Get Ready for the Chapter

Diagnose Readiness | You have two options for checking prerequisite skills.

1 Textbook Option

Take the Quick Check below. Refer to the Quick Review for help.

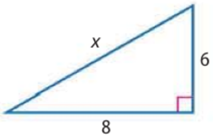
QuickCheck

Simplify.

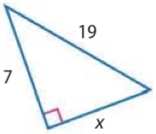
1. $\sqrt{112}$
2. $\frac{\sqrt{24}}{2\sqrt{3}}$
3. $\sqrt{15 \cdot 20}$
4. $\frac{\sqrt{6}}{\sqrt{3}} \cdot \frac{\sqrt{18}}{\sqrt{3}}$
5. $\sqrt{\frac{45}{80}}$
6. $\frac{8\sqrt{2}}{6 - 3\sqrt{8}}$

Find x.

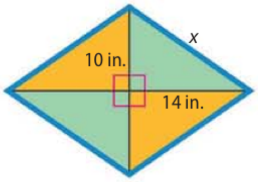
7.



8.



9. **BANNERS** Anna is making a banner out of 4 congruent triangles as shown below. How much blue trim will she need for each side?



Graph the line segment with the given endpoints.

10. $G(3, -4)$ and $H(3, 4)$
11. $E(-3, 5)$ and $F(4, -3)$
12. **COLLEGES** Quinn is visiting a college campus. He notices from his map that several important buildings are located around a grassy area the students call the Quad. If the library is represented on the map by $L(6, 8)$ and the cafeteria is represented by $C(0, 0)$, graph the line segment that represents the shortest path between the two buildings.

QuickReview

Example 1

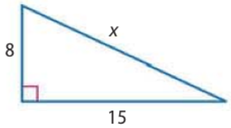
Simplify $\frac{6}{\sqrt{3}}$.

$\frac{6}{\sqrt{3}} = \frac{6}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$ Multiply by $\frac{\sqrt{3}}{\sqrt{3}}$.

$= \frac{6\sqrt{3}}{3}$ or $2\sqrt{3}$ Simplify.

Example 2

Find x.



$a^2 + b^2 = c^2$ Pythagorean Theorem

$8^2 + 15^2 = x^2$ $a = 8$ and $b = 15$

$289 = x^2$ Simplify.

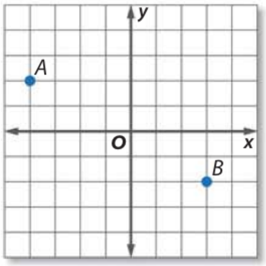
$\sqrt{289} = \sqrt{x^2}$ Take the positive square root of each side.

$17 = x$ Simplify.

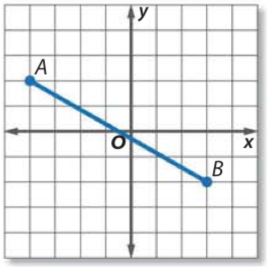
Example 3

Graph the line segment with endpoints $A(-4, 2)$ and $B(3, -2)$.

Plot points A and B.



Connect the points.



2 Online Option

Take an online self-check Chapter Readiness Quiz at connectED.mcgraw-hill.com.

Get Started on the Chapter

You will learn several new concepts, skills, and vocabulary terms as you study Chapter 8. To get ready, identify important terms and organize your resources. You may wish to refer to Chapter 0 to review prerequisite skills.

FOLDABLES StudyOrganizer

Right Angles and Trigonometry Make this Foldable to help you organize your Chapter 8 notes about right angles and trigonometry. Begin with three sheets of notebook paper and one sheet of construction paper.

1 **Stack** the notebook paper on the construction paper.

2 **Fold** the paper diagonally to form a triangle and cut off the excess.

3 **Open** the paper and staple the inside fold to form a booklet.

4 **Label** each page with a lesson number and title.

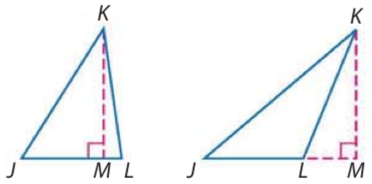
NewVocabulary

English		Español
geometric mean	p. 537	media geométrica
Pythagorean triple	p. 548	triplete pitágorico
trigonometry	p. 568	trigonométria
trigonometric ratio	p. 568	razón trigonométrica
sine	p. 568	seno
cosine	p. 568	coseno
tangent	p. 568	tangente
angle of elevation	p. 580	ángulo de elevación
angle of depression	p. 580	ángulo de depresión
Law of Sines	p. 588	ley de los senos
Law of Cosines	p. 589	ley do los cosenos
vector	p. 600	vector
magnitude	p. 600	magnitud
resultant	p. 601	resultante
component form	p. 602	componente

ReviewVocabulary

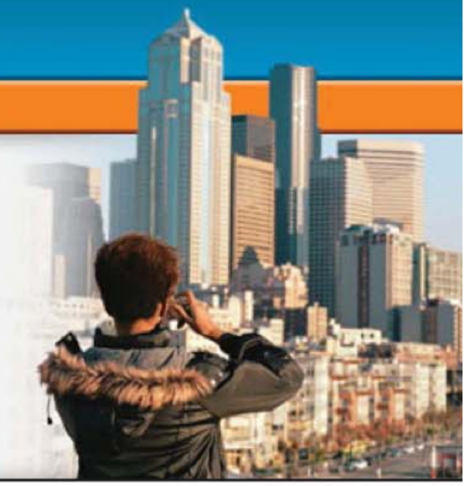
altitude altura a segment drawn from a vertex of a triangle perpendicular to the line containing the other side

Pythagorean Theorem Teorema de Pitágoras If *a* and *b* are the measures of the legs of a right triangle and *c* is the measure of the hypotenuse, then $a^2 + b^2 = c^2$.



$\overline{KM}$ is an altitude of $\triangle JKL$.

LESSON 8-1 Geometric Mean



Then

- You used proportional relationships of corresponding angle bisectors, altitudes, and medians of similar triangles.

Now

1. Find the geometric mean between two numbers.
2. Solve problems involving relationships between parts of a right triangle and the altitude to its hypotenuse.

Why?

- Photographing very tall or very wide objects can be challenging. It can be difficult to include the entire object in your shot without distorting the image. If your camera is set for a vertical viewing angle of 90° and you know the height of the object you wish to photograph, you can use the geometric mean of the distance from the top of the object to your camera level and the distance from the bottom of the object to camera level.

NewVocabulary

geometric mean

Common Core State Standards

Content Standards

G.SRT.4 Prove theorems about triangles.

G.SRT.5 Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

Mathematical Practices

7 Look for and make use of structure.

3 Construct viable arguments and critique the reasoning of others.

Digital Vision/Alamy

1 Geometric Mean

Geometric Mean When the means of a proportion are the same number, that number is called the geometric mean of the extremes. The **geometric mean** between two numbers is the positive square root of their product.

$$\frac{\text{extreme} \rightarrow a}{\text{mean} \rightarrow x} = \frac{x \leftarrow \text{mean}}{b \leftarrow \text{extreme}}$$

KeyConcept Geometric Mean

Words	The geometric mean of two positive numbers a and b is the number x such that $\frac{a}{x} = \frac{x}{b}$. So, $x^2 = ab$ and $x = \sqrt{ab}$.
Example	The geometric mean of $a = 9$ and $b = 4$ is 6, because $6 = \sqrt{9 \cdot 4}$.

Example 1 Geometric Mean

Find the geometric mean between 8 and 10.

$x = \sqrt{ab}$	Definition of geometric mean
$= \sqrt{8 \cdot 10}$	$a = 8$ and $b = 10$
$= \sqrt{(4 \cdot 2) \cdot (2 \cdot 5)}$	Factor.
$= \sqrt{16 \cdot 5}$	Associative Property
$= 4\sqrt{5}$	Simplify.

The geometric mean between 8 and 10 is $4\sqrt{5}$ or about 8.9.

GuidedPractice

Find the geometric mean between each pair of numbers.

1A. 5 and 45

1B. 12 and 15

2 Geometric Means in Right Triangles

Geometric Means in Right Triangles In a right triangle, an altitude drawn from the vertex of the right angle to the hypotenuse forms two additional right triangles. These three right triangles share a special relationship.

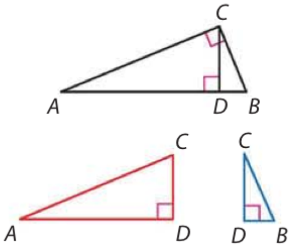
Review Vocabulary

altitude (of a triangle)
a segment from a vertex to the line containing the opposite side and perpendicular to the line containing that side

Theorem 8.1

If the altitude is drawn to the hypotenuse of a right triangle, then the two triangles formed are similar to the original triangle and to each other.

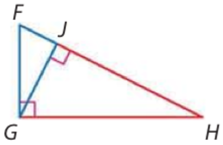
Example If $\overline{CD}$ is the altitude to hypotenuse $\overline{AB}$ of right $\triangle ABC$, then $\triangle ACD \sim \triangle ABC$, $\triangle CBD \sim \triangle ABC$, and $\triangle ACD \sim \triangle CBD$.



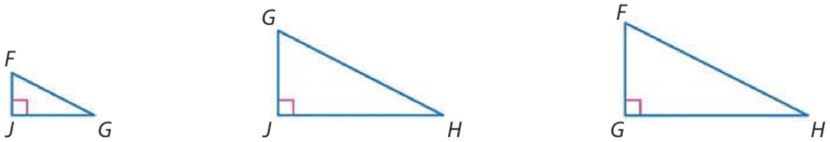
You will prove Theorem 8.1 in Exercise 39.

Example 2 Identify Similar Right Triangles

Write a similarity statement identifying the three similar right triangles in the figure.



Separate the triangle into two triangles along the altitude. Then sketch the three triangles, reorienting the smaller ones so that their corresponding angles and sides are in the same positions as the original triangle.



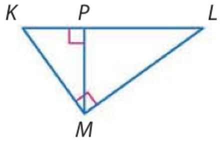
So by Theorem 8.1, $\triangle FJG \sim \triangle GJH \sim \triangle FGH$.

Study Tip

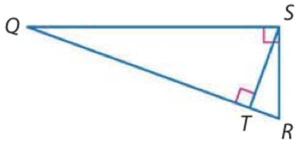
Reorienting Triangles
To reorient the right triangles in Example 2, first match up the right angles. Then match up the shorter sides.

Guided Practice

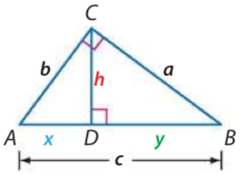
2A.

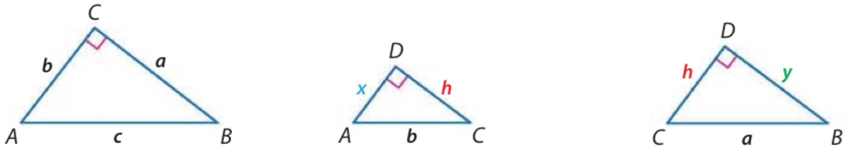


2B.



From Theorem 8.1, you know that altitude $\overline{CD}$ drawn to the hypotenuse of right triangle ABC forms three similar triangles: $\triangle ACB \sim \triangle ADC \sim \triangle CDB$. By the definition of similar polygons, you can write the following proportions comparing the side lengths of these triangles.



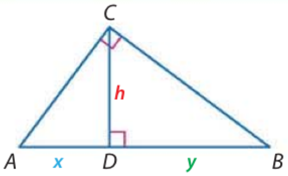


$$\frac{\text{shorter leg}}{\text{longer leg}} = \frac{b}{a} = \boxed{\frac{x}{h} = \frac{h}{y}} \qquad \frac{\text{hypotenuse}}{\text{shorter leg}} = \boxed{\frac{c}{b} = \frac{b}{x}} = \frac{a}{h} \qquad \frac{\text{hypotenuse}}{\text{longer leg}} = \boxed{\frac{c}{a}} = \frac{b}{h} = \boxed{\frac{a}{y}}$$

Notice that the circled relationships involve geometric means. This leads to the theorems at the top of the next page.

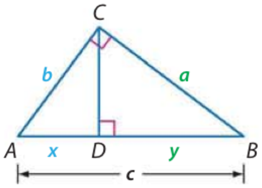
Theorems Right Triangle Geometric Mean Theorems

8.2 Geometric Mean (Altitude) Theorem The altitude drawn to the hypotenuse of a right triangle separates the hypotenuse into two segments. The length of this altitude is the geometric mean between the lengths of these two segments.



Example If $\overline{CD}$ is the altitude to hypotenuse $\overline{AB}$ of right $\triangle ABC$, then $\frac{x}{h} = \frac{h}{y}$ or $h = \sqrt{xy}$.

8.3 Geometric Mean (Leg) Theorem The altitude drawn to the hypotenuse of a right triangle separates the hypotenuse into two segments. The length of a leg of this triangle is the geometric mean between the length of the hypotenuse and the segment of the hypotenuse adjacent to that leg.

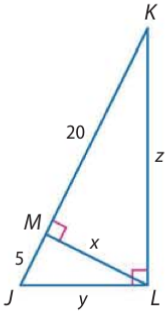


Example If $\overline{CD}$ is the altitude to hypotenuse $\overline{AB}$ of right $\triangle ABC$, then $\frac{c}{b} = \frac{b}{x}$ or $b = \sqrt{xc}$ and $\frac{c}{a} = \frac{a}{y}$ or $a = \sqrt{yc}$.

You will prove Theorems 8.2 and 8.3 in Exercises 40 and 41, respectively.

Example 3 Use Geometric Mean with Right Triangles

Find *x*, *y*, and *z*.



Since *x* is the measure of the altitude drawn to the hypotenuse of right △*JKL*, *x* is the geometric mean of the lengths of the two segments that make up the hypotenuse, *JM* and *MK*.

$x = \sqrt{JM \cdot MK}$ Geometric Mean (Altitude) Theorem

$= \sqrt{5 \cdot 20}$ Substitution

$= \sqrt{100}$ or 10 Simplify.

Since *y* is the measure of leg $\overline{JL}$, *y* is the geometric mean of $\overline{JM}$, the measure of the segment adjacent to this leg, and the measure of the hypotenuse *JK*.

$y = \sqrt{JM \cdot JK}$ Geometric Mean (Leg) Theorem

$= \sqrt{5 \cdot (20 + 5)}$ Substitution

$= \sqrt{125}$ or about 11.2 Use a calculator to simplify.

Since *z* is the measure of leg $\overline{KL}$, *z* is the geometric mean of $\overline{MK}$, the measure of the segment adjacent to $\overline{KL}$, and the measure of the hypotenuse *JK*.

$z = \sqrt{MK \cdot JK}$ Geometric Mean (Leg) Theorem

$= \sqrt{20 \cdot (20 + 5)}$ Substitution

$= \sqrt{500}$ or about 22.4 Use a calculator to simplify.

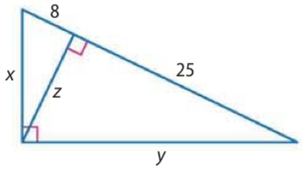
StudyTip

Use a Proportion
In Example 3, the value of *x* could also be found by solving the proportion $\frac{5}{x} = \frac{x}{20}$.

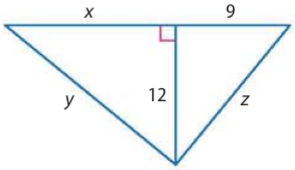
GuidedPractice

Find *x*, *y*, and *z*.

3A.



3B.



You can use geometric mean to measure height indirectly.

Real-World Career

Event Planner

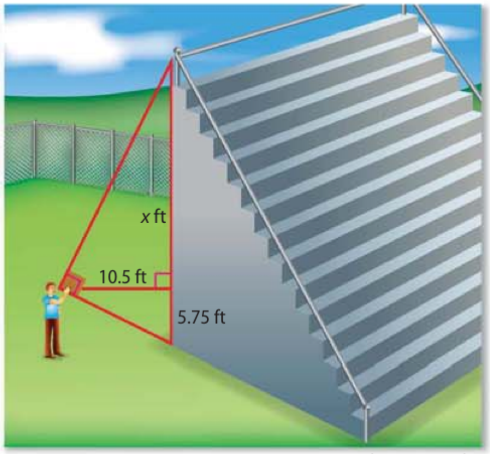
Event planners organize events including choosing a location, arranging for food, and scheduling entertainment. They also coordinate services like transportation and photography.

Most of the skills required for event planning are acquired through on-the-job experience.

Real-World Example 4 Indirect Measurement

ADVERTISING Zach wants to order a banner that will hang over the side of his high school baseball stadium grandstand and reach the ground.

To find this height, he uses a cardboard square to line up the top and bottom of the grandstand. He measures his distance from the grandstand and from the ground to his eye level. Find the height of the grandstand to the nearest foot.



Note: Not drawn to scale.

The distance from Zach to the grandstand is the altitude to the hypotenuse of a right triangle. The length of this altitude is the geometric mean of the two segments that make up the hypotenuse. The shorter segment has the measure of 5.75 feet. Let the unknown measure be x feet.

$10.5 = \sqrt{5.75 \cdot x}$	Geometric Mean (Altitude) Theorem
$110.25 = 5.75x$	Square each side.
$19.17 \approx x$	Divide each side by 5.75.

The height of the grandstand is the total length of the hypotenuse, 5.75 + 19.17, or about 25 feet.

GuidedPractice

4. **SPORTS** A community center needs to estimate the cost of installing a rock climbing wall by estimating the height of the wall. Sue holds a book up to her eyes so that the top and bottom of the wall are in line with the bottom edge and binding of the cover. If her eye level is 5 feet above the ground and she stands 11 feet from the wall, how high is the wall? Draw a diagram and explain your reasoning.

UpperCut Images/Getty Images

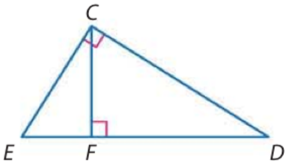
Check Your Understanding

● = Step-by-Step Solutions begin on page R14.

Example 1 **Find the geometric mean between each pair of numbers.**

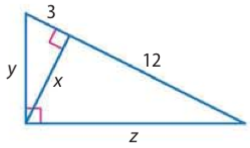
1. 5 and 20 **2.** 36 and 4 **3.** 40 and 15

Example 2 **4.** Write a similarity statement identifying the three similar triangles in the figure.

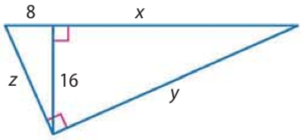


Example 3 **Find *x*, *y*, and *z*.**

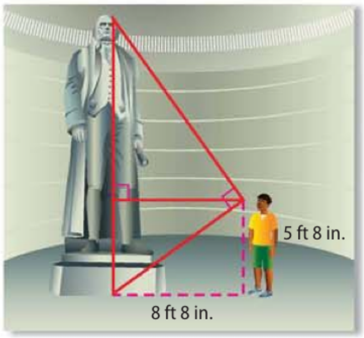
5.



6.



Example 4 **7.** **CCSS MODELING** Corey is visiting the Jefferson Memorial with his family. He wants to estimate the height of the statue of Thomas Jefferson. Corey stands so that his line of vision to the top and base of the statue form a right angle as shown in the diagram. About how tall is the statue?



Note: Not drawn to scale.

Practice and Problem Solving

Extra Practice is on page R8.

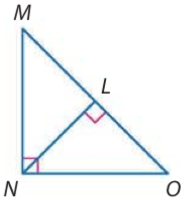
Example 1 **Find the geometric mean between each pair of numbers.**

8. 81 and 4 **9.** 25 and 16 **10.** 20 and 25

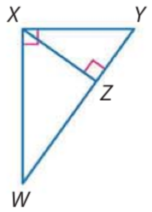
11. 36 and 24 **12.** 12 and 2.4 **13.** 18 and 1.5

Example 2 **Write a similarity statement identifying the three similar triangles in the figure.**

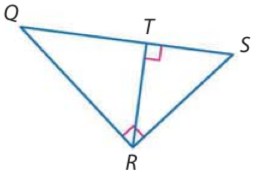
14.



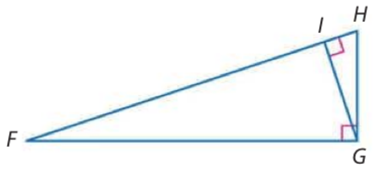
15.



16.

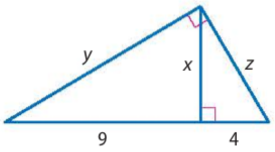


17.



Example 3 **Find x, y, and z.**

18.



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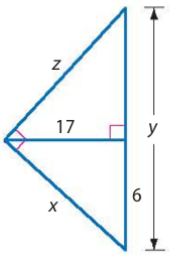

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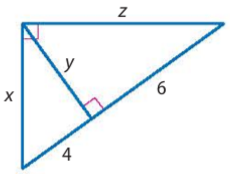

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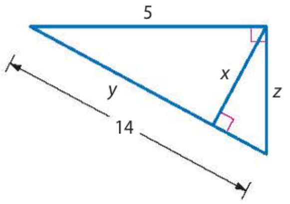
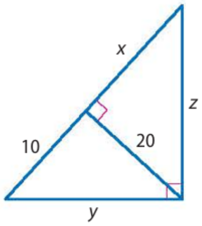
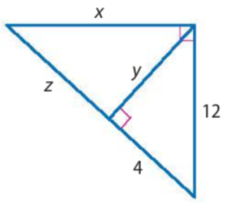


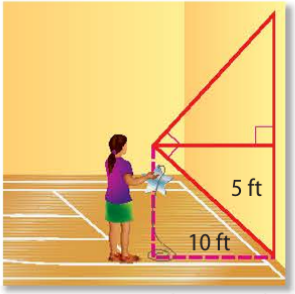
22.



23.

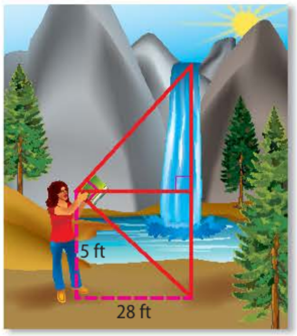


Example 4 **24.** **CCSS MODELING** Evelina is hanging silver stars from the gym ceiling using string for the homecoming dance. She wants the ends of the strings where the stars will be attached to be 7 feet from the floor. Use the diagram to determine how long she should make the strings.



Note: Not drawn to scale.

25. **CCSS MODELING** Makayla is using a book to sight the top of a waterfall. Her eye level is 5 feet from the ground and she is a horizontal distance of 28 feet from the waterfall. Find the height of the waterfall to the nearest tenth of a foot.



Note: Not drawn to scale.

Find the geometric mean between each pair of numbers.

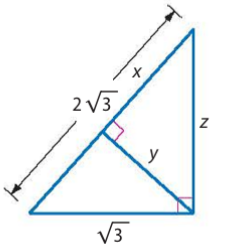
26. $\frac{1}{5}$ and 60

27. $\frac{3\sqrt{2}}{7}$ and $\frac{5\sqrt{2}}{7}$

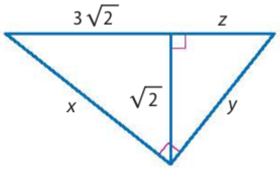
28. $\frac{3\sqrt{5}}{4}$ and $\frac{5\sqrt{5}}{4}$

Find x, y, and z.

29.



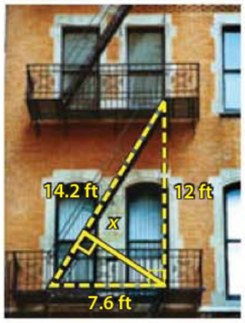
30.



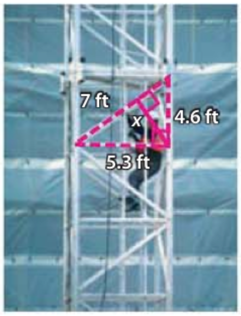
31. **ALGEBRA** The geometric mean of a number and four times the number is 22. What is the number?

Use similar triangles to find the value of *x*.

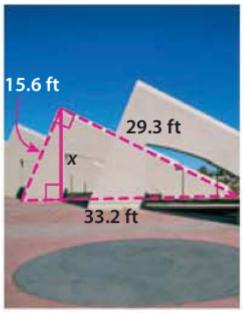
32.



33.

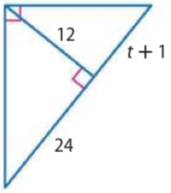


34.



ALGEBRA **Find the value of the variable.**

35.



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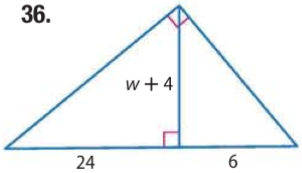

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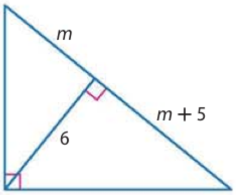
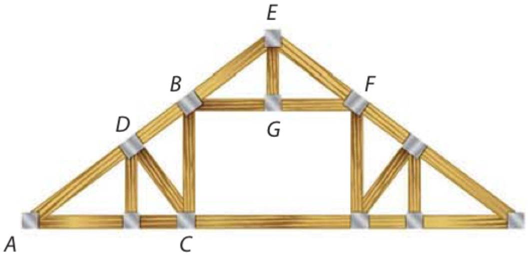


38. **CONSTRUCTION** A room-in-attic truss is a truss design that provides support while leaving area that can be enclosed as living space. In the diagram, $\angle BCA$ and $\angle EGB$ are right angles, $\triangle BEF$ is isosceles, $\overline{CD}$ is an altitude of $\triangle ABC$, and $\overline{EG}$ is an altitude of $\triangle BEF$. If $DB = 5$ feet, $CD = 6$ feet 4 inches, $BF = 10$ feet 10 inches, and $EG = 4$ feet 6 inches, what is AE?



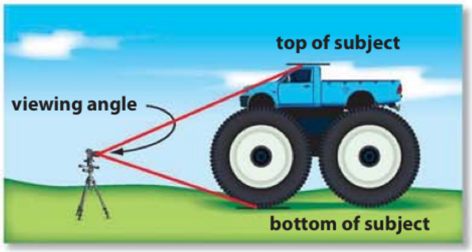
CCSS **ARGUMENTS** **Write a proof for each theorem.**

39. Theorem 8.1

40. Theorem 8.2

41. Theorem 8.3

42. **TRUCKS** In photography, the angle formed by the top of the subject, the camera, and the bottom of the subject is called the viewing angle, as shown at the right. Natalie is taking a picture of Bigfoot #5, which is 15 feet 6 inches tall. She sets her camera on a tripod that is 5 feet above ground level. The vertical viewing angle of her camera is set for 90°.



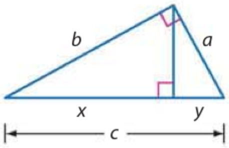
a. Sketch a diagram of this situation.

b. How far away from the truck should Natalie stand so that she perfectly frames the entire height of the truck in her shot?

43 **FINANCE** The average rate of return on an investment over two years is the geometric mean of the two annual returns. If an investment returns 12% one year and 7% the next year, what is the average rate of return on this investment over the two-year period?

(tl)BananaStock/Punchstock, (tc)Image Source/SuperStock, (tr)Richard Cummins/SuperStock

44. **PROOF** Derive the Pythagorean Theorem using the figure at the right and the Geometric Mean (Leg) Theorem.



Determine whether each statement is *always*, *sometimes*, or *never* true. Explain your reasoning.

45 The geometric mean for consecutive positive integers is the mean of the two numbers.

46. The geometric mean for two perfect squares is a positive integer.

47. The geometric mean for two positive integers is another integer.

48. **MULTIPLE REPRESENTATIONS** In this problem, you will investigate geometric mean.

a. **Tabular** Copy and complete the table of five ordered pairs (x, y) such that $\sqrt{xy} = 8$.

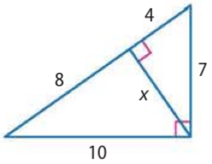
x	y	$\sqrt{xy}$
		8
		8
		8
		8
		8

b. **Graphical** Graph the ordered pairs from your table in a scatter plot.

c. **Verbal** Make a conjecture as to the type of graph that would be formed if you connected the points from your scatter plot. Do you think the graph of any set of ordered pairs that results in the same geometric mean would have the same general shape? Explain your reasoning.

H.O.T. Problems Use Higher-Order Thinking Skills

49. **ERROR ANALYSIS** Aiden and Tia are finding the value x in the triangle shown. Is either of them correct? Explain your reasoning.



Aiden

$\frac{4}{x} = \frac{x}{7}$

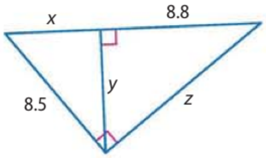
$x \approx 5.3$

Tia

$\frac{4}{x} = \frac{x}{10}$

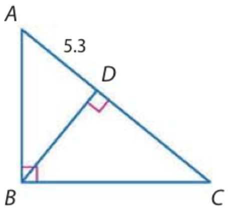
$x \approx 6.3$

50. **CHALLENGE** Refer to the figure at the right. Find x, y, and z.



51. **OPEN ENDED** Find two pairs of whole numbers with a geometric mean that is also a whole number. What condition must be met in order for a pair of numbers to produce a whole-number geometric mean?

52. **CCSS** **REASONING** Refer to the figure at the right. The orthocenter of $\triangle ABC$ is located 6.4 units from point D. Find BC.



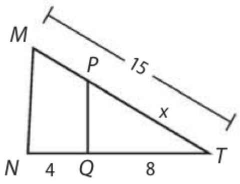
53. **WRITING IN MATH** Compare and contrast the arithmetic and geometric means of two numbers. When will the two means be equal? Justify your reasoning.

Standardized Test Practice

54. What is the geometric mean of 8 and 22 in simplest form?

A $4\sqrt{11}$ **C** $16\sqrt{11}$
B 15 **D** 176

55. SHORT RESPONSE If $\overline{MN} \parallel \overline{PQ}$, use a proportion to find the value of x. Show your work.

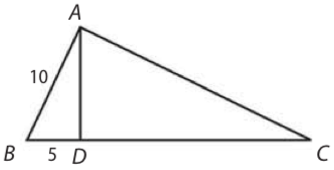


56. ALGEBRA What are the solutions of the quadratic equation $x^2 - 20 = 8x$?

F 2, 10 **H** −1, 20
G 20, 1 **J** −2, 10

57. SAT/ACT In the figure, $\overline{AD}$ is perpendicular to $\overline{BC}$, and $\overline{AB}$ is perpendicular to $\overline{AC}$. What is BC?

