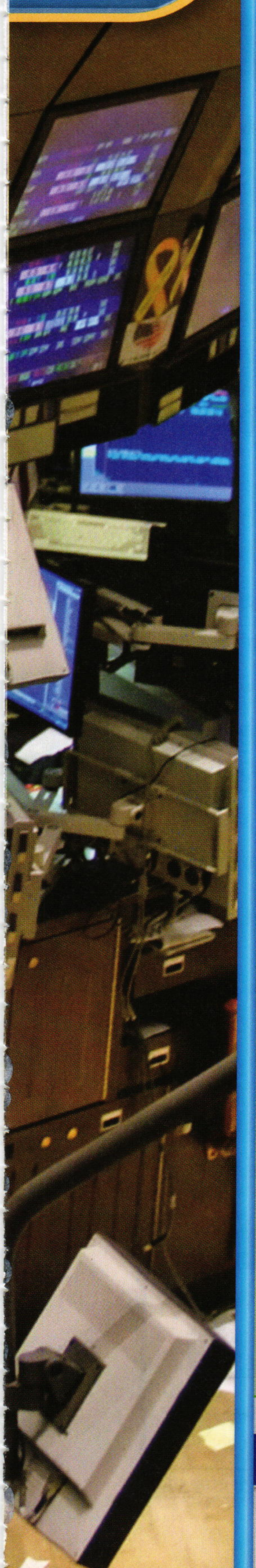


Get Ready for Chapter 9

Diagnose Readiness You have two options for checking Prerequisite Skills.

Text Option

Take the Quick Check below. Refer to the Quick Review for help.

QuickCheck

Use a table of values to graph each equation. (Lesson 3-1)

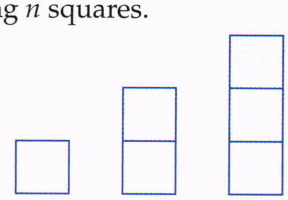
1. $y = x + 3$
2. $y = 2x + 2$
3. $y = -2x - 3$
4. $y = 0.5x - 1$
5. $4x - 3y = 12$
6. $3y = 6 + 9x$
7. **SAVINGS** Jack has \$100 to buy a game system. He plans to save \$10 each week. Graph an equation to show the total amount T Jack will have in w weeks.

Determine whether each trinomial is a perfect square trinomial. Write *yes* or *no*. If so, factor it. (Lesson 8-6)

8. $a^2 + 12a + 36$
9. $x^2 + 5x + 25$
10. $x^2 - 12x + 32$
11. $x^2 + 20x + 100$
12. $4x^2 + 28x + 49$
13. $k^2 - 16k + 64$
14. $a^2 - 22a + 121$
15. $5t^2 - 12t + 25$

Find the next three terms of each arithmetic sequence. (Lessons 3-5)

16. 16, 4, −8, −20, …
17. 2, 10, 18, 26, …
18. −5, −2, 1, 4, …
19. 3, 5, 7, 9, …
20. **GEOMETRY** Write a formula that can be used to find the perimeter of a figure containing n squares.

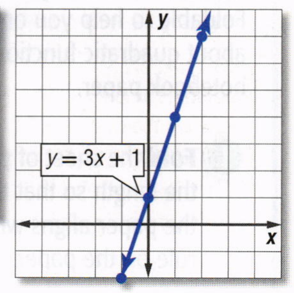


QuickReview

EXAMPLE 1

Use a table of values to graph $y = 3x + 1$.

x	$y = 3x + 1$	y
−1	3(−1) + 1	−2
0	3(0) + 1	1
1	3(1) + 1	4
2	3(2) + 1	7



EXAMPLE 2

Determine whether $x^2 - 10x + 25$ is a perfect square trinomial. Write *yes* or *no*. If so, factor it.

1. Is the first term a perfect square? yes
2. Is the last term a perfect square? yes
3. Is the middle term equal to $-2(1x)(5)$? yes

$x^2 - 10x + 25 = (x - 5)^2$

EXAMPLE 3

Find the next three terms of the arithmetic sequence 5, 9, 13, 17, … .

Find the common difference by subtracting a term from the next term.

$$9 - 5 = 4$$

Add to find the next three terms.

$17 + 4 = 21, 21 + 4 = 25, 25 + 4 = 29$

The next three terms are 21, 25, 29.

Online Option

Math Online Take a self-check Chapter Readiness Quiz at glencoe.com.

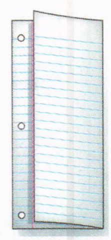
Get Started on Chapter 9

You will learn several new concepts, skills, and vocabulary terms as you study Chapter 9. To get ready, identify important terms and organize your resources. You may wish to refer to **Chapter 0** to review prerequisite skills.

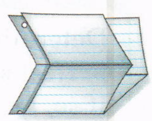
FOLDABLES® Study Organizer

Quadratic and Exponential Functions Make this Foldable to help you organize your Chapter 9 notes about quadratic functions. Begin with a sheet of notebook paper.

1. **Fold** the sheet of paper along the length so that the edge of the paper aligns with the margin rule on the paper.

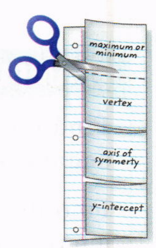


2. **Fold** the sheet twice widthwise to form four sections.



3. **Unfold** the sheet, and cut along the folds on the front flap only.

4. **Label** each section as shown.



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- Study the chapter online
- Explore **Math in Motion**
- Get extra help from your own **Personal Tutor**
- Use **Extra Examples** for additional help
- Take a **Self-Check Quiz**
- **Review Vocabulary** in fun ways

New Vocabulary

English		Español
axis of symmetry	• p. 525 •	eje de simetría
maximum	• p. 525 •	máximo
minimum	• p. 525 •	mínimo
parabola	• p. 525 •	parábola
quadratic function	• p. 525 •	función cuadrática
vertex	• p. 525 •	vértice
double root	• p. 538 •	doble raíz
transformation	• p. 544 •	transformación
completing the square	• p. 552 •	completar el cuadrado
Quadratic Formula	• p. 558 •	Formula cuadrática
discriminant	• p. 561 •	discriminante
exponential function	• p. 567 •	función exponencial
compound interest	• p. 574 •	interés es compuesta
common ratio	• p. 578 •	proporción común
geometric sequence	• p. 578 •	secuencia geométrica

Review Vocabulary

domain • p. 38 • dominio all the possible values of the independent variable, x

leading coefficient • p. 425 • coeficiente delantero the coefficient of the first term of a polynomial written in standard form

range • p. 38 • rango all the possible values of the dependent variable, y

In the function represented by the table, the domain is {0, 2, 4, 6}, and the range is {3, 5, 7, 9}.

x	y
0	3
2	5
4	7
6	9

Multilingual eGlossary glencoe.com

9-1 Graphing Quadratic Functions

Then
You graphed linear functions. (Lesson 3-2)

Now
- Analyze the characteristics of graphs of quadratic functions.
- Graph quadratic functions.

New Vocabulary
quadratic function
standard form
parabola
axis of symmetry
vertex
minimum
maximum
symmetry

Math Online
glencoe.com
- Extra Examples
- Personal Tutor
- Self-Check Quiz
- Homework Help

Why?

The Innovention Fountain in Epcot's Futureworld in Orlando, Florida, is an elaborate display of water, light, and music. The sprayers shoot water in shapes that can be modeled by quadratic equations. You can use the graph of this equation to show the path of the water.

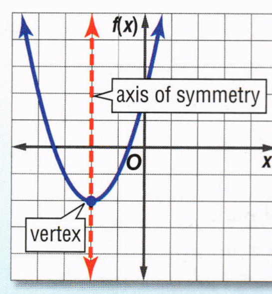
Characteristics of Quadratic Functions You have studied linear functions. There are also nonlinear functions with graphs of other shapes. **Quadratic functions** are nonlinear and can be written in the form $f(x) = ax^2 + bx + c$, where $a \neq 0$. This form is called the **standard form** of a quadratic function.

The shape of the graph of a quadratic function is called a **parabola**. Parabolas are symmetric about a central line called the **axis of symmetry**. The axis of symmetry intersects a parabola at only one point, called the **vertex**.

Key Concept — Quadratic Functions

For Your FOLDABLE

Parent Function:	$f(x) = x^2$
Standard Form:	$f(x) = ax^2 + bx + c$
Type of Graph:	parabola
Axis of Symmetry:	$x = -\frac{b}{2a}$
y-intercept:	c

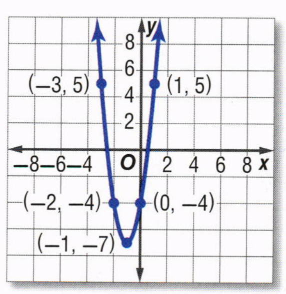


When $a > 0$, the graph of $y = ax^2 + bx + c$ opens upward. The lowest point on the graph is the **minimum**. When $a < 0$, the graph of $y = ax^2 + bx + c$ opens downward. The highest point on the graph is the **maximum**. The maximum or minimum is the vertex.

EXAMPLE 1 Graph a Parabola

Use a table of values to graph $y = 3x^2 + 6x - 4$. State the domain and range.

x	y
1	5
0	−4
−1	−7
−2	−4
−3	5



Graph the ordered pairs, and connect them to create a smooth curve. The parabola extends to infinity. The domain is all real numbers. The range is $\{y \mid y \geq -7\}$, because −7 is the minimum.

Check Your Progress

1. Use a table of values to graph $y = x^2 + 3$. State the domain and range.

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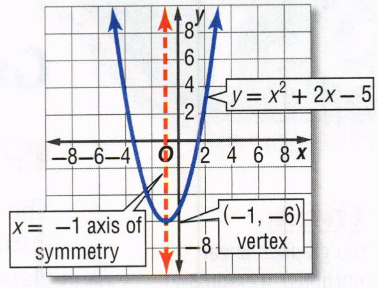
Review Vocabulary

Domain and Range The domain is the set of all of the possible values of the independent variable x. The range is the set of all of the possible values of the dependent variable y.

Figures that possess **symmetry** are those in which each half of the figure matches exactly.

A parabola is symmetric about the axis of symmetry. Every point on the parabola to the left of the axis of symmetry has a corresponding point on the other half.

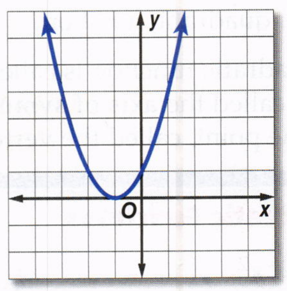


When identifying characteristics from a graph, it is often easiest to locate the vertex first. It is either the maximum or minimum point of the graph.

EXAMPLE 2 Identify Characteristics from Graphs

Find the vertex, the equation of the axis of symmetry, and the y-intercept of each graph.

a.

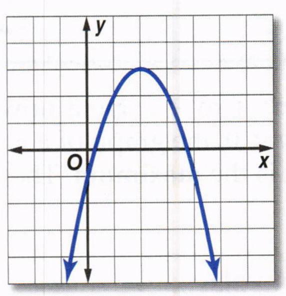


Step 1 Find the vertex.
Because the parabola opens upward, the vertex is located at the minimum point of the parabola. It is located at $(-1, 0)$.

Step 2 Find the axis of symmetry.
The axis of symmetry is the line that goes through the vertex and divides the parabola into congruent halves. It is located at $x = -1$.

Step 3 Find the y-intercept.
The y-intercept is the point where the graph intersects the y-axis. It is located at $(0, 1)$, so the y-intercept is 1.

b.



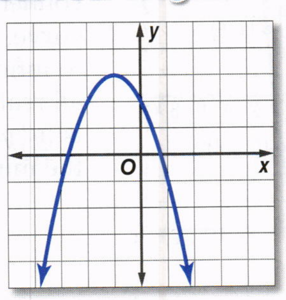
Step 1 Find the vertex.
The parabola opens downward, so the vertex is located at its maximum point, $(2, 3)$.

Step 2 Find the axis of symmetry.
The axis of symmetry is located at $x = 2$.

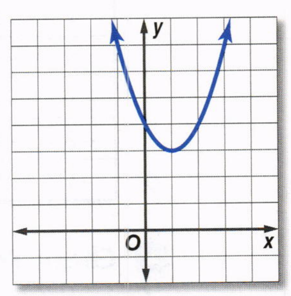
Step 3 Find the y-intercept.
The y-intercept is where the parabola crosses the y-axis. It is located at $(0, -1)$, so the y-intercept is -1.

Check Your Progress

2A.



2B.



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StudyTip

Function Characteristics When identifying characteristics of a function, it is often easiest to locate the axis of symmetry first.

EXAMPLE 3 Identify Characteristics from Functions

Find the vertex, the equation of the axis of symmetry, and the y-intercept of each function.

a. $y = 2x^2 + 4x - 3$

$x = -\frac{b}{2a}$ — **Formula for the equation of the axis of symmetry**

$x = -\frac{4}{2 \cdot 2}$ — **$a = 2$ and $b = 4$**

$x = -1$ — **Simplify.**

The equation for the axis of symmetry is $x = -1$.

To find the vertex, use the value you found for the axis of symmetry as the x-coordinate of the vertex. To find the y-coordinate, substitute that value for x in the original equation.

$y = 2x^2 + 4x - 3$ — **Original equation**

$= 2(-1)^2 + 4(-1) - 3$ — **$x = -1$**

$= -5$ — **Simplify.**

The vertex is at $(-1, -5)$.

The y-intercept always occurs at $(0, c)$. So, the y-intercept is -3.

b. $y = -x^2 + 6x + 4$

$x = -\frac{b}{2a}$ — **Formula for the equation of the axis of symmetry**

$x = -\frac{6}{2(-1)}$ — **$a = -1$ and $b = 6$**

$x = 3$ — **Simplify.**

The equation of the axis of symmetry is $x = 3$.

$y = -x^2 + 6x + 4$ — **Original equation**

$= -(3)^2 + 6(3) + 4$ — **$x = 3$**

$= 13$ — **Simplify.**

The vertex is at $(3, 13)$.

The y-intercept is 4.

StudyTip

***y*-intercept** The y-coordinate of the y-intercept is also the constant term (c) of the quadratic function in standard form.

Check Your Progress

3A. $y = -3x^2 + 6x - 5$

3B. $y = 2x^2 + 2x + 2$

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There are general differences between linear functions and quadratic functions.

	Linear Functions	Quadratic Functions
Standard Form	$y = ax + b$	$y = ax^2 + bx + c; a \neq 0$
Degree	1; Notice that all of the variables are to the first power.	2; Notice that the independent variable, x, is squared in the first term. The coefficient a can not equal 0, or the equation would be linear.
Example	$y = 2x + 6$	$y = 3x^2 + 5x - 4$
Graph	line	parabola

Next you will learn how to identify whether the parabola opens up or down and whether the vertex is a maximum or a minimum point.

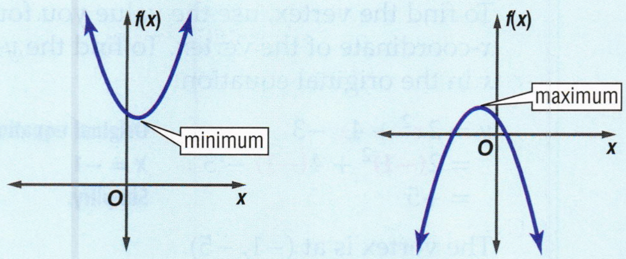
Key Concept — Maximum and Minimum Values

For Your FOLDABLE

Words The graph of $f(x) = ax^2 + bx + c$, where $a \neq 0$:

- opens upward and has a minimum value when $a > 0$, and
- opens downward and has a maximum value when $a < 0$.
- The range of a quadratic function is all real numbers greater than or equal to the minimum, or all real numbers less than or equal to the maximum.

Examples a is positive. a is negative.



Watch Out!

Minimum and Maximum Values Don't forget to find both coordinates of the vertex (x, y). The minimum or maximum value is the y-coordinate.

EXAMPLE 4 Maximum and Minimum Values

Consider $f(x) = -2x^2 - 4x + 6$.

a. Determine whether the function has a *maximum* or *minimum* value.

For $f(x) = -2x^2 - 4x + 6$, $a = -2$, $b = -4$, and $c = 6$.
Because a is negative the graph opens down, so the function has a maximum value.

b. State the maximum or minimum value of the function.

The maximum value is the y-coordinate of the vertex.

The x-coordinate of the vertex is $\frac{-b}{2a}$ or $\frac{4}{2(-2)}$ or -1.

$f(x) = -2x^2 - 4x + 6$	**Original function**
$f(-1) = -2(-1)^2 - 4(-1) + 6$	**$x = -1$**
$f(-1) = 8$	**Simplify.**

The maximum value is 8.

c. State the domain and range of the function.

The domain is all real numbers. The range is all real numbers less than or equal to the maximum value, or $\{y \mid y \leq 8\}$.

Check Your Progress

Consider $g(x) = 2x^2 - 4x - 1$.

4A. Determine whether the function has a *maximum* or *minimum* value.

4B. State the maximum or minimum value.

4C. State the domain and range of the function.

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Math *in Motion*, Animation glencoe.com

Graph Quadratic Functions You have learned how to find several important characteristics of quadratic functions.

Key Concept — Graph Quadratic Functions

For Your FOLDABLE

Step 1 Find the equation of the axis of symmetry.

Step 2 Find the vertex, and determine whether it is a maximum or minimum.

Step 3 Find the y-intercept.

Step 4 Use symmetry to find additional points on the graph, if necessary.

Step 5 Connect the points with a smooth curve.

EXAMPLE 5 Graph Quadratic Functions

Graph $f(x) = x^2 + 4x + 3$.

Step 1 Find the equation of the axis of symmetry.

$x = \frac{-b}{2a}$ — **Formula for the equation of the axis of symmetry**

$x = \frac{-4}{2 \cdot 1}$ — **$a = 1$ and $b = 4$**

$x = -2$ — **Simplify.**

Step 2 Find the vertex, and determine whether it is a maximum or minimum.

$f(x) = x^2 + 4x + 3$ — **Original equation**

$= (-2)^2 + 4(-2) + 3$ — **$x = -2$**

$= -1$ — **Simplify.**

The vertex lies at $(-2, -1)$. Because a is positive the graph opens up, and the vertex is a minimum.

