + 36t - 8 = 0$   
 48.  $6x^2 - 7x - 5 = 0$   
 49. **GEOMETRY** The area of the rectangle shown is  $6x^2 + 11x - 7$  square units. What is the width of the rectangle?


**EXAMPLE 6**

 Factor  $12a^2 + 17a + 6$ 

$a = 12$ ,  $b = 17$ , and  $c = 6$ . Since  $b$  is positive,  $m + p$  is positive. Since  $c$  is positive,  $mp$  is positive. So,  $m$  and  $p$  are both positive. List the factors of  $12(6)$  or 72, where both factors are positive.

Factors of 72	Sum of 17
1, 72	73
2, 36	38
3, 24	27
4, 18	22
6, 12	18
8, 9	17

The correct factors are 8 and 9.

$$\begin{aligned}
 12a^2 + 17a + 6 &= 12a^2 + ma + pa + 6 \\
 &= 12a^2 + 8a + 9a + 6 \\
 &= (12a^2 + 8a) + (9a + 6) \\
 &= 4a(3a + 2) + 3(3a + 2) \\
 &= (3a + 2)(4a + 3)
 \end{aligned}$$

$$\text{So, } 12a^2 + 17a + 6 = (3a + 2)(4a + 3).$$



**8-5 Quadratic Equations: Differences of Squares** (pp. 499–504)

Factor each polynomial.

50.  $y^2 - 81$

51.  $64 - 25x^2$

52.  $16a^2 - 21b^2$

53.  $3x^2 - 3$

Solve each equation by factoring. Check your solutions.

54.  $a^2 - 25 = 0$

55.  $9x^2 - 25 = 0$

56.  $81 - y^2 = 0$

57.  $x^2 - 5 = 20$

58. **EROSION** A boulder falls down a mountain into water 64 feet below. The distance  $d$  that the boulder falls in  $t$  seconds is given by the equation  $d = 16t^2$ . How long does it take the boulder to hit the water?

**EXAMPLE 7**Solve  $x^2 - 4 = 12$  by factoring.

$$x^2 - 4 = 12$$

Original equation

$$x^2 - 16 = 0$$

Subtract 12 from each side.

$$x^2 - (4)^2 = 0$$

$$16 = 4^2$$

$$(x + 4)(x - 4) = 0$$

Factor the difference of squares.

$$x + 4 = 0 \quad \text{or} \quad x - 4 = 0$$

Zero Product Property

$$x = -4$$

$$x = 4$$

Solve each equation.

The solutions are  $-4$  and  $4$ .**8-6 Quadratic Equations: Perfect Squares** (pp. 505–512)Factor each polynomial, if possible. If the polynomial cannot be factored write *prime*.

59.  $x^2 + 12x + 36$

60.  $x^2 + 5x + 25$

61.  $9y^2 - 12y + 4$

62.  $4 - 28a + 49a^2$

63.  $x^4 - 1$

64.  $x^4 - 16x^2$

Solve each equation. Check your solutions.

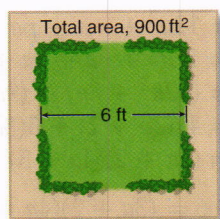
65.  $(x - 5)^2 = 121$

66.  $4c^2 + 4c + 1 = 9$

67.  $4y^2 = 64$

68.  $16d^2 + 40d + 25 = 9$

69. **LANDSCAPING** A sidewalk of equal width is being built around a square yard. What is the width of the sidewalk?

**EXAMPLE 8**Solve  $(x - 9)^2 = 144$ .

$$(x - 9)^2 = 144$$

Original equation

$$x - 9 = \pm\sqrt{144}$$

Square Root Property

$$x - 9 = \pm 12$$

$$144 = 12 \cdot 12$$

$$x = 9 \pm 12$$

Add 9 to each side.

$$x = 9 + 12 \quad \text{or} \quad x = 9 - 12$$

Zero Product Property

$$x = 21$$

$$x = -3$$

Solve.

**CHECK**

$$(x - 9)^2 = 144$$

$$(x - 9)^2 = 144$$

$$(21 - 9)^2 \stackrel{?}{=} 144$$

$$(-3 - 9)^2 \stackrel{?}{=} 144$$

$$(12)^2 \stackrel{?}{=} 144$$

$$(-12)^2 \stackrel{?}{=} 144$$

$$144 = 144 \checkmark$$

$$144 = 144 \checkmark$$



Factor each monomial completely.