A $5\sqrt{2}$
B $5\sqrt{3}$
C 20
D 25
E 75



Spiral Review

58. MAPS Use the map to estimate how long it would take to drive from Chicago to Springfield if you averaged 65 miles per hour. (Lesson 7-7)

Graph the original figure and its dilated image. Then verify that the dilation is a similarity transformation. (Lesson 7-6)

59. $A(-3, 1), B(9, 7), C(3, -2); D(-1, 1), E(3, 3), F(1, 0)$

60. $G(-4, -4), H(-1, 2), J(2, -1); K(-3, -2), L(1, 0)$

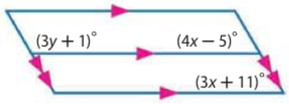
61. $M(7, -4), N(5, -4), P(7, -1); Q(2, -8), R(6, -8), S(2, -2)$

The interior angle measure of a regular polygon is given. Identify the polygon. (Lesson 6-1)

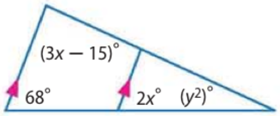
62. 108 **63.** 135

Find x and y in each figure. (Lesson 3-2)

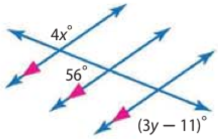
64.



65.

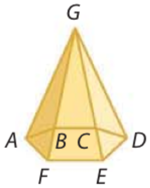


66.



Identify each solid. Name the bases, faces, edges, and vertices. (Lesson 1-7)

67.



68.

69.

Skills Review

Simplify each expression by rationalizing the denominator.

70. $\frac{2}{\sqrt{2}}$ **71.** $\frac{16}{\sqrt{3}}$ **72.** $\frac{\sqrt{6}}{\sqrt{4}}$ **73.** $\frac{3\sqrt{5}}{\sqrt{11}}$ **74.** $\frac{21}{\sqrt{3}}$

EXPLORE 8-2

Geometry Lab
Proofs Without Words

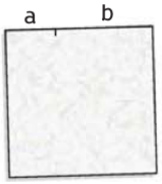
In Chapter 1, you learned that the Pythagorean Theorem relates the measures of the legs and the hypotenuse of a right triangle. You can prove the Pythagorean Theorem by using diagrams without words.

CCSS Common Core State Standards
Content Standards
G.CO.10 Prove theorems about triangles.
Mathematical Practices 4

Activity

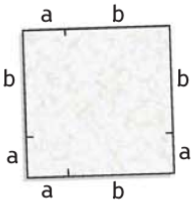
Prove the Pythagorean Theorem by using paper and algebra.

Step 1



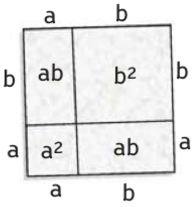
On a piece of tracing paper, mark one side *a* and *b* as shown above.

Step 2



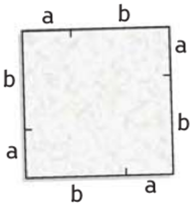
Copy these measures on each of the other sides.

Step 3



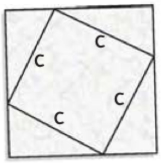
Fold the paper into four sections and label the area of each section.

Step 4



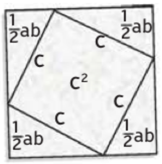
On another piece of tracing paper, mark each side *a* and *b* as shown above.

Step 5



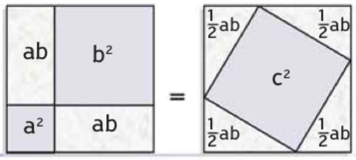
Connect the marks using a straightedge. Let *c* represent the length of each hypotenuse.

Step 6

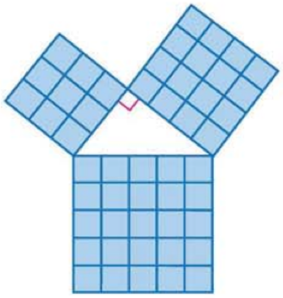


Label the area of each triangle $\frac{1}{2}ab$ and the area of each square c^2.

Step 7 Place the squares side by side and color the corresponding regions that have the same area. For example, $ab = \frac{1}{2}ab + \frac{1}{2}ab$. The parts that are not shaded tell us that $a^2 + b^2 = c^2$.



Analyze the Results



1. Use a ruler to measure *a*, *b*, and *c*. Do these measures confirm that $a^2 + b^2 = c^2$?
2. Repeat the activity with different *a* and *b* values. What do you notice?
3. **WRITING IN MATH** Explain why the diagram at the right is an illustration of the Pythagorean Theorem.
4. **CHALLENGE** Draw a geometric diagram to show that for any positive numbers *a* and *b*, $a + b > \sqrt{a^2 + b^2}$. Explain.

LESSON 8-2

The Pythagorean Theorem and Its Converse

Then

- You used the Pythagorean Theorem to develop the Distance Formula.

Now

1 Use the Pythagorean Theorem.

2 Use the Converse of the Pythagorean Theorem.

Why?

- Tether lines are used to steady an inflatable snowman. Suppose you know the height at which the tether lines are attached to the snowman and how far away you want to anchor the tether in the ground. You can use the converse of the Pythagorean Theorem to adjust the lengths of the tethers to keep the snowman perpendicular to the ground.

NewVocabulary

Pythagorean triple

Common Core State Standards

Content Standards

G.SRT.8 Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems. ★

G.MG.3 Apply geometric methods to solve problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios). ★

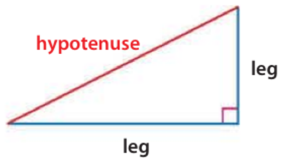
Mathematical Practices

1 Make sense of problems and persevere in solving them.

4 Model with mathematics.

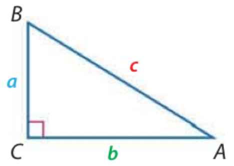
1 The Pythagorean Theorem

The Pythagorean Theorem is perhaps one of the most famous theorems in mathematics. It relates the lengths of the hypotenuse (side opposite the right angle) and legs (sides adjacent to the right angle) of a right triangle.



Theorem 8.4 Pythagorean Theorem

Words	In a right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse.
Symbols	If $\triangle ABC$ is a right triangle with right angle C, then $a^2 + b^2 = c^2$.



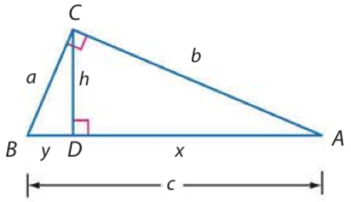
The geometric mean can be used to prove the Pythagorean Theorem.

Proof Pythagorean Theorem

Given: $\triangle ABC$ with right angle at C

Prove: $a^2 + b^2 = c^2$

Proof:



Draw right triangle ABC so C is the right angle. Then draw the altitude from C to $\overline{AB}$. Let $AB = c$, $AC = b$, $BC = a$, $AD = x$, $DB = y$, and $CD = h$. Two geometric means now exist.

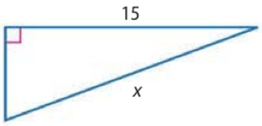
$\frac{c}{a} = \frac{a}{y}$	and	$\frac{c}{b} = \frac{b}{x}$	Geometric Mean (Leg) Theorem
$a^2 = cy$		$b^2 = cx$	Cross products
$a^2 + b^2 = cy + cx$			Add the equations.
$a^2 + b^2 = c(y + x)$			Factor.
$a^2 + b^2 = c \cdot c$			Since $c = y + x$, substitute c for $(y + x)$.
$a^2 + b^2 = c^2$			Simplify.

You can use the Pythagorean Theorem to find the measure of any side of a right triangle given the lengths of the other two sides.

Example 1 Find Missing Measures Using the Pythagorean Theorem

Find x.

a.



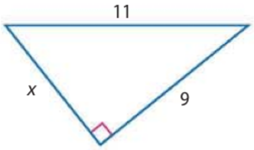
The side opposite the right angle is the hypotenuse, so $c = x$.

$a^2 + b^2 = c^2$	Pythagorean Theorem
$6^2 + 15^2 = x^2$	$a = 6$ and $b = 15$
$261 = x^2$	Simplify.
$\sqrt{261} = x$	Take the positive square root of each side.
$3\sqrt{29} = x$	Simplify.

Study Tip

Positive Square Root When finding the length of a side using the Pythagorean Theorem, use only the positive and not the negative square root, since length cannot be negative.

b.

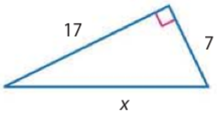


The hypotenuse is 11, so $c = 11$.

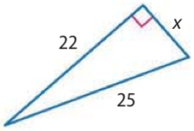
$a^2 + b^2 = c^2$	Pythagorean Theorem
$x^2 + 9^2 = 11^2$	$a = x$ and $b = 9$
$x^2 + 81 = 121$	Simplify.
$x^2 = 40$	Subtract 81 from each side.
$x = \sqrt{40}$ or $2\sqrt{10}$	Take the positive square root of each side and simplify.

Guided Practice

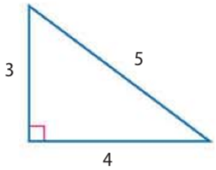
1A.



1B.



A **Pythagorean triple** is a set of three nonzero whole numbers a, b, and c, such that $a^2 + b^2 = c^2$. One common Pythagorean triple is 3, 4, 5; that is, the sides of a right triangle are in the ratio 3:4:5. The most common Pythagorean triples are shown below in the first row. The triples below these are found by multiplying each number in the triple by the same factor.



Study Tip

Pythagorean Triples If the measures of the sides of any right triangle are *not* whole numbers, the measures do not form a Pythagorean triple.

Key Concept Common Pythagorean Triples

3, 4, 5	5, 12, 13	8, 15, 17	7, 24, 25
6, 8, 10	10, 24, 26	16, 30, 34	14, 48, 50
9, 12, 15	15, 36, 39	24, 45, 51	21, 72, 75
$3x$, $4x$, $5x$	$5x$, $12x$, $13x$	$8x$, $15x$, $17x$	$7x$, $24x$, $25x$

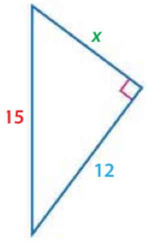
The largest number in each triple is the length of the hypotenuse.

ReadingMath

3-4-5 A right triangle with side lengths 3, 4, and 5 is called a *3-4-5 right triangle.*

Example 2 Use a Pythagorean Triple

Use a Pythagorean triple to find x. Explain your reasoning.



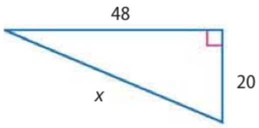
Notice that 15 and 12 are both multiples of 3, because $15 = 3 \cdot 5$ and $12 = 3 \cdot 4$. Since 3, 4, 5 is a Pythagorean triple, the missing leg length x is $3 \cdot 3$ or 9.

CHECK $12^2 + 9^2 \stackrel{?}{=} 15^2$ Pythagorean Theorem

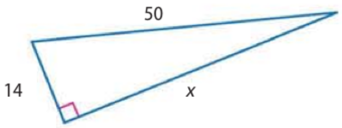
$225 = 225$ ✓ Simplify.

GuidedPractice

2A.



2B.



The Pythagorean Theorem can be used to solve many real-world problems.

Standardized Test Example 3 Use the Pythagorean Theorem

Damon is locked out of his house. The only open window is on the second floor, which is 12 feet above the ground. He needs to borrow a ladder from his neighbor. If he must place the ladder 5 feet from the house to avoid some bushes, what length of ladder does Damon need?

A 7 feet **C** 13 feet

B 11 feet **D** 17 feet

Note: Not drawn to scale.

Read the Test Item

The distance the ladder is from the house, the height the ladder reaches, and the length of the ladder itself make up the lengths of the sides of a right triangle. You need to find the length of the ladder, which is the hypotenuse.

Test-TakingTip

CCSS Sense-Making

Since the hypotenuse of a right triangle is always the longest side, the length of the ladder in Example 3 must be greater than 5 or 12 feet. Since 7 and 11 feet are both less than 12 feet, choices A and B can be eliminated.

Solve the Test Item

Method 1 Use a Pythagorean triple.

The lengths of the legs are 5 and 12. 5, 12, 13 is a Pythagorean triple, so the length of the ladder is 13 feet.

Method 2 Use the Pythagorean Theorem.

Let x represent the length of the ladder.

$5^2 + 12^2 = x^2$ Pythagorean Theorem

$169 = x^2$ Simplify.

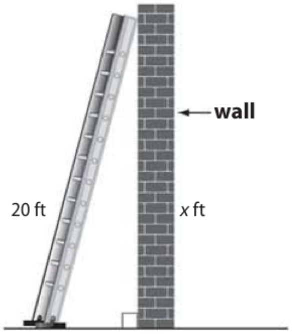
$\sqrt{169} = x$ Take the positive square root of each side.

$13 = x$ Simplify.

So, the answer is choice C.

GuidedPractice

3. According to your company's safety regulations, the distance from the base of a ladder to a wall that it leans against should be at least one fourth of the ladder's total length. You are given a 20-foot ladder to place against a wall at a job site. If you follow the company's safety regulations, what is the maximum distance x up the wall the ladder will reach, to the nearest tenth?



Note: Not drawn to scale.

F 12 feet
H 20.6 feet
G 19.4 feet
J 30.6 feet

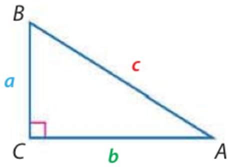
2 Converse of the Pythagorean Theorem

The converse of the Pythagorean Theorem also holds. You can use this theorem to help you determine whether a triangle is a right triangle given the measures of all three sides.

Theorem 8.5 Converse of the Pythagorean Theorem

Words If the sum of the squares of the lengths of the shortest sides of a triangle is equal to the square of the length of the longest side, then the triangle is a right triangle.

Symbols If $a^2 + b^2 = c^2$, then $\triangle ABC$ is a right triangle.



You will prove Theorem 8.5 in Exercise 35.

StudyTip

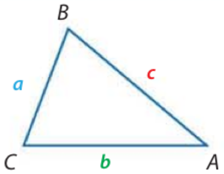
Determining the Longest Side If the measures of any of the sides of a triangle are expressed as radicals, you may wish to use a calculator to determine which length is the longest.

You can also use side lengths to classify a triangle as acute or obtuse.

Theorems Pythagorean Inequality Theorems

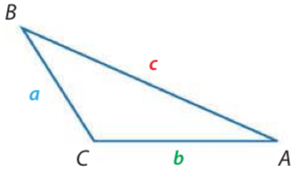
8.6 If the square of the length of the longest side of a triangle is less than the sum of the squares of the lengths of the other two sides, then the triangle is an acute triangle.

Symbols If $c^2 < a^2 + b^2$, then $\triangle ABC$ is acute.



8.7 If the square of the length of the longest side of a triangle is greater than the sum of the squares of the lengths of the other two sides, then the triangle is an obtuse triangle.

Symbols If $c^2 > a^2 + b^2$, then $\triangle ABC$ is obtuse.



You will prove Theorems 8.6 and 8.7 in Exercises 36 and 37, respectively.

ReviewVocabulary

Triangle Inequality Theorem The sum of the lengths of any two sides of a triangle must be greater than the length of the third side.

Example 4 Classify Triangles

Determine whether each set of numbers can be the measures of the sides of a triangle. If so, classify the triangle as *acute*, *right*, or *obtuse*. Justify your answer.

a. 7, 14, 16

Step 1 Determine whether the measures can form a triangle using the Triangle Inequality Theorem.

$7 + 14 > 16$ ✓ $14 + 16 > 7$ ✓ $7 + 16 > 14$ ✓

The side lengths 7, 14, and 16 can form a triangle.

Step 2 Classify the triangle by comparing the square of the longest side to the sum of the squares of the other two sides.

$c^2 \stackrel{?}{=} a^2 + b^2$ Compare c^2 and $a^2 + b^2$.

$16^2 \stackrel{?}{=} 7^2 + 14^2$ Substitution

$256 > 245$ Simplify and compare.

Since $c^2 > a^2 + b^2$, the triangle is obtuse.

b. 9, 40, 41

Step 1 Determine whether the measures can form a triangle.

$9 + 40 > 41$ ✓ $40 + 41 > 9$ ✓ $9 + 41 > 40$ ✓

The side lengths 9, 40, and 41 can form a triangle.

Step 2 Classify the triangle.

$c^2 \stackrel{?}{=} a^2 + b^2$ Compare c^2 and $a^2 + b^2$.

$41^2 \stackrel{?}{=} 9^2 + 40^2$ Substitution

$1681 = 1681$ Simplify and compare.

Since $c^2 = a^2 + b^2$, the triangle is a right triangle.

GuidedPractice

4A. 11, 60, 61

4B. $2\sqrt{3}, 4\sqrt{2}, 3\sqrt{5}$

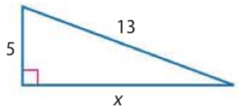
4C. 6.2, 13.8, 20

Check Your Understanding

● = Step-by-Step Solutions begin on page R14.

Example 1 Find *x*.

1.



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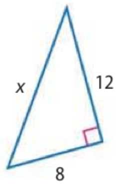

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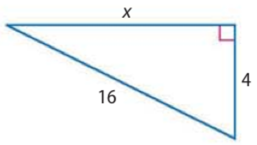
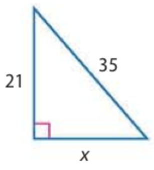


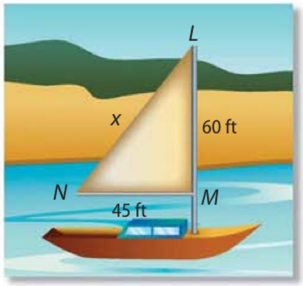
Example 2 **4.** Use a Pythagorean triple to find *x*. Explain your reasoning.



Example 3 **5.** **MULTIPLE CHOICE** The mainsail of a boat is shown. What is the length, in feet, of $\overline{LN}$?

A 52.5 **C** 72.5

B 65 **D** 75



Example 4 **Determine whether each set of numbers can be the measures of the sides of a triangle. If so, classify the triangle as *acute*, *obtuse*, or *right*. Justify your answer.**

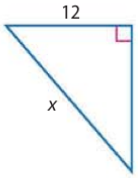
6. 15, 36, 39 **7.** 16, 18, 26 **8.** 15, 20, 24

Practice and Problem Solving

Extra Practice is on page R8.

Example 1 **Find *x*.**

9.



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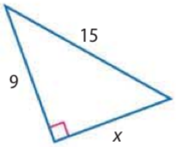

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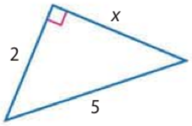

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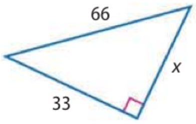

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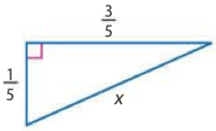

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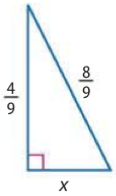
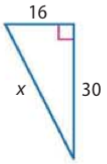


Example 2 **PERSEVERANCE** **Use a Pythagorean Triple to find *x*.**

15



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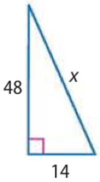

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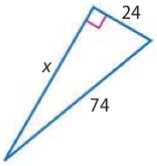

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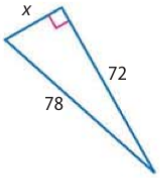
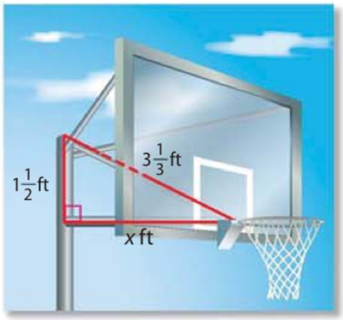
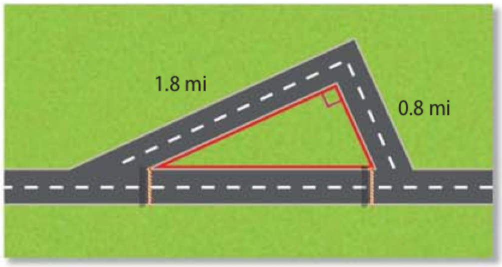


Example 3 **19.** **BASKETBALL** The support for a basketball goal forms a right triangle as shown. What is the length *x* of the horizontal portion of the support?



20. **DRIVING** The street that Khaliah usually uses to get to school is under construction. She has been taking the detour shown. If the construction starts at the point where Khaliah leaves her normal route and ends at the point where she re-enters her normal route, about how long is the stretch of road under construction?



Example 4 **Determine whether each set of numbers can be the measures of the sides of a triangle. If so, classify the triangle as *acute*, *obtuse*, or *right*. Justify your answer.**

21. 7, 15, 21

22. 10, 12, 23

23. 4.5, 20, 20.5

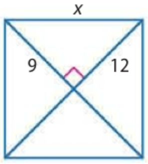
24. 44, 46, 91

25. 4.2, 6.4, 7.6

26. 4, 12, 14

Find x.

27.



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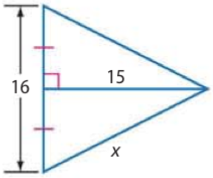

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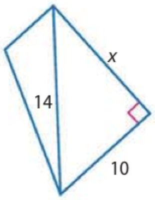


COORDINATE GEOMETRY **Determine whether $\triangle XYZ$ is an *acute*, *right*, or *obtuse* triangle for the given vertices. Explain.**

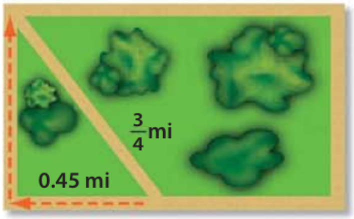
30. $X(-3, -2)$, $Y(-1, 0)$, $Z(0, -1)$

31. $X(-7, -3)$, $Y(-2, -5)$, $Z(-4, -1)$

32. $X(1, 2)$, $Y(4, 6)$, $Z(6, 6)$

33. $X(3, 1)$, $Y(3, 7)$, $Z(11, 1)$

34. **JOGGING** Brett jogs in the park three times a week. Usually, he takes a $\frac{3}{4}$-mile path that cuts through the park. Today, the path is closed, so he is taking the orange route shown. How much farther will he jog on his alternate route than he would have if he had followed his normal path?



35. **PROOF** Write a paragraph proof of Theorem 8.5.

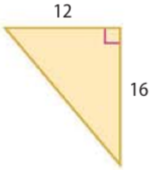
PROOF **Write a two-column proof for each theorem.**

36. Theorem 8.6

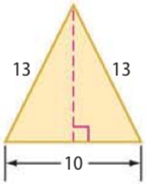
37. Theorem 8.7

CCSS SENSE-MAKING **Find the perimeter and area of each figure.**

38.



39.



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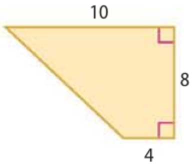


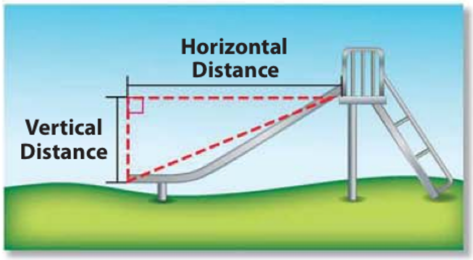
41. **ALGEBRA** The sides of a triangle have lengths x, $x + 5$, and 25. If the length of the longest side is 25, what value of x makes the triangle a right triangle?

42. **ALGEBRA** The sides of a triangle have lengths $2x$, 8, and 12. If the length of the longest side is $2x$, what values of x make the triangle acute?

43. **TELEVISION** The screen aspect ratio, or the ratio of the width to the height, of a high-definition television is 16:9. The size of a television is given by the diagonal distance across the screen. If an HDTV is 41 inches wide, what is its screen size?

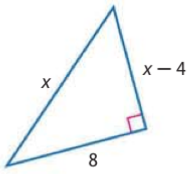
David Samuel Robbins/Photographer's Choice/Getty Images

44. **PLAYGROUND** According to the *Handbook for Public Playground Safety,* the ratio of the vertical distance to the horizontal distance covered by a slide should not be more than about 4 to 7. If the horizontal distance allotted in a slide design is 14 feet, approximately how long should the slide be?



Find *x*.

45



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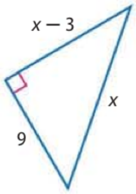

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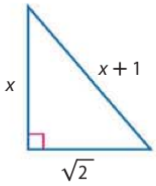


48. **MULTIPLE REPRESENTATIONS** In this problem, you will investigate special right triangles.

a. **Geometric** Draw three different isosceles right triangles that have whole-number side lengths. Label the triangles *ABC*, *MNP*, and *XYZ* with the right angle located at vertex *A*, *M*, and *X*, respectively. Label the leg lengths of each side, and find the exact length of the hypotenuse.

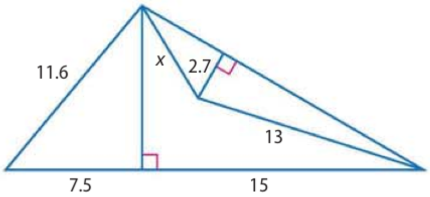
b. **Tabular** Copy and complete the table below.

Triangle	Length				Ratio	
ABC	*BC*		*AB*		$\frac{BC}{AB}$	
MNP	*NP*		*MN*		$\frac{NP}{MN}$	
XYZ	*YZ*		*XY*		$\frac{YZ}{XY}$	

c. **Verbal** Make a conjecture about the ratio of the hypotenuse to a leg of an isosceles right triangle.

H.O.T. Problems Use Higher-Order Thinking Skills

49. **CHALLENGE** Find the value of *x* in the figure at the right.



50. **CCSS** **ARGUMENTS** *True* or *false*? Any two right triangles with the same hypotenuse have the same area. Explain your reasoning.

51. **OPEN ENDED** Draw a right triangle with side lengths that form a Pythagorean triple. If you double the length of each side, is the resulting triangle *acute*, *right*, or *obtuse*? if you halve the length of each side? Explain.

52. **WRITING IN MATH** Research *incommensurable magnitudes*, and describe how this phrase relates to the use of irrational numbers in geometry. Include one example of an irrational number used in geometry.

Standardized Test Practice

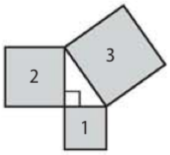
53. Which set of numbers cannot be the measures of the sides of a triangle?

A 10, 11, 20
B 14, 16, 28
C 35, 45, 75
D 41, 55, 98

54. A square park has a diagonal walkway from one corner to another. If the walkway is 120 meters long, what is the approximate length of each side of the park?

F 60 m
G 85 m
H 170 m
J 240 m

55. SHORT RESPONSE If the perimeter of square 2 is 200 units and the perimeter of square 1 is 150 units, what is the perimeter of square 3?



56. SAT/ACT In $\triangle ABC$, $\angle B$ is a right angle and $\angle A$ is 20° greater than $\angle C$. What is the measure of $\angle C$?

A 30
B 35
C 40
D 45
E 70

Spiral Review

Find the geometric mean between each pair of numbers. (Lesson 8-1)

57. 9 and 4

58. 45 and 5

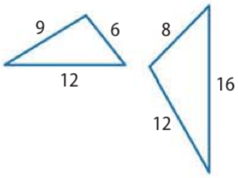
59. 12 and 15

60. 36 and 48

61. SCALE DRAWING Teodoro is creating a scale model of a skateboarding ramp on a 10-by-8-inch sheet of graph paper. If the real ramp is going to be 12 feet by 8 feet, find an appropriate scale for the drawing and determine the ramp's dimensions. (Lesson 7-7)

Determine whether the triangles are similar. If so, write a similarity statement. If not, what would be sufficient to prove the triangles similar? Explain your reasoning. (Lesson 7-3)

62.



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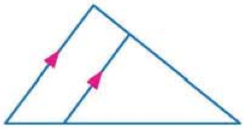

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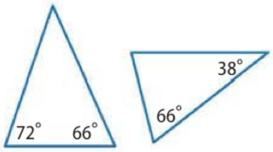
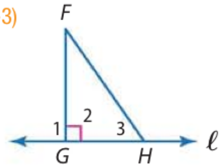


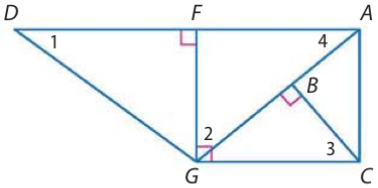
65. PROOF Write a two-column proof. (Lesson 5-3)

Given: $\overline{FG} \perp \ell$
$\overline{FH}$ is any nonperpendicular segment from F to ℓ.

Prove: $FH > FG$



Find each measure if $m\angle DGF = 53$ and $m\angle AGC = 40$. (Lesson 4-2)



66. $m\angle 1$

67. $m\angle 2$

68. $m\angle 3$

69. $m\angle 4$

Find the distance between each pair of parallel lines with the given equations. (Lesson 3-6)

70. $y = 4x$
$y = 4x - 17$

71. $y = 2x - 3$
$2x - y = -4$

72. $y = -0.75x - 1$
$3x + 4y = 20$

Skills Review

Find the value of x.

73. $18 = 3x\sqrt{3}$

74. $24 = 2x\sqrt{2}$

75. $9\sqrt{2} \cdot x = 18\sqrt{2}$

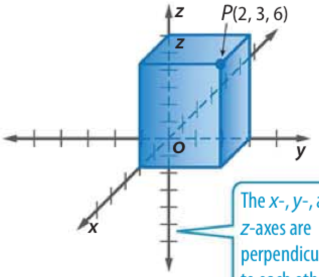
76. $2 = x \cdot \frac{4}{\sqrt{3}}$

EXTEND 8-2

Geometry Lab
Coordinates in Space

You have used ordered pairs of two coordinates to describe the location of a point on the coordinate plane. Because space has three dimensions, a point requires three numbers, or coordinates, to describe its location in space.

A point in space is represented by an **ordered triple** of real numbers (x, y, z). In the figure at the right, the ordered triple (2, 3, 6) locates point P. Notice that a rectangular prism is used to show perspective.



The x-, y-, and z-axes are perpendicular to each other.

Activity 1 Graph a Rectangular Solid

Graph a rectangular solid that has two vertices, $L(4, -5, 2)$ and the origin. Label the coordinates of each vertex.

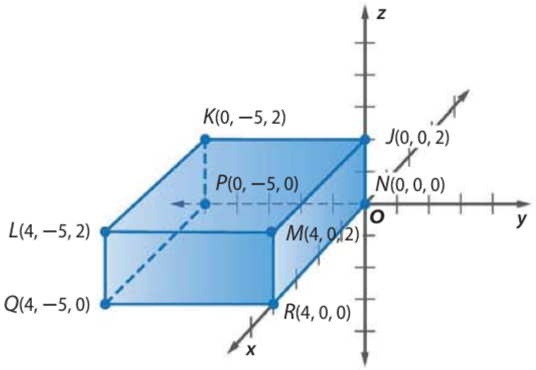
Step 1 Plot the x-coordinate first. Draw a segment from the origin 4 units in the positive direction.