Step 3 Find the y-intercept.

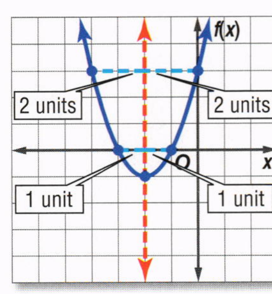
$f(x) = x^2 + 4x + 3$ — **Original equation**

$= (0)^2 + 4(0) + 3$ — **$x = 0$**

$= 3$ — **Simplify.**

The y-intercept is 3.

Step 4 The axis of symmetry divides the parabola into two equal parts. So if there is a point on one side, there is a corresponding point on the other side that is the same distance from the axis of symmetry and has the same y-value.



Step 5 Connect the points with a smooth curve.

StudyTip

Symmetry and Points When locating points that are on opposite sides of the axis of symmetry, they are not only the same distance to the left and right of the axis of symmetry. They are also the same number of spaces up or down from the vertex.

Check Your Progress

Graph each function.

5A. $f(x) = -2x^2 + 2x - 1$

5B. $f(x) = 3x^2 - 6x + 2$

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You have used what you know about quadratic functions, parabolas, and symmetry to create graphs. You can analyze these graphs to solve real-world problems.

Real-World Link

About 1 in 17 high school seniors playing football will go on to play football at an NCAA school.

Source: National Collegiate Athletic Association

Real-World EXAMPLE 6 Use a Graph of a Quadratic Function

SCHOOL SPIRIT The cheerleaders at Lake High School launch T-shirts into the crowd every time the Lakers score a touchdown. The height of the T-shirt can be modeled by the function $h(x) = -16x^2 + 48x + 6$, where $h(x)$ represents the height in feet of the T-shirt after x seconds.

a. Graph the function.

$x = -\frac{b}{2a}$ — Equation of the axis of symmetry

$x = -\frac{48}{2(-16)}$ or $\frac{3}{2}$ — $a = -16$ and $b = 48$

The equation of the axis of symmetry is $x = \frac{3}{2}$. Thus, the x-coordinate for the vertex is $\frac{3}{2}$.

$y = -16x^2 + 48x + 6$ — Original equation

$= -16\left(\frac{3}{2}\right)^2 + 48\left(\frac{3}{2}\right) + 6$ — $x = \frac{3}{2}$

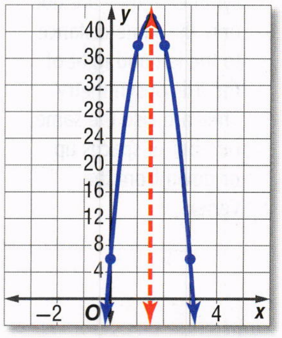
$= -16\left(\frac{9}{4}\right) + 48\left(\frac{3}{2}\right) + 6$ — $\left(\frac{3}{2}\right)^2 = \frac{9}{4}$

$= -36 + 72 + 6$ or 42 — Simplify.

The vertex is at $\left(\frac{3}{2}, 42\right)$.

Let's find another point. Choose an x-value of 0 and substitute. Our new point is at (0, 6). The point paired with it on the other side of the axis of symmetry is (3, 6).

Repeat this and choose an x-value of 1 to get (1, 38) and its corresponding point (2, 38). Connect these points and create a smooth curve.



b. At what height was the T-shirt launched?
The T-shirt is launched when time equals 0, or at the y-intercept. So, the T-shirt was launched 6 feet from the ground.

c. What is the maximum height of the T-shirt? When was the maximum height reached?
The maximum height of the T-shirt occurs at the vertex. So the T-shirt reaches a maximum height of 42 feet. The time was $\frac{3}{2}$ or 1.5 seconds after launch.

Check Your Progress

6. TRACK Emilio is competing in the javelin throw. The height of the javelin can be modeled by the equation $y = -16x^2 + 64x + 6$, where y represents the height in feet of the javelin after x seconds.

A. Graph the path of the javelin.

B. At what height is the javelin thrown?

C. What is the maximum height of the javelin?

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Check Your Understanding

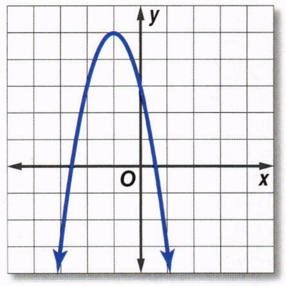
Example 1 p. 525

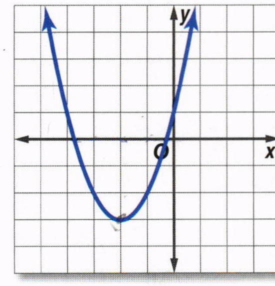
Use a table of values to graph each equation. State the domain and range.

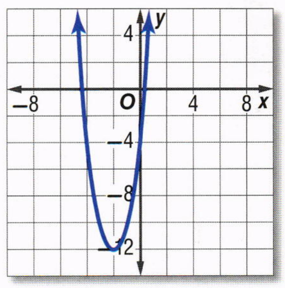
1. $y = 2x^2 + 4x - 6$
2. $y = x^2 + 2x - 1$
3. $y = x^2 - 6x - 3$
4. $y = 3x^2 - 6x - 5$

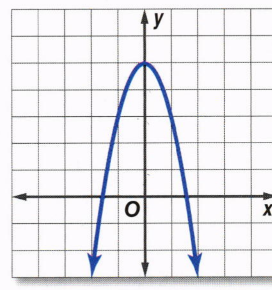
Example 2 p. 526

Find the vertex, the equation of the axis of symmetry, and the y-intercept of each graph.

5. 

6. 

7. 

8. 

Example 3 p. 527

Find the vertex, the equation of the axis of symmetry, and the y-intercept of the graph of each function.

9. $y = -3x^2 + 6x - 1$
10. $y = -x^2 + 2x + 1$
11. $y = x^2 - 4x + 5$
12. $y = 4x^2 - 8x + 9$

Example 4 p. 528

Consider each function.

a. **Determine whether the function has *maximum* or *minimum* value.**

b. **State the maximum or minimum value.**

c. **What are the domain and range of the function?**

13. $y = -x^2 + 4x - 3$
14. $y = -x^2 - 2x + 2$
15. $y = -3x^2 + 6x + 3$
16. $y = -2x^2 + 8x - 6$

Example 5 p. 529

Graph each function.

17. $f(x) = -3x^2 + 6x + 3$
18. $f(x) = -2x^2 + 4x + 1$
19. $f(x) = 2x^2 - 8x - 4$
20. $f(x) = 3x^2 - 6x - 1$

Example 6 p. 530

21. **JUGGLING** A juggler is tossing a ball into the air. The height of the ball in feet can be modeled by the equation $y = -16x^2 + 16x + 5$, where y represents the height of the ball at x seconds.

    a. Graph this equation.

    b. At what height is the ball thrown?

    c. What is the maximum height of the ball?

Practice and Problem Solving

● = **Step-by-Step Solutions** begin on page R12.
Extra Practice begins on page 815.

Example 1 p. 525

Use a table of values to graph each equation. State the domain and range.

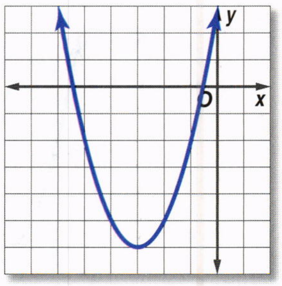
22. $y = x^2 + 4x + 6$ **23.** $y = 2x^2 + 4x + 7$ **24.** $y = 2x^2 - 8x - 5$

25. $y = 3x^2 + 12x + 5$ **26.** $y = 3x^2 - 6x - 2$ **27.** $y = x^2 - 2x - 1$

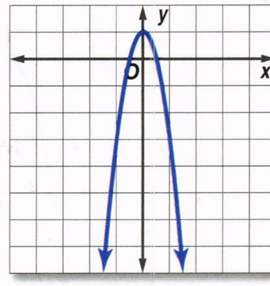
Example 2 p. 526

Find the vertex, the equation of the axis of symmetry, and the y-intercept of each graph.

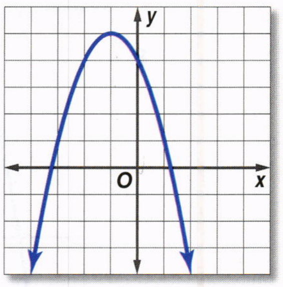
28.



29.



30.



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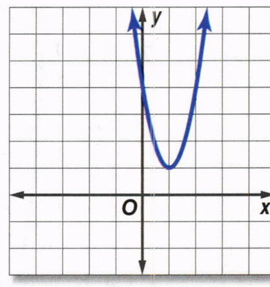

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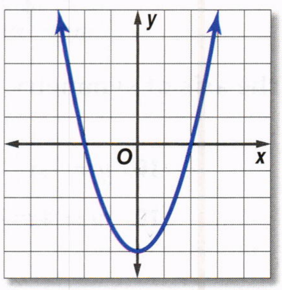

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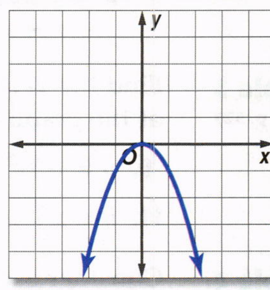


Example 3 p. 527

Find the vertex, the equation of the axis of symmetry, and the y-intercept of each function.

34. $y = x^2 + 8x + 10$ **35** $y = 2x^2 + 12x + 10$ **36.** $y = -3x^2 - 6x + 7$

37. $y = -x^2 - 6x - 5$ **38.** $y = 5x^2 + 20x + 10$ **39.** $y = 7x^2 - 28x + 14$

40. $y = 2x^2 - 12x + 6$ **41.** $y = -3x^2 + 6x - 18$ **42.** $y = -x^2 + 10x - 13$

Example 4 p. 528

Consider each function.

a. **Determine whether the function has a *maximum* or *minimum* value.**

b. **State the maximum or minimum value.**

c. **What are the domain and range of the function?**

43. $y = -2x^2 - 8x + 1$ **44.** $y = x^2 + 4x - 5$ **45.** $y = 3x^2 + 18x - 21$

46. $y = -2x^2 - 16x + 18$ **47.** $y = -x^2 - 14x - 16$ **48.** $y = 4x^2 + 40x + 44$

49. $y = -x^2 - 6x - 5$ **50.** $y = 2x^2 + 4x + 6$ **51.** $y = -3x^2 - 12x - 9$

Example 5 p. 529

Graph each function.

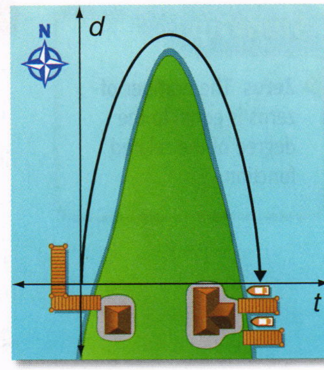
52. $y = -3x^2 + 6x - 4$ **53.** $y = -2x^2 - 4x - 3$ **54.** $y = -2x^2 - 8x + 2$

55. $y = x^2 + 6x - 6$ **56.** $y = x^2 - 2x + 2$ **57.** $y = 3x^2 - 12x + 5$

Example 6
p. 530

58. **BOATING** Miranda has her boat docked on the west side of Casper Point. She is boating over to the Casper Marina. The distance traveled by Miranda over time can be modeled by the equation $d = -16t^2 + 66t$, where d is the number of feet she travels in t minutes.

   a. Graph this equation.
   b. What is the maximum number of feet north that she traveled?
   c. How long did it take her to reach Casper Marina?



GRAPHING CALCULATOR **Graph each equation. Use the TRACE feature to find the vertex on the graph. Round to the nearest thousandth if necessary.**

59. $y = 4x^2 + 10x + 6$

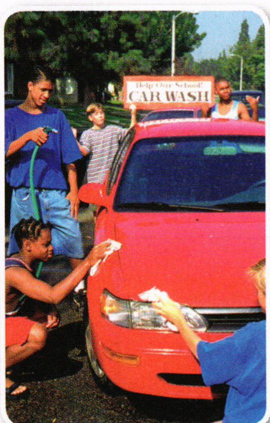
60. $y = 8x^2 - 8x + 8$

61. $y = -5x^2 - 3x - 8$

62. $y = -7x^2 + 12x - 10$

63. **GOLF** The average amateur golfer can hit a ball with an initial velocity of 31.3 meters per second. If the ball is hit straight up, the height can be modeled by the equation $h = -4.9t^2 + 31.3t$, where h is the height of the ball, in meters, after t seconds.

   a. Graph this equation.
   b. At what height is the ball hit?
   c. What is the maximum height of the ball?
   d. How long did it take for the ball to hit the ground?
   e. State a reasonable range and domain for this situation.



Real-World Link

School fundraisers provide revenue for extracurricular activities that are not part of a school budget.

64. **FUNDRAISING** The marching band is selling poinsettias to buy new uniforms. Last year the band charged \$5 each, and they sold 150. They want to increase the price this year, and they expect to lose 10 sales for each \$1 increase. The sales revenue R, in dollars, generated by selling the poinsettias is predicted by the function $R = (5 + p)(150 - 10p)$, where p is the number of \$1 price increases.

   a. Write the function in standard form.
   b. Find the maximum value of the function.
   c. At what price should the poinsettias be sold to generate the most sales revenue? Explain your reasoning.

65. **FOOTBALL** A football is kicked up from ground level at an initial upward velocity of 90 feet per second. The equation $h = -16t^2 + 90t$ gives the height h of the football after t seconds.

   a. What is the height of the ball after one second?
   b. When is the ball 126 feet high?
   c. When is the height of the ball 0 feet? What do these points represent in the context of the situation?

66. **REASONING** Let $f(x) = x^2 - 9$.

   a. What is the domain of $f(x)$?
   b. What is the range of $f(x)$?
   c. For what values of x is $f(x)$ negative?
   d. When x is a real number, what are the domain and range of $f(x) = \sqrt{x^2 - 9}$?

StudyTip

Zeros The number of zeros is equal to the degree of the related function.

67. **MULTIPLE REPRESENTATIONS** In this problem, you will investigate solving quadratic equations using tables.

a. ALGEBRAIC Determine the related function for each equation. Copy and complete the table below.

Equation	Related Function	Zeros	y-Values
$x^2 - x = 12$	?	?	?
$x^2 + 8x = 9$	?	?	?
$x^2 = 14x - 24$	?	?	?
$x^2 + 16x = -28$	?	?	?

b. GRAPHICAL Graph each related function with a graphing calculator.

c. ANALYTICAL Use the table feature on your calculator to determine the zeros of each related function. Record the zeros in the table above. Also record the values of the function one unit less than and one unit more than each zero.

d. VERBAL Compare the signs of the function values for x-values just before and just after a zero. What happens to the sign of the function value before and after a zero?

H.O.T. Problems

Use Higher-Order Thinking Skills

68. **OPEN ENDED** Write a quadratic function for which the graph has an axis of symmetry of $x = -\frac{3}{8}$. Summarize your steps.

69. **FIND THE ERROR** Chase and Jade are finding the axis of symmetry of a parabola. Is either of them correct? Explain your reasoning.

Chase

$y = -x^2 - 4x + 6$

$x = -\frac{b}{2a}$

$x = -\frac{4}{2(-1)}$

$x = 2$

Jade

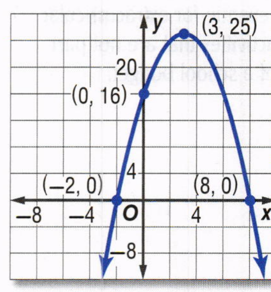
$y = -x^2 - 4x + 6$

$x = -\frac{b}{2a}$

$x = -\frac{-4}{2(-1)}$

$x = -2$

70. **CHALLENGE** Using the axis of symmetry and one x-intercept, write an equation for the graph shown.



71. **REASONING** The graph of a quadratic function has a vertex at (2, 0). One point on the graph is (5, 9). Find another point on the graph. Explain how you found it.

72. **OPEN ENDED** Describe a real-world situation that involves a quadratic equation. Explain what the vertex represents.

73. **REASONING** Provide a counterexample to the following statement. *The vertex of a parabola is always the minimum of the graph.*

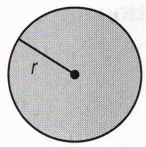
74. **WRITING IN MATH** Explain how to find the axis of symmetry from an equation for a quadratic equation. What other characteristics of the graph can you derive from the equation? Explain.

Standardized Test Practice

75. Which of the following is an equation for the line that passes through (2, −5) and is perpendicular to $2x + 4y = 8$?

A $y = 2x + 10$
B $y = -\frac{1}{2}x - 4$
C $y = 2x - 9$
D $y = -2x - 1$

76. GEOMETRY The area of the circle is 36π square units. If the radius is doubled, what is the area of the new circle?



$A = 36\pi$

F 72π units^2
G 144π units^2
H 1296π units^2
J 9π units^2

77. What is the range of the function $f(x) = -4x^2 - \frac{1}{2}$?

A $\left\{\text{all integers less than or equal to } \frac{1}{2}\right\}$
B {all nonnegative integers}
C {all real numbers}
D $\left\{\text{all real numbers less than or equal to } -\frac{1}{2}\right\}$

78. SHORT RESPONSE Dylan delivers newspapers for extra money. He starts delivering the newspapers at 3:15 P.M. and finishes at 5:05 P.M. How long does it take Dylan to complete his route?