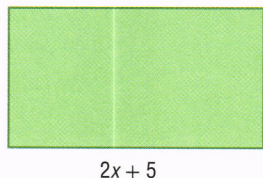
1.  $25x^2y^4$
2.  $17ab^2$
3.  $-18c^5d^3$

4. **GARDENING** Conrado is planting 140 pumpkins in a rectangular arrangement in his garden. In what ways can he arrange them so that he has at least 4 rows of pumpkins, the same number of pumpkins in each row, and at least 6 pumpkins in each row?

Find the greatest common factor of each set of monomials.

5.  $2a, 8a^2, 16a^3$
6.  $7c, 24d$
7.  $50g^2h, 120gh^2$
8.  $8q^2r^2, 36qr$

9. **MULTIPLE CHOICE** The area of the rectangle shown below is  $2x^2 - x - 15$  square units. What is the width of the rectangle?



- A  $x - 5$
- B  $x + 3$
- C  $x - 3$
- D  $2x - 3$

Use the Distributive Property to factor each polynomial.

10.  $5xy - 10x$
11.  $7ab + 14ab^2 + 21a^2b$

Factor each polynomial.

12.  $4x^2 + 8x + x + 2$
13.  $10a^2 - 50a - a + 5$

Solve each equation. Check your solutions.

14.  $y(y - 14) = 0$
15.  $3x(x + 6) = 0$
16.  $a^2 = 12a$

17. **MULTIPLE CHOICE** Chantel is carpeting a room that has an area of  $x^2 - 100$  square feet. If the width of the room is  $x - 10$  feet, what is the length of the room?

- F  $x - 10$  ft
- G  $x + 10$  ft
- H  $x - 100$  ft
- J  $10$  ft

Factor each trinomial.

18.  $x^2 + 7x + 6$
19.  $x^2 - 3x - 28$
20.  $10x^2 - x - 3$
21.  $15x^2 + 7x - 2$
22.  $x^2 - 25$
23.  $4x^2 - 81$
24.  $9x^2 - 12x + 4$
25.  $16x^2 + 40x + 25$

Solve each equation. Check your solutions.

26.  $x^2 - 4x = 21$
27.  $x^2 - 2x - 24 = 0$
28.  $6x^2 - 5x - 6 = 0$
29.  $2x^2 - 13x + 20 = 0$

30. **MULTIPLE CHOICE** Which choice is a factor of  $x^4 - 1$  when it is factored completely?

- A  $x^2 - 1$
- B  $x - 1$
- C  $x$
- D  $x^2$



## Solve Multi-Step Problems

Some problems that you will encounter on standardized tests require you to solve multiple parts in order to come up with the final solution. Use this lesson to practice these types of problems.

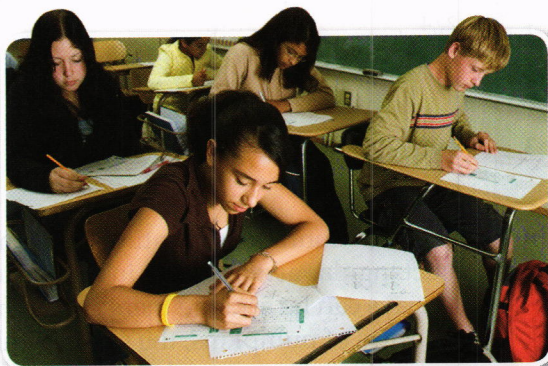
### Strategies for Solving Multi-Step Problems

#### Step 1

Read the problem statement carefully.

**Ask yourself:**

- What am I being asked to solve? What information is given?
- Are there any intermediate steps that need to be completed before I can solve the problem?



#### Step 2

Organize your approach.

- List the steps you will need to complete in order to solve the problem.
- Remember that there may be more than one possible way to solve the problem.

#### Step 3

Solve and check.

- Work as efficiently as possible to complete each step and solve.
- If time permits, check your answer.

### EXAMPLE

**Read the problem. Identify what you need to know. Then use the information in the problem to solve.**

A florist has 80 roses, 50 tulips, and 20 lilies that he wants to use to create bouquets. He wants to create the maximum number of bouquets possible and use all of the flowers. Each bouquet should have the same number of each type of flower. How many roses will be in each bouquet?

- |           |            |
|-----------|------------|
| A 4 roses | C 10 roses |
| B 8 roses | D 15 roses |



Read the problem carefully. You are given the number of roses, tulips, and lilies and told that bouquets will be made using the same number of flowers in each. You need to find the number of roses that will be in each bouquet.