Step 2 To plot the y-coordinate, draw a segment five units in the negative direction.

Step 3 Next, to plot the z-coordinate, draw a segment two units long in the positive direction.

Step 4 Label the coordinate L.

Step 5 Draw the rectangular prism and label each vertex: $L(4, -5, 2)$, $K(0, -5, 2)$, $J(0, 0, 2)$, $M(4, 0, 2)$ $Q(4, -5, 0)$, $P(0, -5, 0)$, $N(0, 0, 0)$, and $R(4, 0, 0)$.



Finding the distance between points and the midpoint of a segment in space is similar to finding distance and a midpoint in the coordinate plane.

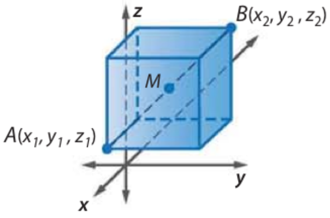
KeyConcept Distance and Midpoint Formulas in Space

If A has coordinates $A(x_1, y_1, z_1)$ and B has coordinates $B(x_2, y_2, z_2)$, then

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}.$$

The midpoint M of $\overline{AB}$ has coordinates

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right).$$



Activity 2 Distance and Midpoint Formulas in Space

Consider $J(2, 4, 9)$ and $K(-4, -5, 11)$.

a. Find JK.

$JK = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$	Distance Formula in Space
$= \sqrt{(-4 - 2)^2 + (-5 - 4)^2 + (11 - 9)^2}$	Substitution
$= \sqrt{121}$	Simplify.
$= 11$	Use a calculator.

b. Determine the coordinates of the midpoint M of $\overline{JK}$.

$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right)$	Midpoint Formula in Space
$= \left(\frac{2 + (-4)}{2}, \frac{4 + (-5)}{2}, \frac{9 + 11}{2}\right)$	Substitution
$= \left(-1, -\frac{1}{2}, 10\right)$	Simplify.

Exercises

Graph a rectangular solid that contains the given point and the origin as vertices. Label the coordinates of each vertex.

1. $A(2, 1, 5)$

2. $P(-1, 4, 2)$

3. $C(-2, 2, 2)$

4. $R(3, -4, 1)$

5. $P(4, 6, -3)$

6. $G(4, 1, -3)$

7. $K(-2, -4, -4)$

8. $W(-1, -3, -6)$

9. $W(3, 3, 4)$

Determine the distance between each pair of points. Then determine the coordinates of the midpoint *M* of the segment joining the pair of points.

10. $D(0, 0, 0)$ and $E(1, 5, 7)$

11. $G(-3, -4, 6)$ and $H(5, -3, -5)$

12. $K(2, 2, 0)$ and $L(-2, -2, 0)$

13. $P(-2, -5, 8)$ and $Q(3, -2, -1)$

14. $A(4, 7, 9)$ and $B(-3, 8, -8)$

15. $W(-12, 8, 10)$ and $Z(-4, 1, -2)$

16. $F\left(\frac{3}{5}, 0, \frac{4}{5}\right)$ and $G(0, 3, 0)$

17. $G(1, -1, 6)$ and $H\left(\frac{1}{5}, -\frac{2}{5}, 2\right)$

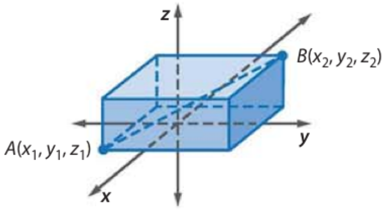
18. $B(\sqrt{3}, 2, 2\sqrt{2})$ and $C(-2\sqrt{3}, 4, 4\sqrt{2})$

19. $S(6\sqrt{3}, 4, 4\sqrt{2})$ and $T(4\sqrt{3}, 5, \sqrt{2})$

20. **PROOF** Write a coordinate proof of the Distance Formula in Space.

Given: A has coordinates $A(x_1, y_1, z_1)$, and B has coordinates $B(x_2, y_2, z_2)$.

Prove: $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$



21. **WRITING IN MATH** Compare and contrast the Distance and Midpoint Formulas on the coordinate plane and in three-dimensional coordinate space.

LESSON 8-3 Special Right Triangles

Then

- You used properties of isosceles and equilateral triangles.

Now

- 1 Use the properties of 45°-45°-90° triangles.
- 2 Use the properties of 30°-60°-90° triangles.

Why?

- As part of a packet for students attending a regional student council meeting, Lyndsay orders triangular highlighters. She wants to buy rectangular boxes for the highlighters and other items, but she is concerned that the highlighters will not fit in the box she has chosen. If she knows the length of a side of the highlighter, Lyndsay can use the properties of special right triangles to determine if it will fit in the box.

Common Core State Standards

Content Standards

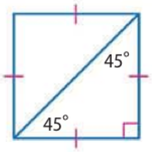
G.SRT.6 Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.

Mathematical Practices

1 Make sense of problems and persevere in solving them.

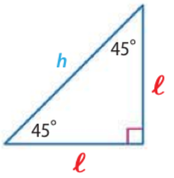
7 Look for and make use of structure.

1 Properties of 45°-45°-90° Triangles



The diagonal of a square forms two congruent isosceles right triangles. Since the base angles of an isosceles triangle are congruent, the measure of each acute angle is 90 ÷ 2 or 45. Such a triangle is also known as a 45°-45°-90° triangle.

You can use the Pythagorean Theorem to find a relationship among the side lengths of a 45°-45°-90° right triangle.



$\ell^2 + \ell^2 = h^2$ Pythagorean Theorem

$2\ell^2 = h^2$ Simplify.

$\sqrt{2\ell^2} = \sqrt{h^2}$ Take the positive square root of each side.

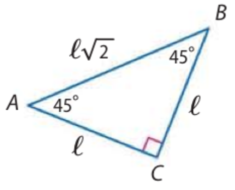
$\ell\sqrt{2} = h$ Simplify.

This algebraic proof verifies the following theorem.

Theorem 8.8 45°-45°-90° Triangle Theorem

In a 45°-45°-90° triangle, the legs ℓ are congruent and the length of the hypotenuse h is $\sqrt{2}$ times the length of a leg.

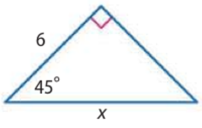
Symbols In a 45°-45°-90° triangle, $\ell = \ell$ and $h = \ell\sqrt{2}$.



Example 1 Find the Hypotenuse Length in a 45°-45°-90° Triangle

Find *x*.

a.

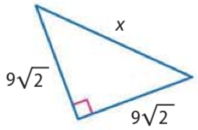


The acute angles of a right triangle are complementary, so the measure of the third angle is 90 − 45 or 45. Since this is a 45°-45°-90° triangle, use Theorem 8.8.

$h = \ell\sqrt{2}$ Theorem 8.8

$x = 6\sqrt{2}$ Substitution

b.



The legs of this right triangle have the same measure, so it is isosceles. Since this is a 45°-45°-90° triangle, use Theorem 8.8.

$h = \ell\sqrt{2}$ Theorem 8.8

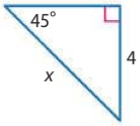
$x = 9\sqrt{2} \cdot \sqrt{2}$ Substitution

$x = 9 \cdot 2$ or 18 $\sqrt{2} \cdot \sqrt{2} = 2$

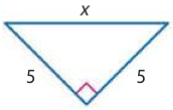
GuidedPractice

Find x.

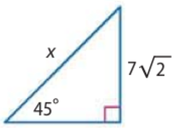
1A.



1B.



1C.

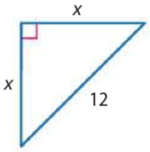


You can also work backward using Theorem 8.8 to find the lengths of the legs of a 45°-45°-90° triangle given the length of its hypotenuse.

Example 2 Find the Leg Lengths in a 45°-45°-90° Triangle

Find x.

The legs of this right triangle have the same measure, x, so it is a 45°-45°-90° triangle. Use Theorem 8.8 to find x.



$h = \ell\sqrt{2}$ 45°-45°-90° Triangle Theorem

$12 = x\sqrt{2}$ Substitution

$\frac{12}{\sqrt{2}} = x$ Divide each side by $\sqrt{2}$.

$\frac{12}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = x$ Rationalize the denominator.

$\frac{12\sqrt{2}}{2} = x$ Multiply.

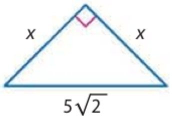
$6\sqrt{2} = x$ Simplify.

ReviewVocabulary

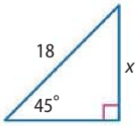
rationalizing the denominator a method used to eliminate radicals from the denominator of a fraction

GuidedPractice

2A.



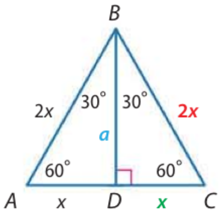
2B.



2 Properties of 30°-60°-90° Triangles

A 30°-60°-90° triangle is another *special* right triangle or right triangle with side lengths that share a special relationship. You can use an equilateral triangle to find this relationship.

When an altitude is drawn from any vertex of an equilateral triangle, two congruent 30°-60°-90° triangles are formed. In the figure shown, $\triangle ABD \cong \triangle CBD$, so $\overline{AD} \cong \overline{CD}$. If $AD = x$, then $CD = x$ and $AC = 2x$. Since $\triangle ABC$ is equilateral, $AB = 2x$ and $BC = 2x$.



StudyTip

Altitudes of Isosceles Triangles Notice that an altitude of an isosceles triangle is also a median of the triangle. In the figure at the right, $\overline{BD}$ bisects $\overline{AC}$.

Use the Pythagorean Theorem to find a, the length of the altitude $\overline{BD}$, which is also the longer leg of $\triangle BDC$.

$a^2 + x^2 = (2x)^2$ Pythagorean Theorem

$a^2 + x^2 = 4x^2$ Simplify.

$a^2 = 3x^2$ Subtract x^2 from each side.

$a = \sqrt{3x^2}$ Take the positive square root of each side.

$a = x\sqrt{3}$ Simplify.

StudyTip

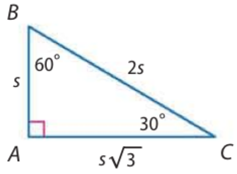
Use Ratios The lengths of the sides of a 30°-60°-90° triangle are in a ratio of 1 to $\sqrt{3}$ to 2 or 1 : $\sqrt{3}$:2.

This algebraic proof verifies the following theorem.

Theorem 8.9 30°-60°-90° Triangle Theorem

In a 30°-60°-90° triangle, the length of the hypotenuse h is 2 times the length of the shorter leg s, and the length of the longer leg ℓ is $\sqrt{3}$ times the length of the shorter leg.

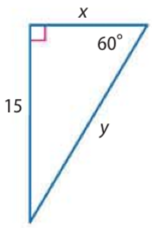
Symbols In a 30°-60°-90° triangle, $h = 2s$ and $\ell = s\sqrt{3}$.



Remember, the shortest side of a triangle is opposite the smallest angle. So the shorter leg in a 30°-60°-90° triangle is opposite the 30° angle, and the longer leg is opposite the 60° angle.

Example 3 Find Lengths in a 30°-60°-90° Triangle

Find x and y.



The acute angles of a right triangle are complementary, so the measure of the third angle in this triangle is 90 − 60 or 30. This is a 30°-60°-90° triangle.

Use Theorem 8.9 to find x, the length of the shorter side.

$\ell = s\sqrt{3}$ Theorem 8.9

$15 = x\sqrt{3}$ Substitution

$\frac{15}{\sqrt{3}} = x$ Divide each side by $\sqrt{3}$.

$\frac{15}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = x$ Rationalize the denominator.

$\frac{15\sqrt{3}}{\sqrt{3} \cdot \sqrt{3}} = x$ Multiply.

$\frac{15\sqrt{3}}{3} = x$ $\sqrt{3} \cdot \sqrt{3} = 3$

$5\sqrt{3} = x$ Simplify.

Now use Theorem 8.9 to find y, the length of the hypotenuse.

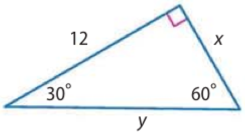
$h = 2s$ Theorem 8.9

$y = 2(5\sqrt{3})$ or $10\sqrt{3}$ Substitution

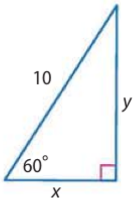
GuidedPractice

Find x and y.

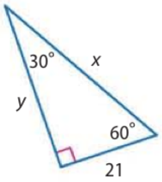
3A.



3B.



3C.



You can use the properties of 30°-60°-90° and 45°-45°-90° triangles to solve real-world problems.

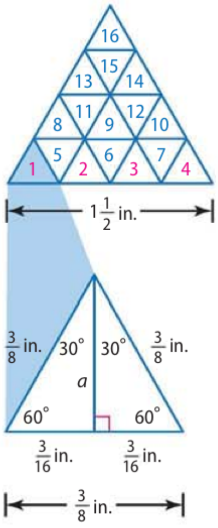
Real-World Example 4 Use Properties of Special Right Triangles

INVENTIONS A company makes crayons that "do not roll off tables" by shaping them as triangular prisms with equilateral bases. Sixteen of these crayons fit into a box shaped like a triangular prism that is $1\frac{1}{2}$ inches wide. The crayons stand on end in the box and the base of the box is equilateral. What are the dimensions of each crayon?

Understand You know that 16 crayons with equilateral triangular bases fit into a prism. You need to find the base length and height of each crayon.

Plan Guess and check to determine the arrangement of 16 crayons that would stack to fill the box. Find the width of one crayon and use the 30°-60°-90° Triangle Theorem to find its altitude.

Solve Make a guess that 4 equilateral crayons will fit across the base of the box. A sketch shows that the total number of crayons it takes to fill the box using 4 crayons across the base is 16. ✓



The width of the box is $1\frac{1}{2}$ inches, so the width of one crayon is $1\frac{1}{2} \div 4$ or $\frac{3}{8}$ inch.

Draw an equilateral triangle representing one crayon. Its altitude forms the longer leg of two 30°-60°-90° triangles. Use Theorem 8.9 to find the approximate length of the altitude *a*.

$$\text{longer leg length} = \text{shorter leg length} \cdot \sqrt{3}$$

$$a = \frac{3}{16} \cdot \sqrt{3} \text{ or about } 0.3$$

Each crayon is $\frac{3}{8}$ or about 0.4 inch by about 0.3 inch.

Check Find the height of the box using the 30°-60°-90° Triangle Theorem. Then divide by four, since the box is four crayons high. The result is a crayon height of about 0.3 inch. ✓

Problem-Solving Tip

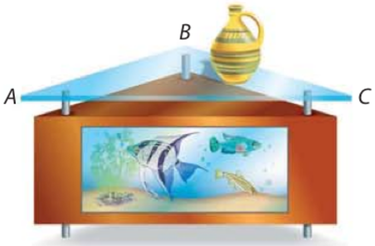
Guess and Check When using the guess and check strategy, it can be helpful to keep a list of those guesses that you have already tried and know do not work.

In Example 4, suppose your first guess had been that the box was 5 crayons wide.

The sketch of this possibility reveals that this leads to a stack of 25, not 16 crayons.

Guided Practice

4. FURNITURE The top of the aquarium coffee table shown is an isosceles right triangle. The table's longest side, $\overline{AC}$, measures 107 centimeters. What is the distance from vertex *B* to side $\overline{AC}$? What are the lengths of the other two sides?

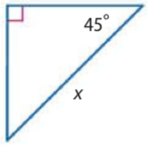


Check Your Understanding

● = Step-by-Step Solutions begin on page R14.

Examples 1-2 **Find x.**

1.



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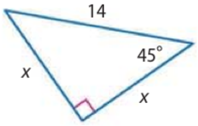

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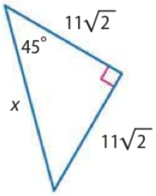
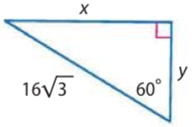


Example 3 **Find x and y.**

4.



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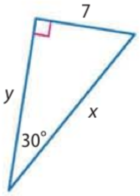

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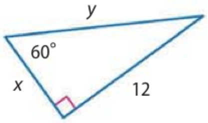


Example 4

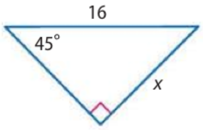
7. **ART** Paulo is mailing an engraved plaque that is $3\frac{1}{4}$ inches high to the winner of a chess tournament. He has a mailer that is a triangular prism with 4-inch equilateral triangle bases as shown in the diagram. Will the plaque fit through the opening of the mailer? Explain.

Practice and Problem Solving

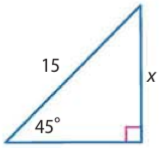
Extra Practice is on page R8.

Examples 1-2 **CCSS SENSE-MAKING** **Find x.**

8.



9.



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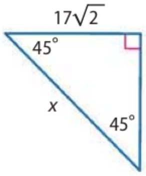

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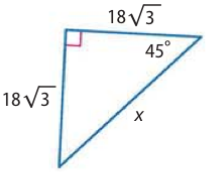

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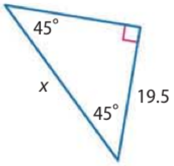

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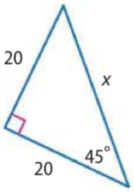


14. If a 45°-45°-90° triangle has a hypotenuse length of 9, find the leg length.

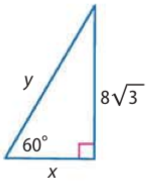
15. Determine the length of the leg of a 45°-45°-90° triangle with a hypotenuse length of 11.

16. What is the length of the hypotenuse of a 45°-45°-90° triangle if the leg length is 6 centimeters?

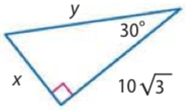
17. Find the length of the hypotenuse of a 45°-45°-90° triangle with a leg length of 8 centimeters.

Example 3 **Find x and y.**

18.



19.



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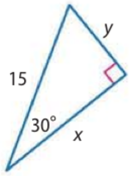

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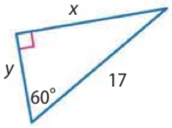

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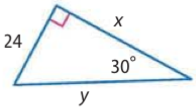
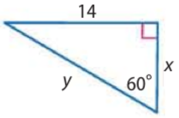


23.

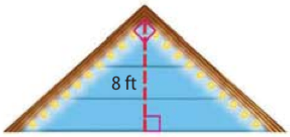


24. An equilateral triangle has an altitude length of 18 feet. Determine the length of a side of the triangle.

25. Find the length of the side of an equilateral triangle that has an altitude length of 24 feet.

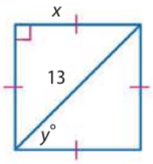
Example 4 **26.** CCSS **MODELING** Refer to the beginning of the lesson. Each highlighter is an equilateral triangle with 9-centimeter sides. Will the highlighter fit in a 10-centimeter by 7-centimeter rectangular box? Explain.

27. **EVENT PLANNING** Grace is having a party, and she wants to decorate the gable of the house as shown. The gable is an isosceles right triangle and she knows that the height of the gable is 8 feet. What length of lights will she need to cover the gable below the roof line?



Find x and y.

28.



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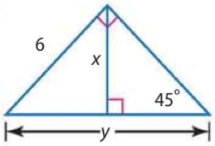

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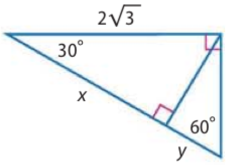

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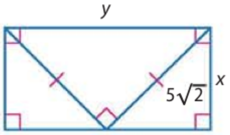

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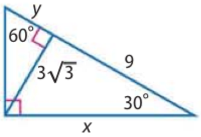

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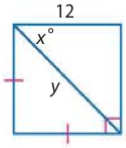
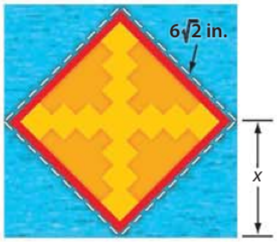
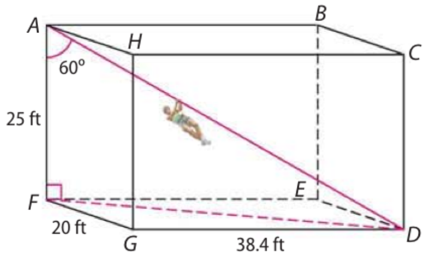


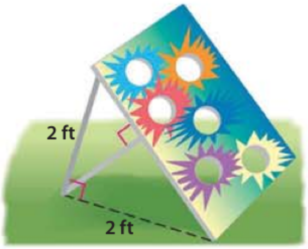
34. **QUILTS** The quilt block shown is made up of a square and four isosceles right triangles. What is the value of x? What is the side length of the entire quilt block?



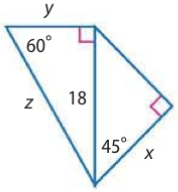
(35) **ZIP LINE** Suppose a zip line is anchored in one corner of a course shaped like a rectangular prism. The other end is anchored in the opposite corner as shown. If the zip line makes a 60° angle with post $\overline{AF}$, find the zip line's length, AD.



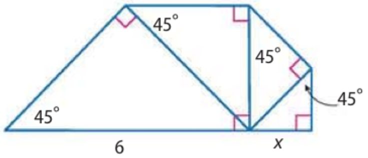
36. **GAMES** Kei is building a bean bag toss for the school carnival. He is using a 2-foot back support that is perpendicular to the ground 2 feet from the front of the board. He also wants to use a support that is perpendicular to the board as shown in the diagram. How long should he make the support?



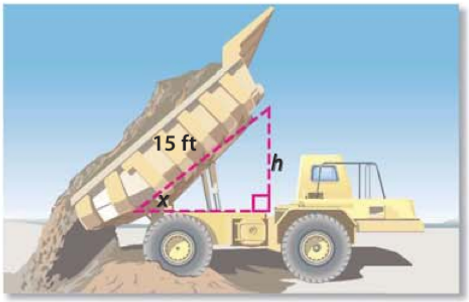
37. Find x, y, and z.



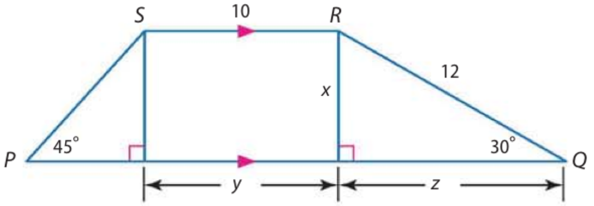
38. Each triangle in the figure is a 45°-45°-90° triangle. Find x.



39. **CCSS MODELING** The dump truck shown has a 15-foot bed length. What is the height of the bed h when angle x is 30°? 45°? 60°?



40. Find x, y, and z, and the perimeter of trapezoid $PQRS$.

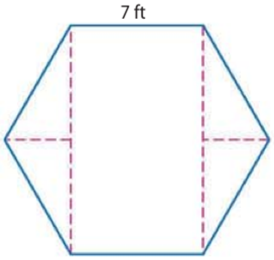


41. **COORDINATE GEOMETRY** $\triangle XYZ$ is a 45°-45°-90° triangle with right angle Z. Find the coordinates of X in Quadrant I for $Y(-1, 2)$ and $Z(6, 2)$.

42. **COORDINATE GEOMETRY** $\triangle EFG$ is a 30°-60°-90° triangle with $m\angle F = 90$. Find the coordinates of E in Quadrant III for $F(-3, -4)$ and $G(-3, 2)$. $\overline{FG}$ is the longer leg.

43. **COORDINATE GEOMETRY** $\triangle JKL$ is a 45°-45°-90° triangle with right angle K. Find the coordinates of L in Quadrant IV for $J(-3, 5)$ and $K(-3, -2)$.

44. **EVENT PLANNING** Eva has reserved a gazebo at a local park for a party. She wants to be sure that there will be enough space for her 12 guests to be in the gazebo at the same time. She wants to allow 8 square feet of area for each guest. If the floor of the gazebo is a regular hexagon and each side is 7 feet, will there be enough room for Eva and her friends? Explain. (*Hint:* Use the Polygon Interior Angle Sum Theorem and the properties of special right triangles.)



45 **MULTIPLE REPRESENTATIONS** In this problem, you will investigate ratios in right triangles.

a. **Geometric** Draw three similar right triangles with a 50° angle. Label one triangle *ABC* where angle *A* is the right angle and *B* is the 50° angle. Label a second triangle *MNP* where *M* is the right angle and *N* is the 50° angle. Label the third triangle *XYZ* where *X* is the right angle and *Y* is the 50° angle.

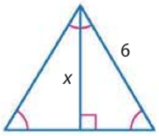
b. **Tabular** Copy and complete the table below.

Triangle	Length				Ratio	
ABC	*AC*		*BC*		$\frac{AC}{BC}$	
MNP	*MP*		*NP*		$\frac{MP}{NP}$	
XYZ	*XZ*		*YZ*		$\frac{XZ}{YZ}$	

c. **Verbal** Make a conjecture about the ratio of the leg opposite the 50° angle to the hypotenuse in any right triangle with an angle measuring 50°.

H.O.T. Problems Use Higher-Order Thinking Skills

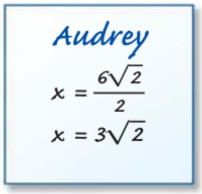
46. **CCSS CRITIQUE** Carmen and Audrey want to find *x* in the triangle shown. Is either of them correct? Explain.



Carmen

$x = \frac{6\sqrt{3}}{2}$

$x = 3\sqrt{3}$



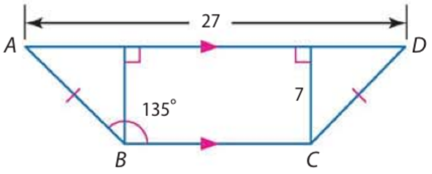
Audrey

$x = \frac{6\sqrt{2}}{2}$

$x = 3\sqrt{2}$

47. **OPEN ENDED** Draw a rectangle that has a diagonal twice as long as its width. Then write an equation to find the length of the rectangle.

48. **CHALLENGE** Find the perimeter of quadrilateral *ABCD*.



49. **REASONING** The ratio of the measure of the angles of a triangle is 1:2:3. The length of the shortest side is 8. What is the perimeter of the triangle?

50. **WRITING IN MATH** Why are some right triangles considered *special?*

Standardized Test Practice

51. If the length of the longer leg in a 30°-60°-90° triangle is $5\sqrt{3}$, what is the length of the shorter leg?

A 3
B 5
C $5\sqrt{2}$
D 10

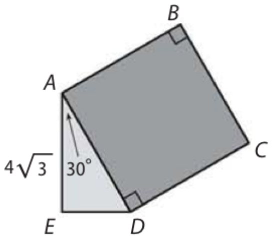
52. ALGEBRA Solve $\sqrt{5 - 4x} - 6 = 7$.

F -44
G -41
H 41
J 44

53. SHORT RESPONSE $\triangle XYZ$ is a 45°-45°-90° triangle with right angle Y. Find the coordinates of X in Quadrant III for $Y(-3, -3)$ and $Z(-3, 7)$.

54. SAT/ACT In the figure, below, square $ABCD$ is attached to $\triangle ADE$ as shown. If $m\angle EAD$ is 30° and AE is equal to $4\sqrt{3}$, then what is the area of square $ABCD$?

A $8\sqrt{3}$
B 16
C 64
D 72
E $64\sqrt{2}$

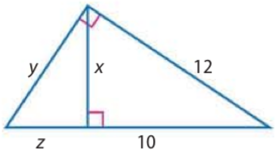


Spiral Review

55. SPORTS Dylan is making a ramp for bike jumps. The ramp support forms a right angle. The base is 12 feet long, and the height is 9 feet. What length of plywood does Dylan need for the ramp? (Lesson 8-2)

Find x, y, and z. (Lesson 8-1)

56.



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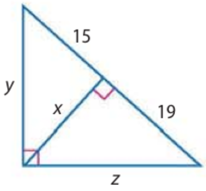

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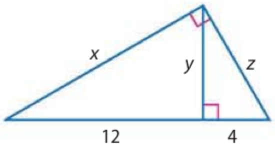


Find the measures of the angles of each triangle. (Lesson 7-1)

59. The ratio of the measures of the three angles is 2:5:3.

60. The ratio of the measures of the three angles is 6:9:10.

61. The ratio of the measures of the three angles is 5:7:8.

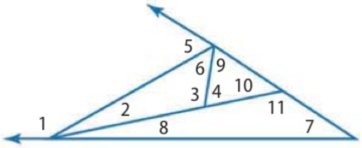
Use the Exterior Angle Inequality Theorem to list all of the angles that satisfy the stated condition. (Lesson 5-3)

62. measures less than $m\angle 5$

63. measures greater than $m\angle 6$

64. measures greater than $m\angle 10$

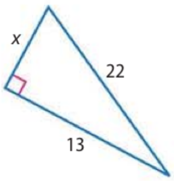
65. measures less than $m\angle 11$



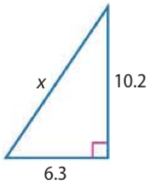
Skills Review

Find x.

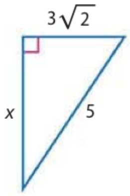
66.



67.



68.



EXPLORE **8-4**

Graphing Technology Lab
Trigonometry

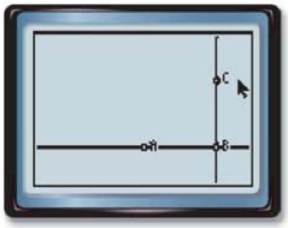
You have investigated patterns in the measures of special right triangles. *Trigonometry* is the study of the patterns in all right triangles. You can use the Cabri™ Jr. application on a graphing calculator to investigate these patterns.

CCSS Common Core State Standards
Content Standards
G.SRT.6 Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.
Mathematical Practices 5

Activity Investigate Trigonometric Ratios

Step 1 Use the line tool on the **F2** menu to draw a horizontal line. Label the points on the line *A* and *B*.

Step 2 Press **F2** and choose the **Perpendicular** tool to create a perpendicular line through point *B*. Draw and label a point *C* on the perpendicular line.

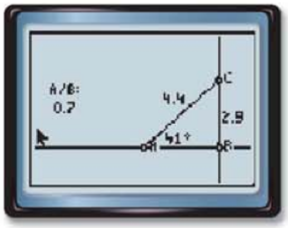


Steps 1 and 2

Step 3 Use the **Segment** tool on the **F2** menu to draw $\overline{AC}$.