Spiral Review

Determine whether each trinomial is a perfect square trinomial. Write *yes* or *no*. If so, factor it. (Lesson 8-6)

79. $4x^2 + 4x + 1$
80. $4x^2 - 20x + 25$
81. $9x^2 + 8x + 16$

Factor each polynomial if possible. If the polynomial cannot be factored, write *prime*. (Lesson 8-5)

82. $n^2 - 16$
83. $x^2 + 25$
84. $9 - 4a^2$

Find each product. (Lesson 7-7)

85. $(b - 7)(b + 3)$
86. $(c - 6)(c - 5)$
87. $(2x - 1)(x + 9)$

88. MULTIPLE BIRTHS The number of quadruplet births Q in the United States in recent years can be modeled by $Q = -0.5t^3 + 11.7t^2 - 21.5t + 218.6$, where t represents the number of years since 1992. For what values of t does this model no longer allow for realistic predictions? Explain your reasoning. (Lesson 7-4)

Use elimination to solve each system of equations. (Lesson 6-4)

89. $2x + y = 5$
$3x - 2y = 4$

90. $4x - 3y = 12$
$x + 2y = 14$

91. $2x - 3y = 2$
$5x + 4y = 28$

92. HEALTH About 20% of the time you sleep is spent in rapid eye movement (REM), which is associated with dreaming. If an adult sleeps 7 to 8 hours, how much time is spent in REM sleep? (Lesson 5-4)

Skills Review

Find the x-intercept of the graph of each equation. (Lesson 3-1)

93. $x + 2y = 10$
94. $2x - 3y = 12$
95. $3x - y = -18$

EXTEND 9-1

Algebra Lab

Rate of Change of a Quadratic Function

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Math in Motion, Animation

Objective
Investigate the rate of change for a quadratic function.

A model rocket is launched from the ground with an upward velocity of 144 feet per second. The function $y = -16x^2 + 144x$ models the height y of the rocket in feet after x seconds. Using this function, we can investigate the rate of change of a quadratic function.

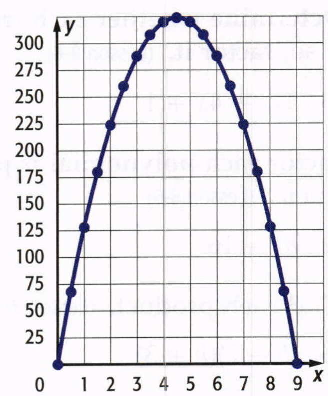
ACTIVITY

Step 1 Copy the table below.

x	0	0.5	1.0	1.5	...	9.0
y	0					
Rate of Change	–					

Step 2 Find the value of y for each value of x from 0 through 9.

Step 3 Graph the ordered pairs (x, y) on grid paper. Connect the points with a smooth curve. Notice that the function *increases* when $0 < x < 4.5$ and *decreases* when $4.5 < x < 9$.



Step 4 Recall that the *rate of change* is the change in y divided by the change in x. Find the rate of change for each half second interval of x and y.

Exercises

Use the quadratic function $y = x^2$.

1. Make a table, similar to the one in the Activity, for the function using $x = -4, -3, -2, -1, 0, 1, 2, 3,$ and 4. Find the values of y for each x-value.

2. Graph the ordered pairs on grid paper. Connect the points with a smooth curve. Describe where the function is increasing and where it is decreasing.

3. Find the rate of change for each column starting with $x = -3$. Compare the rates of change when the function is increasing and when it is decreasing.

4. **CHALLENGE** If an object is dropped from 100 feet in the air and air resistance is ignored, the object will fall at a rate that can be modeled by the equation $f(x) = -16x^2 + 100$, where $f(x)$ represents the object's height in feet after x seconds. Make a table like that in Exercise 1, selecting appropriate values for x. Fill in the x-values, the y-values, and rates of change. Compare the rates of change. Describe any patterns that you see.

9-2 Solving Quadratic Equations by Graphing

Then

You solved quadratic equations by factoring. (Lesson 8-3)

Now

- Solve quadratic equations by graphing.
- Estimate solutions of quadratic equations by graphing.

New Vocabulary

double root

Math Online

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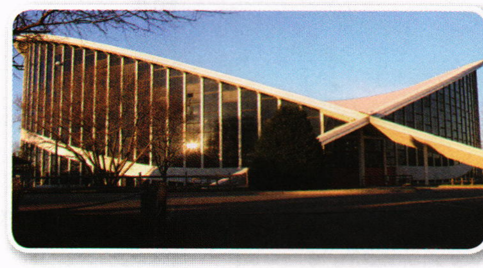
- Extra Examples
- Personal Tutor
- Self-Check Quiz
- Homework Help
- Math in Motion

Why?

Dorton Arena at the state fairgrounds in Raleigh, North Carolina, has a shape created by two intersecting parabolas. The shape of one of the parabolas can be modeled by the equation $y = -x^2 + 127x$, where x represents the width of the parabola in feet, and y represents the length of the parabola in feet.

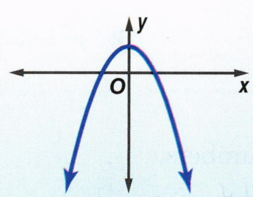


The x-intercepts of the graph of this function can be used to determine the distance between the points where the parabola meets the ground.

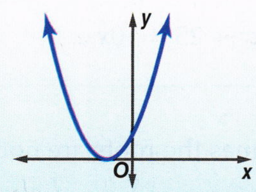
Solve by Graphing A quadratic equation can be written in the standard form $ax^2 + bc + c = 0$, where $a \neq 0$. To write a quadratic function as an equation, replace y or $f(x)$ with 0. Recall that the solutions or roots of an equation can be identified by finding the x-intercepts of the related graph. Quadratic equations may have two, one, or no solutions.

Key Concept — Solutions of Quadratic Equations

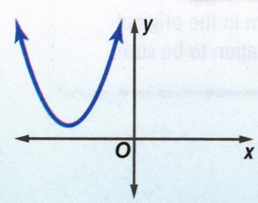
For Your FOLDABLE



two unique real solutions



one unique real solution



no real solutions

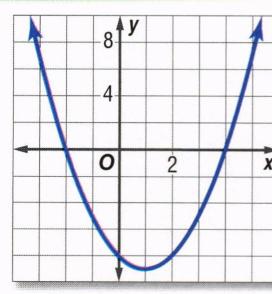
Math *in Motion*, Animation glencoe.com

EXAMPLE 1 Two Roots

Solve $x^2 - 2x - 8 = 0$ by graphing.

Graph the related function $f(x) = x^2 - 2x - 8$.

The x-intercepts of the graph appear to be at -2 and 4, so the solutions are -2 and 4.



CHECK Check each solution in the original equation.

$x^2 - 2x - 8 = 0$	**Original equation**	$x^2 - 2x - 8 = 0$
$(-2)^2 - 2(-2) - 8 \stackrel{?}{=} 0$	**$x = -2$ or $x = 4$**	$(4)^2 - 2(4) - 8 \stackrel{?}{=} 0$
$0 = 0$ ✓	**Simplify.**	$0 = 0$ ✓

Check Your Progress **Solve each equation by graphing.**

1A. $-x^2 - 3x + 18 = 0$

1B. $x^2 - 4x + 3 = 0$

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The solutions in Example 1 were two distinct numbers. Sometimes the two roots are the same number, called a **double root**.

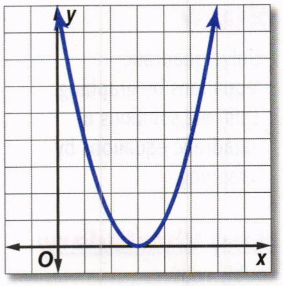
EXAMPLE 2 Double Root

Solve $x^2 - 6x = -9$ by graphing.

Step 1 Rewrite the equation in standard form.

$$x^2 - 6x = -9 \quad \text{Original equation}$$
$$x^2 - 6x + 9 = 0 \quad \text{Add 9 to each side.}$$

Step 2 Graph the related function $f(x) = x^2 - 6x + 9$.



Step 3 Locate the x-intercepts of the graph. Notice that the vertex of the parabola is the only x-intercept. Therefore, there is only one solution, 3.

CHECK Solve by factoring.

$$x^2 - 6x + 9 = 0 \quad \text{Original equation}$$
$$(x - 3)(x - 3) = 0 \quad \text{Factor.}$$
$$x - 3 = 0 \quad \text{or} \quad x - 3 = 0 \quad \text{Zero Product Property}$$
$$x = 3 \qquad\qquad x = 3 \quad \text{Add 3 to each side.}$$

The only solution is 3.

Watch Out!

Exact Solutions Solutions found from the graph of an equation may appear to be exact. Check them in the original equation to be sure.

Check Your Progress

Solve each equation by graphing.

2A. $x^2 + 25 = 10x$

2B. $x^2 = -8x - 16$

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Sometimes the roots are not real numbers.

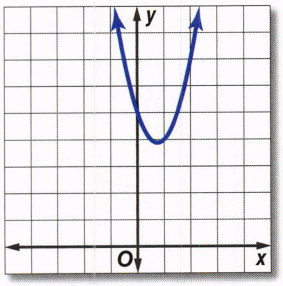
EXAMPLE 3 No Real Roots

Solve $2x^2 - 3x + 5 = 0$ by graphing.

Step 1 Rewrite the equation in standard form.

This equation is written in standard form.

Step 2 Graph the related function $f(x) = 2x^2 - 3x + 5$.



Step 3 Locate the x-intercepts of the graph. This graph has no x-intercepts. Therefore, this equation has no real number solutions. The solution set is $\varnothing$.

CHECK Solve by factoring.

There are no factors of 10 that have a sum of -3, so the expression is not factorable. Thus, the equation has no real number solutions.

Check Your Progress

Solve each equation by graphing.

3A. $-x^2 - 3x = 5$

3B. $-2x^2 - 8 = 6x$

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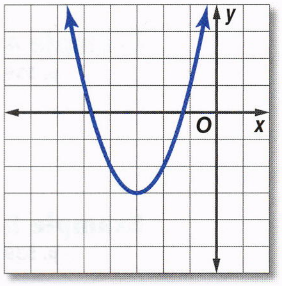
> **StudyTip**
>
> **Location of Zeros** Since quadratic functions are continuous, there must be a zero between two x-values for which the corresponding y-values have opposite signs.

Estimate Solutions The real roots found thus far have been integers. However, the roots of quadratic equations are usually not integers. In these cases, use estimation to approximate the roots of the equation.

EXAMPLE 4 Approximate Roots with a Table

Solve $x^2 + 6x + 6 = 0$ by graphing. If integral roots cannot be found, estimate the roots to the nearest tenth.



Graph the related function $f(x) = x^2 + 6x + 6$.

The x-intercepts are located between -5 and -4 and between -2 and -1.

Make a table using an increment of 0.1 for the x-values located between -5 and -4 and between -2 and -1.

Look for a change in the signs of the function values. The function value that is closest to zero is the best approximation for a zero of the function.

x	−4.9	−4.8	−4.7	−4.6	−4.5	−4.4	−4.3	−4.2	−4.1
y	0.61	0.24	−0.11	−0.44	−0.75	−1.04	−1.31	−1.56	−1.79

x	−1.9	−1.8	−1.7	−1.6	−1.5	−1.4	−1.3	−1.2	−1.1
y	−1.79	−1.56	−1.31	−1.04	−0.75	−0.44	−0.11	0.24	0.61

For each table, the function value that is closest to zero when the sign changes is -0.11. Thus, the roots are approximately -4.7 and -1.3.

Check Your Progress

4. Solve $2x^2 + 6x - 3 = 0$ by graphing. If integral roots cannot be found, estimate the roots to the nearest tenth.

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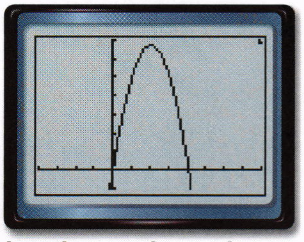
Approximating the x-intercepts of graphs is helpful for real-world applications.

> **Real-World Link**
>
> The game of soccer, called "football" outside of North America, began in 1863 in Britain when the Football Association was founded. Soccer is played on every continent of the world.
>
> **Source:** Sports Know How

Real-World EXAMPLE 5 Approximate Roots with a Calculator

SOCCER A goalie kicks a soccer ball with an upward velocity of 65 feet per second, and her foot meets the ball 1 foot off the ground. The quadratic function $h = -16t^2 + 65t + 1$ represents the height of the ball h in feet after t seconds. Approximately how long is the ball in the air?



[−4, 7] scl: 1 by [−10, 70] scl: 10

You need to find the roots of the equation $-16t^2 + 65t + 1 = 0$. Use a graphing calculator to graph the related function $f(x) = -16t^2 + 65t + 1$.

The positive x-intercept of the graph is approximately 4. Therefore, the ball is in the air for approximately 4 seconds.

Check Your Progress

5. If the goalie kicks the soccer ball with an upward velocity of 55 feet per second and his foot meets the ball 2 feet off the ground, approximately how long is the ball in the air?

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Check Your Understanding

Examples 1–3 pp. 537–538

Solve each equation by graphing.

1. $x^2 + 3x - 10 = 0$
2. $2x^2 - 8x = 0$
3. $x^2 + 4x = -4$
4. $x^2 + 12 = -8x$

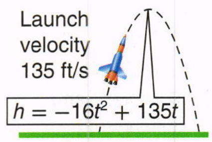
Example 4 p. 539

Solve each equation by graphing. If integral roots cannot be found, estimate the roots to the nearest tenth.

5. $-x^2 - 5x + 1 = 0$
6. $-9 = x^2$
7. $x^2 = 25$
8. $x^2 - 8x = -9$

Example 5 p. 539

9. **SCIENCE FAIR** Ricky built a model rocket. Its flight can be modeled by the equation shown, where h is the height of the rocket in feet after t seconds. About how long was Ricky's rocket in the air?



= **Step-by-Step Solutions** begin on page R12.
Extra Practice begins on page 815.

Practice and Problem Solving

Examples 1–3 pp. 537–538

Solve each equation by graphing.

10. $x^2 + 7x + 14 = 0$
11. $x^2 + 2x - 24 = 0$
12. $x^2 - 16x + 64 = 0$
13. $x^2 - 5x + 12 = 0$
14. $x^2 + 14x = -49$
15. $x^2 = 2x - 1$
16. $x^2 - 10x = -16$
17. $-2x^2 - 8x = 13$
18. $2x^2 - 16x = -30$
19. $2x^2 = -24x - 72$
20. $-3x^2 + 2x = 15$
21. $x^2 = -2x + 80$

Example 4 p. 539

Solve each equation by graphing. If integral roots cannot be found, estimate the roots to the nearest tenth.

22. $x^2 + 2x - 9 = 0$
23. $x^2 - 4x = 20$
24. $x^2 + 3x = 18$
25. $2x^2 - 9x = -8$
26. $3x^2 = -2x + 7$
27. $5x = 25 - x^2$

Example 5 p. 539

28. **SOFTBALL** Sofia hits a softball straight up. The equation $h = -16t^2 + 90t$ models the height h, in feet, of the ball after t seconds. How long is the ball in the air?

29. **RIDES** A skyrocket roller coaster takes riders straight up and then returns straight down. The equation $h = -16t^2 + 185t$ models the height h, in feet, of the coaster after t seconds. How long is it until the coaster returns to the bottom?

Use factoring to determine how many times the graph of each function intersects the x-axis. Identify each zero.

30. $y = x^2 - 8x + 16$
31. $y = x^2 + 3x + 4$
32. $y = x^2 + 2x - 24$
33. $y = x^2 + 12x + 32$

34. **NUMBER THEORY** Use a quadratic equation to find two numbers that have a sum of 9 and a product of 20.

35. **NUMBER THEORY** Use a quadratic equation to find two numbers that have a sum of 1 and a product of −12.

36. **GOLF** The height of a golf ball in the air can be modeled by the equation $h = -16t^2 + 60t + 3$, where h is the height in feet of the ball after t seconds.

    a. How long was the ball in the air?

    b. What is the ball's maximum height?

    c. When will the ball reach its maximum height?

Real-World Link

In the 1998 Winter Games in Japan, snowboarding became an Olympic event for the first time. A total of four competitions were held that were divided into two categories: men's and women's halfpipe and men's and women's giant slalom.

Source: About, Inc.

37. **SNOWBOARDING** Stefanie is in a snowboarding competition. The equation $h = -16t^2 + 30t + 10$ models Stefanie's height h, in feet, in the air after t seconds.

a. How long is Stefanie in the air?

b. When will Stefanie reach a height of 15 feet?

c. To earn bonus points in the competition, you must reach a height of 20 feet. Will Stefanie earn bonus points?

38. **MULTIPLE REPRESENTATIONS** In this problem, you will explore how to further interpret the relationship between quadratic functions and graphs.

a. **GRAPHICAL** Graph $y = x^2$.

b. **ANALYTICAL** Name the vertex and two other points on the graph.

c. **GRAPHICAL** Graph $y = x^2 + 2$, $y = x^2 + 4$, and $y = x^2 + 6$ on the same coordinate plane as the previous graph.

d. **ANALYTICAL** Name the vertex and two points from each of these graphs that have the same x-coordinates as the first graph.

e. **ANALYTICAL** What conclusion can you draw from this?

GRAPHING CALCULATOR **Approximate the zeros of each cubic function by graphing. If integral zeros cannot be found, estimate the zeros to the nearest tenth.**

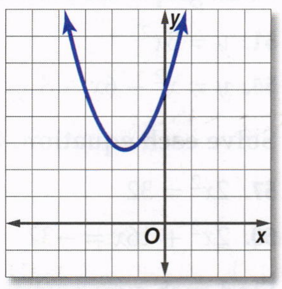
39. $f(x) = x^3 - 3x^2 - 6x + 8$

40. $g(x) = x^3 - 4x^2 + 5x - 12$

H.O.T. Problems

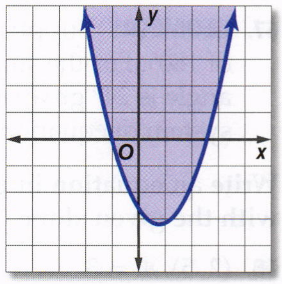
Use Higher-Order Thinking Skills

41. **FIND THE ERROR** Iku and Zachary are finding the number of real zeros of the function graphed at the right. Iku says that the function has no real zeros because there are no x-intercepts. Zachary says that the function has one real zero because the graph has a y-intercept. Is either of them correct? Explain your reasoning.



42. **OPEN ENDED** Describe a real-world situation in which a thrown object travels in the air. Write an equation that models the height of the object with respect to time, and determine how long the object travels in the air.

43. **REASONING** The graph shown is that of a *quadratic inequality*. Analyze the graph, and determine whether the y-value of a solution of the inequality is *sometimes*, *always*, or *never* greater than 2. Explain.



44. **CHALLENGE** Write a quadratic equation that has the roots described.

a. one double root

b. one rational (nonintegral) root and one integral root

c. two distinct integral roots that are additive opposites.