**Step 1** Find the GCF of the number of roses, tulips, and lilies.

**Step 2** Use the GCF to determine how many bouquets will be made.

**Step 3** Divide the total number of roses by the number of bouquets.

**Step 1** Write the prime factorization of each number of flowers to find the GCF.

$$80 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 5$$

$$50 = 2 \cdot 5 \cdot 5$$

$$20 = 2 \cdot 2 \cdot 5$$

$$\text{GCF} = 2 \cdot 5 = 10$$

**Step 2** The GCF of the number of roses, tulips, and lilies tells you how many bouquets can be made because each bouquet will contain the same number of flowers. So, the florist can make a total of 10 bouquets.

**Step 3** Divide the number of roses by the number of bouquets to find the number of roses in each bouquet.

$$\frac{80}{10} = 8$$

So, there will be 8 roses in each bouquet. The answer is B.

## Exercises

Read each problem. Identify what you need to know. Then use the information in the problem to solve.

1. Which of the following values is not a solution to  $x^3 - 3x^2 - 25x + 75 = 0$ ?

A  $x = 5$

C  $x = -3$

B  $x = 3$

D  $x = -5$

2. There are 12 teachers, 90 students, and 36 parent volunteers going on a field trip. Mrs. Bartholomew wants to divide everyone into equal groups with the same number of teachers, students, and parents in each group. If she makes as many groups as possible, how many students will be in each group?

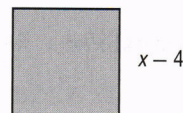
F 6

H 12

G 9

J 15

3. What is the area of the square?



A  $x^2 + 16$

B  $4x - 16$

C  $x^2 - 8x - 16$

D  $x^2 - 8x + 16$

4. Students are selling magazines to raise money for a field trip. They make \$2.75 for each magazine they sell. If they want to raise \$600, what is the least amount of magazines they need to sell?

F 121

H 202

G 177

J 219



## Multiple Choice

Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.

1. A baker has the number of cookies shown in the table below available to put into gift baskets. She wants to put the same number of each cookie into each basket, and each basket should have each type of cookie. If she puts the greatest possible number of cookies in each basket, how many baskets can she make?

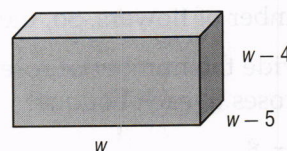
Type of Cookie	Number
Chocolate Chip	54
Peanut Butter	45
Oatmeal Raisin	36
Sugar	60

- A 18                      C 12  
B 16                      D 10
2. Refer to the information given in Exercise 1. How many of each type of cookie will be in each of the baker's gift baskets?
- F 2                      H 4  
G 3                      J 6
3. Factor the polynomial  $mn + 5m - 3n - 15$ .
- A  $(mn - 3)(5)$                       C  $(m - 5)(n + 3)$   
B  $(n - 3)(m + 5)$                       D  $(m - 3)(n + 5)$
4. Which of the following is a solution to  $x^2 + 6x - 112 = 0$ ?
- F -14                      H 6  
G -8                      J 12
5. Which of the following polynomials is prime?
- A  $5x^2 + 34x + 24$   
B  $4x^2 + 22x + 10$   
C  $4x^2 + 38x + 70$   
D  $5x^2 + 3x + 4$

6. Which of the following is not a factor of the polynomial  $45a^2 - 80b^2$ ?

F 5  
G  $3a - 4b$   
H  $2a - 5b$   
J  $3a + 4b$

7. A rectangular gift box has dimensions that can be represented as shown in the figure. The volume of the box is 672 cubic inches. Which of the following is *not* a dimension of the box?



- A 6 in.  
B 7 in.  
C 8 in.  
D 12 in.
8. Factor the polynomial  $y^2 + 9y + 20$ .
- F  $(y + 2)(y + 10)$   
G  $(y + 4)(y + 5)$   
H  $(y + 3)(y + 8)$   
J  $(y + 5)(y + 2)$
9. Which of the following numbers is less than zero?
- A  $1.03 \times 10^{-21}$   
B  $7.5 \times 10^2$   
C  $8.21543 \times 10^{10}$   
D none of the above

## Test-Taking Tip

**Question 4** If time permits, be sure to check your answer. Substitute it into the equation to see if you get a true number sentence.