Step 4 Find and label the measures of $\overline{BC}$ and $\overline{AC}$ using the **Distance** and **Length** tool under **Measure** on the **F5** menu. Use the **Angle** tool to find the measure of ∠*A*.

Step 5 Calculate and display the ratio $\frac{BC}{AC}$ using the **Calculate** tool on the **F5** menu. Label the ratio as *A*/*B*.



Steps 3 through 5

Step 6 Press CLEAR. Then use the arrow keys to move the cursor close to point *B*. When the arrow is clear, press and hold the ALPHA key. Drag *B* and observe the ratio.

Analyze the Results

1. Discuss the effect on $\frac{BC}{AC}$ by dragging point *B* on $\overline{BC}$, $\overline{AC}$, and ∠*A*.

2. Use the calculate tool to find the ratios $\frac{AB}{AC}$ and $\frac{BC}{AB}$. Then drag *B* and observe the ratios.

3. **MAKE A CONJECTURE** The *sine*, *cosine*, and *tangent* functions are trigonometric functions based on angle measures. Make a note of $m\angle A$. Exit Cabri Jr. and use SIN, COS, and TAN on the calculator to find *sine*, *cosine* and *tangent* for $m\angle A$. Compare the results to the ratios you found in the activity. Make a conjecture about the definitions of sine, cosine, and tangent.

LESSON 8-4 Trigonometry

Then	Now	Why?
You used the Pythagorean Theorem to find missing lengths in right triangles.	**1** Find trigonometric ratios using right triangles. **2** Use trigonometric ratios to find angle measures in right triangles.	The steepness of a hiking trail is often expressed as a *percent of grade*. The steepest part of Bright Angel Trail in the Grand Canyon National Park has about a 15.7% grade. This means that the trail rises or falls 15.7 feet over a horizontal distance of 100 feet. You can use trigonometric ratios to determine that this steepness is equivalent to an angle of about 9°.

NewVocabulary

trigonometry
trigonometric ratio
sine
cosine
tangent
inverse sine
inverse cosine
inverse tangent

Common Core State Standards

Content Standards

G.SRT.6 Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.

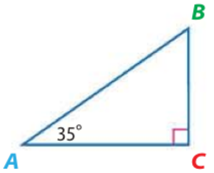
G.SRT.7 Explain and use the relationship between the sine and cosine of complementary angles.

Mathematical Practices

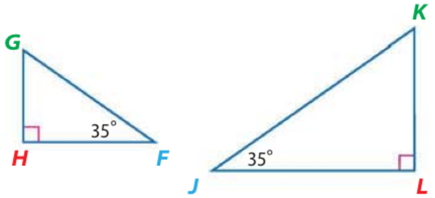
1 Make sense of problems and persevere in solving them.
5 Use appropriate tools strategically.

1 Trigonometric Ratios

The word **trigonometry** comes from two Greek terms, *trigon*, meaning triangle, and *metron*, meaning measure. The study of trigonometry involves triangle measurement. A **trigonometric ratio** is a ratio of the lengths of two sides of a right triangle. One trigonometric ratio of $\triangle ABC$ is $\frac{AC}{AB}$.



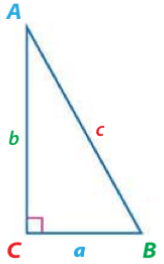
By AA Similarity, a right triangle with a given acute angle measure is similar to every other right triangle with the same acute angle measure. So, trigonometric ratios are constant for a given angle measure.



$\triangle ABC \sim \triangle FGH \sim \triangle JKL$, so $\frac{AC}{AB} = \frac{FH}{FG} = \frac{JL}{JK}$

The names of the three most common trigonometric ratios are given below.

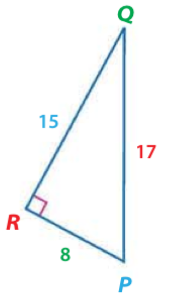
KeyConcept Trigonometric Ratios

Words	Symbols	
If $\triangle ABC$ is a right triangle with acute $\angle A$, then the **sine** of $\angle A$ (written sin A) is the ratio of the length of the leg opposite $\angle A$ (opp) to the length of the hypotenuse (hyp).	$\sin A = \frac{\text{opp}}{\text{hyp}}$ or $\frac{a}{c}$ $\sin B = \frac{\text{opp}}{\text{hyp}}$ or $\frac{b}{c}$	
If $\triangle ABC$ is a right triangle with acute $\angle A$, then the **cosine** of $\angle A$ (written cos A) is the ratio of the length of the leg adjacent $\angle A$ (adj) to the length of the hypotenuse (hyp).	$\cos A = \frac{\text{adj}}{\text{hyp}}$ or $\frac{b}{c}$ $\cos B = \frac{\text{adj}}{\text{hyp}}$ or $\frac{a}{c}$	
If $\triangle ABC$ is a right triangle with acute $\angle A$, then the **tangent** of $\angle A$ (written tan A) is the ratio of the length of the leg opposite $\angle A$ (opp) to the length of the leg adjacent $\angle A$ (adj).	$\tan A = \frac{\text{opp}}{\text{adj}}$ or $\frac{a}{b}$ $\tan B = \frac{\text{opp}}{\text{adj}}$ or $\frac{b}{a}$	

Douglas Peebles Photography/Alamy

Example 1 Find Sine, Cosine, and Tangent Ratios

Express each ratio as a fraction and as a decimal to the nearest hundredth.



a. sin *P*

$$\sin P = \frac{\text{opp}}{\text{hyp}}$$

$$= \frac{15}{17} \text{ or about } 0.88$$

b. cos *P*

$$\cos P = \frac{\text{adj}}{\text{hyp}}$$

$$= \frac{8}{17} \text{ or about } 0.47$$

c. tan *P*

$$\tan P = \frac{\text{opp}}{\text{adj}}$$

$$= \frac{15}{8} \text{ or about } 1.88$$

d. sin *Q*

$$\sin Q = \frac{\text{opp}}{\text{hyp}}$$

$$= \frac{8}{17} \text{ or about } 0.47$$

e. cos *Q*

$$\cos Q = \frac{\text{adj}}{\text{hyp}}$$

$$= \frac{15}{17} \text{ or about } 0.88$$

f. tan *Q*

$$\tan Q = \frac{\text{opp}}{\text{adj}}$$

$$= \frac{8}{15} \text{ or about } 0.53$$

StudyTip

Memorizing Trigonometric Ratios

SOH-CAH-TOA is a mnemonic device for learning the ratios for sine, cosine, and tangent using the first letter of each word in the ratios.

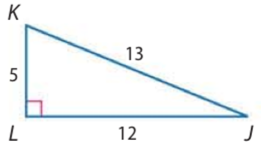
$$\sin A = \frac{\text{opp}}{\text{hyp}}$$

$$\cos A = \frac{\text{adj}}{\text{hyp}}$$

$$\tan A = \frac{\text{opp}}{\text{adj}}$$

GuidedPractice

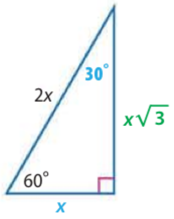
1. Find sin *J*, cos *J*, tan *J*, sin *K*, cos *K*, and tan *K*. Express each ratio as a fraction and as a decimal to the nearest hundredth.



Special right triangles can be used to find the sine, cosine, and tangent of 30°, 60°, and 45° angles.

Example 2 Use Special Right Triangles to Find Trigonometric Ratios

Use a special right triangle to express the tangent of 30° as a fraction and as a decimal to the nearest hundredth.



Draw and label the side lengths of a 30°-60°-90° right triangle, with *x* as the length of the shorter leg.

The side opposite the 30° angle has a measure of *x*.

The side adjacent to the 30° angle has a measure of $x\sqrt{3}$.

$$\tan 30° = \frac{\text{opp}}{\text{adj}}$$ Definition of tangent ratio

$$= \frac{\cancel{x}}{\cancel{x}\sqrt{3}}$$ Substitution

$$= \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$$ Simplify and rationalize the denominator.

$$= \frac{\sqrt{3}}{3} \text{ or about } 0.58$$ Simplify and use a calculator.

GuidedPractice

2. Use a special right triangle to express the cosine of 45° as a fraction and as a decimal to the nearest hundredth.

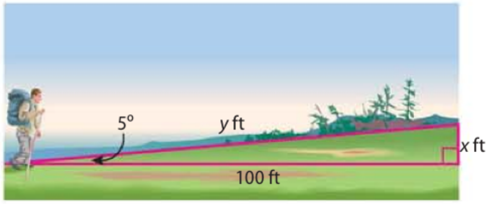
Real-WorldLink

The grade of a trail often changes many times. Average grade is the average of several consecutive running grades of a trail. Maximum grade is the smaller section of a trail that exceeds the trail's typical running grade. Trails often have maximum grades that are much steeper than a trail's average running grade.

Source: Federal Highway Administration

Real-World Example 3 Estimate Measures Using Trigonometry

HIKING A certain part of a hiking trail slopes upward at about a 5° angle. After traveling a horizontal distance of 100 feet along this part of the trail, what would be the change in a hiker's vertical position? What distance has the hiker traveled along the path?



Let $m\angle A = 5$. The vertical change in the hiker's position is x, the measure of the leg opposite $\angle A$. The horizontal distance traveled is 100 feet, the measure of the leg adjacent to $\angle A$. Since the length of the leg opposite and the leg adjacent to a given angle are involved, write an equation using a tangent ratio.

$$\tan A = \frac{\text{opp}}{\text{adj}}$$ Definition of tangent ratio

$$\tan 5° = \frac{x}{100}$$ Substitution

$$100 \cdot \tan 5° = x$$ Multiply each side by 100.

Use a calculator to find x.

100 TAN 5 ENTER **8.748866353**

The hiker is about 8.75 feet higher than when he started walking.

The distance y traveled along the path is the length of the hypotenuse, so you can use a cosine ratio to find this distance.

$$\cos A = \frac{\text{adj}}{\text{hyp}}$$ Definition of cosine ratio

$$\cos 5° = \frac{100}{y}$$ Substitution

$$y \cdot \cos 5° = 100$$ Multiply each side by y.

$$y = \frac{100}{\cos 5°}$$ Divide each side by cos 5°.

StudyTip

Graphing Calculator Be sure your graphing calculator is in degree mode rather than radian mode.

Use a calculator to find y.

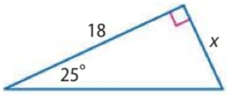
100 ÷ COS 5 ENTER **100.3819838**

The hiker has traveled a distance of about 100.38 feet along the path.

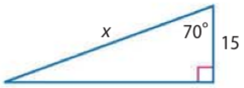
GuidedPractice

Find x to the nearest hundredth.

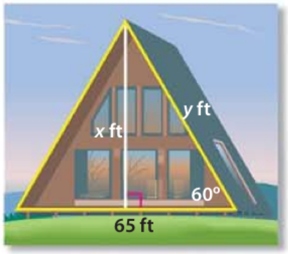
3A.



3B.



3C. ARCHITECTURE The front of the vacation cottage shown is an isosceles triangle. What is the height x of the cottage above its foundation? What is the length y of the roof? Explain your reasoning.



J.A. Kraulis/Masterfile

2 Use Inverse Trigonometric Ratios

In Example 2, you found that tan 30° ≈ 0.58. It follows that if the tangent of an acute angle is 0.58, then the angle measures approximately 30.

If you know the sine, cosine, or tangent of an acute angle, you can use a calculator to find the measure of the angle, which is the inverse of the trigonometric ratio.

ReadingMath

Inverse Trigonometric Ratios The expression $\sin^{-1} x$ is read *the inverse sine of x* and is interpreted as the angle with sine x. Be careful not to confuse this notation with the notation for negative exponents— $\sin^{-1} x \neq \frac{1}{\sin x}$. Instead, this notation is similar to the notation for an inverse function, $f^{-1}(x)$.

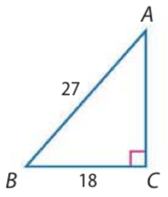
KeyConcept Inverse Trigonometric Ratios

Words	If $\angle A$ is an acute angle and the sine of A is x, then the **inverse sine** of x is the measure of $\angle A$.
Symbols	If $\sin A = x$, then $\sin^{-1} x = m\angle A$.
Words	If $\angle A$ is an acute angle and the cosine of A is x, then the **inverse cosine** of x is the measure of $\angle A$.
Symbols	If $\cos A = x$, then $\cos^{-1} x = m\angle A$.
Words	If $\angle A$ is an acute angle and the tangent of A is x, then the **inverse tangent** of x is the measure of $\angle A$.
Symbols	If $\tan A = x$, then $\tan^{-1} x = m\angle A$.

So if tan 30° ≈ 0.58, then $\tan^{-1} 0.58 \approx 30°$.

Example 4 Find Angle Measures Using Inverse Trigonometric Ratios

Use a calculator to find the measure of ∠A to the nearest tenth.



The measures given are those of the leg opposite ∠A and the hypotenuse, so write an equation using the sine ratio.

$\sin A = \frac{18}{27}$ or $\frac{2}{3}$ $\quad \sin A = \frac{\text{opp}}{\text{hyp}}$

If $\sin A = \frac{2}{3}$, then $\sin^{-1} \frac{2}{3} = m\angle A$. Use a calculator.

KEYSTROKES: 2nd [SIN $^{-1}$] (2 ÷ 3) ENTER 41.8103149

So, $m\angle A \approx 41.8°$.

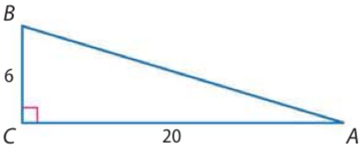
StudyTip

Tools Use a graphing calculator. The second functions of the SIN, COS, and TAN keys are usually the inverses.

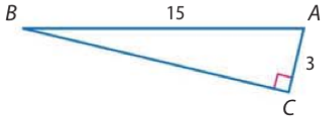
GuidedPractice

Use a calculator to find the measure of ∠A to the nearest tenth.

4A.



4B.

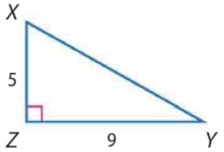


When you use given measures to find the unknown angle and side measures of a right triangle, this is known as *solving a right triangle*. To solve a right triangle, you need to know

- two side lengths or
- one side length and the measure of one acute angle.

Example 5 Solve a Right Triangle

Solve the right triangle. Round side measures to the nearest tenth and angle measures to the nearest degree.



Step 1 Find $m\angle X$ by using a tangent ratio.

$\tan X = \frac{9}{5}$ — $\tan X = \frac{\text{opp}}{\text{adj}}$

$\tan^{-1} \frac{9}{5} = m\angle X$ — Definition of inverse tangent

$60.9453959 \approx m\angle X$ — Use a calculator.

So, $m\angle X \approx 61$.

Step 2 Find $m\angle Y$ using Corollary 4.1, which states that the acute angles of a right triangle are complementary.

$m\angle X + m\angle Y = 90$ — Corollary 4.1

$61 + m\angle Y \approx 90$ — $m\angle X \approx 61$

$m\angle Y \approx 29$ — Subtract 61 from each side.

So, $m\angle Y \approx 29$.

StudyTip

Alternative Methods Right triangles can often be solved using different methods. In Example 5, $m\angle Y$ could have been found using a tangent ratio, and $m\angle X$ and a sine ratio could have been used to find XY.

Step 3 Find XY by using the Pythagorean Theorem.

$(XZ)^2 + (ZY)^2 = (XY)^2$ — Pythagorean Theorem

$5^2 + 9^2 = (XY)^2$ — Substitution

$106 = (XY)^2$ — Simplify.

$\sqrt{106} = XY$ — Take the positive square root of each side.

$10.3 \approx XY$ — Use a calculator.

So $XY \approx 10.3$.

WatchOut!

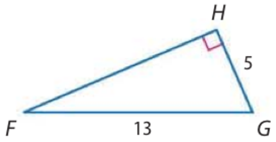
Approximation If using calculated measures to find other measures in a right triangle, be careful not to round values until the last step. So in the following equation, use $\tan^{-1} \frac{9}{5}$ instead of its approximate value, 61°.

$$XY = \frac{9}{\sin X} = \frac{9}{\sin\left(\tan^{-1} \frac{9}{5}\right)} \approx 10.3$$

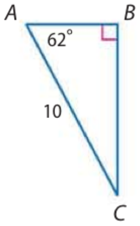
GuidedPractice

Solve each right triangle. Round side measures to the nearest tenth and angle measures to the nearest degree.

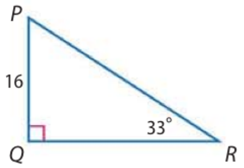
5A.



5B.



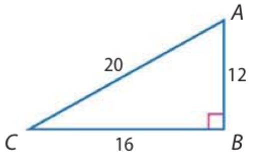
5C.



Check Your Understanding

● = Step-by-Step Solutions begin on page R14.

Example 1 **Express each ratio as a fraction and as a decimal to the nearest hundredth.**



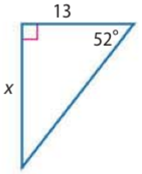
1. sin A **2.** tan C **3.** cos A

4. tan A **5.** cos C **6.** sin C

Example 2 **7.** Use a special right triangle to express sin 60° as a fraction and as a decimal to the nearest hundredth.

Example 3 **Find x. Round to the nearest hundredth.**

8.



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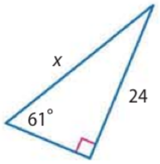

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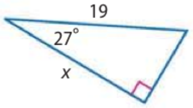
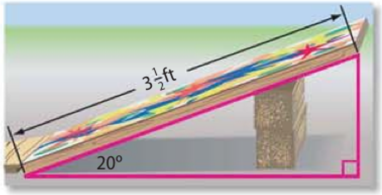
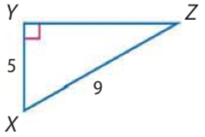


11. **SPORTS** David is building a bike ramp. He wants the angle that the ramp makes with the ground to be 20°. If the board he wants to use for his ramp is $3\frac{1}{2}$ feet long, about how tall will the ramp need to be at the highest point?



Example 4 **CCSS TOOLS** **Use a calculator to find the measure of ∠Z to the nearest tenth.**

12.



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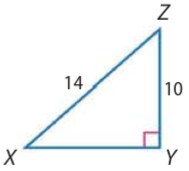

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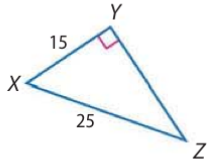
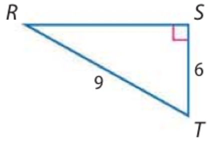


Example 5 **15.** Solve the right triangle. Round side measures to the nearest tenth and angle measures to the nearest degree.

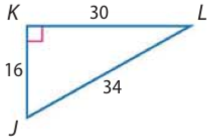


Practice and Problem Solving

Extra Practice is on page R8.

Example 1 **Find sin J, cos J, tan J, sin L, cos L, and tan L. Express each ratio as a fraction and as a decimal to the nearest hundredth.**

16.



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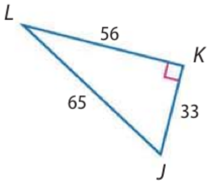

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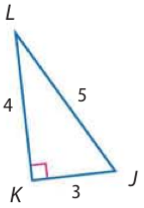

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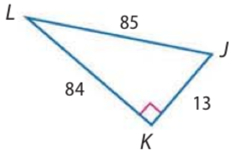

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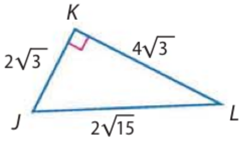

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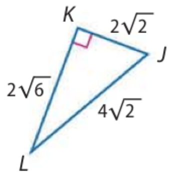
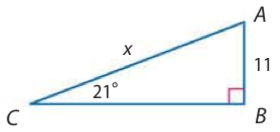


Example 2 **Use a special right triangle to express each trigonometric ratio as a fraction and as a decimal to the nearest hundredth.**

22. tan 60°
23. cos 30°
24. sin 45°
25. sin 30°
26. tan 45°
27. cos 60°

Example 3 **Find x. Round to the nearest tenth.**

28.



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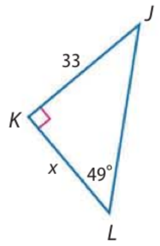

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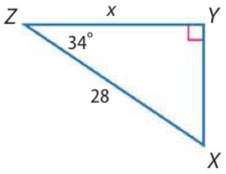

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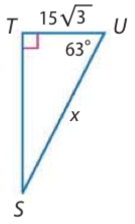

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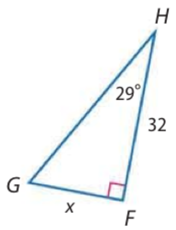

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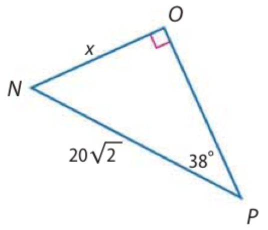
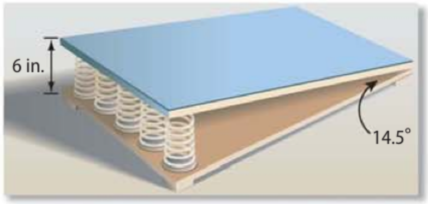


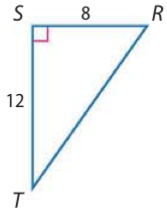
34. **GYMNASTICS** The springboard that Eric uses in his gymnastics class has 6-inch coils and forms an angle of 14.5° with the base. About how long is the springboard?



㉟ **ROLLER COASTERS** The angle of ascent of the first hill of a roller coaster is 55°. If the length of the track from the beginning of the ascent to the highest point is 98 feet, what is the height of the roller coaster when it reaches the top of the first hill?

Example 4 **TOOLS** **Use a calculator to find the measure of ∠T to the nearest tenth.**

36.



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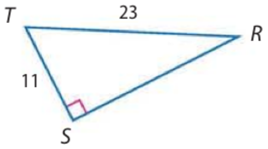

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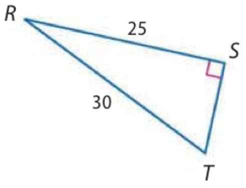

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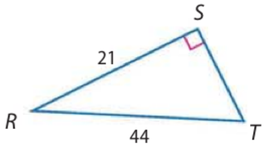

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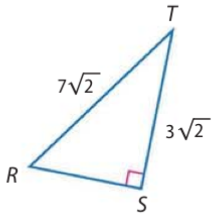

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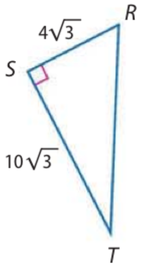
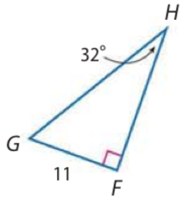


Example 5 **Solve each right triangle. Round side measures to the nearest tenth and angle measures to the nearest degree.**

42.



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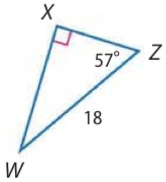

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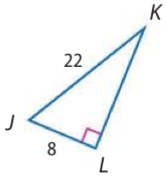

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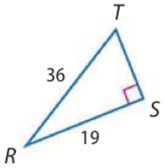


46. **BACKPACKS** Ramón has a rolling backpack that is $3\frac{3}{4}$ feet tall when the handle is extended. When he is pulling the backpack, Ramon's hand is 3 feet from the ground. What angle does his backpack make with the floor? Round to the nearest degree.

COORDINATE GEOMETRY **Find the measure of each angle to the nearest tenth of a degree using the Distance Formula and an inverse trigonometric ratio.**

47. $\angle K$ in right triangle JKL with vertices $J(-2, -3)$, $K(-7, -3)$, and $L(-2, 4)$

48. $\angle Y$ in right triangle XYZ with vertices $X(4, 1)$, $Y(-6, 3)$, and $Z(-2, 7)$

49. $\angle A$ in right triangle ABC with vertices $A(3, 1)$, $B(3, -3)$, and $C(8, -3)$

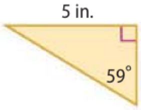
50. **SCHOOL SPIRIT** Hana is making a pennant for each of the 18 girls on her basketball team. She will use $\frac{1}{2}$-inch seam binding to finish the edges of the pennants.

a. What is the total length of seam binding needed to finish all of the pennants?

b. If seam binding is sold in 3-yard packages at a cost of $1.79, how much will it cost?

CCSS SENSE-MAKING **Find the perimeter and area of each triangle. Round to the nearest hundredth.**

51.



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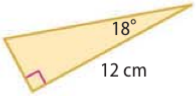

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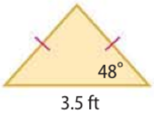
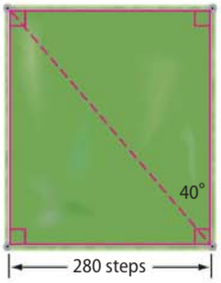


54. Find the tangent of the greater acute angle in a triangle with side lengths of 3, 4, and 5 centimeters.

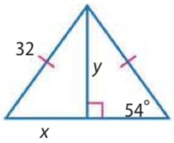
55. Find the cosine of the smaller acute angle in a triangle with side lengths of 10, 24, and 26 inches.

56. **ESTIMATION** Ethan and Tariq want to estimate the area of the field that their team will use for soccer practice. They know that the field is rectangular, and they have paced off the width of the field as shown. They used the fence posts at the corners of the field to estimate that the angle between the length of the field and the diagonal is about 40°. If they assume that each of their steps is about 18 inches, what is the area of the practice field in square feet? Round to the nearest square foot.



Find x and y. Round to the nearest tenth.

57



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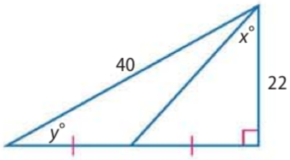

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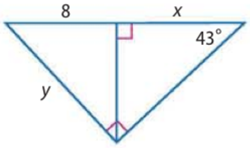
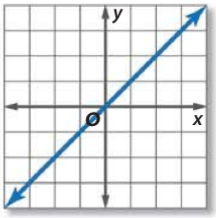


60. **COORDINATE GEOMETRY** Show that the slope of a line at 225° from the x-axis is equal to the tangent of 225°.



61. **MULTIPLE REPRESENTATIONS** In this problem, you will investigate an algebraic relationship between the sine and cosine ratios.

a. Geometric Draw three right triangles that are not similar to each other. Label the triangles ABC, MNP, and XYZ, with the right angles located at vertices B, N, and Y, respectively. Measure and label each side of the three triangles.

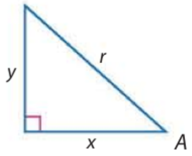
b. Tabular Copy and complete the table below.

Triangle	Trigonometric Ratios				Sum of Ratios Squared	
ABC	cos A		sin A		$(\cos A)^2 + (\sin A)^2 =$	
	cos C		sin C		$(\cos C)^2 + (\sin C)^2 =$	
MNP	cos M		sin M		$(\cos M)^2 + (\sin M)^2 =$	
	cos P		sin P		$(\cos P)^2 + (\sin P)^2 =$	
XYZ	cos X		sin X		$(\cos X)^2 + (\sin X)^2 =$	
	cos Z		sin Z		$(\cos Z)^2 + (\sin Z)^2 =$	

c. Verbal Make a conjecture about the sum of the squares of the cosine and sine of an acute angle of a right triangle.

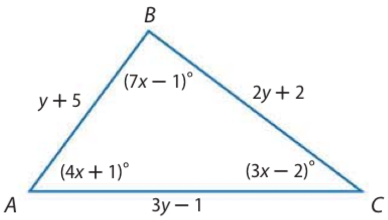
d. Algebraic Express your conjecture algebraically for an angle X.

e. Analytical Show that your conjecture is valid for angle A in the figure at the right using the trigonometric functions and the Pythagorean Theorem.



H.O.T. Problems Use Higher-Order Thinking Skills

62. **CHALLENGE** Solve $\triangle ABC$. Round to the nearest whole number.



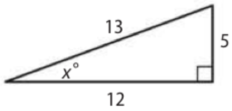
63. **REASONING** Are the values of sine and cosine for an acute angle of a right triangle always less than 1? Explain.

64. **REASONING** What is the relationship between the sine and cosine of complementary angles? Explain your reasoning and use the relationship to find cos 50 if sin 40 ≈ 0.64.

65. **WRITING IN MATH** Explain how you can use ratios of the side lengths to find the angle measures of the acute angles in a right triangle.

Standardized Test Practice

66. What is the value of tan x?



A $\tan x = \frac{13}{5}$

B $\tan x = \frac{12}{5}$

C $\tan x = \frac{5}{13}$

D $\tan x = \frac{5}{12}$

67. ALGEBRA Which of the following has the same value as $2^{-12} \times 2^3$?

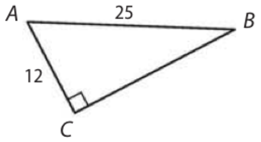
F 2^{-36}

G 4^{-9}

H 2^{-9}

J 2^{-4}

68. GRIDDED RESPONSE If $AC = 12$ and $AB = 25$, what is the measure of $\angle B$ to the nearest tenth?



69. SAT/ACT The area of a right triangle is 240 square inches. If the base is 30 inches long, how many inches long is the hypotenuse?

A 5

B 8

C 16

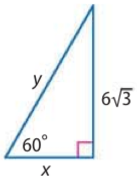
D $2\sqrt{241}$

E 34

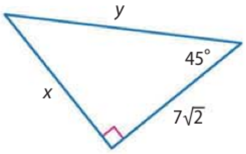
Spiral Review

Find x and y. (Lesson 8-3)

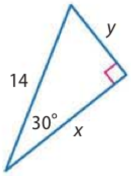
70.



71.



72.



Determine whether each set of numbers can be the measures of the sides of a triangle. If so, classify the triangle as *acute*, *obtuse*, or *right*. Justify your answer. (Lesson 8-2)

73. 8, 15, 17

74. 11, 12, 24

75. 13, 30, 35

76. 18, 24, 30

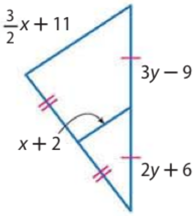
77. 3.2, 5.3, 8.6

78. $6\sqrt{3}$, 14, 17

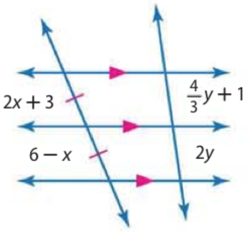
79. MAPS The scale on the map of New Mexico is 2 centimeters = 160 miles. The width of New Mexico through Albuquerque on the map is 4.1 centimeters. How long would it take to drive across New Mexico if you drove at an average of 60 miles per hour? (Lesson 7-7)

ALGEBRA **Find x and y.** (Lesson 7-4)

80.