45. **CHALLENGE** Find the roots of $x^2 = 2.25$ without using a calculator. Explain your strategy.

46. **WRITING IN MATH** Explain how to approximate the roots of a quadratic equation when the roots are not integers.

Standardized Test Practice

47. Adrahan earned 50 out of 80 points on a test. What percentage did Adrahan score on the test?

A 62.5%
B 6.25%
C 1.6%
D 16%

48. Ernesto needs to loosen a bolt. He needs a wrench that is smaller than a $\frac{7}{8}$-inch wrench, but larger than a $\frac{3}{4}$-inch wrench. Which of the following sizes should Ernesto use?

F $\frac{11}{16}$ inch
G $\frac{5}{8}$ inch
H $\frac{13}{16}$ inch
J $\frac{3}{8}$ inch

49. EXTENDED RESPONSE Two boats leave a dock. One boat travels 4 miles east and then 5 miles north. The second boat travels 12 miles south and 9 miles west. Draw a diagram that represents the paths traveled by the boats. How far apart are the boats in miles?

50. The formula $s = \frac{1}{2}at^2$ represents the distance s in meters that a free-falling object will fall near a planet or the Moon in a given time t in seconds. Solve the formula for a, the acceleration due to gravity.

A $a = \frac{1}{2}t^2 - s$
B $a = 2s - t^2$
C $a = s - \frac{1}{2}t^2$
D $a = \frac{2s}{t^2}$

Spiral Review

Write the equation of the axis of symmetry, and find the coordinates of the vertex of the graph of each function. Identify the vertex as a maximum or minimum. Then graph the function. (Lesson 9-1)

51. $y = 3x^2$
52. $y = -4x^2 - 5$
53. $y = -x^2 + 4x - 7$
54. $y = x^2 - 6x - 8$
55. $y = 3x^2 + 2x + 1$
56. $y = -4x^2 - 8x + 5$

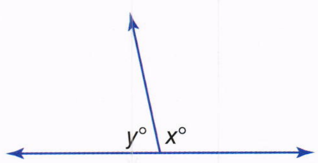
Solve each equation. Check the solutions. (Lesson 8-6)

57. $2x^2 = 32$
58. $(x - 4)^2 = 25$
59. $4x^2 - 4x + 1 = 16$
60. $2x^2 + 16x = -32$
61. $(x + 3)^2 = 5$
62. $4x^2 - 12x = -9$

Find each sum or difference. (Lesson 7-5)

63. $(3n^2 - 3) + (4 + 4n^2)$
64. $(2d^2 - 7d - 3) - (4d^2 + 7)$
65. $(2b^3 - 4b^2 + 4) - (3b^4 + 5b^2 - 9)$
66. $(8 - 4h^2 + 6h^4) + (5h^2 - 3 + 2h^3)$

67. GEOMETRY Supplementary angles are two angles with measures that have a sum of 180°. For the supplementary angles in the figure, the measure of the larger angle is 24° greater than the measure of the smaller angle. Write and solve a system of equations to find these measures. (Lesson 6-5)



Write an equation in point-slope form for the line that passes through each point with the given slope. (Lesson 4-3)

68. $(2, 5)$, $m = 3$
69. $(-3, 6)$, $m = -7$
70. $(-1, -2)$, $m = -\frac{1}{2}$

Skills Review

Graph each function. (Lesson 9-1)

71. $y = x^2 + 5$
72. $y = x^2 - 8$
73. $y = 2x^2 - 7$
74. $y = -x^2 + 2$
75. $y = -0.5x^2 - 3$
76. $y = (-x)^2 + 1$

EXTEND
9-2 Graphing Technology Lab
Quadratic Inequalities

Math Online glencoe.com
• Other Calculator Keystrokes
• Graphing Technology Personal Tutor

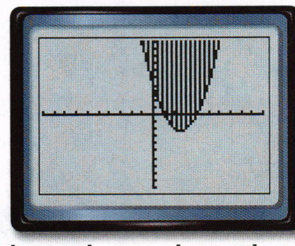
Recall that the graph of a linear inequality consists of the boundary and the shaded half plane. The solution set of the inequality lies in the shaded region of the graph. Graphing quadratic inequalities is similar to graphing linear inequalities.

ACTIVITY 1 Shade Inside a Parabola

Graph $y \geq x^2 - 5x + 4$ in the standard viewing window.

First, clear all functions from the Y= list.

To graph $y \geq x^2 - 5x + 4$, enter the equation in the Y= list. Then use the left arrow to select =. Press ENTER until shading above the line is selected.



[−10, 10] scl: 1 by [−10, 10] scl: 1

KEYSTROKES: ◀ ◀ ENTER ENTER ▶ ▶ X,T,θ,n x^2 − 5 X,T,θ,n + 4 GRAPH

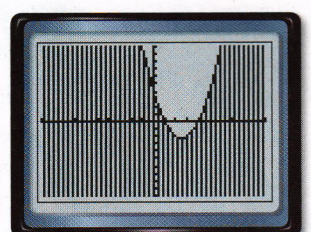
All ordered pairs for which y is *greater than or equal* to $x^2 - 5x + 4$ lie *above or on* the line and are solutions.

A similar procedure will be used to graph an inequality in which the shading is outside of the parabola.

ACTIVITY 2 Shade Outside a Parabola

Graph $y - 4 \leq x^2 - 5x$ in the standard viewing window.

First, clear the graph that is displayed.



[−10, 10] scl: 1 by [−10, 10] scl: 1

KEYSTROKES: Y= CLEAR

Then rewrite $y - 4 \leq x^2 - 5x$ as $y \leq x^2 - 5x + 4$, and graph it.

KEYSTROKES: ◀ ◀ ENTER ENTER ENTER ▶ ▶ X,T,θ,n x^2 − 5 X,T,θ,n + 4 GRAPH

All ordered pairs for which y is *less than or equal* to $x^2 - 5x + 4$ lie *below or on* the line and are solutions.

Exercises

1. Compare and contrast the two graphs shown above.
2. Graph $y - 2x + 6 \geq 5x^2$ in the standard viewing window. Name three solutions of the inequality.
3. Graph $y - 6x \leq -x^2 - 3$ in the standard viewing window. Name three solutions of the inequality.

9-3 Transformations of Quadratic Functions

Then

You graphed quadratic functions by using the vertex and axis of symmetry. (Lesson 9-1)

Now

- Apply translations to quadratic functions.
- Apply dilations and reflections to quadratic functions.

New Vocabulary

transformation
translation
dilation
reflection

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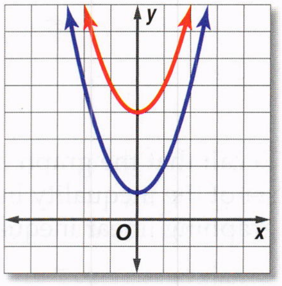
- Extra Examples
- Personal Tutor
- Self-Check Quiz
- Homework Help
- Math in Motion

Why?

The graphs of the parabolas shown at the right are the same size and shape, but notice that the vertex of the red parabola is higher on the y-axis than the vertex of the blue parabola. Shifting a parabola up and down is an example of a transformation.



Translations A **transformation** changes the position or size of a figure. One type of transformation, a **translation**, moves a figure up, down, left, or right. When a constant c is added to or subtracted from the parent function, the graph of the resulting function $f(x) \pm c$ is the graph of the parent function translated up or down.

The parent function of the family of quadratics is $f(x) = x^2$. All other quadratic functions have graphs that are transformations of the graph of $f(x) = x^2$.

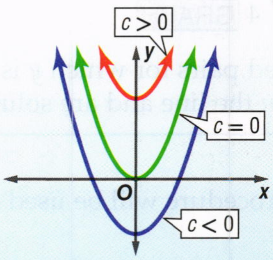
Key Concept — Vertical Translations

For Your FOLDABLE

The graph of $g(x) = x^2 + c$ is the graph of $f(x) = x^2$ translated vertically.

If $c > 0$, the graph of $f(x) = x^2$ is translated $|c|$ units **up**.

If $c < 0$, the graph of $f(x) = x^2$ is translated $|c|$ units **down**.



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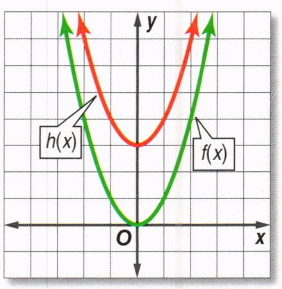
EXAMPLE 1 Describe and Graph Translations

Describe how the graph of each function is related to the graph of $f(x) = x^2$.

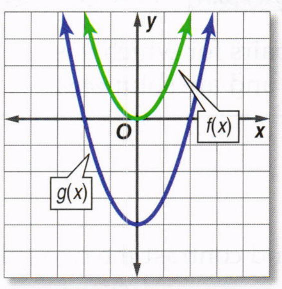
a. $h(x) = x^2 + 3$

The value of c is 3, and $3 > 0$. Therefore, the graph of $y = x^2 + 3$ is a translation of the graph of $y = x^2$ up 3 units.



b. $g(x) = x^2 - 4$

The value of c is -4, and $-4 < 0$. Therefore, the graph of $y = x^2 - 4$ is a translation of the graph of $y = x^2$ down 4 units.



Check Your Progress

1A. $f(x) = x^2 - 7$ **1B.** $g(x) = 5 + x^2$ **1C.** $h(x) = -5 + x^2$ **1D.** $f(x) = x^2 + 1$

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Dilations and Reflections Another type of transformation is a dilation. A **dilation** makes the graph narrower than the parent graph or wider than the parent graph. When the parent function $f(x) = x^2$ is multiplied by a constant a, the graph of the resulting function $f(x) = ax^2$ is either stretched or compressed vertically.

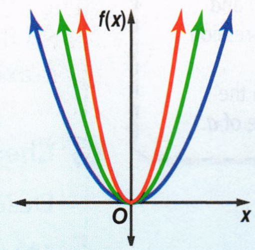
Key Concept — Dilations

For Your FOLDABLE

The graph of $g(x) = ax^2$ is the graph of $f(x) = x^2$ stretched or compressed vertically.

If $|a| > 1$, the graph of $f(x) = x^2$ is stretched vertically.

If $0 < |a| < 1$, the graph of $f(x) = x^2$ is compressed vertically.



StudyTip

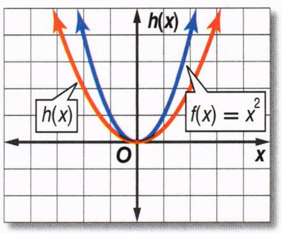
Compress or Stretch When the graph of a quadratic function is stretched vertically, the shape of the graph is narrower than that of the parent function. When it is compressed vertically, the graph is wider than the parent function.

EXAMPLE 2 Describe and Graph Dilations

Describe how the graph of each function is related to the graph of $f(x) = x^2$.

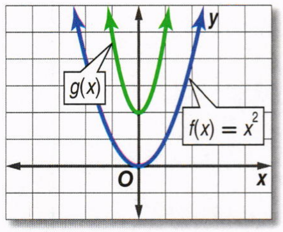
a. $h(x) = \frac{1}{2}x^2$

The function can be written $h(x) = ax^2$, where $a = \frac{1}{2}$. Since $0 < \frac{1}{2} < 1$, the graph of $y = \frac{1}{2}x^2$ is a dilation of the graph of $y = x^2$ that is compressed vertically.



b. $g(x) = 3x^2 + 2$

The function $g(x) = ax^2 + c$, where $a = 3$ and $c = 2$. Since $2 > 0$ and $3 > 1$, the graph of $y = 3x^2 + 2$ translates the graph $y = x^2$ up 2 units and stretches it vertically.



Check Your Progress

2A. $j(x) = 2x^2$

2B. $h(x) = 5x^2 - 2$

2C. $g(x) = \frac{1}{3}x^2 + 2$

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A **reflection** flips a figure across a line. When $f(x) = x^2$ or the variable x is multiplied by -1, the graph is reflected across the x- or y-axis.

StudyTip

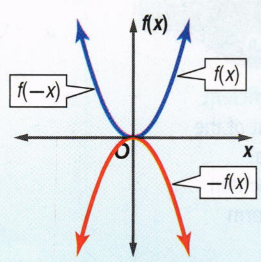
Reflection A reflection of $f(x) = x^2$ across the y-axis results in the same function, because $f(-x) = (-x)^2 = x^2$.

Key Concept — Reflections

For Your FOLDABLE

The graph of $-f(x)$ is the reflection of the graph of $f(x) = x^2$ across the x-axis.

The graph of $f(-x)$ is the reflection of the graph of $f(x) = x^2$ across the y-axis.



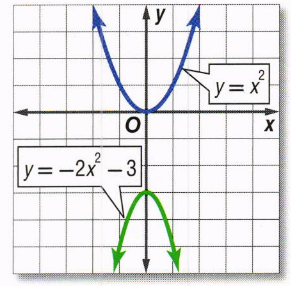
Watch Out!

Transformations The graph of $f(x) = -ax^2$ can result in two transformations of the graph of $f(x) = x^2$: a reflection across the x-axis if $a > 0$ and either a compression or expansion depending on the absolute value of a.

EXAMPLE 3 Describe and Graph Reflections

Describe how the graph of $g(x) = -2x^2 - 3$ is related to the graph of $f(x) = x^2$.

Three separate transformations are occurring. The negative sign of the coefficient of x^2 causes a reflection across the x-axis. Then a dilation occurs and finally a translation down 3 units.



So the graph of $y = -2x^2 - 3$ is reflected across the x-axis, compressed, and translated down 3 units.

Check Your Progress

Describe how the graph of each function is related to the graph of $f(x) = x^2$.

3A. $h(x) = 2(-x)^2 - 9$

3B. $g(x) = -\frac{1}{5}x^2 + 3$

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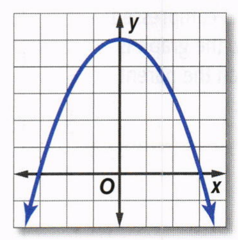
You can use what you know about the characteristics of graphs of quadratic equations to match an equation with a graph.

STANDARDIZED TEST EXAMPLE 4

Which is an equation for the function shown in the graph?



A $y = \frac{1}{2}x^2 - 5$

B $y = -2x^2 - 5$

C $y = -\frac{1}{2}x^2 + 5$

D $y = 2x^2 + 5$

Read the Test Item

You are given the graph of a parabola. You need to find an equation of the graph.

Solve the Test Item

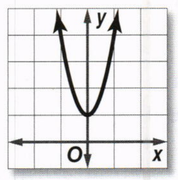
Notice that the graph opens downward. Therefore, the graph of $y = x^2$ has been reflected across the x-axis. The leading coefficient should be negative, so eliminate choices A and D.

The parabola is translated up 5 units, so $c = 5$. Look at the equations. Only choices C and D have $c = 5$. The answer is C.

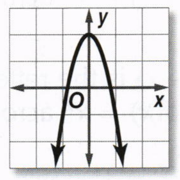
Check Your Progress

4. Which is the graph of $y = -3x^2 + 1$?

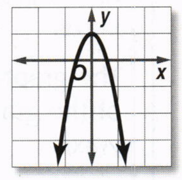
F



G



H



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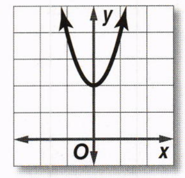

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Review Vocabulary

leading coefficient the coefficient of the first term of a polynomial written in standard form (Lesson 7-4)

Check Your Understanding

Examples 1–3
pp. 544–546

Describe how the graph of each function is related to the graph of $f(x) = x^2$.

1. $g(x) = x^2 - 11$ **2.** $h(x) = \frac{1}{2}x^2$ **3.** $h(x) = -x^2 + 8$

4. $g(x) = x^2 + 6$ **5.** $g(x) = -4x^2$ **6.** $h(x) = -x^2 - 2$

Example 4
p. 546

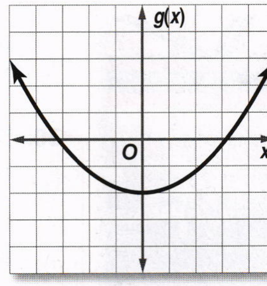
7. MULTIPLE CHOICE Which is an equation for the function shown in the graph?

A $g(x) = \frac{1}{5}x^2 + 2$

B $g(x) = -5x^2 - 2$

C $g(x) = \frac{1}{5}x^2 - 2$

D $g(x) = -\frac{1}{5}x^2 - 2$



Practice and Problem Solving

● = **Step-by-Step Solutions** begin on page R12.
Extra Practice begins on page 815.

Examples 1–3
pp. 544–546

Describe how the graph of each function is related to the graph of $f(x) = x^2$.

8. $g(x) = -10 + x^2$ **9.** $h(x) = -7 - x^2$

10. $g(x) = 2x^2 + 8$ **11.** $h(x) = 6 + \frac{2}{3}x^2$

12. $g(x) = -5 - \frac{4}{3}x^2$ **13.** $h(x) = 3 + \frac{5}{2}x^2$

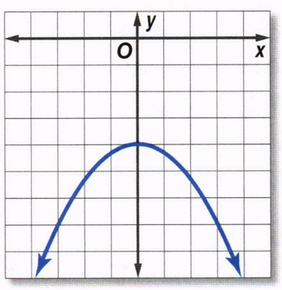
14. $g(x) = 0.25x^2 - 1.1$ **15.** $h(x) = 1.35x^2 + 2.6$

16. $g(x) = \frac{3}{4}x^2 + \frac{5}{6}$ **17.** $h(x) = 1.01x^2 - 6.5$

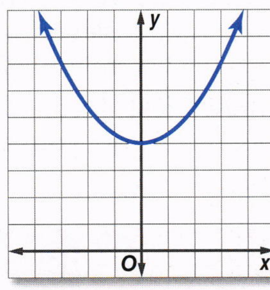
Example 4
p. 546

Match each equation to its graph.