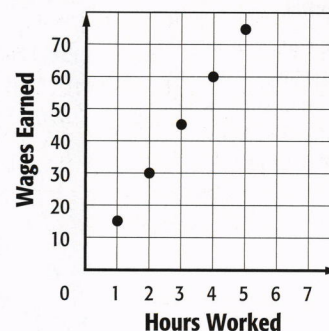
## Short Response/Gridded Response

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

- 10. GRIDDED RESPONSE** Mr. Branson bought a total of 9 tickets to the zoo. He bought children tickets at the rate of \$6.50 and adult tickets for \$9.25 each. If he spent \$69.50 altogether, how many adult tickets did Mr. Branson purchase?
- 11.** What is the domain of the following relation?  
 $\{(2, -1), (4, 3), (7, 6)\}$
- 12.** Lawrence just added 15 more songs to his MP3 player, making the total number of songs more than 84. Draw a number line that represents the original number of songs he had on his MP3 player.
- 13. GRIDDED RESPONSE** Carlos bought a rare painting in 1995 for \$14,200. By 2003, the painting was worth \$17,120. Assuming that the relationship is linear, write an equation in slope-intercept form that represents the value  $V$  of the painting after  $t$  years.
- 14.** The equation  $h = -16t^2 + 40t + 3$  models the height  $h$  in feet of a soccer ball after  $t$  seconds. What is the height of the ball after 2 seconds?
- 15.** Marcel spent \$24.50 on peanuts and walnuts for a party. He bought 1.5 pounds more peanuts than walnuts. How many pounds of peanuts and walnuts did he buy?

Product	Price per pound
Peanuts $p$	\$3.80
Cashews $c$	\$6.90
Walnuts $w$	\$5.60

- 16. GRIDDED RESPONSE** The amount of money that Humberto earns varies directly as the number of hours that he works as shown in the graph. How much money will he earn for working 40 hours next week? Express your answer in dollars.



## Extended Response

Record your answers on a sheet of paper. Show your work.

- 17.** The height in feet of a model rocket  $t$  seconds after being launched into the air is given by the function  $h(t) = -16t^2 + 200t$ .
- Write the expression that shows the height of the rocket in factored form.
  - At what time(s) is the height of the rocket equal to zero feet above the ground? Describe the real world meaning of your answer.
  - What is the greatest height reached by the model rocket? When does this occur?

### Need Extra Help?

If you missed Question...

Go to Lesson or Page...

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
8-1	8-1	8-2	8-3	8-4	8-5	8-6	8-3	7-3	6-5	1-6	5-1	4-2	8-4	2-9	3-4	8-2



# CHAPTER 9

# Quadratic and Exponential Functions

## Then


In Chapter 8, you solved quadratic equations by factoring and by using the Square Root Property.

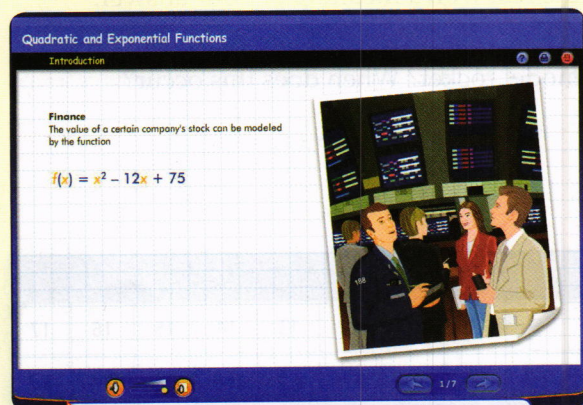
## Now

In Chapter 9, you will:

- Solve quadratic equations by graphing, completing the square, and using the Quadratic Formula.
- Graph exponential functions.
- Identify geometric sequences.

## Why?

 **FINANCE** The value of a certain company's stock can be modeled by the function  $f(x) = x^2 - 12x + 75$ . By graphing this quadratic function, we can make an educated guess as to how the stock will perform in the near future.

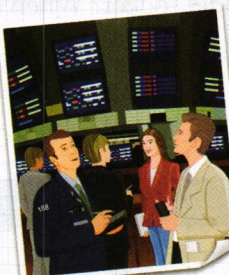


Quadratic and Exponential Functions

Introduction

**Finance**  
The value of a certain company's stock can be modeled by the function

$$f(x) = x^2 - 12x + 75$$



**Math in Motion, Animation** [glencoe.com](http://glencoe.com)