81.



Skills Review

Solve each proportion. Round to the nearest tenth if necessary.

82. $2.14 = \frac{x}{12}$

83. $0.05x = 13$

84. $0.37 = \frac{32}{x}$

85. $0.74 = \frac{14}{x}$

86. $1.66 = \frac{x}{23}$

87. $0.21 = \frac{33}{x}$

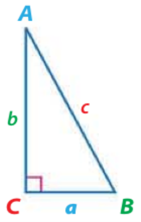
EXTEND 8-4

Graphing Technology Lab
Secant, Cosecant, and Cotangent

In the previous lesson, you used the trigonometric functions sine, cosine, and tangent to find angle relationships in right angles. In this activity, you will use the reciprocals of those functions, cosecant, secant, and cotangent, to explore angle and side relationships in right triangles.

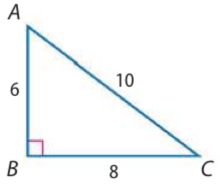
CCSS Common Core State Standards
Content Standards
G.SRT.6 Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.
Mathematical Practices 5

KeyConcept Reciprocal Trigonometric Ratios

Words	Symbols	
The **cosecant** of ∠*A* (written csc *A*) is the reciprocal of sin *A*.	$\csc A = \frac{1}{\sin A}$ or $\frac{c}{a}$	
The **secant** of ∠*A* (written sec *A*) is the reciprocal of cos *A*.	$\sec A = \frac{1}{\cos A}$ or $\frac{c}{b}$	
The **cotangent** of ∠*A* (written cot *A*) is the reciprocal of tan *A*.	$\cot A = \frac{1}{\tan A}$ or $\frac{b}{a}$	

Activity Find Trigonometric Values

Step 1 Draw and label a right triangle with the dimensions shown at the right.



Step 2 Use your graphing calculator to find the values for sin *A*, cos *A*, and tan *A*.

Step 3 Next, find the value for csc *A* by dividing 1 by SIN *A*. Repeat step 3 to find sec *A* and cot *A* .

Step 4 Copy the table below and record your results. Next, find the value of each trigonometric function for angle *C*.

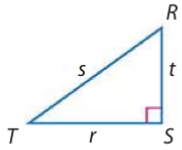
Angle	sin	cos	tan	csc	sec	cot
A						
C						

Exercises

1. Find the values of the six trigonometric functions for a 45° angle in a 45°-45°-90° triangle with legs that are 4 cm.

2. In △*FGH*, $\tan F = \frac{5}{12}$. Find cot *F* and sin *F* if ∠*G* is a right angle.

3. Find the values of the six trigonometric functions for angle *T* in △*RST* if $m\angle R = 36°$. Round to the nearest hundredth.



CHAPTER 8 Mid-Chapter Quiz

Lessons 8-1 through 8-4

Find the geometric mean between each pair of numbers. (Lesson 8-1)

1. 12 and 3

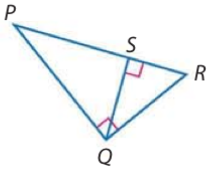
2. 63 and 7

3. 45 and 20

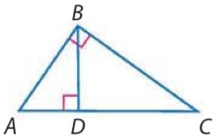
4. 50 and 10

Write a similarity statement identifying the three similar triangles in each figure. (Lesson 8-1)

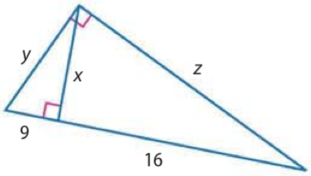
5.



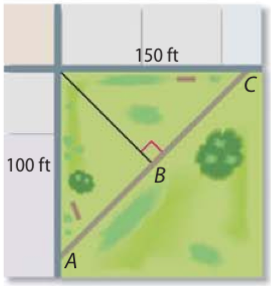
6.



7. Find *x*, *y*, and *z*. (Lesson 8-1)

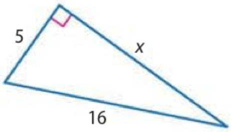


8. **PARKS** There is a small park in a corner made by two perpendicular streets. The park is 100 ft by 150 ft, with a diagonal path, as shown below. What is the length of path $\overline{AC}$? (Lesson 8-2)

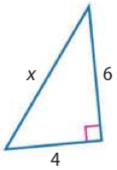


Find *x*. Round to the nearest hundredth. (Lesson 8-2)

9.



10.



11. **MULTIPLE CHOICE** Which of the following sets of numbers is not a Pythagorean triple? (Lesson 8-2)

A 9, 12, 15

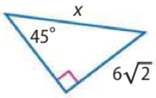
C 15, 36, 39

B 21, 72, 75

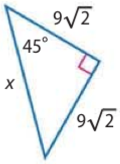
D 8, 13, 15

Find *x*. (Lesson 8-3)

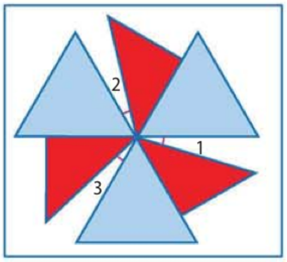
12.



13.



14. **DESIGN** Jamie designed a pinwheel to put in her garden. In the pinwheel, the blue triangles are congruent equilateral triangles, each with an altitude of 4 inches. The red triangles are congruent isosceles right triangles. The hypotenuse of a red triangle is congruent to a side of the blue triangle. (Lesson 8-3)

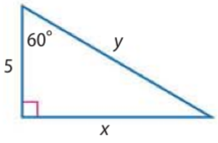


a. If angles 1, 2, and 3 are congruent, find the measure of each angle.

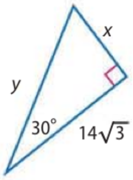
b. Find the perimeter of the pinwheel.

Find *x* and *y*. (Lesson 8-3)

15.



16.



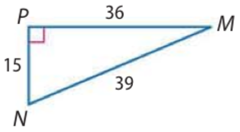
Express each ratio as a fraction and as a decimal to the nearest hundredth. (Lesson 8-4)

17. tan *M*

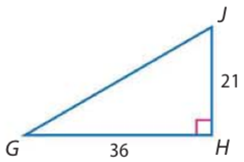
18. cos *M*

19. cos *N*

20. sin *N*



21. Solve the right triangle. Round angle measures to the nearest degree and side measures to the nearest tenth. (Lesson 8-4)



LESSON 8-5

Angles of Elevation and Depression

Then

- You used similar triangles to measure distances indirectly.

Now

- 1 Solve problems involving angles of elevation and depression.
- 2 Use angles of elevation and depression to find the distance between two objects.

Why?

- To make a field goal, a kicker must kick the ball with enough force and at an appropriate angle of elevation to ensure that the ball will reach the goal post at a level high enough to make it over the horizontal bar. This angle must change depending on the initial placement of the ball away from the base of the goalpost.

New Vocabulary

angle of elevation
angle of depression

Common Core State Standards

Content Standards

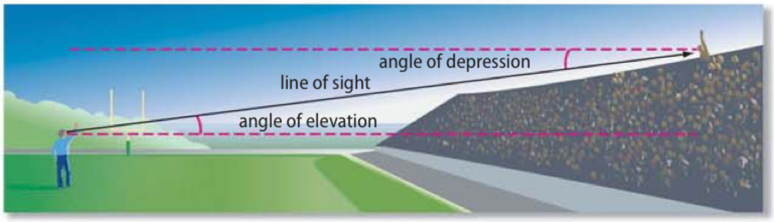
G.SRT.8 Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems. ★

Mathematical Practices

4 Model with mathematics.
1 Make sense of problems and persevere in solving them.

1 Angles of Elevation and Depression

An **angle of elevation** is the angle formed by a horizontal line and an observer's line of sight to an object above the horizontal line. An **angle of depression** is the angle formed by a horizontal line and an observer's line of sight to an object below the horizontal line.

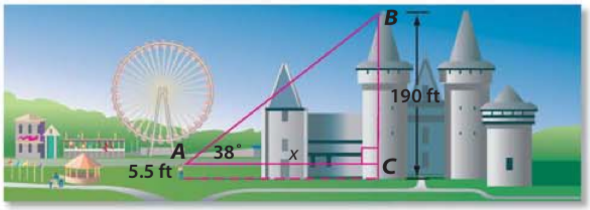


Horizontal lines are parallel, so the angle of elevation and the angle of depression in the diagram are congruent by the Alternate Interior Angles Theorem.

Example 1 Angle of Elevation

VACATION Leah wants to see a castle in an amusement park. She sights the top of the castle at an angle of elevation of 38°. She knows that the castle is 190 feet tall. If Leah is 5.5 feet tall, how far is she from the castle to the nearest foot?

Make a sketch to represent the situation.



Since Leah is 5.5 feet tall, $BC = 190 - 5.5$ or 184.5 feet. Let x represent the distance from Leah to the castle, AC.

$\tan A = \frac{BC}{AC}$ $\quad$ $\tan = \frac{\text{opposite}}{\text{adjacent}}$

$\tan 38° = \frac{184.5}{x}$ $\quad$ $m\angle A = 38$, $BC = 184.5$, $AC = x$

$x = \frac{184.5}{\tan 38°}$ $\quad$ Solve for x.

$x \approx 236.1$ $\quad$ Use a calculator.

Leah is about 236 feet from the castle.

Juice Images/Corbis

GuidedPractice

1. **FOOTBALL** The cross bar of a goalpost is 10 feet high. If a field goal attempt is made 25 yards from the base of the goalpost that clears the goal by 1 foot, what is the smallest angle of elevation at which the ball could have been kicked to the nearest degree?

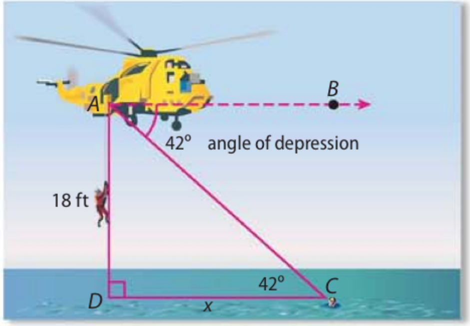
Example 2 Angle of Depression

EMERGENCY **A search and rescue team is airlifting people from the scene of a boating accident when they observe another person in need of help. If the angle of depression to this other person is 42° and the helicopter is 18 feet above the water, what is the horizontal distance from the rescuers to this person to the nearest foot?**

WatchOut!

Angles of Elevation and Depression To avoid mislabeling, remember that angles of elevation and depression are always formed with a horizontal line and never with a vertical line.

Make a sketch of the situation.



Note: Art not drawn to scale.

Since $\overrightarrow{AB}$ and $\overline{DC}$ are parallel, $m\angle BAC = m\angle ACD$ by the Alternate Interior Angles Theorem.

Let x represent the horizontal distance from the rescuers to the person DC.

$\tan C = \frac{AD}{DC}$ $\quad$ $\tan = \frac{\text{opposite}}{\text{adjacent}}$

$\tan 42° = \frac{18}{x}$ $\quad$ $C = 42$, $AD = 18$, and $DC = x$

$x \tan 42° = 18$ $\quad$ Multiply each side by x.

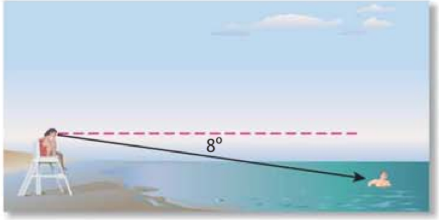
$x = \frac{18}{\tan 42°}$ $\quad$ Divide each side by tan 42°.

$x \approx 20.0$ $\quad$ Use a calculator.

The horizontal distance from the rescuers to the person is 20.0 feet.

GuidedPractice

2. **LIFEGUARDING** A lifeguard is watching a beach from a line of sight 6 feet above the ground. She sees a swimmer at an angle of depression of 8°. How far away from the tower is the swimmer?



The Art Gallery Collection/Alamy

Math History Link

Eratosthenes **(276–194 B.C.)** Eratosthenes was a mathematician and astronomer who was born in Cyrene, which is now Libya. He used the angle of elevation of the Sun at noon in the cities of Alexandria and Syene (now Egypt) to measure the circumference of Earth.

Source: *Encyclopaedia Britannica*

2 Two Angles of Elevation or Depression

Angles of elevation or depression to two different objects can be used to estimate the distance between those objects. Similarly, the angles from two different positions of observation to the same object can be used to estimate the object's height.

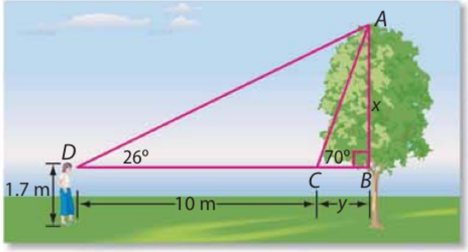
Real-WorldLink

In the United States, lumber volume is measured in board-feet, which is defined as a piece of wood containing 144 cubic inches. Woodland owners often estimate the lumber volume of trees they own to determine how many to cut and sell.

Source: The Ohio State University School of Natural Resources

Example 3 Use Two Angles of Elevation or Depression

TREE REMOVAL **To estimate the height of a tree she wants removed, Mrs. Long sights the tree's top at a 70° angle of elevation. She then steps back 10 meters and sights the top at a 26° angle. If Mrs. Long's line of sight is 1.7 meters above the ground, how tall is the tree to the nearest meter?**



Understand $\triangle ABC$ and $\triangle ABD$ are right triangles. The height of the tree is the sum of Mrs. Long's height and AB.

Plan Since her initial distance from the tree is not given, write and solve a system of equations using both triangles. Let $AB = x$ and $CB = y$. So $DB = y + 10$ and the height of the tree is $x + 1.7$.

Solve Use $\triangle ABC$.

$\tan 70° = \frac{x}{y}$ $\quad$ $\tan = \frac{\text{opposite}}{\text{adjacent}}$; $m\angle ACB = 70$

$y \tan 70° = x$ $\quad$ Multiply each side by y.

Use $\triangle ABD$.

$\tan 26° = \frac{x}{y + 10}$ $\quad$ $\tan = \frac{\text{opposite}}{\text{adjacent}}$; $m\angle D = 26$

$(y + 10) \tan 26° = x$ $\quad$ Multiply each side by $y + 10$.

Substitute the value for x from $\triangle ABD$ in the equation for $\triangle ABC$ and solve for y.

$$y \tan 70° = x$$

$$y \tan 70° = (y + 10) \tan 26°$$

$$y \tan 70° = y \tan 26° + 10 \tan 26°$$

$$y \tan 70° - y \tan 26° = 10 \tan 26°$$

$$y(\tan 70° - \tan 26°) = 10 \tan 26°$$

$$y = \frac{10 \tan 26°}{\tan 70° - \tan 26°}$$

Use a calculator to find that $y \approx 2.16$. Using the equation from $\triangle ABC$, $x = 2.16 \tan 70°$ or about 5.9.

The height of the tree is 5.9 + 1.7 or 7.6, which is about 8 meters.

Check Substitute the value for y in the equation from $\triangle ABD$.

$x = (2.16 + 10) \tan 26°$ or about 5.9. This is the same value found using the equation from $\triangle ABC$. ✓

StudyTip

Indirect Measurement When using the angles of depression to two different objects to calculate the distance between them, it is important to remember that the two objects must lie in the same horizontal plane. In other words, one object cannot be higher or lower than the other.

GuidedPractice

3. **SKYSCRAPERS** Two buildings are sited from atop a 200-meter skyscraper. Building A is sited at a 35° angle of depression, while Building B is sighted at a 36° angle of depression. How far apart are the two buildings to the nearest meter?

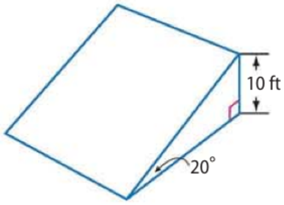
Compassionate Eye Foundation/Colorblind Images/Digital Vision/Getty Images

Check Your Understanding

● = Step-by-Step Solutions begin on page R14.

Example 1

1. **BIKING** Lenora wants to build the bike ramp shown. Find the length of the base of the ramp.

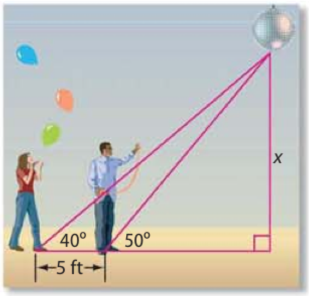


Example 2

2. **BASEBALL** A fan is seated in the upper deck of a stadium 200 feet away from home plate. If the angle of depression to the field is 62°, at what height is the fan sitting?

Example 3

3. **CCSS MODELING** Annabelle and Rich are setting up decorations for their school dance. Rich is standing 5 feet directly in front of Annabelle under a disco ball. If the angle of elevation from Annabelle to the ball is 40° and Rich to the ball is 50°, how high is the disco ball?

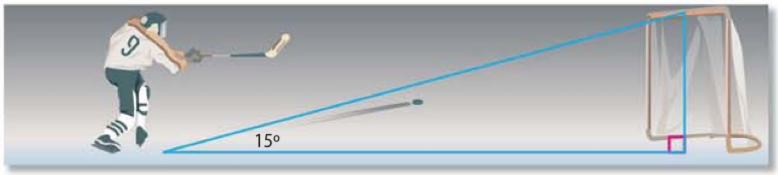


Practice and Problem Solving

Extra Practice is on page R8.

Example 1

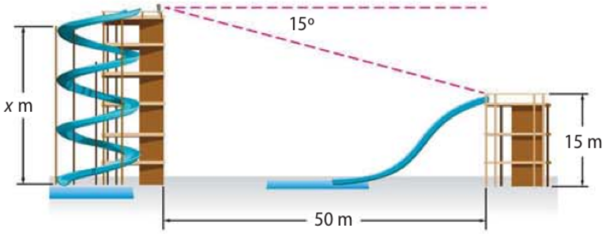
4. **HOCKEY** A hockey player takes a shot 20 feet away from a 5-foot goal. If the puck travels at a 15° angle of elevation toward the center of the goal, will the player score?



⑤ **MOUNTAINS** Find the angle of elevation to the peak of a mountain for an observer who is 155 meters from the mountain if the observer's eye is 1.5 meters above the ground and the mountain is 350 meters tall.

Example 2

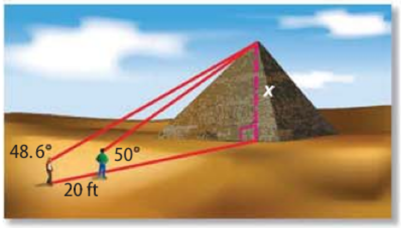
6. **WATERPARK** Two water slides are 50 meters apart on level ground. From the top of the taller slide, you can see the top of the shorter slide at an angle of depression of 15°. If you know that the top of the other slide is approximately 15 meters above the ground, about how far above the ground are you? Round to the nearest tenth of a meter.



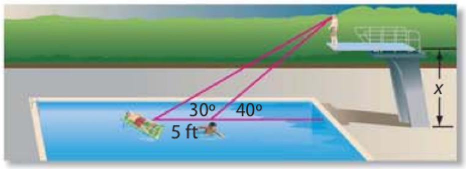
7. **AVIATION** Due to a storm, a pilot flying at an altitude of 528 feet has to land. If he has a horizontal distance of 2000 feet to land, at what angle of depression should he land?

Example 3

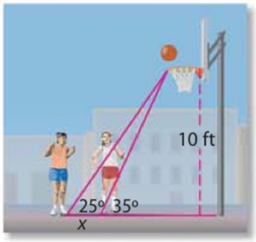
8. **PYRAMIDS** Miko and Tyler are visiting the Great Pyramid in Egypt. From where Miko is standing, the angle of elevation to the top of the pyramid is 48.6°. From Tyler's position, the angle of elevation is 50°. If they are standing 20 feet apart, and they are each 5 feet 6 inches tall, how tall is the pyramid?



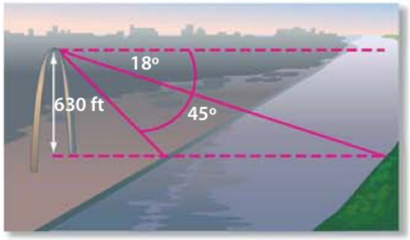
9 **DIVING** Austin is standing on the high dive at the local pool. Two of his friends are in the water as shown. If the angle of depression to one of his friends is 40°, and 30° to his other friend who is 5 feet beyond the first, how tall is the platform?



10. **BASKETBALL** Claire and Marisa are both waiting to get a rebound during a basketball game. If the height of the basketball hoop is 10 feet, the angle of elevation between Claire and the goal is 35°, and the angle of elevation between Marisa and the goal is 25°, how far apart are they standing?

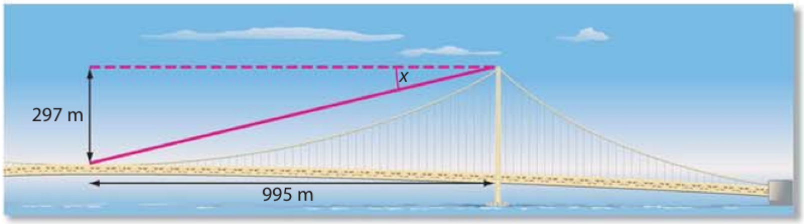


11. **RIVERS** Hugo is standing in the top of St. Louis' Gateway Arch, looking down on the Mississippi River. The angle of depression to the closer bank is 45° and the angle of depression to the farther bank is 18°. The arch is 630 feet tall. Estimate the width of the river at that point.



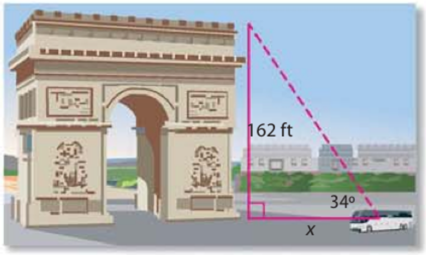
12. **MODELING** The Unzen Volcano in Japan has a magma reservoir located 15 kilometers beneath the Chijiwa Bay, located east of the volcano. A magma channel, which connects the reservoir to the volcano, rises at a 40° angle of elevation toward the volcano. What length of magma channel is below sea level?

13. **BRIDGES** Suppose you are standing in the middle of the platform of the world's longest suspension bridge, the Akashi Kaikyo Bridge. If the height from the top of the platform holding the suspension cables is 297 meters, and the length from the platform to the center of the bridge is 995 meters, what is the angle of depression from the center of the bridge to the platform?

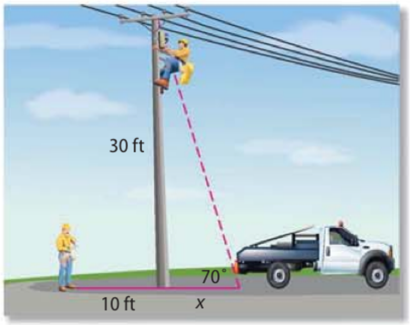


14. **LIGHTHOUSES** Little Gull Island Lighthouse shines a light from a height of 91 feet with a 6° angle of depression. Plum Island Lighthouse, 1800 feet away, shines a light from a height of 34 feet with a 2° angle of depression. Which light will reach a boat that sits exactly between Little Gull Island Lighthouse and Plum Island Lighthouse?

15. **TOURISM** From the position of the bus on the street, the L'arc de Triomphe is at an angle of 34°. If the arc is 162 feet tall, how far away is the bus? Round to the nearest tenth.

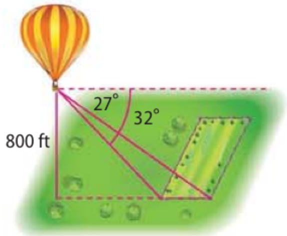


16. **MAINTENANCE** Two telephone repair workers arrive at a location to restore electricity after a power outage. One of the workers climbs up the telephone pole while the other worker stands 10 feet to left of the pole. If the terminal box is located 30 feet above ground on the pole and the angle of elevation from the truck to the repair worker is 70°, how far is the worker on the ground standing from the truck?



17 **PHOTOGRAPHY** A digital camera with a panoramic lens is described as having a view with an angle of elevation of 38°. If the camera is on a 3-foot tripod aimed directly at a 124-foot-tall monument, how far from the monument should you place the tripod to see the entire monument in your photograph?

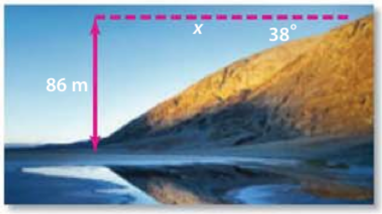
18. **CCSS MODELING** As a part of their weather unit, Anoki's science class took a hot air balloon ride. As they passed over a fenced field, the angle of depression of the closer side of the fence was 32°, and the angle of depression of the farther side of the fence was 27°. If the height of the balloon was 800 feet, estimate the width of the field.



19. **MARATHONS** The Badwater Ultramarathon is a race that begins at the lowest point in California, Death Valley, and ends at the highest point of the state, Mount Whitney. The race starts at a depth of 86 meters below sea level and ends 2530 meters above sea level.

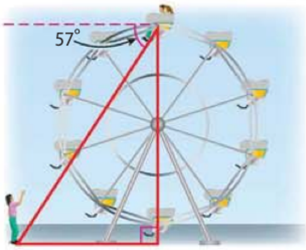
a. Determine the angle of elevation to Mount Whitney if the horizontal distance from the base to the peak is 1200 meters.

b. If the angle of depression to Death Valley is 38°, what is the horizontal distance from sea level?



20. **AMUSEMENT PARKS** India, Enrique, and Trina went to an amusement park while visiting Japan. They went on a Ferris wheel that was 100 meters in diameter and on an 80-meter cliff-dropping slide.

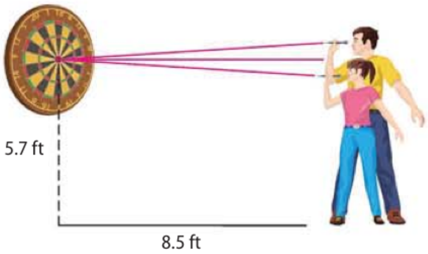
a. When Enrique and Trina are at the topmost point on the Ferris wheel shown below, how far are they from India?



b. If the cliff-dropping ride has an angle of depression of 46°, how long is the slide?

(l)Smari/Photodisc/Getty Images, (r)Ed Freeman/Stone/Getty Images

21. **DARTS** Kelsey and José are throwing darts from a distance of 8.5 feet. The center of the bull's-eye on the dartboard is 5.7 feet from the floor. José throws from a height of 6 feet, and Kelsey throws from a height of 5 feet. What are the angles of elevation or depression from which each must throw to get a bull's-eye? Ignore other factors such as air resistance, velocity, and gravity.



22. **MULTIPLE REPRESENTATIONS** In this problem, you will investigate relationships between the sides and angles of triangles.

   a. **Geometric** Draw three triangles. Make one acute, one obtuse, and one right. Label one triangle *ABC*, a second *MNP*, and the third *XYZ*. Label the side lengths and angle measures of each triangle.

   b. **Tabular** Copy and complete the table below.

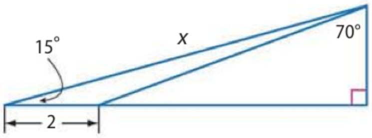
Triangle	Ratios		
ABC	$\frac{\sin A}{BC} =$	$\frac{\sin B}{CA} =$	$\frac{\sin C}{AB} =$
MNP	$\frac{\sin M}{NP} =$	$\frac{\sin N}{PM} =$	$\frac{\sin P}{MN} =$
XYZ	$\frac{\sin X}{YZ} =$	$\frac{\sin Y}{ZX} =$	$\frac{\sin Z}{XY} =$

   c. **Verbal** Make a conjecture about the ratio of the sine of an angle to the length of the leg opposite that angle for a given triangle.

H.O.T. Problems Use Higher-Order Thinking Skills

23. **ERROR ANALYSIS** Terrence and Rodrigo are trying to determine the relationship between angles of elevation and depression. Terrence says that if you are looking up at someone with an angle of elevation of 35°, then they are looking down at you with an angle of depression of 55°, which is the complement of 35°. Rodrigo disagrees and says that the other person would be looking down at you with an angle of depression equal to your angle of elevation, or 35°. Is either of them correct? Explain.

24. **CHALLENGE** Find the value of *x*. Round to the nearest tenth.



25. **REASONING** Classify the statement below as *true* or *false*. Explain.

As a person moves closer to an object he or she is sighting, the angle of elevation increases.

26. **WRITE A QUESTION** A classmate finds the angle of elevation of an object, but she is trying to find the angle of depression. Write a question to help her solve the problem.

27. **WRITING IN MATH** Describe a way that you can estimate the height of an object without using trigonometry by choosing your angle of elevation. Explain your reasoning.

Standardized Test Practice

28. Ryan wanted to know the height of a cell-phone tower neighboring his property. He walked 80 feet from the base of the tower and measured the angle of elevation to the top of the tower at 54°. If Ryan is 5 feet tall, what is the height of the cell-phone tower?

A 52 ft **C** 110 ft

B 63 ft **D** 115 ft

29. SHORT RESPONSE A searchlight is 6500 feet from a weather station. If the angle of elevation to the spot of light on the clouds above the station is 45°, how high is the cloud ceiling?

30. ALGEBRA What is the solution of this system of equations?

$$\begin{aligned} 2x - 4y &= -12 \\ -x + 4y &= 8 \end{aligned}$$

F (4, 4) **H** (−4, −4)

G (−4, 1) **J** (1, −4)

31. SAT/ACT A triangle has sides in the ratio of 5 : 12 : 13. What is the measure of the triangle's smallest angle in degrees?

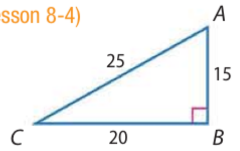
A 13.34 **D** 42.71

B 22.62 **E** 67.83

C 34.14

Spiral Review

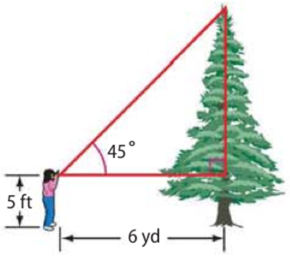
Express each ratio as a fraction and as a decimal to the nearest hundredth. (Lesson 8-4)



32. sin C **33.** tan A **34.** cos C

35. tan C **36.** cos A **37.** sin A

38. LANDSCAPING Imani needs to determine the height of a tree. Holding a drafter's 45° triangle so that one leg is horizontal, she sights the top of the tree along the hypotenuse, as shown at the right. If she is 6 yards from the tree and her eyes are 5 feet from the ground, find the height of the tree. (Lesson 8-3)

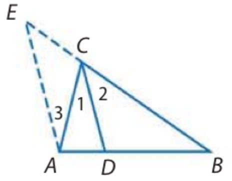


PROOF **Write a two-column proof.** (Lesson 7-5)

39. Given: $\overline{CD}$ bisects $\angle ACB$.

By construction, $\overline{AE} \parallel \overline{CD}$.