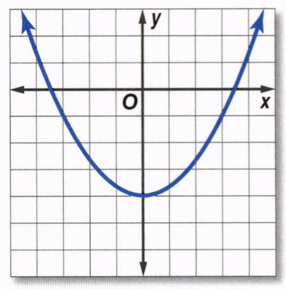
A



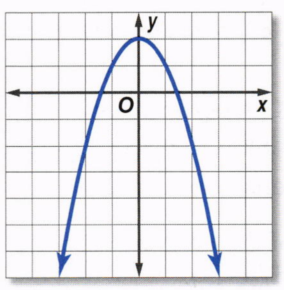
B



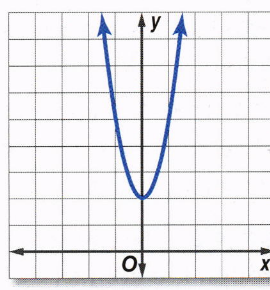
C



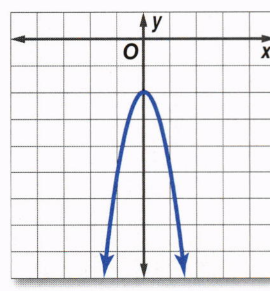
D



E



F



18. $y = \frac{1}{3}x^2 - 4$ **19.** $y = -\frac{1}{3}x^2 - 4$ **20.** $y = \frac{1}{3}x^2 + 4$

21. $y = -3x^2 - 2$ **22.** $y = -x^2 + 2$ **23.** $y = 3x^2 + 2$

Real-World Link

There are over 365 species of squirrels or squirrel-like mammals in the world. The three most common are the flying squirrel, the tree squirrel, and the ground squirrel.

Source: The Squirrel Place

24. SQUIRRELS A squirrel 12 feet above the ground drops an acorn from a tree. The function $h = -16t^2 + 12$ models the height of the acorn above the ground in feet after t seconds. Graph the function and compare this graph to the graph of its parent function.

List the functions in order from the most stretched vertically to the least stretched vertically.

25. $g(x) = 2x^2, h(x) = \frac{1}{2}x^2$

26. $g(x) = -3x^2, h(x) = \frac{2}{3}x^2$

27. $g(x) = -4x^2, h(x) = 6x^2, f(x) = 0.3x^2$

28. $g(x) = -x^2, h(x) = \frac{5}{3}x^2, f(x) = -4.5x^2$

29 **ROCKS** A rock drops from a cliff 20,000 inches above the ground. Another rock drops from a cliff 30,000 inches above the ground.

a. Write two functions that model the heights h of the rocks after t seconds.

b. Which rock will reach the ground first?

Real-World Link

This fountain is in the East Concourse of Midfield Terminal in Detroit, Michigan. The streams are choreographed to create feelings ranging from tranquil to energetic.

Source: The WET Design

30. **SPRINKLERS** The path of water from a sprinkler can be modeled by quadratic functions. The following functions model paths for three different sprinklers.

Sprinkler A: $y = -0.35x^2 + 3.5$ Sprinkler B: $y = -0.21x^2 + 1.7$

Sprinkler C: $y = -0.08x^2 + 2.4$

a. Which sprinkler will send water the farthest? Explain.

b. Which sprinkler will send water the highest? Explain.

c. Which sprinkler will produce the narrowest path? Explain.

Describe the transformations to obtain the graph of $g(x)$ from the graph of $f(x)$.

31. $f(x) = x^2 + 3$
$g(x) = x^2 - 2$

32. $f(x) = x^2 - 4$
$g(x) = x^2 + 7$

33. $f(x) = -6x^2$
$g(x) = -3x^2$

34. **MULTIPLE REPRESENTATIONS** In this problem, you will investigate another type of transformation using your graphing calculator.

a. GRAPHICAL Graph the following family of equations: $y = x^2$, $y = (x - 2)^2$, $y = (x - 4)^2$, $y = (x + 3)^2$, and $y = (x + 5)^2$ on the same screen. Describe how the graphs of the functions are related to the graph of $f(x) = x^2$.

b. ALGEBRAIC Write a concept for quadratic functions, similar to the concept for vertical translations, to describe the effect of a value being added to or subtracted from x inside the parentheses.

c. ANALYTICAL Predict where the graphs of $y = (x - 7)^2$ and $y = (x + 4)^2$ will be located. Verify your answer by graphing each equation.

H.O.T. Problems

Use Higher-Order Thinking Skills

35. **REASONING** Are the following statements *sometimes*, *always*, or *never* true? Explain.

a. The graph of $y = x^2 + c$ has its vertex at the origin.

b. The graphs of $y = ax^2$ and of $y = -ax^2$ are the same width.

c. The graph of $y = x^2 + c$ opens downward.

36. **CHALLENGE** Write a function of the form $y = ax^2 + c$ with a graph that passes through the points $(-2, 3)$ and $(4, 15)$.

37. **REASONING** Determine whether all quadratic functions that are reflected across the y-axis produce the same graph. Explain your answer.

38. **OPEN ENDED** Write a quadratic function that opens downward and is wider than the parent graph.

39. **WRITING IN MATH** Describe how the values of a and c affect the graphical and tabular representations for the functions $y = ax^2$, $y = x^2 + c$, and $y = ax^2 + c$.

Standardized Test Practice

40. SHORT RESPONSE A plumber charges a flat fee of \$55 and \$30 for each hour of work. Write a function that represents the total charge C, in terms of the number of hours h worked.

41. Which *best* describes the graph of $y = 2x^2$?

- **A** a line with a y-intercept of (0, 2) and an x-intercept at the origin
- **B** a parabola with a minimum point at (0, 0) and that is twice as wide as the graph of $y = x^2$ when $y = 2$
- **C** a parabola with a maximum point at (0, 0) and that is half as wide as the graph of $y = x^2$ when $y = 2$
- **D** a parabola with a minimum point at (0, 0) and that is half as wide as the graph of $y = x^2$ when $y = 2$

42. Candace is 5 feet tall. If 1 inch is about 2.54 centimeters, how tall is Candace to the nearest centimeter?

- **F** 123 cm
- **G** 26 cm
- **H** 13 cm
- **J** 152 cm

43. While in England, Imani spent 49.60 British pounds on a pair of jeans. If this is equivalent to \$100 in U.S. currency, how many British pounds would Imani have spent on a sweater that cost \$60?

- **A** 8.26 pounds
- **B** 29.76 pounds
- **C** 2976 pounds
- **D** 19.84 pounds

Spiral Review

Solve each equation by graphing. (Lesson 9-2)

44. $x^2 + 6 = 0$

45. $x^2 - 10x = -24$

46. $x^2 + 5x + 4 = 0$

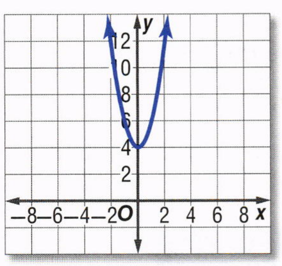
47. $2x^2 - x = 3$

48. $2x^2 - x = 15$

49. $12x^2 = -11x + 15$

Find the vertex, the equation of the axis of symmetry, and the y-intercept of each graph. (Lesson 9-1)

50.



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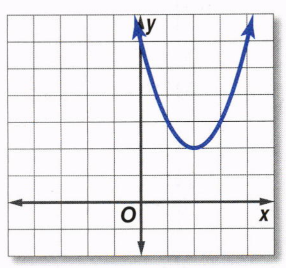

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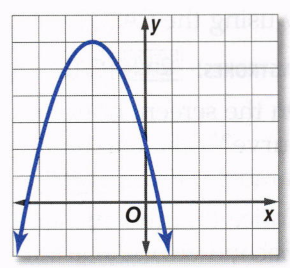


53. CLASS TRIP Mr. Wong's American History class will take taxis from their hotel in Washington, D.C., to the Lincoln Memorial. The fare is \$2.75 for the first mile and \$1.25 for each additional mile. If the distance is m miles and t taxis are needed, write an expression for the cost to transport the group. (Lesson 7-6)

Solve each inequality. Check your solution. (Lesson 5-3)

54. $-3t + 6 \leq -3$

55. $59 > -5 - 8f$

56. $-2 - \frac{d}{5} < 23$

Skills Check

Determine whether each trinomial is a perfect square trinomial. If so, factor it. (Lesson 8-6)

57. $16x^2 - 24x + 9$

58. $9x^2 + 6x + 1$

59. $25x^2 - 60x + 36$

60. $x^2 - 8x + 81$

61. $36x^2 - 84x + 49$

62. $4x^2 - 3x + 9$

EXTEND
9-3
Graphing Technology Lab
Systems of Linear and Quadratic Equations

Math Online glencoe.com
- Other Calculator Keystrokes
- Graphing Technology Personal Tutor

You can use a graphing calculator to solve systems involving linear and quadratic equations.

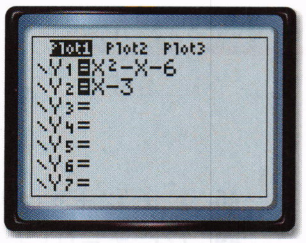
ACTIVITY 1

Use a graphing calculator to solve the system of equations.

$$y = x^2 - x - 6$$
$$y = x - 3$$

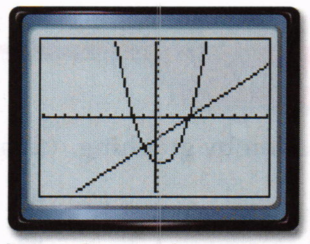
Step 1 Enter each equation in the Y= list. Enter the quadratic equation as Y1 and the linear equation as Y2.

KEYSTROKES: X,T,θ,n x^2 − X,T,θ,n − 6 ENTER X,T,θ,n − 3



Step 2 Graph the system. **KEYSTROKE:** Graph

The solutions of the system are the intersection points. The graphs intersect at two points. So, there are two solutions.



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Step 3 Find the first intersection of the graphs by using the CALC menu.

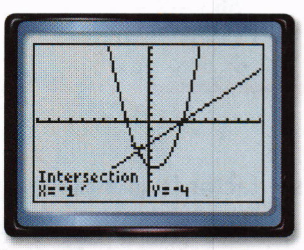
KEYSTROKES: 2nd [CALC] 5

On the screen, notice the question "First Curve?" The cursor should be on the parabola. Press ENTER.

Notice that the question changes to "Second curve?" and the cursor jumps to the line. Press ENTER.

Use the arrow keys to move the cursor as close as possible to the intersection point in Quadrant III. Press ENTER again.

The intersection is the point at (−1, −4).



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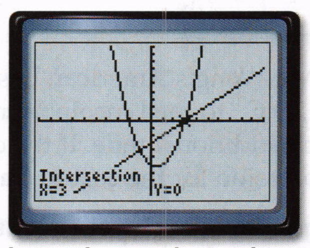
Step 4 Move the cursor to the second intersection. Find the second intersection by repeating Step 3.

The intersection is at (3, 0).

Therefore, the solutions of the system of equations are (−1, −4) and (3, 0).



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ACTIVITY 2

Use a graphing calculator to solve the system of equations.

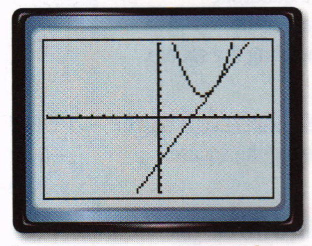
$y = x^2 - 8x + 19$
$y = 2x - 6$

Step 1 Enter each equation in the Y= list.

Enter the quadratic equation as Y1 and the linear equation as Y2.

Step 2 Graph the system.

In this case, the graphs of the equations intersect at only one point. Therefore, there is only one solution of this system of equations.



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Step 3 Find the intersection of the graphs of the equations.

The intersection is the point at about (5, 4).

Thus, the solution of the system of equations is about (5, 4).

ACTIVITY 3

Use a graphing calculator to solve the system of equations.

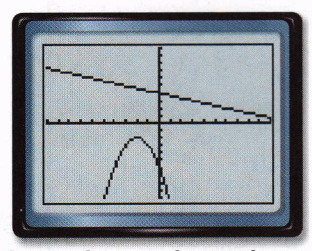
$y = -x^2 - 4x - 6$
$y = -\frac{1}{3}x + 4$

Step 1 Enter each equation in the Y= list.

Enter the quadratic equation as Y1 and the linear equation as Y2.

Step 2 Graph the system.

The graphs of the equations do not intersect. Thus, this system of equations has no solution.



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Exercises

Use factoring to solve each system of equations. Then use a graphing calculator to check your solutions.

1. $y = x^2 + 7x + 12$
$y = 2x + 8$

2. $y = x^2 - x - 20$
$y = 3x + 12$

3. $y = 3x^2 - x - 2$
$y = -2x + 2$

Use a graphing calculator to solve each system of equations.

4. $y = x^2$
$y = 2x$

5. $y = -x^2 - 6x - 3$
$y = 6$

6. $y = -x^2 + 4$
$y = \frac{1}{2}x + 5$

7. $y = x^2 + 5x + 4$
$y = -x - 8$

8. $y = \frac{1}{2}x^2 - 4$
$y = 3x + 4$

9. $y = x^2$
$y = -2x - 1$

9-4

Solving Quadratic Equations by Completing the Square

Then

You solved quadratic equations by using the square root property. (Lesson 8-6)

Now

- Complete the square to write perfect square trinomials.
- Solve quadratic equations by completing the square.

New Vocabulary

completing the square

Math Online

glencoe.com

- Extra Examples
- Personal Tutor
- Self-Check Quiz
- Homework Help
- Math in Motion

Why?

In competitions, skateboarders may launch themselves from a half pipe into the air to perform tricks. The equation $h = -16t^2 + 20t + 12$ can be used to model their height, in feet, after t seconds.

To find how long a skateboarder is in the air if he is 25 feet above the half pipe, you can solve $25 = -16t^2 + 20t + 12$ by using a method called completing the square.

Complete the Square In Lesson 8-6, you solved equations by taking the square root of each side. This method worked only because the expression on the left-hand side was a perfect square. In perfect square trinomials in which the leading coefficient is 1, there is a relationship between the **coefficient of the x-term** and the **constant term**.

$$\begin{aligned}(x+5)^2 &= x^2 + 2(5)(x) + 5^2 \\ &= x^2 + 10x + 25\end{aligned}$$

Notice that $\left(\frac{10}{2}\right)^2 = 25$. To get the constant term, divide the coefficient of the x-term by 2 and square the result. Any quadratic expression in the form $x^2 + bx$ can be made into a perfect square by using a method called **completing the square**.

Key Concept — Completing the Square

For Your FOLDABLE

Words To complete the square for any quadratic expression of the form $x^2 + bx$, follow the steps below.

Step 1 Find one half of b, the coefficient of x.

Step 2 Square the result in Step 1.

Step 3 Add the result of Step 2 to $x^2 + bx$.

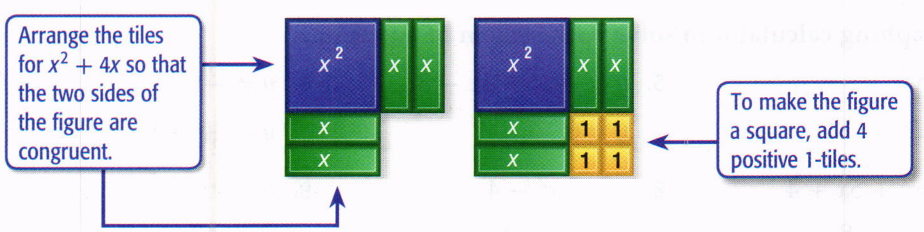
Symbols $x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)^2$

Math *in Motion*, Animation glencoe.com

EXAMPLE 1 Complete the Square

Find the value of c that makes $x^2 + 4x + c$ a perfect square trinomial.

Method 1 Use algebra tiles.



StudyTip

Algorithms An algorithm is a series of steps for carrying out a procedure or solving a problem.

Method 2 Use complete the square algorithm.

Step 1 Find $\frac{1}{2}$ of 4. $\frac{4}{2} = 2$

Step 2 Square the result in Step 1. $2^2 = 4$

Step 3 Add the result of Step 2 to $x^2 + 4x$. $x^2 + 4x + 4$

Thus, $c = 4$. Notice that $x^2 + 4x + 4 = (x + 2)^2$.

Check Your Progress

1. Find the value of c that makes $r^2 - 8r + c$ a perfect square trinomial.

Personal Tutor glencoe.com

Solve Equations by Completing the Square You can complete the square to solve quadratic equations. First, you must isolate the x^2- and bx-terms.

EXAMPLE 2 Solve an Equation by Completing the Square

Solve $x^2 - 6x + 12 = 19$ by completing the square.

$x^2 - 6x + 12 = 19$	Original equation
$x^2 - 6x = 7$	Subtract 12 from each side.
$x^2 - 6x + 9 = 7 + 9$	Since $\left(\frac{-6}{2}\right)^2 = 9$, add 9 to each side.
$(x - 3)^2 = 16$	Factor $x^2 - 6x + 9$.
$x - 3 = \pm 4$	Take the square root of each side.
$x = 3 \pm 4$	Add 3 to each side.
$x = 3 + 4$ or $x = 3 - 4$	Separate the solutions.
$= 7$ $\quad = -1$	The solutions are 7 and -1.

Check Your Progress

2. Solve $x^2 - 12x + 3 = 8$ by completing the square.

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To solve a quadratic equation in which the leading coefficient is not 1, divide each term by the coefficient. Then isolate the x^2- and x-terms and complete the square.

Watch Out!

Leading Coefficient Remember that the leading coefficient has to be 1 before you can complete the square.

EXAMPLE 3 Equation with $a \neq 1$

Solve $-2x^2 + 8x - 18 = 0$ by completing the square.

$-2x^2 + 8x - 18 = 0$	Original equation
$\frac{-2x^2 + 8x - 18}{-2} = \frac{0}{-2}$	Divide each side by -2.
$x^2 - 4x + 9 = 0$	Simplify.
$x^2 - 4x = -9$	Subtract 9 from each side.
$x^2 - 4x + 4 = -9 + 4$	Since $\left(\frac{-4}{2}\right)^2 = 4$, add 4 to each side.
$(x - 2)^2 = -5$	Factor $x^2 - 4x + 4$.

No real number has a negative square. So, this equation has no real solutions.

Check Your Progress

3. Solve $3x^2 - 9x - 3 = 21$ by completing the square.

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Real-World Link

The oldest public high school rivalry takes place between Wellesley High School and Needham Heights High School in Massachusetts. The first football game between them took place on Thanksgiving morning in 1882 in Needham.