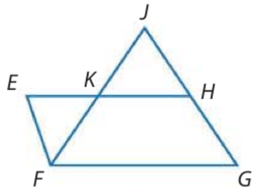
Prove: $\frac{AD}{DB} = \frac{AC}{BC}$



40. Given: $\overline{JF}$ bisects $\angle EFG$.

$\overline{EH} \parallel \overline{FG}$, $\overline{EF} \parallel \overline{HG}$

Prove: $\frac{EK}{KF} = \frac{GJ}{JF}$



COORDINATE GEOMETRY **Find the coordinates of the centroid of each triangle.** (Lesson 5-2)

41. $A(2, 2)$, $B(7, 8)$, $C(12, 2)$ **42.** $X(-3, -2)$, $Y(1, -12)$, $Z(-7, -7)$

43. $A(-1, 11)$, $B(-5, 1)$, $C(-9, 6)$ **44.** $X(4, 0)$, $Y(-2, 4)$, $Z(0, 6)$

Skills Review

Solve each proportion.

45. $\frac{1}{5} = \frac{x}{10}$ **46.** $\frac{2x}{11} = \frac{3}{8}$ **47.** $\frac{4x}{16} = \frac{62}{118}$ **48.** $\frac{12}{21} = \frac{45}{10x}$

LESSON 8-6 The Law of Sines and Law of Cosines

Then

- You used trigonometric ratios to solve right triangles.

Now

- **1** Use the Law of Sines to solve triangles.
- **2** Use the Law of Cosines to solve triangles.

Why?

- You have learned that the height or length of a tree can be calculated using *right triangle trigonometry* if you know the angle of elevation to the top of the tree and your distance from the tree. Some trees, however, grow at an angle or lean due to weather damage. To calculate the length of such trees, you must use other forms of trigonometry.

Ben Mangor/SuperStock

NewVocabulary
Law of Sines
Law of Cosines

Common Core State Standards

Content Standards

G.SRT.9 Derive the formula $A = \frac{1}{2}ab \sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.

G.SRT.10 Prove the Laws of Sines and Cosines and use them to solve problems.

Mathematical Practices

4 Model with mathematics.
1 Make sense of problems and persevere in solving them.

1 Law of Sines

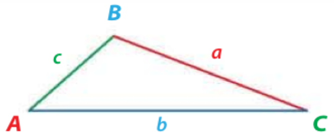
In Lesson 8-4, you used trigonometric ratios to find side lengths and acute angle measures in *right* triangles. To find measures for nonright triangles, the definitions of sine and cosine can be extended to obtuse angles.

The **Law of Sines** can be used to find side lengths and angle measures for any triangle.

Theorem 8.10 Law of Sines

If $\triangle ABC$ has lengths a, b, and c, representing the lengths of the sides opposite the angles with measures A, B, and C, then

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}.$$



You will prove one of the proportions for Theorem 8.10 in Exercise 45.

You can use the Law of Sines to solve a triangle if you know the measures of two angles and any side (AAS or ASA).

Example 1 Law of Sines (AAS)

Find x. Round to the nearest tenth.

We are given the measures of two angles and a nonincluded side, so use the Law of Sines to write a proportion.

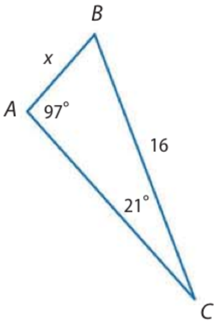
$$\frac{\sin A}{a} = \frac{\sin C}{c}$$ Law of Sines

$$\frac{\sin 97°}{16} = \frac{\sin 21°}{x}$$ $m\angle A = 97$, $a = 16$, $m\angle C = 21$, $c = x$

$$x \sin 97° = 16 \sin 21°$$ Cross Products Property

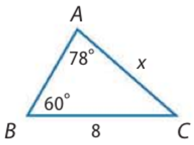
$$x = \frac{16 \sin 21°}{\sin 97°}$$ Divide each side by sin 97°.

$$x \approx 5.8$$ Use a calculator.

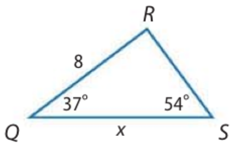


GuidedPractice

1A.



1B.



StudyTip

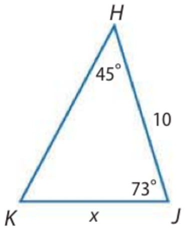
Ambiguous Case You can sometimes use the Law of Sines to solve a triangle if you know the measures of two sides and a nonincluded angle (SSA). However, these three measures do not always determine exactly one triangle. You will learn more about this *ambiguous case* in Extend 8-6.

If given ASA, use the Triangle Angle Sum Theorem to first find the measure of the third angle.

Example 2 Law of Sines (ASA)

Find x. Round to the nearest tenth.

By the Triangle Angle Sum Theorem, $m\angle K = 180 - (45 + 73)$ or 62.



$\frac{\sin H}{h} = \frac{\sin K}{k}$ Law of Sines

$\frac{\sin 45°}{x} = \frac{\sin 62°}{10}$ $m\angle H = 45$, $h = x$, $m\angle K = 62$, $k = 10$

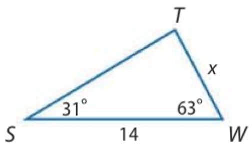
$10 \sin 45° = x \sin 62°$ Cross Products Property

$\frac{10 \sin 45°}{\sin 62°} = x$ Divide each side by sin 62°.

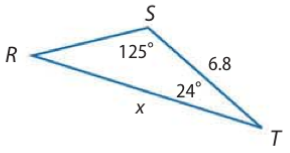
$x \approx 8.0$ Use a calculator.

GuidedPractice

2A.



2B.



2 Law of Cosines

When the Law of Sines cannot be used to solve a triangle, the Law of Cosines may apply.

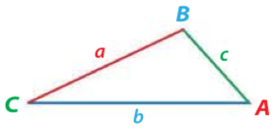
Theorem 8.11 Law of Cosines

If $\triangle ABC$ has lengths a, b, and c, representing the lengths of the sides opposite the angles with measures A, B, and C, then

$a^2 = b^2 + c^2 - 2bc \cos A$,

$b^2 = a^2 + c^2 - 2ac \cos B$, and

$c^2 = a^2 + b^2 - 2ab \cos C$.



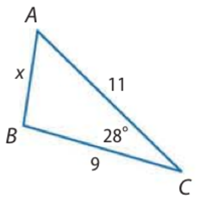
You will prove one of the equations for Theorem 8.11 in Exercise 46.

You can use the **Law of Cosines** to solve a triangle if you know the measures of two sides and the included angle (SAS).

Example 3 Law of Cosines (SAS)

Find x. Round to the nearest tenth.

We are given the measures of two sides and their included angle, so use the Law of Cosines.



$c^2 = a^2 + b^2 - 2ab \cos C$ Law of Cosines

$x^2 = 9^2 + 11^2 - 2(9)(11) \cos 28°$ Substitution

$x^2 = 202 - 198 \cos 28°$ Simplify.

$x = \sqrt{202 - 198 \cos 28°}$ Take the square root of each side.

$x \approx 5.2$ Use a calculator.

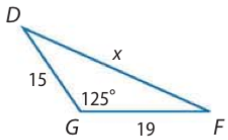
WatchOut!

Order of operations Remember to follow the order of operations when simplifying expressions. Multiplication or division must be performed before addition or subtraction. So, 202 − 198 cos 28° *cannot* be simplified to 4 cos 28°.

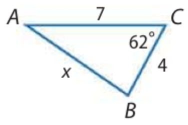
GuidedPractice

Find x. Round to the nearest tenth.

3A.



3B.



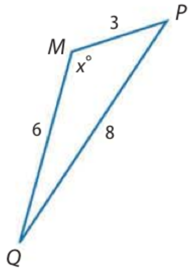
StudyTip

Obtuse Angles There are also values for sin A, cos A, and tan A when $A \geq 90°$. Values of the ratios for these angles can be found using the trigonometric functions on your calculator.

You can also use the Law of Cosines if you know three side measures (SSS).

Example 4 Law of Cosines (SSS)

Find x. Round to the nearest degree.



$m^2 = p^2 + q^2 - 2pq \cos M$ Law of Cosines

$8^2 = 6^2 + 3^2 - 2(6)(3) \cos x°$ Substitution

$64 = 45 - 36 \cos x°$ Simplify.

$19 = -36 \cos x°$ Subtract 45 from each side.

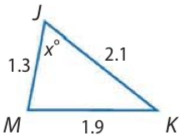
$\frac{19}{-36} = \cos x°$ Divide each side by −36.

$x = \cos^{-1}\left(-\frac{19}{36}\right)$ Use the inverse cosine ratio.

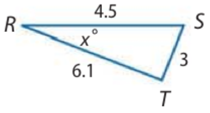
$x \approx 122$ Use a calculator.

GuidedPractice

4A.



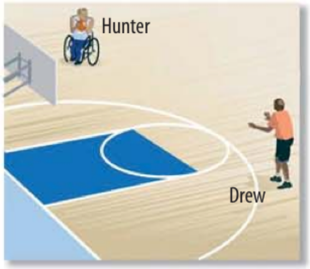
4B.



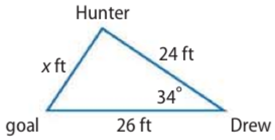
You can use the Law of Sines and Law of Cosines to solve direct and indirect measurement problems.

Real-World Example 5 Indirect Measurement

BASKETBALL **Drew and Hunter are playing basketball. Drew passes the ball to Hunter when he is 26 feet from the goal and 24 feet from Hunter. How far is Hunter from the goal if the angle from the goal to Drew and then to Hunter is 34°?**



Draw a diagram. Since we know two sides of a triangle and the included angle, use the Law of Cosines.



$x^2 = 24^2 + 26^2 - 2(24)(26) \cos 34°$ Law of Cosines

$x = \sqrt{1252 - 1248 \cos 34°}$ Simplify and take the positive square root of each side.

$x \approx 15$ Use a calculator.

Hunter is about 15 feet from the goal when he takes his shot.

Real-World Link

The first game of basketball was played at a YMCA in Springfield, Massachusetts, on December 1, 1891. James Naismith, a physical education instructor, invented the sport using a soccer ball and two half-bushel peach baskets, which is how the name *basketball* came about.

Source: *Encyclopaedia Britannica.*

SuperStock

GuidedPractice

5. **LANDSCAPING** At 10 feet away from the base of a tree, the angle the top of a tree makes with the ground is 61°. If the tree grows at an angle of 78° with respect to the ground, how tall is the tree to the nearest foot?

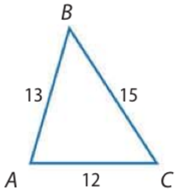
When solving right triangles, you can use sine, cosine, or tangent. When solving other triangles, you can use the Law of Sines or the Law of Cosines, depending on what information is given.

ReadingMath

Solve a Triangle Remember that to *solve* a triangle means to find all of the missing side measures and/or angle measures.

Example 6 Solve a Triangle

Solve triangle *ABC*. Round to the nearest degree.



Since $13^2 + 12^2 \neq 15^2$, this is not a right triangle. Since the measures of all three sides are given (SSS), begin by using the Law of Cosines to find $m\angle A$.

$a^2 = b^2 + c^2 - 2bc \cos A$ Law of Cosines

$15^2 = 12^2 + 13^2 - 2(12)(13) \cos A$ $a = 15$, $b = 12$, and $c = 13$

$225 = 313 - 312 \cos A$ Simplify.

$-88 = -312 \cos A$ Subtract 313 from each side.

$\frac{-88}{-312} = \cos A$ Divide each side by −312.

$m\angle A = \cos^{-1} \frac{88}{312}$ Use the inverse cosine ratio.

$m\angle A \approx 74$ Use a calculator.

Use the Law of Sines to find $m\angle B$.

$\frac{\sin A}{a} = \frac{\sin B}{b}$ Law of Sines

$\frac{\sin 74°}{15} \approx \frac{\sin B}{12}$ $m\angle A \approx 74$, $a = 15$, and $b = 12$

$12 \sin 74° = 15 \sin B$ Cross Products Property

$\frac{12 \sin 74°}{15} = \sin B$ Divide each side by 15.

$m\angle B = \sin^{-1} \frac{12 \sin 74°}{15}$ Use the inverse sine ratio.

$m\angle B \approx 50$ Use a calculator.

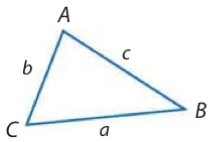
By the Triangle Angle Sum Theorem, $m\angle C \approx 180 - (74 + 50)$ or 56. Therefore $m\angle A \approx 74$, $m\angle B \approx 50$, and $m\angle C \approx 56$.

WatchOut

Rounding When you round a numerical solution and then use it in later calculations, your answers may be inaccurate. Wait until after you have completed all of your calculations to round.

GuidedPractice

Solve triangle *ABC* using the given information. Round angle measures to the nearest degree and side measures to the nearest tenth.



6A. $b = 10.2$, $c = 9.3$, $m\angle A = 26$

6B. $a = 6.4$, $m\angle B = 81$, $m\angle C = 46$

ConceptSummary Solving a Triangle

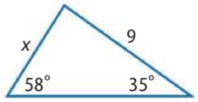
To solve . . .	Given	Begin by using . . .
a right triangle	leg-leg (LL) hypotenuse-leg (HL) acute angle-hypotenuse (AH) acute angle-leg (AL)	tangent ratio sine or cosine ratio sine or cosine ratio sine, cosine, or tangent ratios
any triangle	angle-angle-side (AAS) angle-side-angle (ASA) side-angle-side (SAS) side-side-side (SSS)	Law of Sines Law of Sines Law of Cosines Law of Cosines

Check Your Understanding

● = Step-by-Step Solutions begin on page R14.

Examples 1–2 **Find *x*. Round angle measures to the nearest degree and side measures to the nearest tenth.**

1.



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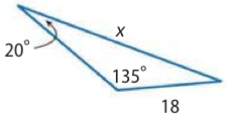

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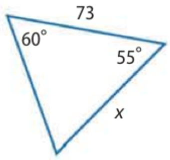

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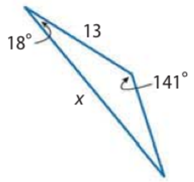
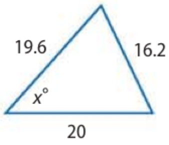


Examples 3–4 **5.**

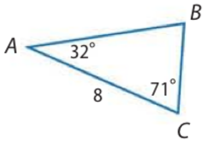
6.



Example 5 **7. SAILING** Determine the length of the bottom edge, or foot, of the sail.

Example 6 **CCSS STRUCTURE** **Solve each triangle. Round angle measures to the nearest degree and side measures to the nearest tenth.**

8.



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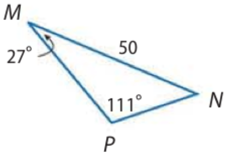

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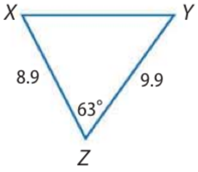


11. Solve $\triangle DEF$ if $DE = 16$, $EF = 21.6$, $FD = 20$.

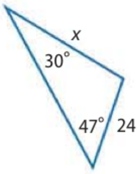
Paul Miles/Axiom Photographic Agency/Getty Images

Practice and Problem Solving

Extra Practice is on page R8.

Examples 1–2 **Find x. Round side measures to the nearest tenth.**

12.



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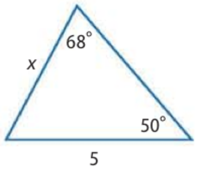

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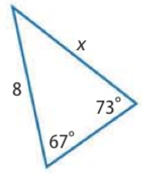

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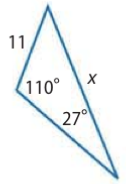

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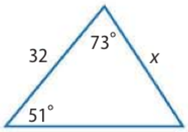

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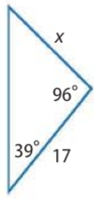

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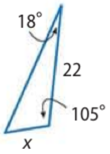

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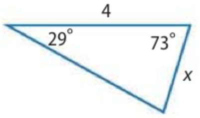

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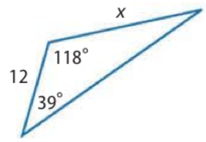
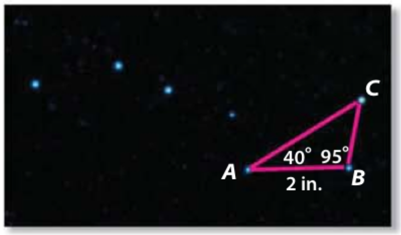
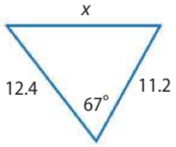


21. CCSS **MODELING** Angelina is looking at the Big Dipper through a telescope. From her view, the cup of the constellation forms a triangle that has measurements shown on the diagram at the right. Use the Law of Sines to determine distance between A and C.



Examples 3–4 **Find x. Round angle measures to the nearest degree and side measures to the nearest tenth.**

22.



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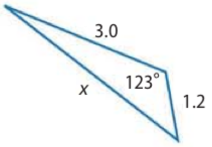

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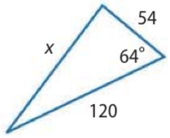

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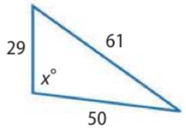

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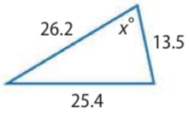

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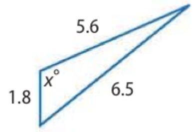
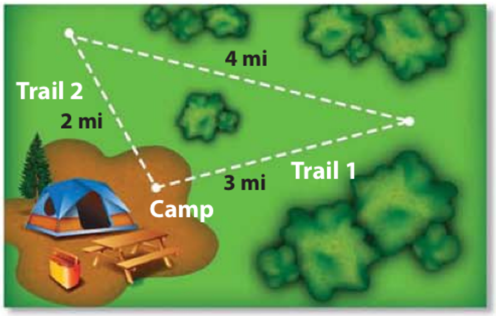
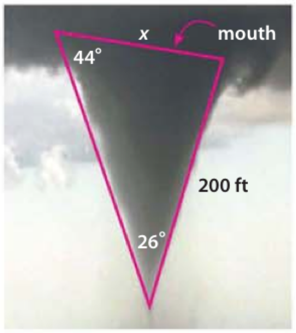


28. **HIKING** A group of friends who are camping decide to go on a hike. According to the map shown at the right, what is the measure of the angle between Trail 1 and Trail 2?

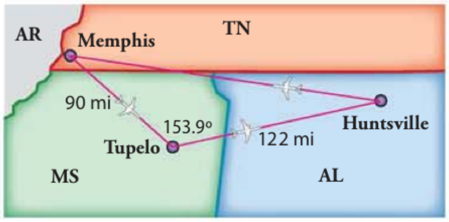


Example 5

29. **TORNADOES** Find the width of the mouth of the tornado shown below.



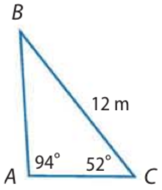
30. **TRAVEL** A pilot flies 90 miles from Memphis, Tennessee, to Tupelo, Mississippi, to Huntsville, Alabama, and finally back to Memphis. How far is Memphis from Huntsville?



Example 6

CCSS STRUCTURE **Solve each triangle. Round angle measures to the nearest degree and side measures to the nearest tenth.**

31.



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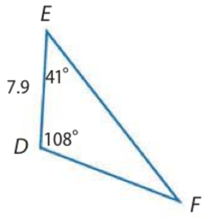

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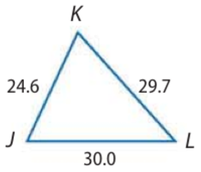

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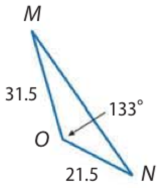

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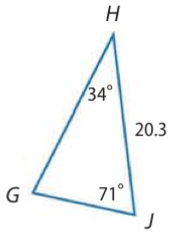

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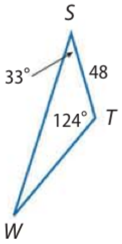

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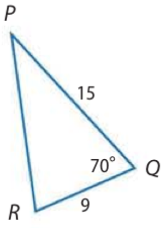

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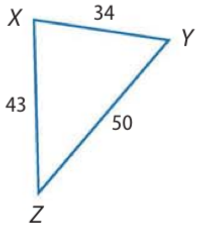

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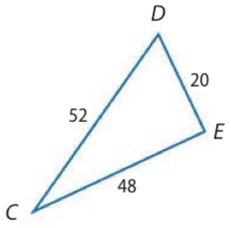
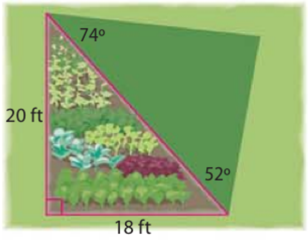


40. Solve $\triangle JKL$ if $JK = 33$, $KL = 56$, $LJ = 65$.

41. Solve $\triangle ABC$ if $m\angle B = 119$, $m\angle C = 26$, $CA = 15$.

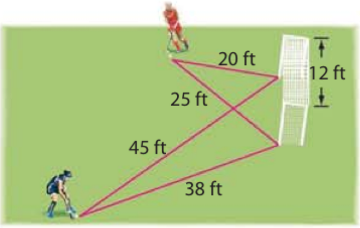
42. Solve $\triangle XYZ$ if $XY = 190$, $YZ = 184$, $ZX = 75$.

43. **GARDENING** Crystal has an organic vegetable garden. She wants to add another triangular section so that she can start growing tomatoes. If the garden and neighboring space have the dimensions shown, find the perimeter of the new garden to the nearest foot.



Jason Lafontaine/Painet Inc.

44. **FIELD HOCKEY** Alyssa and Nari are playing field hockey. Alyssa is standing 20 feet from one goal post and 25 feet from the opposite post. Nari is standing 45 feet from one goal post and 38 feet from the other post. If the goal is 12 feet wide, which player has a greater chance to make a shot? What is the measure of the player's angle?



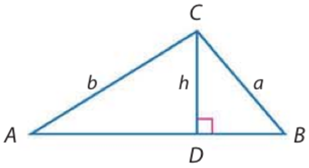
45. **PROOF** Justify each statement for the derivation of the Law of Sines.

Given: $\overline{CD}$ is an altitude of $\triangle ABC$.

Prove: $\frac{\sin A}{a} = \frac{\sin B}{b}$

Proof:

Statements	Reasons
$\overline{CD}$ is an altitude of $\triangle ABC$	Given
$\triangle ACD$ and $\triangle CBD$ are right	Def. of altitude
a. $\sin A = \frac{h}{b}, \sin B = \frac{h}{a}$	**a.** ___?___
b. $b \sin A = h, a \sin B = h$	**b.** ___?___
c. $b \sin A = a \sin B$	**c.** ___?___
d. $\frac{\sin A}{a} = \frac{\sin B}{b}$	**d.** ___?___



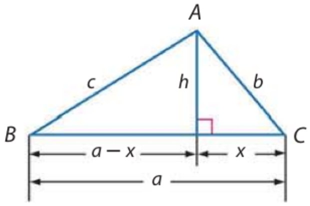
46. **PROOF** Justify each statement for the derivation of the Law of Cosines.

Given: h is an altitude of $\triangle ABC$.

Prove: $c^2 = a^2 + b^2 - 2ab \cos C$

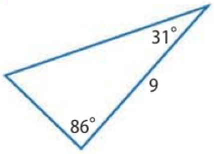
Proof:

Statements	Reasons
h is an altitude of $\triangle ABC$	Given
Altitude h separates $\triangle ABC$ into two right triangles	Def. of altitude
a. $c^2 = (a - x)^2 + h^2$	**a.** ___?___
b. $c^2 = a^2 - 2ax + x^2 + h^2$	**b.** ___?___
c. $x^2 + h^2 = b^2$	**c.** ___?___
d. $c^2 = a^2 - 2ax + b^2$	**d.** ___?___
e. $\cos C = \frac{x}{b}$	**e.** ___?___
f. $b \cos C = x$	**f.** ___?___
g. $c^2 = a^2 - 2a(b \cos C) + b^2$	**g.** ___?___
h. $c^2 = a^2 + b^2 - 2ab \cos C$	**h.** ___?___



CCSS **SENSE-MAKING** **Find the perimeter of each figure. Round to the nearest tenth.**

47.



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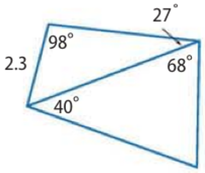

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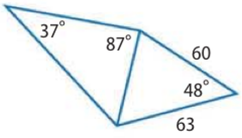

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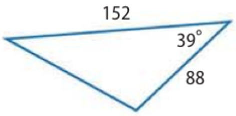
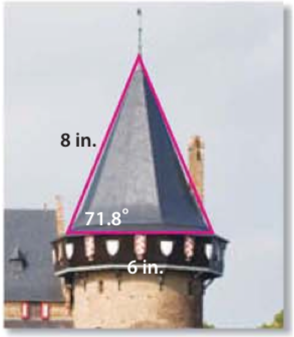


51. **MODELS** Vito is working on a model castle. Find the length of the missing side (in inches) using the diagram at the right.



52. **COORDINATE GEOMETRY** Find the measure of the largest angle in $\triangle ABC$ with coordinates $A(-3, 6)$, $B(4, 2)$, and $C(-5, 1)$. Explain your reasoning.

53. **MULTIPLE REPRESENTATIONS** In this problem, you will use trigonometry to find the area of a triangle.

a. **Geometric** Draw an acute, scalene $\triangle ABC$ including an altitude of length h originating at vertex A.

b. **Algebraic** Use trigonometry to represent h in terms of $m\angle B$.

c. **Algebraic** Write an equation to find the area of $\triangle ABC$ using trigonometry.

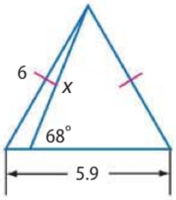
d. **Numerical** If $m\angle B$ is 47, $AB = 11.1$, $BC = 14.1$, and $CA = 10.4$, find the area of $\triangle ABC$. Round to the nearest tenth.

e. **Analytical** Write an equation to find the area of $\triangle ABC$ using trigonometry in terms of a different angle measure.

H.O.T. Problems Use Higher-Order Thinking Skills

54. **CCSS CRITIQUE** Colleen and Mike are planning a party. Colleen wants to sew triangular decorations and needs to know the perimeter of one of the triangles to buy enough trim. The triangles are isosceles with angle measurements of 64° at the base and side lengths of 5 inches. Colleen thinks the perimeter is 15.7 inches and Mike thinks it is 15 inches. Is either of them correct?

55. **CHALLENGE** Find the value of x in the figure at the right.



56. **REASONING** Explain why the Pythagorean Theorem is a specific case of the Law of Cosines.

57. **OPEN ENDED** Draw and label a triangle that can be solved:

a. using only the Law of Sines.

b. using only the Law of Cosines.

58. **WRITING IN MATH** What methods can you use to solve a triangle?

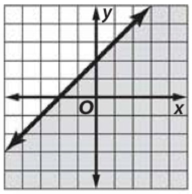
Lenz, G/Arco Images/age fotostock

Standardized Test Practice

59. For $\triangle ABC$, $m\angle A = 42$, $m\angle B = 74$, and $a = 3$, what is the value of b?

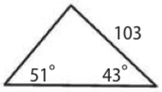
A 4.3
B 3.8
C 2.1
D 1.5

60. ALGEBRA Which inequality *best* describes the graph below?

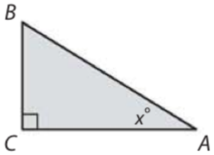


F $y \geq -x + 2$
G $y \leq x + 2$
H $y \geq -3x + 2$
J $y \leq 3x + 2$

61. SHORT RESPONSE What is the perimeter of the triangle shown below? Round to the nearest tenth.



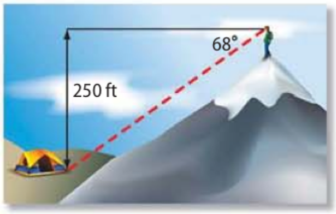
62. SAT/ACT If $\sin x = 0.6$ and $AB = 12$, what is the area of $\triangle ABC$?



A 9.6 units^2
B 28.8 units^2
C 31.2 units^2
D 34.6 units^2
E 42.3 units^2

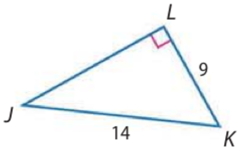
Spiral Review

63. HIKING A hiker is on top of a mountain 250 feet above sea level with a 68° angle of depression. She can see her camp from where she is standing. How far is her camp from the top of the mountain? (Lesson 8-5)

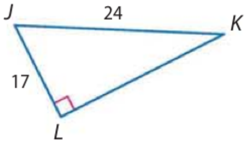


Use a calculator to find the measure of $\angle J$ to the nearest degree. (Lesson 8-4)

64.



65.



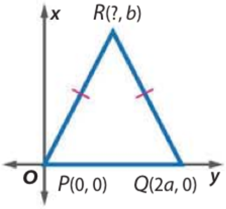
Determine whether the polygons are *always*, *sometimes*, or *never* similar. Explain your reasoning. (Lesson 7-2)

66. a right triangle and an isosceles triangle

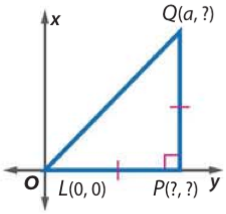
67. an equilateral triangle and a scalene triangle

Name the missing coordinates of each triangle. (Lesson 4-8)

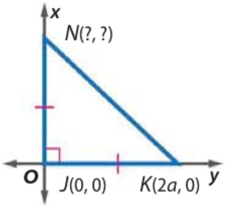
68.



69.



70.



Skills Review

Find the distance between each pair of points. Round to the nearest tenth.

71. $A(5, 1)$ and $C(-3, -3)$

72. $J(7, 11)$ and $K(-1, 5)$

73. $W(2, 0)$ and $X(8, 6)$

EXTEND 8-6

Geometry Lab
The Ambiguous Case

CCSS Common Core State Standards

Content Standards

G.SRT.11 Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

Mathematical Practices 2

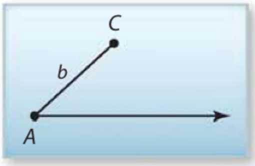
From your work with congruent triangles, you know that three measures determine a unique triangle when the measures are

- three sides (SSS),
- two sides and an included angle (SAS),
- two angles and an included side (ASA), or
- two angles and a nonincluded side (AAS).

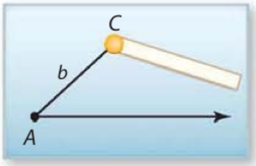
A unique triangle is not necessarily determined by three angles (AAA) or by two sides and a nonincluded angle. In this lab, you will investigate how many triangles are determined by this last case (SSA), called the **ambiguous case**.

Activity 1 The Ambiguous Case (SSA): $\angle A$ is Acute

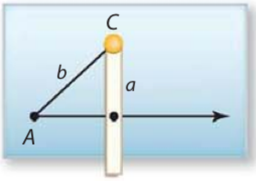
Step 1 On a 5″ × 8″ notecard, draw and label $\overline{AC}$ and a ray extending from A to form an acute angle. Label side $\overline{AC}$ as b.



Step 2 Using a brass fastener, attach one end of a half-inch strip of cardstock to the notecard at C. The strip should be longer than b. This represents side a.



Step 3 Position side a so that it is perpendicular to the ray. Make a black mark on the strip at the point where it touches the ray.



Model and Analyze

1. If a has the given length, how many triangles can be formed? (*Hint*: Rotate the strip to see if the mark can intersect the ray at any other locations to form a different triangle.)
2. Show that if side a is perpendicular to the third side of the triangle, then $a = b \sin A$.

Determine the number of triangles that can be formed given each of the modifications to a in Activity 1.

3. $a < b \sin A$ (*Hint*: Make a green mark above the black mark on the strip, and try to form triangle(s) using this new length for a.)
4. $a = b$ (*Hint*: Rotate the strip so that it lies on top of $\overline{AC}$ and mark off this length in red. Then rotate the strip to try to form triangle(s) using this new length for a.)
5. $a < b$ and $a > b \sin A$ (*Hint*: Make a blue mark between the black and the red marks. Then rotate the strip to try to form triangle(s) using this new length for a.)
6. $a > b$ (*Hint*: Rotate the strip to try to form triangle(s) using the entire length of the strip as the length for a.)

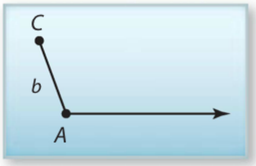
Use your results from Exercises 1–6 to determine whether the given measures define *0, 1, 2,* or *infinitely many* acute triangles. Justify your answers.

7. $a = 14, b = 16, m\angle A = 55$
8. $a = 7, b = 11, m\angle A = 68$
9. $a = 22, b = 25, m\angle A = 39$
10. $a = 13, b = 12, m\angle A = 81$
11. $a = 10, b = 10, m\angle A = 45$
12. $a = 6, b = 9, m\angle A = 24$

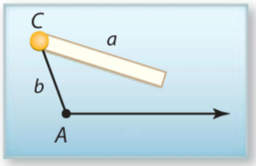
In the next activity, you will investigate how many triangles are determined for the ambiguous case when the angle given is obtuse.

Activity 2 The Ambiguous Case (SSA): ∠*A* is Obtuse

Step 1 On a 5″ × 8″ notecard, draw and label $\overline{AC}$ and a ray extending from *A* to form an obtuse angle. Label side $\overline{AC}$ as *b*.



Step 2 Using a brass fastener, attach one end of a half-inch strip of cardstock to the notecard at *C*. The strip should be longer than *b*. This represents side *a*.



Model and Analyze

13. How many triangles can be formed if $a = b$? if $a < b$? if $a > b$?

Use your results from Exercise 13 to determine whether the given measures define *0, 1, 2,* or *infinitely many* obtuse triangles. Justify your answers.

14. $a = 10, b = 8, m\angle A = 95$

15. $a = 13, b = 17, m\angle A = 100$

16. $a = 15, b = 15, m\angle A = 125$

17. Explain why three angle measures do not determine a unique triangle. How many triangles are determined by three angles measures?

Determine whether the given measures define *0, 1, 2,* or *infinitely many* triangles. Justify your answers.

18. $a = 25, b = 21, m\angle A = 39$

19. $m\angle A = 41, m\angle B = 68, m\angle C = 71$

20. $a = 17, b = 15, m\angle A = 128$

21. $a = 13, b = 17, m\angle A = 52$

22. $a = 5, b = 9, c = 6$

23. $a = 10, b = 15, m\angle A = 33$

24. **OPEN ENDED** Give measures for *a*, *b*, and an acute ∠*A* that define

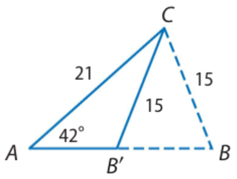
a. 0 triangles.

b. exactly one triangle.

c. two triangles.

25. **CHALLENGE** Find both solutions for $\triangle ABC$ if $a = 15$, $b = 21$, $m\angle A = 42$. Round angle measures to the nearest degree and side measures to the nearest tenth.

- For Solution 1, assume that ∠*B* is acute, and use the Law of Sines to find $m\angle B$. Then find $m\angle C$. Finally, use the Law of Sines again to find *c*.
- For Solution 2, assume that ∠*B* is obtuse. Let this obtuse angle be ∠*B*′. Use $m\angle B$ you found in Solution 1 and the diagram shown to find $m\angle B'$. Then find $m\angle C$. Finally, use the Law of Sines to find *c*.



LESSON 8-7 Vectors

Then

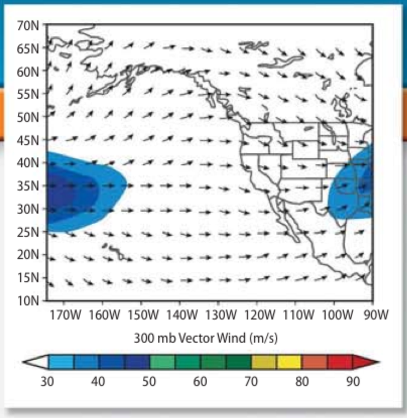
- You used trigonometry to find side lengths and angle measures of right triangles.

Now

1. Perform vector operations geometrically.
2. Perform vector operations on the coordinate plane.

Why?

- Meteorologists use vectors to represent weather patterns. For example, *wind vectors* are used to indicate wind direction and speed.



NewVocabulary

vector
magnitude
direction
resultant
parallelogram method
triangle method
standard position
component form

Common Core State Standards

Content Standards

G.GPE.6 Find the point on a directed line segment between two given points that partitions the segment in a given ratio.

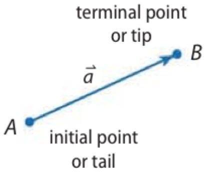
Mathematical Practices

1. Make sense of problems and persevere in solving them.
4. Model with mathematics.

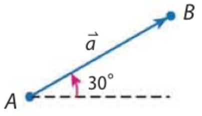
1 Geometric Vector Operations

Some quantities are described by a real number known as a *scalar*, which describes the *magnitude* or size of the quantity. Other quantities are described by a **vector**, which describes both the magnitude and *direction* of the quantity. For example, a speed of 5 miles per hour is a scalar, while a velocity of 5 miles per hour due north is a vector.

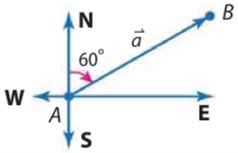
A vector can be represented by a directed line segment with an initial point and a terminal point. The vector shown, with initial point A and terminal point B, can be called $\overrightarrow{AB}$, $\vec{a}$, or **a**.



The **magnitude** of $\overrightarrow{AB}$, denoted $|\overrightarrow{AB}|$, is the length of the vector from its initial point to its terminal point. The **direction** of a vector can be expressed as the angle it forms with the horizontal or as a measurement between 0° and 90° east or west of the north-south line.



The direction of $\vec{a}$ is 30° relative to the horizontal.



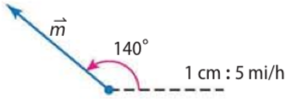
The direction of $\vec{a}$ is 60° east of north.

Example 1 Represent Vectors Geometrically

Use a ruler and a protractor to draw each vector. Include a scale on each diagram.

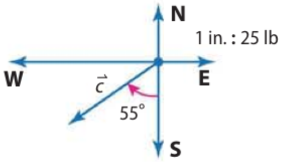
a. $\vec{m}$ **= 15 miles per hour at 140° to the horizontal**

Using a scale of 1 cm : 5 mi/h, draw and label a 15 ÷ 5 or 3-centimeter arrow at a 140° angle to the horizontal.



b. $\vec{c}$ **= 55 pounds of force 55° west of south**

Using a scale of 1 in: 25 lbs, draw and label a 55 ÷ 25 or 2.2-inch arrow 55° west of the north-south line on the south side.



GuidedPractice

1A. $\vec{b}$ = 40 feet per second at 35° to the horizontal

1B. $\vec{t}$ = 12 kilometers per hour at 85° east of north

The sum of two or more vectors is a single vector called the **resultant**.

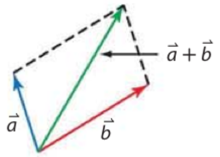
KeyConcept Vector Addition

To find the resultant of $\vec{a}$ and $\vec{b}$, use one of the following methods.

Parallelogram Method

Step 1 Translate $\vec{b}$ so that the tail of $\vec{b}$ touches the tail of $\vec{a}$.

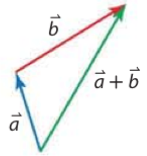
Step 2 Complete the parallelogram. The resultant is the indicated diagonal of the parallelogram.



Triangle Method

Step 1 Translate $\vec{b}$ so that the tail of $\vec{b}$ touches the tip of $\vec{a}$.

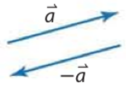
Step 2 Draw the resultant vector from the tail of $\vec{a}$ to the tip of $\vec{b}$.



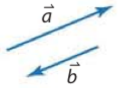
StudyTip

Types of Vectors

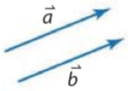
Parallel vectors have the same or opposite direction but not necessarily the same magnitude.



Opposite vectors have the same magnitude but *opposite* direction.



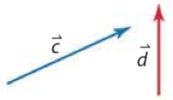
Equivalent vectors have the same magnitude and direction.



Example 2 Find the Resultant of Two Vectors

Copy the vectors. Then find $\vec{c}-\vec{d}$.

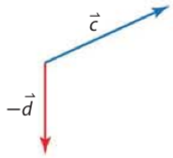
Subtracting a vector is equivalent to adding its opposite vector.



Parallelogram Method

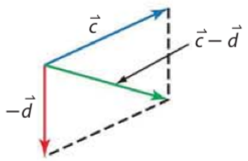
Step 1

Copy $\vec{c}$ and $\vec{d}$. Draw $-\vec{d}$, and translate it so that its tail touches the tail of $\vec{c}$.



Step 2

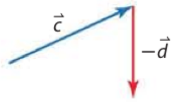
Complete the parallelogram. Then draw the diagonal.



Triangle Method

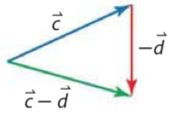
Step 1

Copy $\vec{c}$ and $\vec{d}$. Draw $-\vec{d}$, and translate it so that its tail touches the tip of $\vec{c}$.



Step 2

Draw the resultant vector from the tail of $\vec{c}$ to the tip of $-\vec{d}$.



Both methods produce the same resultant vector $\vec{c}-\vec{d}$. You can use a ruler and a protractor to measure the magnitude and direction of each vector to verify your results.

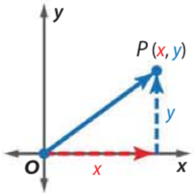
GuidedPractice

2A. Find $\vec{c}+\vec{d}$.

2B. Find $\vec{d}-\vec{c}$.

2 Vectors on the Coordinate Plane

Vectors can also be represented on the coordinate plane.



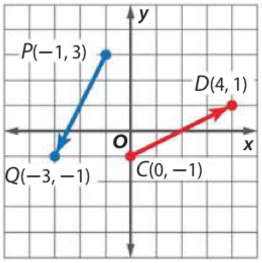
A vector is in **standard position** if its initial point is at the origin. In this position, a vector can be uniquely described by its terminal point $P(x, y)$.

To describe a vector with any initial point, you can use the **component form** $\langle x, y\rangle$, which describes the vector in terms of its horizontal component x and vertical component y.

To write the component form of a vector with initial point (x_1, y_1) and terminal point (x_2, y_2), find $\langle x_2 - x_1, y_2 - y_1\rangle$.

Example 3 Write a Vector in Component Form

Write the component form of $\overrightarrow{CD}$.



$\overrightarrow{CD} = \langle x_2 - x_1, y_2 - y_1\rangle$ Component form of a vector

$= \langle 4 - 0, 1 - (-1)\rangle$ $(x_1, y_1) = (0, -1)$ and $(x_2, y_2) = (4, 1)$

$= \langle 4, 2\rangle$ Simplify.

Guided Practice

3. Write the component form of $\overrightarrow{PQ}$.

The magnitude of a vector on the coordinate plane can be found by using the Distance Formula, and the direction can be found by using trigonometric ratios.

Example 4 Find the Magnitude and Direction of a Vector

Find the magnitude and direction of $\vec{r} = \langle -4, -5\rangle$.

Step 1 Use the Distance Formula to find the magnitude.

$|\vec{r}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ Distance Formula

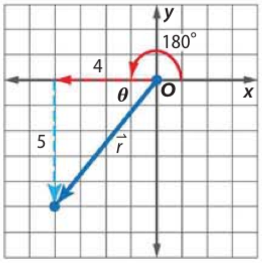
$= \sqrt{(-4 - 0)^2 + (-5 - 0)^2}$ $(x_1, y_1) = (0, 0)$ and $(x_2, y_2) = (-4, -5)$

$= \sqrt{41}$ or about 6.4 Simplify.

Study Tip

Direction Angles Vectors in standard position that lie in the third or fourth quadrants will have direction angles greater than 180°.

Step 2 Use trigonometry to find the direction.



Graph $\vec{r}$, its horizontal component, and its vertical component. Then use the inverse tangent function to find θ.

$\tan\theta = \frac{5}{4}$ $\tan\theta = \frac{\text{opp}}{\text{adj}}$

$\theta = \tan^{-1}\frac{5}{4}$ Def. of inverse tangent

$\theta \approx 51.3°$ Use a calculator.

The direction of $\vec{r}$ is the angle that it makes with the positive x-axis, which is about 180° + 51.3° or 231.3°.

So, the magnitude of $\vec{r}$ is about 6.4 units and the direction is at an angle of about 231.3° to the horizontal.

Guided Practice

4. Find the magnitude and direction of $\vec{p} = \langle -1, 4\rangle$.

You can use the properties of real numbers to add vectors, subtract vectors, and multiply vectors by scalars.

KeyConcept Vector Operations

If $\langle a, b\rangle$ and $\langle c, d\rangle$ are vectors and k is a scalar, then the following are true.

Vector Addition	$\langle a, b\rangle + \langle c, d\rangle = \langle a + c, b + d\rangle$
Vector Subtraction	$\langle a, b\rangle - \langle c, d\rangle = \langle a - c, b - d\rangle$
Scalar Multiplication	$k\langle a, b\rangle = \langle ka, kb\rangle$

Example 5 Operations with Vectors

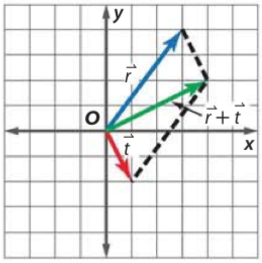
Find each of the following for $\vec{r} = \langle 3, 4\rangle$, $\vec{s} = \langle 5, -1\rangle$, and $\vec{t} = \langle 1, -2\rangle$. Check your answers graphically.

a. $\vec{r} + \vec{t}$

Solve Algebraically

$$\begin{aligned}\vec{r} + \vec{t} &= \langle 3, 4\rangle + \langle 1, -2\rangle\\ &= \langle 3 + 1, 4 + (-2)\rangle\\ &= \langle 4, 2\rangle\end{aligned}$$

Check Graphically

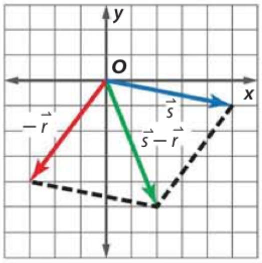


StudyTip

Vector Subtraction To represent vector subtraction graphically, graph the opposite of the vector that is being subtracted. For instance, in Example 5b, the opposite of $\vec{r} = \langle 3, 4\rangle$ is $-\vec{r} = \langle -3, -4\rangle$.

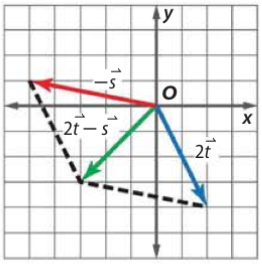
b. $\vec{s} - \vec{r}$

$$\begin{aligned}\vec{s} - \vec{r} &= \vec{s} + (-\vec{r})\\ &= \langle 5, -1\rangle + \langle -3, -4\rangle\\ &= \langle 5 + (-3), -1 + (-4)\rangle\\ &= \langle 2, -5\rangle\end{aligned}$$



c. $2\vec{t} - \vec{s}$

$$\begin{aligned}2\vec{t} - \vec{s} &= 2\vec{t} + (-\vec{s})\\ &= 2\langle 1, -2\rangle + \langle -5, 1\rangle\\ &= \langle 2, -4\rangle + \langle -5, 1\rangle\\ &= \langle 2 + (-5), -4 + 1\rangle\\ &= \langle -3, -3\rangle\end{aligned}$$



StudyTip

Scalar Multiplication The graph of a vector $k\langle a, b\rangle$ is a dilation of the vector $\langle a, b\rangle$ with scale factor k. For instance, in Example 5c, $2\vec{t} = \langle 2, -4\rangle$ is a dilation of $\vec{t} = \langle 1, -2\rangle$ with scale factor 2.

GuidedPractice

5A. $\vec{t} - \vec{r}$ **5B.** $\vec{s} + 2\vec{t}$ **5C.** $\vec{s} - \vec{t}$

You can use vectors to solve real-world problems.

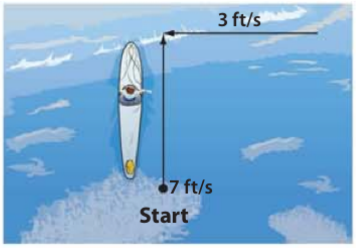
Real-WorldLink

Approximately 47% of kayakers participate in the sport one to three times per year.

Source: Outdoor Industry Association

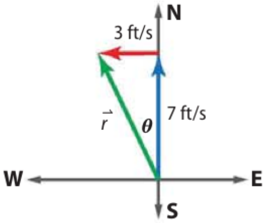
Real-World Example 6 Vector Applications

KAYAKING **Trey is paddling due north in a kayak at 7 feet per second. The river is moving with a velocity of 3 feet per second due west. What is the resultant speed and direction of the kayak to an observer on shore?**



Step 1 Draw a diagram. Let $\vec{r}$ represent the resultant vector.

The component form of the vector representing the paddling velocity is $\langle 0, 7\rangle$, and the component form of the vector representing the velocity of the river is $\langle -3, 0\rangle$.



The resultant vector is $\langle 0, 7\rangle + \langle -3, 0\rangle$ or $\langle -3, 7\rangle$. This vector represents the resultant velocity of the kayak, and its magnitude represents the resultant speed.

Step 2 Use the Distance Formula to find the resultant speed.

$|\vec{r}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ Distance Formula

$= \sqrt{(-3 - 0)^2 + (7 - 0)^2}$ $(x_1, y_1) = (0, 0)$ and $(x_2, y_2) = (-3, 7)$

$= \sqrt{58}$ or about 7.6 Simplify.

Step 3 Use trigonometry to find the resultant direction.

$\tan\theta = \frac{3}{7}$ $\tan\theta = \frac{\text{opp}}{\text{adj}}$

$\theta = \tan^{-1}\frac{3}{7}$ Def. of inverse tangent

$\theta \approx 23.2^\circ$ Use a calculator.

The direction of $\vec{r}$ is about 23.2° west of north.

Therefore, the resultant speed of the kayak is about 7.6 feet per second at an angle of about 23.2° west of north.

GuidedPractice

6. **KAYAKING** Suppose Trey starts paddling due south at a speed of 8 feet per second. If the river is flowing at a velocity of 2 feet per second due west, what is the resultant speed and direction of the kayak?

Javier Pierini/Digital Vision/Getty Images

Check Your Understanding

● = Step-by-Step Solutions begin on page R14.

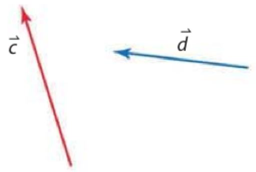
Example 1 **Use a ruler and a protractor to draw each vector. Include a scale on each diagram.**

1. $\vec{w}$ = 75 miles per hour 40° east of south

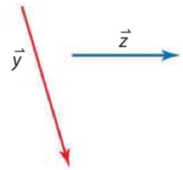
2. $\vec{h}$ = 46 feet per second 170° to the horizontal

Example 2 **Copy the vectors. Then find each sum or difference.**

3. $\vec{c} + \vec{d}$

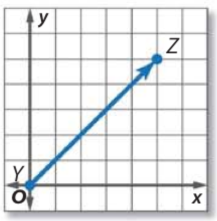


4. $\vec{y} - \vec{z}$



Example 3 **Write the component form of each vector.**

⑤



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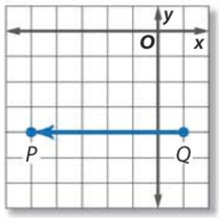


Example 4 **Find the magnitude and direction of each vector.**

7. $\vec{t} = \langle 2, -4 \rangle$

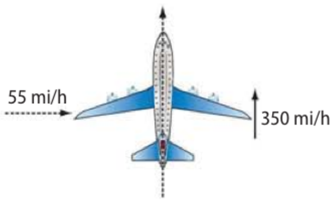
8. $\vec{f} = \langle -6, -5 \rangle$

Example 5 **Find each of the following for $\vec{a} = \langle -4, 1 \rangle$, $\vec{b} = \langle -1, -3 \rangle$, and $\vec{c} = \langle 3, 5 \rangle$. Check your answers graphically.**

9. $\vec{c} + \vec{a}$

10. $2\vec{b} - \vec{a}$

Example 6 **11.** **CCSS MODELING** A plane is traveling due north at a speed of 350 miles per hour. If the wind is blowing from the west at a speed of 55 miles per hour, what is the resultant speed and direction that the airplane is traveling?



Practice and Problem Solving

Extra Practice is on page R8.

Example 1 **Use a ruler and a protractor to draw each vector. Include a scale on each diagram.**

12. $\vec{g}$ = 60 inches per second at 145° to the horizontal

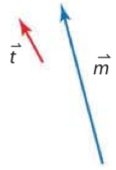
13. $\vec{n}$ = 8 meters at an angle of 24° west of south

14. $\vec{a}$ = 32 yards per minute at 78° to the horizontal

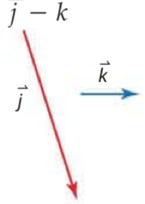
15. $\vec{k}$ = 95 kilometers per hour at angle of 65° east of north

Example 2 **Copy the vectors. Then find each sum or difference.**

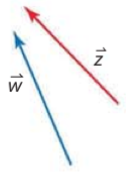
16. $\vec{t} - \vec{m}$



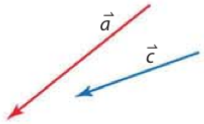
17. $\vec{j} - \vec{k}$



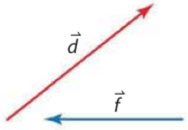
18. $\vec{w} + \vec{z}$



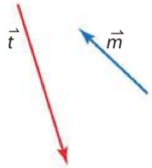
19. $\vec{c} + \vec{a}$



20. $\vec{d} - \vec{f}$

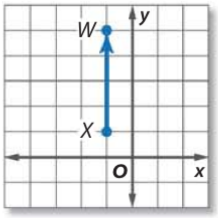


21. $\vec{t} - \vec{m}$



Example 3 **Write the component form of each vector.**

22.



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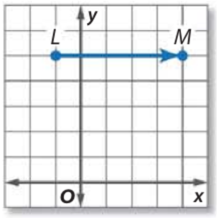

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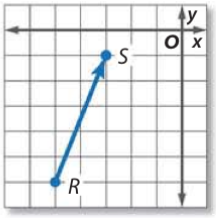

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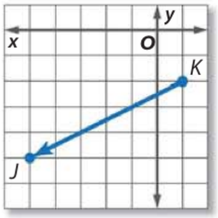

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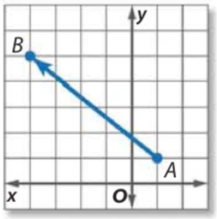

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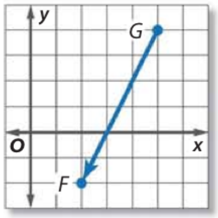
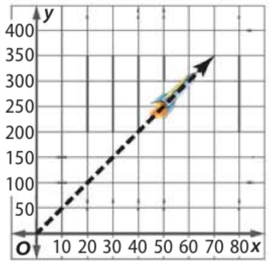


28. **FIREWORKS** The ascent of a firework shell can be modeled using a vector. Write a vector in component form that can be used to describe the path of the firework shown.



Example 4 **CCSS SENSE-MAKING** **Find the magnitude and direction of each vector.**

29. $\vec{c} = \langle 5, 3\rangle$

30. $\vec{m} = \langle 2, 9\rangle$

31. $\vec{z} = \langle -7, 1\rangle$

32. $\vec{d} = \langle 4, -8\rangle$

(33) $\vec{k} = \langle -3, -6\rangle$

34. $\vec{q} = \langle -9, -4\rangle$

Example 5 **Find each of the following for $\vec{a} = \langle -3, -5\rangle$, $\vec{b} = \langle 2, 4\rangle$, and $\vec{c} = \langle 3, -1\rangle$. Check your answers graphically.**

35. $\vec{b} + \vec{c}$

36. $\vec{c} + \vec{a}$

37. $\vec{b} - \vec{c}$

38. $\vec{a} - \vec{c}$

39. $2\vec{c} - \vec{a}$

40. $2\vec{b} + \vec{c}$

41. **HIKING** Amy hiked due east for 2 miles and then hiked due south for 3 miles.

- **a.** Draw a diagram to represent the situation, where $\vec{r}$ is the resultant vector.
- **b.** How far and in what direction is Amy from her starting position?

42. **EXERCISE** A runner's velocity is 6 miles per hour due east, with the wind blowing 2 miles per hour due north.

- **a.** Draw a diagram to represent the situation, where $\vec{r}$ is the resultant vector.
- **b.** What is the resultant velocity of the runner?

Find each of the following for $\vec{f} = \langle -4, -2 \rangle$, $\vec{g} = \langle 6, 1 \rangle$, and $\vec{h} = \langle 2, -3 \rangle$.

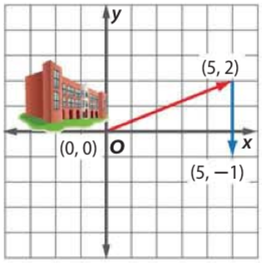
43. $\vec{f} + \vec{g} + \vec{h}$

44. $\vec{h} - 2\vec{f} + \vec{g}$

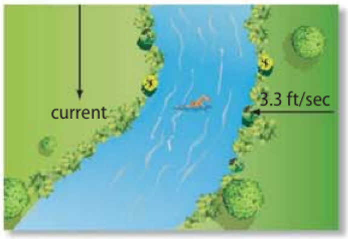
45. $2\vec{g} - 3\vec{f} + \vec{h}$

46. **HOMECOMING** Nikki is on a committee to help plan her school's homecoming parade. The parade starts at the high school and continues as shown.

- **a.** Find the magnitude and direction of the vector formed with an initial point at the school and terminal point at the end of the parade.
- **b.** Find the length of the parade if 1 unit = 0.25 mile.



47. **SWIMMING** Jonas is swimming from the east bank to the west bank of a stream at a speed of 3.3 feet per second. The stream is 80 feet wide and flows south. If Jonas crosses the stream in 20 seconds, what is the speed of the current?



H.O.T. Problems Use Higher-Order Thinking Skills

48. **CHALLENGE** Find the coordinates of point *P* on $\overleftrightarrow{AB}$ that partitions the segment into the given ratio *AP* to *PB*.

- **a.** $A(0, 0)$, $B(0, 6)$, 2 to 1
- **b.** $A(0, 0)$, $B(-15, 0)$, 2 to 3

49. **CCSS PRECISION** Are parallel vectors *sometimes*, *always*, or *never* opposite vectors? Explain.

PROOF **Prove each vector property. Let $\vec{a} = \langle x_1, y_1 \rangle$ and $\vec{b} = \langle x_2, y_2 \rangle$.**

50. commutative: $\vec{a} + \vec{b} = \vec{b} + \vec{a}$

51. scalar multiplication: $k(\vec{a} + \vec{b}) = k\vec{a} + k\vec{b}$, where *k* is a scalar

52. **OPEN ENDED** Draw a set of parallel vectors.

- **a.** Find the sum of the two vectors. What is true of the direction of the vector representing the sum?
- **b.** Find the difference of the two vectors. What is true of the direction of the vector representing the difference?

53. **WRITING IN MATH** Compare and contrast the parallelogram and triangle methods of adding vectors.

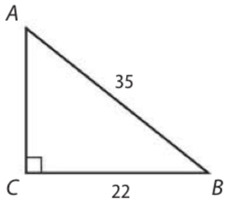
Standardized Test Practice

54. EXTENDED RESPONSE Sydney parked her car and hiked along two paths described by the vectors ⟨2, 3⟩ and ⟨5, −1⟩.

a. What vector represents her hike along both paths?

b. When she got to the end of the second path, how far is she from her car if the numbers represent miles?

55. In right triangle *ABC* shown below, what is the measure of ∠*A* to the nearest tenth of a degree?



A 32.2

B 38.9

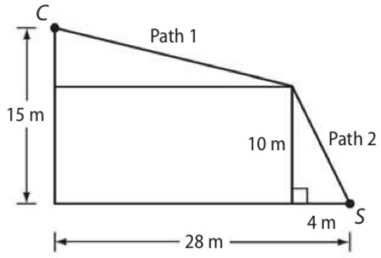
C 51.1

D 57.8

56. PROBABILITY A die is rolled. Find the probability of rolling a number greater than 4.

F 0.17 **G** 0.33 **H** 0.5 **J** 0.67

57. SAT/ACT Caleb followed the two paths shown below to get to his house *C* from a store *S*. What is the total distance of the two paths, in meters, from *C* to *S*?