Source: USA Football

Real-World EXAMPLE 4 Solve a Problem by Completing the Square

JERSEYS The senior class at Bay High School buys jerseys to wear to the football games. The cost of the jerseys can be modeled by the equation $C = 0.1x^2 + 2.4x + 25$, where C is the amount it costs to buy x jerseys. How many jerseys can they purchase for \$430?

The seniors have \$430, so set the equation equal to 430 and complete the square.

$$0.1x^2 + 2.4x + 25 = 430 \quad \text{Original equation}$$

$$\frac{0.1x^2 + 2.4x + 25}{0.1} = \frac{430}{0.1} \quad \text{Divide each side by 0.1.}$$

$$x^2 + 24x + 250 = 4300 \quad \text{Simplify.}$$

$$x^2 + 24x + 250 - 250 = 4300 - 250 \quad \text{Subtract 250 from each side.}$$

$$x^2 + 24x = 4050 \quad \text{Simplify.}$$

$$x^2 + 24x + 144 = 4050 + 144 \quad \text{Since } \left(\frac{24}{2}\right)^2 = 144\text{, add 144 to each side.}$$

$$x^2 + 24x + 144 = 4194 \quad \text{Simplify.}$$

$$(x + 12)^2 = 4194 \quad \text{Factor } x^2 + 24x + 144.$$

$$x + 12 = \pm\sqrt{4194} \quad \text{Take the square root of each side.}$$

$$x = -12 \pm\sqrt{4194} \quad \text{Subtract 12 from each side.}$$

Use a calculator to approximate each value of x.

$$x = -12 + \sqrt{4194} \quad \text{or} \quad x = -12 - \sqrt{4194} \quad \text{Separate the solutions.}$$

$$\approx 52.8 \qquad\qquad \approx -76.8 \quad \text{Evaluate.}$$

Since you cannot buy a negative number of jerseys, the negative solution is not reasonable. The seniors can afford to buy 52 jerseys.

Check Your Progress

4. If the senior class were able to raise \$620, how many jerseys could they buy?

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Check Your Understanding

Example 1 pp. 552–553

Find the value of c that makes each trinomial a perfect square.

1. $x^2 - 18x + c$

2. $x^2 + 22x + c$

3. $x^2 + 9x + c$

4. $x^2 - 7x + c$

Examples 2 and 3 p. 553

Solve each equation by completing the square. Round to the nearest tenth if necessary.

5. $x^2 + 4x = 6$

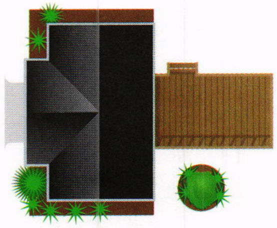
6. $x^2 - 8x = -9$

7. $4x^2 + 9x - 1 = 0$

8. $-2x^2 + 10x + 22 = 4$

Example 4 p. 554

9. **CONSTRUCTION** Collin is building a deck on the back of his family's house. He has enough lumber for the deck to be 144 square feet. The length should be 10 feet more than its width. What should the dimensions of the deck be?



Practice and Problem Solving

= **Step-by-Step Solutions** begin on page R12.
Extra Practice begins on page 815.

Example 1 pp. 552–553

Find the value of c that makes each trinomial a perfect square.

10. $x^2 + 26x + c$ **11.** $x^2 - 24x + c$ **12.** $x^2 - 19x + c$

13. $x^2 + 17x + c$ **14.** $x^2 + 5x + c$ **15.** $x^2 - 13x + c$

16. $x^2 - 22x + c$ **17.** $x^2 - 15x + c$ **18.** $x^2 + 24x + c$

Examples 2 and 3 p. 553

Solve each equation by completing the square. Round to the nearest tenth if necessary.

19. $x^2 + 6x - 16 = 0$ **20.** $x^2 - 2x - 14 = 0$

21. $x^2 - 8x - 1 = 8$ **22.** $x^2 + 3x + 21 = 22$

23. $x^2 - 11x + 3 = 5$ **24.** $5x^2 - 10x = 23$

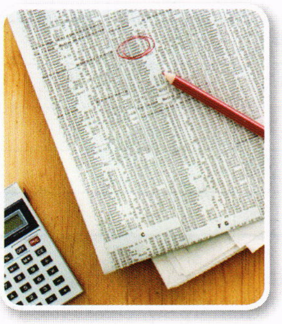
25. $2x^2 - 2x + 7 = 5$ **26.** $3x^2 + 12x + 81 = 15$

27. $4x^2 + 6x = 12$ **28.** $4x^2 + 5 = 10x$

29. $-2x^2 + 10x = -14$ **30.** $-3x^2 - 12 = 14x$

Example 4 p. 554

31. FINANCIAL LITERACY The price p in dollars for a particular stock can be modeled by the quadratic equation $p = 3.5t - 0.05t^2$, where t represents the number of days after the stock is purchased. When is the stock worth \$60?



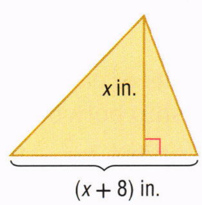
Real-World Link

According to a Junior Achievement survey, 25% of 13- and 14-year-olds own stocks in their own name. Most got their shares as gifts.

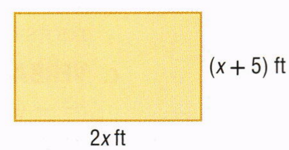
Source: *Money Magazine*

GEOMETRY **Find the value of x for each figure. Round to the nearest tenth if necessary.**

32. $A = 45 \text{ in}^2$



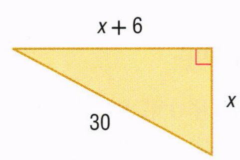
33. $A = 110 \text{ ft}^2$



34. NUMBER THEORY The product of two consecutive even integers is 224. Find the integers.

35. NUMBER THEORY The product of two consecutive negative odd integers is 483. Find the integers.

36. GEOMETRY Find the area of the triangle below.

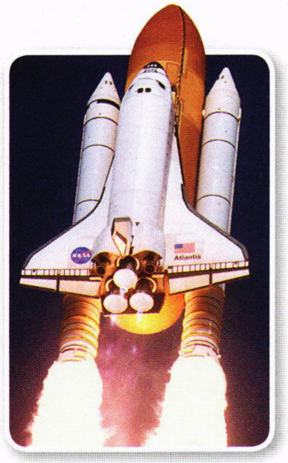


Solve each equation by completing the square. Round to the nearest tenth if necessary.

37. $0.2x^2 - 0.2x - 0.4 = 0$ **38.** $0.5x^2 = 2x - 0.3$

39. $2x^2 - \frac{11}{5}x = -\frac{3}{10}$ **40.** $\frac{2}{3}x^2 - \frac{4}{3}x = \frac{5}{6}$

41. $\frac{1}{4}x^2 + 2x = \frac{3}{8}$ **42.** $\frac{2}{5}x^2 + 2x = \frac{1}{5}$



Real-World Link

The National Aeronautics and Space Administration (NASA) began operations on October 1, 1958. NASA employs about 8000 people and has an annual budget of $100 million.

Source: NASA

43. **ASTRONOMY** The height of an object t seconds after it is dropped is given by the equation $h = -\frac{1}{2}gt^2 + h_0$, where h_0 is the initial height and g is the acceleration due to gravity. The acceleration due to gravity near the surface of Mars is 3.73 m/s^2, while on Earth it is 9.8 m/s^2. Suppose an object is dropped from an initial height of 120 meters above the surface of each planet.

a. On which planet would the object reach the ground first?

b. How long would it take the object to reach the ground on each planet? Round each answer to the nearest tenth.

c. Do the times that it takes the object to reach the ground seem reasonable? Explain your reasoning.

44. Find all values of c that make $x^2 + cx + 100$ a perfect square trinomial.

45. Find all values of c that make $x^2 + cx + 225$ a perfect square trinomial.

46. **PAINTING** Before she begins painting a picture, Donna stretches her canvas over a wood frame. The frame has a length of 60 inches and a width of 4 inches. She has enough canvas to cover 480 square inches. Donna decides to increase the dimensions of the frame. If the increase in the length is 10 times the increase in the width, what will the dimensions of the frame be?

47. **MULTIPLE REPRESENTATIONS** In this problem, you will investigate a property of quadratic equations.

a. **TABULAR** Copy the table shown and complete the second column.

b. **ALGEBRAIC** Set each trinomial equal to zero, and solve the equation by completing the square. Complete the last column of the table with the number of roots of each equation.

Trinomial	$b^2 - 4ac$	Number of Roots
$x^2 - 8x + 16$	0	1
$2x^2 - 11x + 3$		
$3x^2 + 6x + 9$		
$x^2 - 2x + 7$		
$x^2 + 10x + 25$		
$x^2 + 3x - 12$		

c. **VERBAL** Compare the number of roots of each equation to the result in the $b^2 - 4ac$ column. Is there a relationship between these values? If so, describe it.

d. **ANALYTICAL** Predict how many solutions $2x^2 - 9x + 15 = 0$ will have. Verify your prediction by solving the equation.

H.O.T. Problems

Use Higher-Order Thinking Skills

48. **CHALLENGE** Given $y = ax^2 + bx + c$ with $a \neq 0$, derive the equation for the axis of symmetry by completing the square and rewriting the equation in the form $y = a(x - h)^2 + k$.

49. **REASONING** Determine the number of solutions $x^2 + bx = c$ has if $c < -\left(\frac{b}{2}\right)^2$. Explain.

50. **WHICH ONE DOESN'T BELONG?** Identify the expression that does not belong with the other three. Explain your reasoning.

$n^2 - n + \frac{1}{4}$	$n^2 + n + \frac{1}{4}$	$n^2 - \frac{2}{3}n + \frac{1}{9}$	$n^2 + \frac{1}{3}n + \frac{1}{9}$

51. **OPEN ENDED** Write a quadratic equation for which the only solution is 4.

52. **WRITING IN MATH** Compare and contrast the following strategies for solving $x^2 - 5x - 7 = 0$: completing the square, graphing, and factoring.

Standardized Test Practice

53. The length of a rectangle is 3 times its width. The area of the rectangle is 75 square feet. Find the length of the rectangle in feet.

A 25 **B** 15 **C** 10 **D** 5

54. PROBABILITY At a festival, winners of a game draw a token for a prize. There is one token for each prize. The prizes include 9 movie passes, 8 stuffed animals, 5 hats, 10 jump ropes, and 4 glow necklaces. What is the probability that the first person to draw a token will win a movie pass?

F $\frac{9}{61}$ **G** $\frac{1}{9}$ **H** $\frac{1}{4}$ **J** $\frac{1}{36}$

55. GRIDDED RESPONSE The population of a town can be modeled by $P = 22{,}000 + 125t$, where P represents the population and t represents the number of years from 2000. How many years after 2000 will the population be 26,000?

56. Percy delivers pizzas for Pizza King. He is paid \$6 an hour plus \$2.50 for each pizza he delivers. Percy earned \$280 last week. If he worked a total of 30 hours, how many pizzas did he deliver?

A 250 pizzas
B 184 pizzas
C 40 pizzas
D 34 pizzas

Spiral Review

Describe how the graph of each function is related to the graph of $f(x) = x^2$. (Lesson 9-3)

57. $g(x) = -12 + x^2$ **58.** $h(x) = 2 - x^2$ **59.** $g(x) = 2x^2 + 5$

60. $h(x) = -6 + \frac{2}{3}x^2$ **61.** $g(x) = 6 + \frac{4}{3}x^2$ **62.** $h(x) = -1 - \frac{3}{2}x^2$

63. RIDES A popular amusement park ride whisks riders to the top of a 250-foot tower and drops them. A function for the height of a rider is $h = -16t^2 + 250$, where h is the height and t is the time in seconds. The ride stops the descent of the rider 40 feet above the ground. Write an equation that models the drop of the rider. How long does it take to fall from 250 feet to 40 feet? (Lesson 9-2)

Simplify. Assume that no denominator is equal to zero. (Lesson 7-2)

64. $\frac{a^6}{a^3}$ **65.** $\frac{4^7}{4^5}$ **66.** $\frac{c^3d^4}{cd^7}$

67. $\left(\frac{4h^{-2}g}{2g^5}\right)^0$ **68.** $\frac{5q^{-2}t^6}{10q^2t^{-4}}$ **69.** $b^3(m^{-3})(b^{-6})$

Solve each open sentence. (Lesson 5-5)

70. $|y - 2| > 7$ **71.** $|z + 5| < 3$ **72.** $|2b + 7| \leq -6$

73. $|3 - 2y| \geq 8$ **74.** $|9 - 4m| < -1$ **75.** $|5c - 2| \leq 13$

Skills Check

Evaluate $\sqrt{b^2 - 4ac}$ for each set of values. Round to the nearest tenth if necessary. (Lesson 1-2)

76. $a = 2, b = -5, c = 2$ **77.** $a = 1, b = 12, c = 11$ **78.** $a = -9, b = 10, c = -1$

79. $a = 1, b = 7, c = -3$ **80.** $a = 2, b = -4, c = -6$ **81.** $a = 3, b = 1, c = 2$

9-5 Solving Quadratic Equations by Using the Quadratic Formula

Then
You solved quadratic equations by completing the square. (Lesson 9-4)

Now
- Solve quadratic equations by using the Quadratic Formula.
- Use the discriminant to determine the number of solutions of a quadratic equation.

New Vocabulary
Quadratic Formula
discriminant

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- Extra Examples
- Personal Tutor
- Self-Check Quiz
- Homework Help

Why?

For adult women, the normal systolic blood pressure P in millimeters of mercury (mm Hg) can be modeled by $P = 0.01a^2 + 0.05a + 107$, where a is age in years. This equation can be used to approximate the age of a woman with a certain systolic blood pressure. However, it would be difficult to solve by factoring, graphing, or completing the square.

Quadratic Formula Completing the square of the quadratic equation $ax^2 + bx + c = 0$ produces a formula that allows you to find the solutions of *any* quadratic equation that is written in standard form. This formula is called the **Quadratic Formula**.

Key Concept — The Quadratic Formula

For Your FOLDABLE

The solutions of a quadratic equation $ax^2 + bx + c = 0$, where $a \neq 0$, are given by the Quadratic Formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

You will derive this formula in Lesson 10-2.

EXAMPLE 1 Use the Quadratic Formula

Solve $x^2 - 12x = -20$ by using the Quadratic Formula.

Step 1 Rewrite the equation in standard form.

$$x^2 - 12x = -20 \quad \text{Original equation}$$

$$x^2 - 12x + 20 = 0 \quad \text{Add 20 to each side.}$$

Step 2 Apply the Quadratic Formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \text{Quadratic Formula}$$

$$= \frac{-(-12) \pm \sqrt{(-12)^2 - 4(1)(20)}}{2(1)} \quad a = 1, b = -12, \text{ and } c = 20$$

$$= \frac{12 \pm \sqrt{144 - 80}}{2} \quad \text{Multiply.}$$

$$= \frac{12 \pm \sqrt{64}}{2} \text{ or } \frac{12 \pm 8}{2} \quad \text{Subtract and take the square root.}$$

$$x = \frac{12 - 8}{2} \text{ or } x = \frac{12 + 8}{2} \quad \text{Separate the solutions.}$$

$$= 2 \qquad = 10 \quad \text{Simplify.}$$

The solutions are 2 and 10.

Check Your Progress

1. Solve $2x^2 + 9x = 18$ by using the Quadratic Formula.

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The solutions of quadratic equations are not always integers.

EXAMPLE 2 Use the Quadratic Formula

Solve each equation by using the Quadratic Formula. Round to the nearest tenth if necessary.

a. $3x^2 + 5x - 12 = 0$

For this equation, $a = 3$, $b = 5$, and $c = -12$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \text{Quadratic Formula}$$

$$= \frac{-(5) \pm \sqrt{(5)^2 - 4(3)(-12)}}{2(3)} \quad a = 3, b = 5, \text{ and } c = -12$$

$$= \frac{-5 \pm \sqrt{25 + 144}}{6} \quad \text{Multiply.}$$

$$= \frac{-5 \pm \sqrt{169}}{6} \text{ or } \frac{-5 \pm 13}{6} \quad \text{Add and simplify.}$$

$$x = \frac{-5 - 13}{6} \text{ or } x = \frac{-5 + 13}{6} \quad \text{Separate the solutions.}$$

$$= -3 \qquad = \frac{4}{3} \quad \text{Simplify.}$$

The solutions are -3 and $\frac{4}{3}$.

b. $10x^2 - 5x = 25$

Step 1 Rewrite the equation in standard form.

$$10x^2 - 5x = 25 \quad \text{Original equation}$$

$$10x^2 - 5x - 25 = 0 \quad \text{Subtract 25 from each side.}$$

Step 2 Apply the Quadratic Formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \text{Quadratic Formula}$$

$$= \frac{-(-5) \pm \sqrt{(-5)^2 - 4(10)(-25)}}{2(10)} \quad a = 10, b = -5, \text{ and } c = -25$$

$$= \frac{5 \pm \sqrt{25 + 1000}}{20} \quad \text{Multiply.}$$

$$= \frac{5 \pm \sqrt{1025}}{20} \quad \text{Add.}$$

$$= \frac{5 - \sqrt{1025}}{20} \text{ or } \frac{5 + \sqrt{1025}}{20} \quad \text{Separate the solutions.}$$

$$\approx -1.4 \qquad \approx 1.9 \quad \text{Simplify.}$$

The solutions are about -1.4 and 1.9.

StudyTip

Exact Answers In Example 2, the number $\sqrt{1025}$ is irrational, so the calculator can only give you an approximation of its value. So, the exact answer in Example 2 is $\frac{5 \pm \sqrt{1025}}{20}$. The numbers -1.4 and 1.9 are approximations.

Check Your Progress

2A. $4x^2 - 24x + 35 = 0$

2B. $3x^2 - 2x - 9 = 0$

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You can solve quadratic equations by using many different methods. No one way is always best.

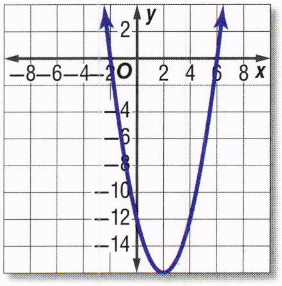
EXAMPLE 3 Solve Quadratic Equations Using Different Methods

Solve $x^2 - 4x = 12$.

Method 1 Graphing

Rewrite the equation in standard form.

$x^2 - 4x = 12$	**Original equation**
$x^2 - 4x - 12 = 0$	**Subtract 12 from each side.**



Graph the related function $f(x) = x^2 - 4x - 12$.
Locate the x-intercepts of the graph.
The solutions are -2 and 6.

Method 2 Factoring

$x^2 - 4x = 12$	**Original equation**
$x^2 - 4x - 12 = 0$	**Subtract 12 from each side.**
$(x - 6)(x + 2) = 0$	**Factor.**
$x - 6 = 0$ or $x + 2 = 0$	**Zero Product Property**
$x = 6 \qquad x = -2$	**Solve for x.**

Method 3 Completing the Square

The equation is in the correct form to complete the square, since the leading coefficient is 1 and the x^2 and x terms are isolated.

$x^2 - 4x = 12$	**Original equation**
$x^2 - 4x + 4 = 12 + 4$	**Since $\left(\frac{-4}{2}\right)^2 = 4$, add 4 to each side.**
$(x - 2)^2 = 16$	**Factor $x^2 - 4x + 4$.**
$x - 2 = \pm 4$	**Take the square root of each side.**
$x = 2 \pm 4$	**Add 2 to each side.**
$x = 2 + 4$ or $x = 2 - 4$	**Separate the solutions.**
$= 6 \qquad = -2$	**Simplify.**

Method 4 Quadratic Formula

From Method 1, the standard form of the equation is $x^2 - 4x - 12 = 0$.

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	**Quadratic Formula**
$= \dfrac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-12)}}{2(1)}$	**$a = 1$, $b = -4$, and $c = -12$**
$= \dfrac{4 \pm \sqrt{16 + 48}}{2}$	**Multiply.**
$= \dfrac{4 \pm \sqrt{64}}{2}$ or $\dfrac{4 \pm 8}{2}$	**Add and simplify.**
$x = \dfrac{4 - 8}{2}$ or $x = \dfrac{4 + 8}{2}$	**Separate the solutions.**
$= -2 \qquad = 6$	**Simplify.**

Watch Out!

Solutions No matter what method is used to solve a quadratic equation, all of the methods should produce the same solution(s).

Check Your Progress

Solve each equation.

3A. $2x^2 - 17x + 8 = 0$

3B. $4x^2 - 4x - 11 = 0$

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Concept Summary — Solving Quadratic Equations

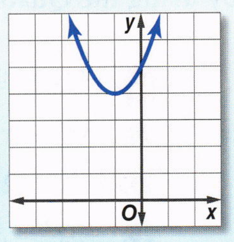
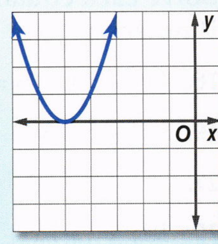
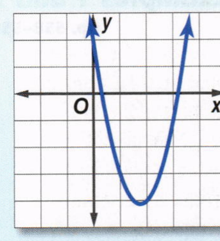
For Your FOLDABLE

Method	When to Use
Factoring	Use when the constant term is 0 or if the factors are easily determined. Not all equations are factorable.
Graphing	Use when an approximate solution is sufficient.
Using Square Roots	Use when an equation can be written in the form $x^2 = n$. Can only be used if the equation has no x-term.
Completing the Square	Can be used for any equation $ax^2 + bx + c = 0$, but is simplest to apply when b is even and $a = 1$.
Quadratic Formula	Can be used for any equation $ax^2 + bx + c = 0$.

The Discriminant In the Quadratic Formula, the expression under the radical sign, $b^2 - 4ac$, is called the **discriminant**. The discriminant can be used to determine the number of real solutions of a quadratic equation.

Key Concept — Using the Discriminant

For Your FOLDABLE

Equation	$x^2 + 2x + 5 = 0$	$x^2 + 10x + 25 = 0$	$2x^2 - 7x + 2 = 0$
Discriminant	$b^2 - 4ac = -16$ negative	$b^2 - 4ac = 0$ zero	$b^2 - 4ac = 33$ positive
Graph of Related Function	 0 x-intercepts	 1 x-intercept	 2 x-intercepts
Real Solutions	0	1	2

StudyTip

Discriminant Recall that when the left side of the standard form of an equation is a perfect square trinomial, there is only one solution. Therefore, the discriminant of a perfect square trinomial will always be zero.

EXAMPLE 4 Use the Discriminant

State the value of the discriminant of $4x^2 + 5x = -3$. Then determine the number of real solutions of the equation.

Step 1 Rewrite in standard form. $4x^2 - 5x = -3 \longrightarrow 4x^2 - 5x + 3 = 0$

Step 2 Find the discriminant.

$$b^2 - 4ac = (-5)^2 - 4(4)(3) \quad a = 4, b = -5, \text{ and } c = 3$$
$$= -23 \quad \text{Simplify.}$$

Since the discriminant is negative, the equation has no real solutions.

Check Your Progress

4A. $2x^2 + 11x + 15 = 0$ **4B.** $9x^2 - 30x + 25 = 0$

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Check Your Understanding

Examples 1 and 2 pp. 558–559

Solve each equation by using the Quadratic Formula. Round to the nearest tenth if necessary.

1. $x^2 - 2x - 15 = 0$
2. $x^2 - 10x + 16 = 0$
3. $x^2 - 8x = -10$
4. $x^2 + 3x = 12$
5. $10x^2 - 31x + 15 = 0$
6. $5x^2 + 5 = -13x$

Example 3 p. 560

Solve each equation. State which method you used.

7. $2x^2 + 11x - 6 = 0$
8. $2x^2 - 3x - 6 = 0$
9. $9x^2 = 25$
10. $x^2 - 9x = -19$

Example 4 p. 561

State the value of the discriminant for each equation. Then determine the number of real solutions of the equation.

11. $x^2 - 9x + 21 = 0$
12. $2x^2 - 11x + 10 = 0$
13. $9x^2 + 24x = -16$
14. $3x^2 - x = 8$

15. **TRAMPOLINE** Eva is jumping on a trampoline. Her height h in feet can be modeled by the equation $h = -16t^2 + 2.4t + 6$, where t is time in seconds. Use the discriminant to determine if Eva will ever reach a height of 20 feet. Explain.

Practice and Problem Solving

● = **Step-by-Step Solutions** begin on page R12.
Extra Practice begins on page 815.

Examples 1 and 2 pp. 558–559

Solve each equation by using the Quadratic Formula. Round to the nearest tenth if necessary.

16. $4x^2 + 5x - 6 = 0$
17. $x^2 + 16 = 0$
18. $6x^2 - 12x + 1 = 0$
19. $5x^2 - 8x = 6$
20. $2x^2 - 5x = -7$
21. $5x^2 + 21x = -18$
22. $81x^2 = 9$
23. $8x^2 + 12x = 8$
24. $4x^2 = -16x - 16$
25. $10x^2 = -7x + 6$
26. $-3x^2 = 8x - 12$
27. $2x^2 = 12x - 18$

28. **AMUSEMENT PARKS** The Demon Drop at Cedar Point in Ohio takes riders to the top of a tower and drops them 60 feet. A function that approximates this ride is $h = -16t^2 + 64t - 60$, where h is the height in feet and t is the time in seconds. About how many seconds does it take for riders to drop from 60 feet to 0 feet?

Example 3 p. 560

Solve each equation. State which method you used.

29. $2x^2 - 8x = 12$
30. $3x^2 - 24x = -36$
31. $x^2 - 3x = 10$
32. $4x^2 + 100 = 0$
33. $x^2 = -7x - 5$
34. $12 - 12x = -3x^2$

Example 4 p. 561

State the value of the discriminant for each equation. Then determine the number of real solutions of the equation.

35. $0.2x^2 - 1.5x + 2.9 = 0$
36. $2x^2 - 5x + 20 = 0$
37. $x^2 - \frac{4}{5}x = 3$
38. $0.5x^2 - 2x = -2$
39. $2.25x^2 - 3x = -1$
40. $2x^2 = \frac{5}{2}x + \frac{3}{2}$

41. **INTERNET** The percent of U.S. households with high-speed Internet h can be estimated by $h = -0.2n^2 + 7.2n + 1.5$, where n is the number of years since 1990.
    a. Use the Quadratic Formula to determine when 20% of the population will have high-speed Internet.
    b. Is a quadratic equation a good model for this information? Explain.

Real-World Link

The risk of motor vehicle crashes is higher among 16- to 19-year-olds than any other age group. Per mile driven, 16- to 19-year-olds are four times more likely than older drivers to crash.

Source: Centers for Disease Control and Prevention

42. **TRAFFIC** The equation $d = 0.05v^2 + 1.1v$ models the distance d in feet it takes a car traveling at a speed of v miles per hour to come to a complete stop. The speed limit on some highways is 65 miles per hour. If Hannah's car stopped after 250 feet, was she speeding? Explain your reasoning.

Without graphing, determine the number of x-intercepts of the graph of the related function for each equation.

43. $4.25x + 3 = -3x^2$ **44.** $x^2 + \frac{2}{25} = \frac{3}{5}x$ **45.** $0.25x^2 + x = -1$

Solve each equation by using the Quadratic Formula. Round to the nearest tenth if necessary.

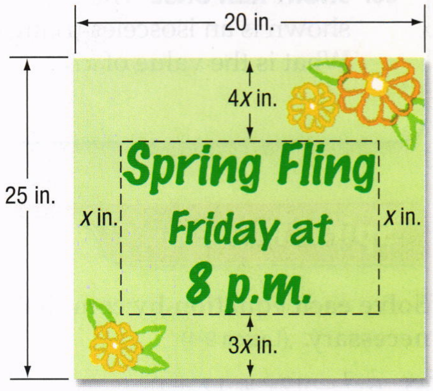
46. $-2x^2 - 7x = -1.5$ **47.** $2.3x^2 - 1.4x = 6.8$ **48.** $x^2 - 2x = 5$

49. **POSTER** Bartolo is making a poster for the dance. He wants to cover three fourths of the area with text.

a. Write an equation for the area of the section with text.

b. Solve the equation by using the Quadratic Formula.

c. What should be the margins of the poster?



50. **MULTIPLE REPRESENTATIONS** In this problem, you will investigate *exponential* functions.

a. **TABULAR** Copy and complete the table.

b. **GRAPHICAL** Construct a graph from the information given in the table using the points (time, number of bacteria). Is the graph linear or quadratic?

c. **ANALYTICAL** What happens to the number of bacteria after every hour? Write a function that models the pattern in the table.

Time (hours)	Number of Bacteria
0	$1 = 2^0$
1	$2 = 2^1$
2	$4 = 2^2$
3	
4	
5	
6	

H.O.T. Problems

Use Higher-Order Thinking Skills

51. **CHALLENGE** Find all values of k such that $2x^2 - 3x + 5k = 0$ has two solutions.

52. **REASONING** Use factoring techniques to determine the number of real zeros of $f(x) = x^2 - 8x + 16$. Compare this method to using the discriminant.

REASONING **Determine whether there are *two*, *one*, or *no* real solutions.**

53. The graph of a quadratic function does not have an x-intercept.

54. The graph of a quadratic function is tangent at the x-axis.

55. The graph of a quadratic function intersects the x-axis twice.

56. Both a and b are greater than 0 and c is less than 0 in a quadratic equation.

57. **OPEN ENDED** Write a quadratic function that has a positive discriminant, one with a negative discriminant, and one with a zero discriminant.

58. **WRITING IN MATH** Describe the advantages and disadvantages of each method of solving quadratic equations. Which method do you prefer, and why?

Standardized Test Practice

59. If n is an even integer, which expression represents the product of three consecutive even integers?

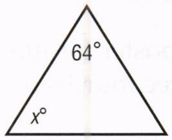
A $n(n + 1)(n + 2)$

B $(n + 1)(n + 2)(n + 3)$

C $3n + 2$

D $n(n + 2)(n + 4)$

60. SHORT RESPONSE The triangle shown is an isosceles triangle. What is the value of x?



61. Which statement best describes the graph of $x = 5$?

F It is parallel to the x-axis.

G It is parallel to the y-axis.

H It passes through the point (2, 5).

J It has a y-intercept of 5.

62. What are the solutions of the quadratic equation $6h^2 + 6h = 72$?

A 3 or −4

B −3 or 4

C no solution

D 12 or −48

Spiral Review

Solve each equation by completing the square. Round to the nearest tenth if necessary. (Lesson 9-4)

63. $6x^2 - 17x + 12 = 0$

64. $x^2 - 9x = -12$

65. $4x^2 = 20x - 25$

Describe the transformations needed to obtain the graph of $g(x)$ from the graph of $f(x)$. (Lesson 9-3)

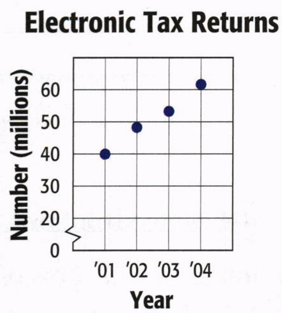
66. $f(x) = 4x^2$
$g(x) = 2x^2$

67. $f(x) = x^2 + 5$
$g(x) = x^2 - 1$

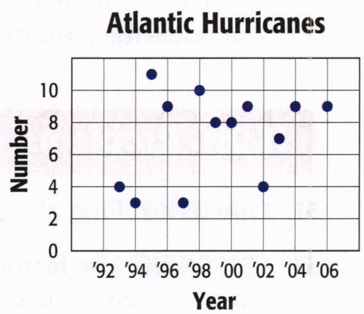
68. $f(x) = x^2 - 6$
$g(x) = x^2 + 3$

Determine whether each graph shows a *positive correlation*, a *negative correlation*, or *no correlation*. If there is a positive or negative correlation, describe its meaning in the situation. (Lesson 4-4)

69.



70.



71. ENTERTAINMENT Coach Washington wants to take her softball team out for pizza and soft drinks after the last game of the season. A large pizza costs \$12 and a pitcher of a soft drink costs \$3. She does not want to spend more than \$60. Write an inequality that represents this situation and graph the solution set. (Lesson 5-6)

Skills Check

Evaluate $a(b^x)$ for each of the given values. (Lesson 1-2)

72. $a = 1, b = 2, x = 4$

73. $a = 4, b = 1, x = 7$

74. $a = 5, b = 3, x = 0$

75. $a = 0, b = 6, x = 8$

76. $a = -2, b = 3, x = 1$

77. $a = -3, b = 5, x = 2$

Graphing Technology Lab
Cubic Functions

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- Other Calculator Keystrokes
- Graphing Technology Personal Tutor

You have studied linear functions and monomials. Some functions can be defined by the sums of monomials. One function that can be defined this way is a cubic function. A **cubic equation** has the form $ax^3 + bx^2 + cx + d = 0$, where $a \neq 0$. All cubic equations have at least one but no more than three real roots.

ACTIVITY

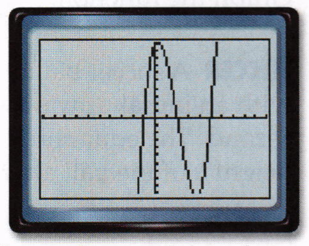
Solve $x^3 - 6x^2 + 3x + 10 = 0$ by graphing.

Step 1 Enter the related function in the Y= list.

KEYSTROKES: [Y=] [X,T,θ,n] [^] 3 [−] 6 [X,T,θ,n] [x^2] [+] 3 [X,T,θ,n] [+] 10

Step 2 Graph the function in the standard viewing window.

KEYSTROKES: [Zoom] 6



[−10, 10] scl: 1 by [−10, 10] scl: 1

Step 3 Find the zeros of the function by determining where the graph crosses the x-axis. Notice that this graph crosses the x-axis three times. Therefore, there are 3 real solutions for the equation.

KEYSTROKES: [2nd] [CALC] 2

Press the left arrow to move to the left of the intercept closest to the origin.

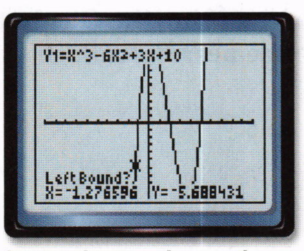
Press [ENTER].

Press the right arrow to move to the right of the intercept.

Do not go past another intercept. Press [ENTER].

Notice the arrows above the intercept. The intercept you are finding should be between these two arrows.

Press the left arrow to move as close as possible to the intercept. Press [ENTER].



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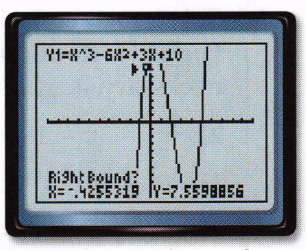

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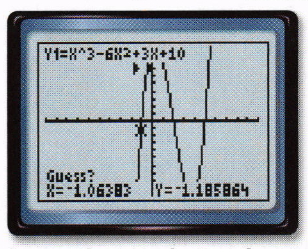

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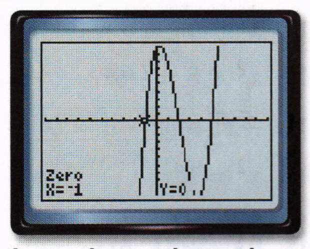


[−10, 10] scl:1 by [−10, 10] scl:1

One root is $x = -1$.

Step 4 Repeat Step 3 for each additional root.

The solutions for $x^3 - 6x^2 + 3x + 10 = 0$ are $x = -1$, 2, and 5.

Exercises

Solve each equation by graphing.

1. $x^3 - 4x^2 - 9x + 36 = 0$

2. $x^3 - 6x^2 - 6x - 7 = 0$

3. $x^3 + x^2 + x - 3 = 0$

4. $x^3 - 5x^2 - 2x + 24 = 0$

CHAPTER 9

Mid-Chapter Quiz

Lessons 9-1 through 9-5

Use a table of values to graph each equation. State the domain and range. (Lesson 9-1)

1. $y = x^2 + 3x + 1$

2. $y = 2x^2 - 4x + 3$

3. $y = -x^2 - 3x - 3$

4. $y = -3x^2 - x + 1$

Consider $y = x^2 - 5x + 4$. (Lesson 9-1)

5. Write the equation of the axis of symmetry.

6. Find the coordinates of the vertex. Is it a maximum or minimum point?

7. Graph the function.

8. SOCCER A soccer ball is kicked from ground level with an initial upward velocity of 90 feet per second. The equation $h = -16t^2 + 90t$ gives the height h of the ball after t seconds. (Lesson 9-1)

a. What is the height of the ball after one second?

b. How many seconds will it take for the ball to reach its maximum height?

c. When is the height of the ball 0 feet? What do these points represent in this situation?