A 10.8 m

B 24.5 m

C 31.8 m

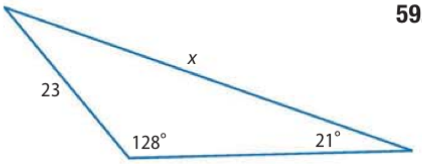
D 35.3 m

E 38.4 m

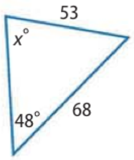
Spiral Review

Find *x*. Round angle measures to the nearest degree and side measures to the nearest tenth. (Lesson 8-6)

58.



59.



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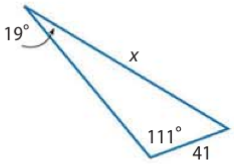
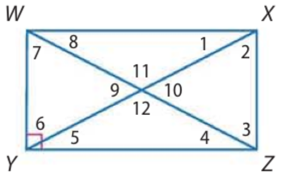


61. SOCCER Adelina is in a soccer stadium 80 feet above the field. The angle of depression to the field is 12°. What is the horizontal distance between Adelina and the soccer field? (Lesson 8-5)

Quadrilateral *WXYZ* is a rectangle. Find each measure if $m\angle 1 = 30$. (Lesson 6-4)



62. $m\angle 2$

63. $m\angle 8$

64. $m\angle 12$

65. $m\angle 5$

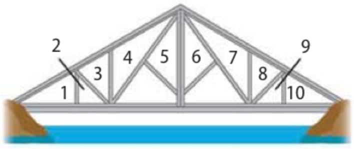
66. $m\angle 6$

67. $m\angle 3$

Skills Review

Assume that segments and angles that appear to be congruent in each figure are congruent. Indicate which triangles are congruent.

68.



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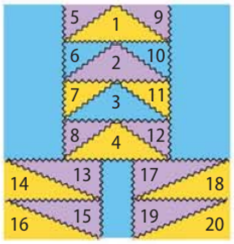

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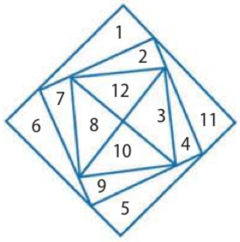


EXTEND 8-7 Geometry Lab
Adding Vectors

You can use scale drawings to represent vectors and solve problems.

Activity

A small aircraft flies due south at an average speed of 175 miles per hour. The wind is blowing 30° south of west at 25 miles per hour. What is the resultant velocity and direction of the plane?

Step 1 **Choose a scale.**

Since it is not reasonable to represent the vectors using their actual sizes, you can use a scale drawing. For this activity, let 2 inches represent 100 miles.

Step 2 **Make a scale drawing.**

Use a ruler and protractor to make a scale drawing of the two vectors.

Step 3 **Find the resultant.**

Find the resultant of the two vectors by using the triangle method or the parallelogram method.

Step 4 **Measure the resultant.**

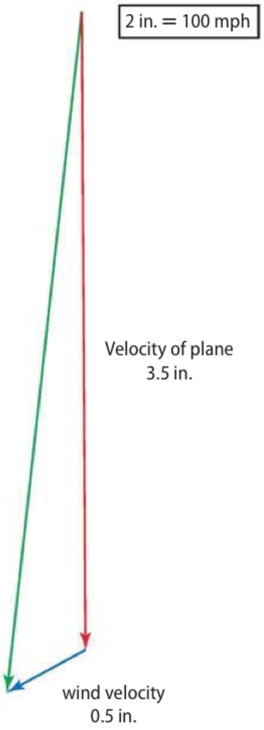
Measure the length and angle of the resultant. The resultant length is $3\frac{3}{4}$ inches, and it makes a 7° angle with the vector representing the velocity of the plane.

Step 5 **Find the magnitude and direction of the resultant.**

Use the scale with the length that you measured in Step 4 to calculate the magnitude of the plane's resultant velocity.

$$3\frac{3}{4}\,\cancel{\text{in.}} \times \frac{100\text{ mph}}{2\,\cancel{\text{in.}}} = 187.5\text{ mph}$$

The resultant velocity of the plane is 187.5 miles per hour 7° west of south.



Exercises

Make a scale drawing to solve each problem.

1. **BIKING** Lance is riding his bike west at a velocity of 10 miles per hour. The wind is blowing 5 miles per hour 20° north of east. What is Lance's resultant velocity and direction?

2. **CANOEING** Bianca is traveling due north across a river in a canoe with a current of 3 miles per hour due west. If Bianca can canoe at a rate of 7 miles per hour, what is her resultant velocity and direction?

CHAPTER 8 Study Guide and Review

Study Guide

KeyConcepts

Geometric Mean (Lesson 8-1)

- For two positive numbers a and b, the geometric mean is the positive number x where $a : x = x : b$ is true.

Pythagorean Theorem (Lesson 8-2)

- Let $\triangle ABC$ be a right triangle with right angle C. Then $a^2 + b^2 = c^2$.

Special Right Triangles (Lesson 8-3)

- The measures of the sides of a 45°-45°-90° triangle are x, x, and $x\sqrt{2}$.
- The measures of the sides of a 30°-60°-90° triangle are x, $2x$, and $x\sqrt{3}$.

Trigonometry (Lesson 8-4)

- $\sin A = \frac{\text{opposite leg}}{\text{hypotenuse}}$
- $\cos A = \frac{\text{adjacent leg}}{\text{hypotenuse}}$
- $\tan A = \frac{\text{opposite leg}}{\text{adjacent leg}}$

Angles of Elevation and Depression (Lesson 8-5)

- An angle of elevation is the angle formed by a horizontal line and the line of sight to an object above.
- An angle of depression is the angle formed by a horizontal line and the line of sight to an object below.

Laws of Sines and Cosines (Lesson 8-6)

Let $\triangle ABC$ be any triangle.

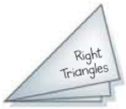
- Law of Sines: $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$
- Law of Cosines: $a^2 = b^2 + c^2 - 2bc \cos A$
  $b^2 = a^2 + c^2 - 2ac \cos B$
  $c^2 = a^2 + b^2 - 2ab \cos C$

Vectors (Lesson 8-7)

- A vector is a quantity with both magnitude and direction.

FOLDABLES StudyOrganizer

Be sure the Key Concepts are noted in your Foldable.



KeyVocabulary

angle of depression (p. 580)
angle of elevation (p. 580)
component form (p. 602)
cosine (p. 568)
direction (p. 600)
geometric mean (p. 537)
inverse cosine (p. 571)
inverse sine (p. 571)
inverse tangent (p. 571)
Law of Cosines (p. 589)
Law of Sines (p. 588)
magnitude (p. 600)
Pythagorean triple (p. 548)
resultant (p. 601)
sine (p. 568)
standard position (p. 602)
tangent (p. 568)
trigonometric ratio (p. 568)
trigonometry (p. 568)
vector (p. 600)

VocabularyCheck

State whether each sentence is *true* or *false*. If *false*, replace the underlined word or phrase to make a true sentence.

1. The arithmetic mean of two numbers is the positive square root of the product of the numbers.
2. Extended ratios can be used to compare three or more quantities.
3. To find the length of the hypotenuse of a right triangle, take the square root of the difference of the squares of the legs.
4. An angle of elevation is the angle formed by a horizontal line and an observer's line of sight to an object below the horizon.
5. The sum of two vectors is the resultant.
6. Magnitude is the angle a vector makes with the x-axis.
7. A vector is in standard position when the initial point is at the origin.
8. The component form of a vector describes the vector in terms of change in x and change in y.
9. The Law of Sines can be used to find an angle measure when given three side lengths.
10. A trigonometric ratio is a ratio of the lengths of two sides of a right triangle.

Lesson-by-Lesson Review

8-1 Geometric Mean

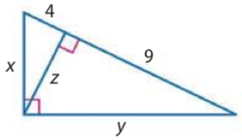
Find the geometric mean between each pair of numbers.

11. 9 and 4

12. $\sqrt{20}$ and $\sqrt{80}$

13. $\frac{8\sqrt{2}}{3}$ and $\frac{4\sqrt{2}}{3}$

14. Find x, y, and z.



15. **DANCES** Mike is hanging a string of lights on his barn for a square dance. Using a book to sight the top and bottom of the barn, he can see he is 15 feet from the barn. If his eye level is 5 feet from the ground, how tall is the barn?

Example 1

Find the geometric mean between 10 and 15.

$x = \sqrt{ab}$ Definition of geometric mean

$= \sqrt{10 \cdot 15}$ $a = 10$ and $b = 15$

$= \sqrt{(5 \cdot 2) \cdot (3 \cdot 5)}$ Factor.

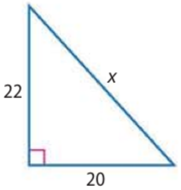
$= \sqrt{25 \cdot 6}$ Associative Property

$= 5\sqrt{6}$ Simplify.

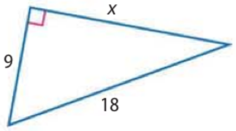
8-2 The Pythagorean Theorem and Its Converse

Find x.

16.



17.



Determine whether each set of numbers can be the measures of the sides of a triangle. If so, classify the triangle as *acute*, *obtuse*, or *right*. Justify your answer.

18. 7, 24, 25

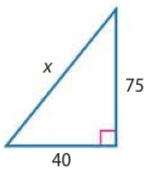
19. 13, 15, 16

20. 65, 72, 88

21. **SWIMMING** Alexi walks 27 meters south and 38 meters east to get around a lake. Her sister swims directly across the lake. How many meters to the nearest tenth did Alexi's sister save by swimming?

Example 2

Find x.



The side opposite the right angle is the hypotenuse, so $c = x$.

$a^2 + b^2 = c^2$ Pythagorean Theorem

$40^2 + 75^2 = x^2$ $a = 40$ and $b = 75$

$7225 = x^2$ Simplify.

$\sqrt{7225} = x$ Take the positive square root of each side.

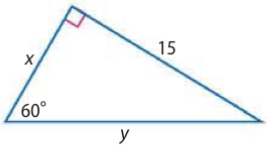
$85 = x$ Simplify.

Study Guide and Review *Continued*

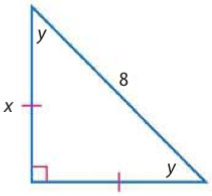
8-3 Special Right Triangles

Find x and y.

22.



23.

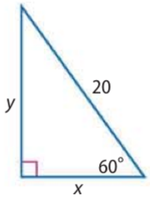


24. **CLIMBING** Jason is adding a climbing wall to his little brother's swing-set. If he starts building 5 feet out from the existing structure, and wants it to have a 60° angle, how long should the wall be?

Example 3

Find x and y.

The measure of the third angle in this triangle is 90 – 60 or 30. This is a 30°-60°-90° triangle.



$h = 2s$ 30°-60°-90° Triangle Theorem

$20 = 2x$ Substitute.

$10 = x$ Divide.

Now find y, the length of the longer leg.

$\ell = s\sqrt{3}$ 30°-60°-90° Triangle Theorem

$y = 10\sqrt{3}$ Substitute.

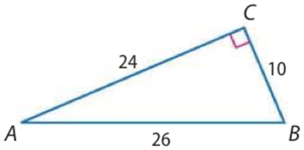
8-4 Trigonometry

Express each ratio as a fraction and as a decimal to the nearest hundredth.

25. $\sin A$ **26.** $\tan B$

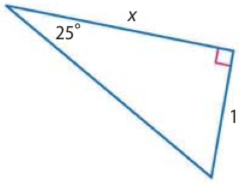
27. $\sin B$ **28.** $\cos A$

29. $\tan A$ **30.** $\cos B$

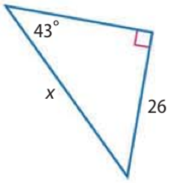


Find x.

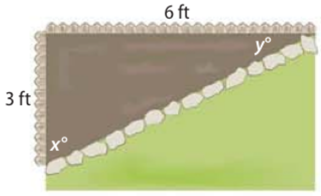
31.



32.

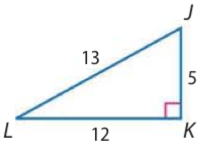


33. **GARDENING** Sofia wants to put a flower bed in the corner of her yard by laying a stone border that starts 3 feet from the corner of one fence and ends 6 feet from the corner of the other fence. Find the angles, x and y, the fence make with the border.



Example 4

Express each ratio as a fraction and as a decimal to the nearest hundredth.



a. $\sin L$

$\sin L = \frac{5}{13}$ or about 0.38 $\sin L = \frac{\text{opp}}{\text{hyp}}$

b. $\cos L$

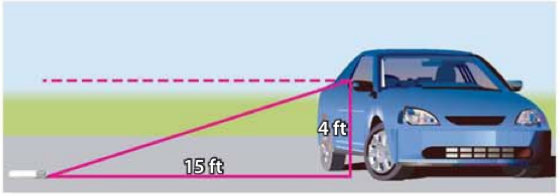
$\cos L = \frac{12}{13}$ or about 0.92 $\cos L = \frac{\text{adj}}{\text{hyp}}$

c. $\tan L$

$\tan L = \frac{5}{12}$ or 0.42 $\tan L = \frac{\text{opp}}{\text{adj}}$

8-5 Angles of Elevation and Depression

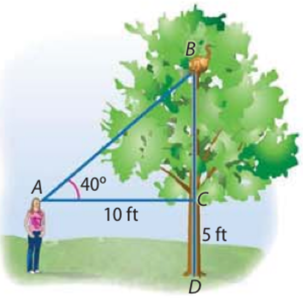
34. **JOBS** Tom delivers papers on a rural route from his car. If he throws a paper from a height of 4 feet, and it lands 15 feet from the car, at what angle of depression did he throw the paper to the nearest degree?



35. **TOWER** There is a cell phone tower in the field across from Jen's house. If Jen walks 50 feet from the tower, and finds the angle of elevation from her position to the top of the tower to be 60°, how tall is the tower?

Example 5

Sarah's cat climbed up a tree. If she sights her cat at an angle of elevation of 40°, and her eyes are 5 feet off the ground, how high up from the ground is her cat?



To find the how high the cat is up the tree, find *CB*.

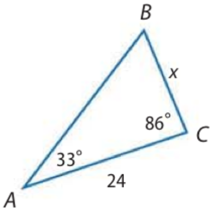
$\tan 40 = \frac{CB}{10}$	$\tan = \frac{\text{opposite}}{\text{adjacent}}$
$10(\tan 40) = CB$	Multiply each side by 10.
$8.4 = CB$	Simplify.

Since Sarah's eyes are 5 feet from the ground, add 5 to 8.4. Sarah's cat is 13.4 feet up.

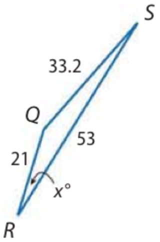
8-6 The Law of Sines and Law of Cosines

Find *x*. Round angle measures to the nearest degree and side measures to the nearest tenth.

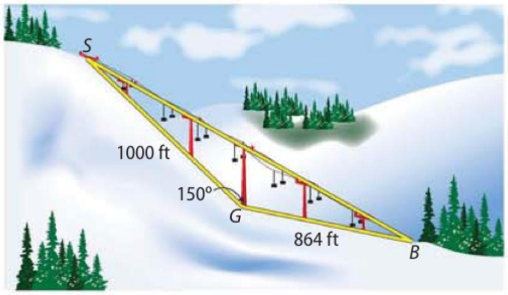
36.



37.

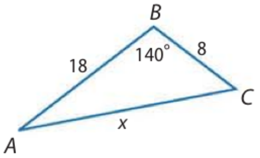


38. **SKIING** At Crazy Ed's Ski resort, Ed wants to put in another ski lift for the skiers to ride from the base to the summit of the mountain. The run over which the ski lift will go is represented by the figure below. The length of the lift is represented by *SB*. If Ed needs twice as much cable as the length of $\overline{SB}$, how much cable does he need?



Example 6

Find *x*. Round to the nearest tenth.

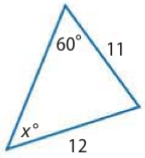


We are given the measures of two sides and their included angle, so use the Law of Cosines.

$b^2 = a^2 + c^2 - 2ac \cos B$	Law of Cosines
$x^2 = 8^2 + 18^2 - 2(8)(18) \cos 140°$	Substitution
$x^2 = 388 - 288 \cos 140°$	Simplify.
$x = \sqrt{388 - 288 \cos 140°} \approx 24.7$	Take the square root of each side.

Example 7

Find *x*. Round to the nearest tenth.

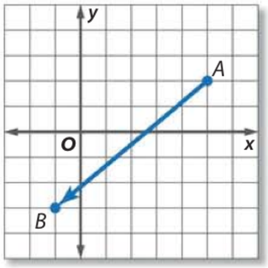


$\frac{\sin A}{a} = \frac{\sin C}{c}$	Law of Sines
$\frac{\sin 60}{12} = \frac{\sin x}{11}$	Substitution
$11 \sin 60° = 12 \sin x$	Cross Products Property
$\frac{11 \sin 60}{12} = \sin x$	Divide each side by 12.

$x = \sin^{-1} \frac{11 \sin 60}{12}$ or about 52.5°

8-7 Vectors

39. Write the component form of the vector shown.

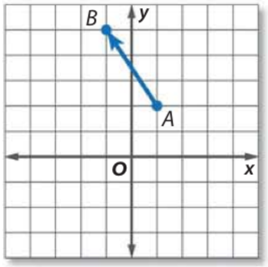


40. Copy the vectors to find $\vec{a} + \vec{b}$.

41. Given that $\vec{s}$ is $\langle 2, -6\rangle$ and $\vec{t}$ is $\langle -10, 7\rangle$, find the component form of $\vec{s} + \vec{t}$.

Example 8

Find the magnitude and direction of $\overrightarrow{AB}$ for $A(1, 2)$ and $B(-1, 5)$.



Use the Distance Formula to find the magnitude.

$\overrightarrow{AB} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	Distance Formula
$= \sqrt{(-1 - 1)^2 + (5 - 2)^2}$	Substitute.
$= \sqrt{13}$ or about 3.6	Simplify.

Draw a right triangle with hypotenuse $\overrightarrow{AB}$ and acute angle A.

$\tan A = \left\lvert \frac{5-2}{-1-1} \right\rvert$ or $\frac{3}{2}$	$\tan = \frac{\text{opp}}{\text{adj}}$; length cannot be negative.
$m\angle A = \tan^{-1}\left(-\frac{3}{2}\right)$	Def. of inverse tangent
≈ -56.3	Use a calculator.

The direction of $\overrightarrow{AB}$ is 180 − 56.3 or 123.7°.

CHAPTER 8 Practice Test

Find the geometric mean between each pair of numbers.

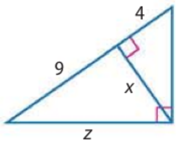
1. 7 and 11

2. 12 and 9

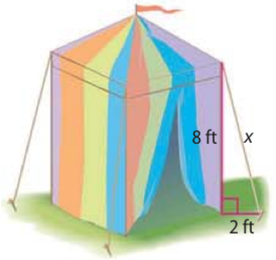
3. 14 and 21

4. $4\sqrt{3}$ and $10\sqrt{3}$

5. Find x, y, and z.

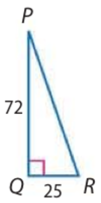


6. FAIRS Blake is setting up his tent at a renaissance fair. If the tent is 8 feet tall, and the tether can be staked no more than two feet from the tent, how long should the tether be?

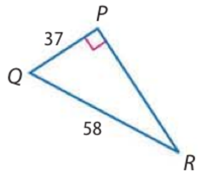


Use a calculator to find the measure of ∠R to the nearest tenth.

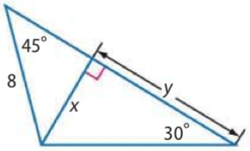
7.



8.



9. Find x and y.



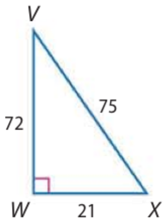
Express each ratio as a fraction and as a decimal to the nearest hundredth.

10. $\cos X$

11. $\tan X$

12. $\tan V$

13. $\sin V$



Find the magnitude and direction of each vector.

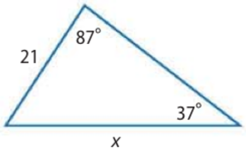
14. $\overrightarrow{JK}$: $J(-6, -4)$ and $K(-10, -4)$

15. $\overrightarrow{RS}$: $R(1, 0)$ and $S(-2, 3)$

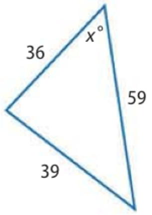
16. SPACE Anna is watching a space shuttle launch 6 miles from Cape Canaveral in Florida. When the angle of elevation from her viewing point to the shuttle is 80°, how high is the shuttle, if it is going straight up?

Find x. Round angle measures to the nearest degree and side measures to the nearest tenth.

17.



18.



19. MULTIPLE CHOICE Which of the following is the length of the leg of a 45°-45°-90° triangle with a hypotenuse of 20?

A 10

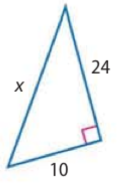
B $10\sqrt{2}$

C 20

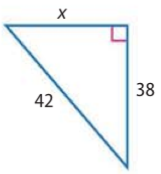
D $20\sqrt{2}$

Find x.

20.



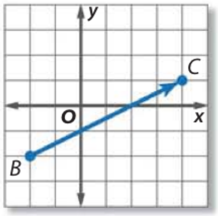
21.



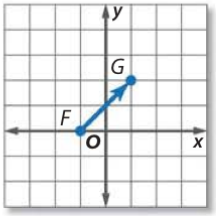
22. WHALE WATCHING Isaac is looking through binoculars on a whale watching trip when he notices a sea otter in the distance. If he is 20 feet above sea level in the boat, and the angle of depression is 30°, how far away from the boat is the otter to the nearest foot?

Write the component form of each vector.

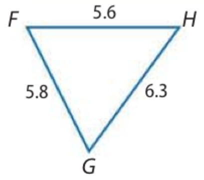
23.



24.



25. Solve △FGH. Round to the nearest degree.



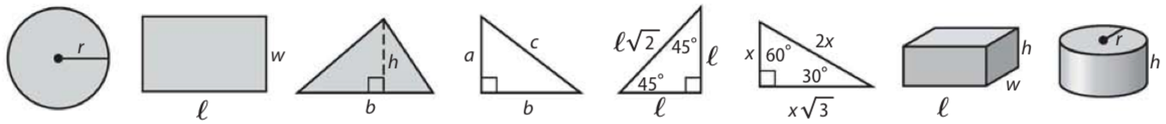
CHAPTER 8

Preparing for Standardized Tests

Use a Formula

Sometimes it is necessary to use a formula to solve problems on standardized tests. In some cases you may even be given a sheet of formulas that you are permitted to reference while taking the test.

Formulas



$C = 2\pi r$
$A = \pi r^2$

$A = \ell w$

$A = \frac{1}{2}bh$

$a^2 + b^2 = c^2$

Special Right Triangles

$V = \ell wh$

$V = \pi r^2 h$

There are 360 degrees in a circle.
The sum of the measures of the angles of a triangle is 180.

Strategies for Using a Formula

Step 1

Read the problem statement carefully.

Ask yourself:

- What am I being asked to solve?
- What information is given in the problem?
- Are there any formulas that I can use to help me solve the problem?

Step 2

Solve the problem.

- Substitute the known quantities that are given in the problem statement into the formula.
- Simplify to solve for the unknown values in the formula.

Step 3

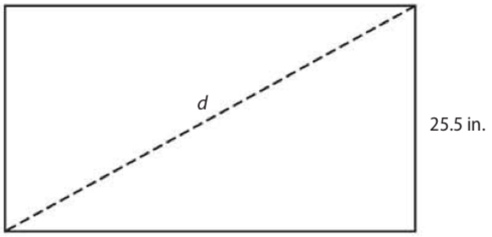
Check your solution.

- Determine a reasonable range of values for the answer.
- Check to make sure that your answer makes sense.
- If time permits, check your answer.

Standardized Test Example

Read the problem. Identify what you need to know. Then use the information in the problem to solve.

The ratio of the width to the height of a high-definition television is 16:9. This is also called the *aspect ratio* of the television. The size of a television is given in terms of the diagonal distance across the screen. If an HD television is 25.5 inches tall, what is its screen size?



A 48 inches

B 50 inches

C 51 inches

D 52 inches

Read the problem statement carefully. You are given the height of the screen and the ratio of the width to the height. You are asked to find the diagonal distance of the screen. You can use the **Pythagorean Theorem** to solve the problem.

Find the width of the screen. Set up and solve a proportion using the aspect ratio 16:9.

$\frac{16}{9} = \frac{w}{25.5}$ ⟵ width of the screen ⟵ height of the screen

$9w = 408$ Cross Products Property

$w = 45\frac{1}{3}$ Divide each side by 9.

So, the width of the screen is $45\frac{1}{3}$ inches. Use the Pythagorean Theorem to solve for the diagonal distance.

$c^2 = a^2 + b^2$ Pythagorean Theorem

$c^2 = (25.5)^2 + \left(45\frac{1}{3}\right)^2$ Substitute for a and b.

$c \approx 52.01$ Simplify. Take the square root of both sides to solve for c.

The diagonal distance of the screen is about 52 inches. So, the answer is D.

Exercises

Read each problem. Identify what you need to know. Then use the information in the problem to solve.

1. Christine is flying a kite on the end of a taut string. The kite is 175 feet above the ground and is a horizontal distance of 130 feet from where Christine is standing. How much kite string has Christine let out? Round to the nearest foot.

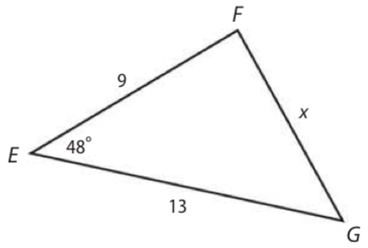
A 204 ft

B 218 ft

C 225 ft

D 236

2. What is the value of x below to the nearest tenth?



F 9.7 **G** 10.2 **H** 10.5 **J** 11.1

CHAPTER 8

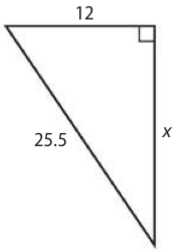
Standardized Test Practice

Cumulative, Chapters 1 through 8

Multiple Choice

Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.

1. What is the value of x in the figure below?



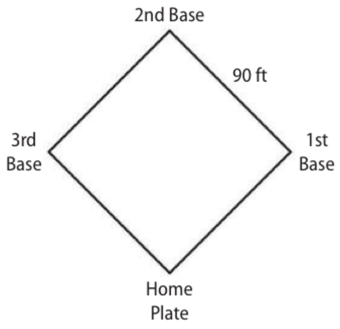
A 22.5

B 23

C 23.5

D 24

2. A baseball diamond is a square with 90-ft sides. What is the length from 3rd base to 1st base? Round to the nearest tenth.



F 155.9 ft

G 141.6 ft

H 127.3 ft

J 118.2 ft

Test-Taking Tip

Question 1 Some test items require the use of a formula to solve them. Use the Pythagorean Theorem to find x.

3. The scale of a map is 1 inch = 4.5 kilometers. What is the distance between two cities that are 2.4 inches apart on the map?

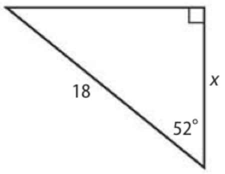
A 10.8 kilometers

B 11.1 kilometers

C 11.4 kilometers

D 11.5 kilometers

4. What is the value of x in the figure below? Round to the nearest tenth.



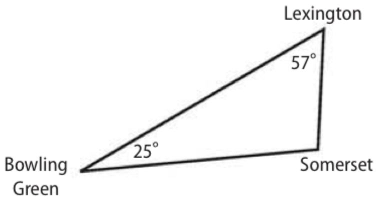
F 10.5

G 11.1

H 13.6

J 14.2

5. What type of triangle is formed by the locations of Lexington, Somerset, and Bowling Green?



A acute

B equiangular

C obtuse

D right

6. Grant is flying a kite on the end of a string that is 350 feet long. The angle elevation from Grant to the kite is 74°. How high above the ground is the kite? Round your answer to the nearest tenth if necessary.

F 336.4 ft

G 295.6 ft

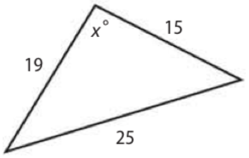
H 141.2 ft

J 96.5 ft

Short Response/Gridded Response

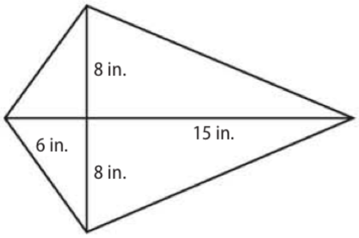
Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

7. **GRIDDED RESPONSE** Find x in the figure below. Round your answer to the nearest tenth if necessary.



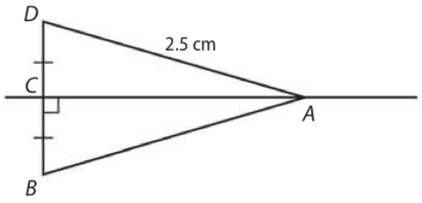
8. Amy is paddling her canoe across a lake at a speed of 10 feet per second headed due north. The wind is blowing 40° east of north with a velocity of 2.8 feet per second. What is Amy's resultant velocity? Express your answer as a vector. Show your work.

9. Janice used a 16-inch dowel and a 21-inch dowel to build a kite as shown below. What is the perimeter of her kite?



10. **GRIDDED RESPONSE** A model airplane takes off at an angle of elevation of 30°. How high will the plane be after traveling 100 feet horizontally? Round to the nearest tenth. Show your work.

11. According to the Perpendicular Bisector Theorem, what is the length of segment AB below?



12. Find the slope of the line that contains the points (7, 2) and (3, 4).

13. If $EG = 15$ meters, what is the length of segment FG?

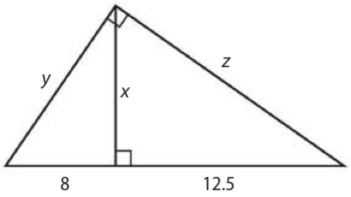
14. What is the contrapositive of the statement below?

If a quadrilateral is a rectangle, then it is a parallelogram.

Extended Response

Record your answers on a sheet of paper. Show your work.

15. Refer to the triangle shown below.



a. Find x to the nearest tenth.

b. Find y to the nearest tenth.

c. Find z to the nearest tenth.

Need ExtraHelp?

If you missed Question...	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Go to Lesson...	8-2	8-3	7-7	8-4	4-1	8-5	8-6	8-7	6-6	8-5	5-1	3-3	1-2	2-3	8-1