Solve each equation by graphing. If integral roots cannot be found, estimate the roots to the nearest tenth. (Lesson 9-2)

9. $x^2 + 5x + 6 = 0$

10. $x^2 + 8 = -6x$

11. $-x^2 + 3x - 1 = 0$

12. $x^2 = 12$

13. BASEBALL Juan hits a baseball. The equation $h = -16t^2 + 120t$ models the height h, in feet, of the ball after t seconds. How long is the ball in the air? (Lesson 9-2)

14. CONSTRUCTION Christopher is repairing the roof on a shed. He accidentally dropped a box of nails from a height of 14 feet. This is represented by the equation $h = -16t^2 + 14$, where h is the height in feet and t is the time in seconds. Describe how the graph is related to $h = t^2$. (Lesson 9-3)

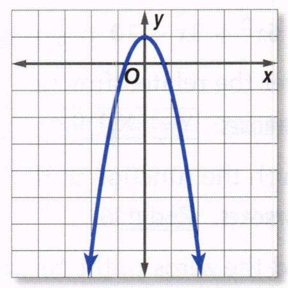
Describe how the graph of each function is related to the graph of $f(x) = x^2$. (Lesson 9-3)

15. $g(x) = x^2 + 3$

16. $h(x) = 2x^2$

17. $g(x) = x^2 - 6$

18. MULTIPLE CHOICE Which is an equation for the function shown in the graph? (Lesson 9-3)



A $y = -2x^2$

B $y = 2x^2 + 1$

C $y = x^2 - 1$

D $y = -2x^2 + 1$

Solve each equation by completing the square. Round to the nearest tenth. (Lesson 9-4)

19. $x^2 + 4x + 2 = 0$

20. $x^2 - 2x - 10 = 0$

21. $2x^2 + 4x - 5 = 7$

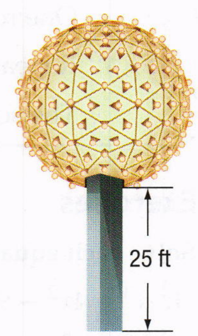
Solve each equation by using the Quadratic Formula. Round to the nearest tenth if necessary. (Lesson 9-5)

22. $x^2 - 3x - 18 = 0$

23. $x^2 - 10x = -24$

24. $2x^2 + 5x - 3 = 0$

25. PARTIES Della's parents are throwing a Sweet 16 party for her. At 10:00, a ball will slide 25 feet down a pole and light up. A function that models the drop is $h = -t^2 + 5t + 25$, where h is height in feet of the ball after t seconds. How many seconds will it take for the ball to reach the bottom of the pole? (Lesson 9-5)



9-6 Exponential Functions

Then
You simplified numerical expressions involving exponents. (Lesson 1-2)

Now
- Graph exponential functions.
- Identify data that display exponential behavior.

New Vocabulary
exponential function

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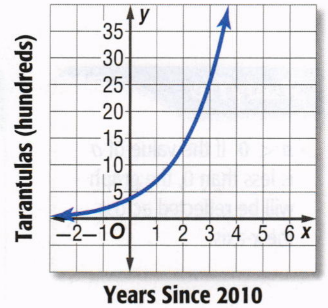
- Extra Examples
- Personal Tutor
- Self-check Quiz
- Homework Help

Why?

Tarantulas can appear scary with their large hairy bodies and legs, but they are harmless to humans. The graph shows a tarantula spider population that increases over time. Notice that the graph is neither linear nor quadratic.



The graph represents the function $y = 3(2)^x$. This is an example of an *exponential* function.

Graph Exponential Functions An **exponential function** is a function of the form $y = ab^x$, where $a \neq 0$, $b > 0$, and $b \neq 1$. Notice that the base is a constant and the exponent is a variable. Exponential functions are nonlinear and nonquadratic functions.

Key Concept — Exponential Function

For Your FOLDABLE

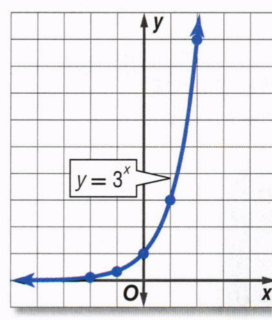
Words An exponential function is a function that can be described by an equation of the form $y = ab^x$, where $a \neq 0$, $b > 0$, and $b \neq 1$.

Examples $y = 2(3)^x$ $\quad$ $y = 4^x$ $\quad$ $y = \left(\frac{1}{2}\right)^x$

EXAMPLE 1 Graph with $a > 0$ and $b > 1$

a. Graph $y = 3^x$. Find the y-intercept, and state the domain and range.

x	3^x	y
−2	3^{-2}	$\frac{1}{9}$
−1	3^{-1}	$\frac{1}{3}$
0	3^0	1
1	3^1	3
2	3^2	9



Graph the ordered pairs, and connect the points with a smooth curve. The graph crosses the y-axis at 1, so the y-intercept is 1. The domain is all real numbers, and the range is all positive real numbers.

b. Use the graph to approximate the value of $3^{0.7}$.

The graph represents all real values of x and their corresponding values of y for $y = 3^x$. So, when $x = 0.7$, y is about 2. Use a calculator to confirm this value: $3^{0.7} \approx 2.157669$.

Check Your Progress

1A. Graph $y = 7^x$. Find the y-intercept, and state the domain and range.

1B. Use the graph to approximate the value of $y = 7^{0.5}$ to the nearest tenth. Use a calculator to confirm the value.

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The graphs of functions of the form $y = ab^x$, where $a > 0$ and $b > 1$, all have the same shape as the graph in Example 1. The greater the base or b-value, the faster the graph rises as you move from left to right on the graph. The graphs of functions of the form $y = ab^x$, where $a > 0$ and $0 < b < 1$, also have the same general shape.

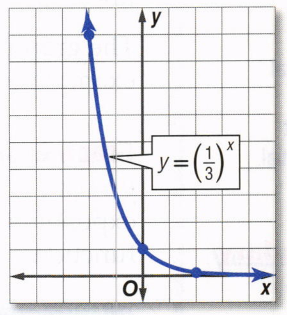
StudyTip

$a < 0$ If the value of a is less than 0, the graph will be reflected across the x-axis.

EXAMPLE 2 Graph with $a > 0$ and $0 < b < 1$

a. Graph $y = \left(\frac{1}{3}\right)^x$. Find the y-intercept, and state the domain and range.

x	$\left(\frac{1}{3}\right)^x$	y
-2	$\left(\frac{1}{3}\right)^{-2}$	9
0	$\left(\frac{1}{3}\right)^0$	1
2	$\left(\frac{1}{3}\right)^2$	$\frac{1}{9}$



The y-intercept is 1. The domain is all real numbers, and the range is all positive real numbers. Notice that as x increases, the y-values decrease less rapidly.

b. Use the graph to approximate the value of $\left(\frac{1}{3}\right)^{-1.5}$.

When $x = -1.5$, the value of y is about 5. Use a calculator to confirm this value:

KEYSTROKES: [( ] 1 [÷] 3 [ )] [^] -1.5 [ENTER] 5.196152.

Check Your Progress

2A. Graph $y = \left(\frac{1}{2}\right)^x - 1$. Find the y-intercept, and state the domain and range.

2B. Use the graph to approximate the value of $\left(\frac{1}{2}\right)^{-2.5} - 1$ to the nearest tenth. Use a calculator to confirm the value.

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Exponential functions occur in many real world situations.

Real-World Link

The United States is the largest soda consumer in the world. In a recent year, the United States accounted for one third of the world's total soda consumption.

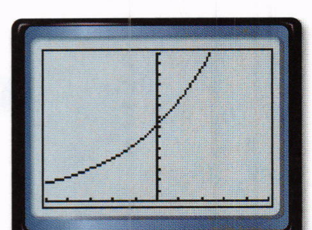
Source: Worldwatch Institute

Real-World EXAMPLE 3 Use Exponential Functions to Solve Problems

SODA The consumption of soda has increased each year since 2000. The function $C = 179(1.029)^t$ models the amount of soda consumed in the world, where C is the amount consumed in billions of liters and t is the number of years since 2000.

a. Graph the function. What values of C and t are meaningful in the context of the problem?

Since t represents time, $t > 0$. At $t = 0$, the consumption is 179 billion liters. Therefore, in the context of this problem, $C > 179$ is meaningful.



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b. How much soda was consumed in 2005?

$C = 179(1.029)^t$ **Original equation**

$= 179(1.029)^5$ **$t = 5$**

≈ 206.5 **Use a calculator.**

The world soda consumption in 2005 was approximately 206.5 billion liters.

Check Your Progress

3. A certain bacteria population doubles every 20 minutes. Beginning with 10 cells in a culture, the population can be represented by the function $B = 10(2)^t$, where B is the number of bacteria cells and t is the time in 20 minute increments. How many will there be after 2 hours?

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Identify Exponential Behavior Recall from Lesson 3-3 that linear functions have a constant rate of change. Exponential functions do not have constant rates of change, but they do have constant ratios.

EXAMPLE 4 Identify Exponential Behavior

Determine whether the set of data shown below displays exponential behavior. Write *yes* or *no*. Explain why or why not.

x	0	5	10	15	20	25
y	64	32	16	8	4	2

Method 1 Look for a pattern.

The domain values are at regular intervals of 5. Look for a common factor among the range values.

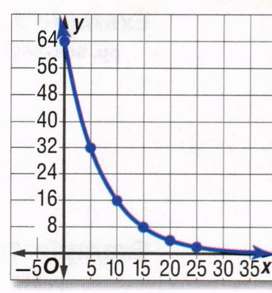
$$64 \quad 32 \quad 16 \quad 8 \quad 4 \quad 2$$
$$\times\tfrac{1}{2} \quad \times\tfrac{1}{2} \quad \times\tfrac{1}{2} \quad \times\tfrac{1}{2} \quad \times\tfrac{1}{2}$$

The range values differ by the common factor of $\frac{1}{2}$.

Since the domain values are at regular intervals and the range values differ by a positive common factor, the data are probably exponential. Its equation may involve $\left(\frac{1}{2}\right)^x$.

Method 2 Graph the data.

Plot the points and connect them with a smooth curve. The graph shows a rapidly decreasing value of y as x increases. This is a characteristic of exponential behavior in which the base is between 0 and 1.



Check Your Progress

4. Determine whether the set of data shown below displays exponential behavior. Write *yes* or *no*. Explain why or why not.

x	0	3	6	9	12	15
y	12	16	20	24	28	32

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Problem-Solving Tip

Make an Organized List Making an organized list of x-values and corresponding y-values is helpful in graphing the function. It can also help you identify patterns in the data.

Study Tip

Checking Answers The graph of an exponential function may resemble part of the graph of a quadratic function. Be sure to check for a pattern as well as to look at a graph.

Check Your Understanding

Examples 1 and 2 pp. 567–568

Graph each function. Find the y-intercept and state the domain and range. Then use the graph to determine the approximate value of the given expression to the nearest tenth. Use a calculator to confirm the value.

1. $y = 2^x; 2^{1.5}$
2. $y = -5^x; -5^{0.5}$
3. $y = -\left(\frac{1}{5}\right)^x; -\left(\frac{1}{5}\right)^{-0.5}$
4. $y = 3\left(\frac{1}{4}\right)^x; 3\left(\frac{1}{4}\right)^{0.5}$

Graph each function. Find the y-intercept, and state the domain and range.

5. $f(x) = 6^x + 3$
6. $f(x) = 2 - 2^x$

Example 3 pp. 568–569

7. **BIOLOGY** The function $f(t) = 100(1.05)^t$ models the growth of a fruit fly population, where $f(t)$ is the number of flies and t is time in days.
   a. What values for the domain and range are reasonable in the context of this situation? Explain.
   b. After two weeks, approximately how many flies are in this population?

Example 4 p. 569

Determine whether the set of data shown below displays exponential behavior. Write *yes* or *no*. Explain why or why not.

8.

x	1	2	3	4	5	6
y	−4	−2	0	2	4	6

9.

x	2	4	6	8	10	12
y	1	4	16	64	256	1024

Practice and Problem Solving

= **Step-by-Step Solutions** begin on page R12.
Extra Practice begins on page 815.

Examples 1 and 2 pp. 567–568

Graph each function. Find the y-intercept and state the domain and range. Then use the graph to determine the approximate value of the given expression to the nearest tenth. Use a calculator to confirm the value.

10. $y = 2 \cdot 8^x, 2(8)^{-0.5}$
11. $y = 2 \cdot \left(\frac{1}{6}\right)^x; 2\left(\frac{1}{6}\right)^{1.5}$
12. $y = \left(\frac{1}{12}\right)^x; \left(\frac{1}{12}\right)^{0.5}$
13. $y = -3 \cdot 9^x, -3(9)^{-0.5}$
14. $y = -4 \cdot 10^x, -4(10)^{-0.5}$
15. $y = 3 \cdot 11^x, 3(11)^{-0.2}$

Graph each function. Find the y-intercept and state the domain and range.

16. $y = 4^x + 3$
17. $y = \frac{1}{2}(2^x - 8)$
18. $y = 5(3^x) + 1$
19. $y = -2(3^x) + 5$

Example 3 pp. 568–569

20. **BIOLOGY** A population of bacteria in a culture increases according to the model $p = 300(2.7)^{0.02t}$, where t is the number of hours and $t = 0$ corresponds to 9:00 A.M.
   a. Use this model to estimate the number of bacteria at 11 A.M.
   b. Graph the function and name the p-intercept. Describe what the p-intercept represents, and describe a reasonable domain and range for this situation.

Example 4 p. 569

Determine whether the set of data shown below displays exponential behavior. Write *yes* or *no*. Explain why or why not.

21.

x	−4	0	4	8	12
y	2	−4	8	−16	32

22.

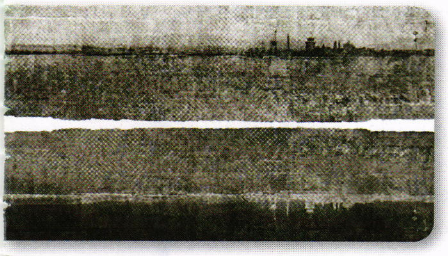
x	−6	−3	0	3
y	5	10	15	20

23.

x	−8	−6	−4	−2
y	0.25	0.5	1	2

24.

x	20	30	40	50	60
y	1	0.4	0.16	0.064	0.0256



Real-World Link

The world's largest photograph, named The Great Picture, was created by a group of photographers known as The Legacy Project. The photograph has an area of 3375 square feet.

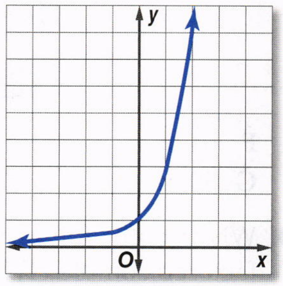
Source: Photoshop Support

25. **PHOTOGRAPHY** Jameka is enlarging a photograph to make a poster for school. She will enlarge the picture repeatedly at 150%. The function $P = 1.5^x$ models the new size of the picture being enlarged, where x is the number of enlargements. How many times as big is the picture after 4 enlargements?

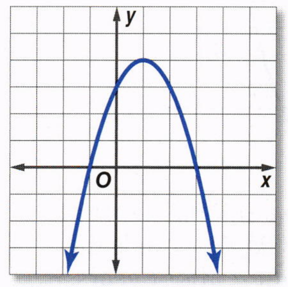
26. **FINANCIAL LITERACY** Daniel invested \$500 into a savings account. The equation $A = 500(1.005)^{12t}$ models the value of Daniel's investment A after t years. How much will Daniel's investment be worth in 8 years?

Identify each function as *linear, quadratic,* or *exponential.*

27.



28.



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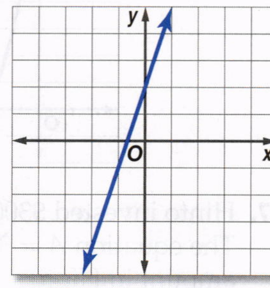


30. $y = 4^x$

31. $y = 2x(x - 1)$

32. $5x + y = 8$

33. **GRADUATION** The number of graduates at a high school has increased by a factor of 1.055 every year since 2001. In 2001, 110 students graduated. The function $N = 110(1.055)^t$ models the number of students N expected to graduate t years after 2001. How many students will graduate in 2012?

Describe the graph of each equation as a transformation of the graph of $y = 2^x$.

34. $y = 2^x + 6$

35. $y = 3(2)^x$

36. $y = -\frac{1}{4}(2)^x$

37. $y = -3 + 2^x$

38. $y = \left(\frac{1}{2}\right)^x$

39. $y = -5(2)^x$

40. **DEER** The deer population at a national park doubles every year. In 2000, there were 25 deer in the park. The function $N = 25(2)^t$ models the number of deer N in the park t years after 2000. What will the deer population be in 2015?

H.O.T. Problems

Use Higher-Order Thinking Skills

41. **CHALLENGE** Write an exponential function for which the graph passes through the points at $(0, 3)$ and $(1, 6)$.

42. **REASONING** Determine whether the graph of $y = ab^x$, where $a \neq 0$, $b > 0$, and $b \neq 1$, *sometimes, always,* or *never* has an x-intercept. Explain your reasoning.

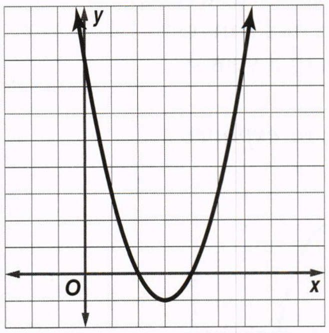
43. **OPEN ENDED** Find an exponential function that represents a real-world situation, and graph the function. Analyze the graph.

44. **REASONING** Compare and contrast a function of the form $y = ab^x + c$, where $a \neq 0$, $b > 0$, and $b \neq 1$ and a quadratic function of the form $y = ax^2 + c$.

45. **WRITING IN MATH** Explain how to determine whether a set of data displays exponential behavior.

Standardized Test Practice

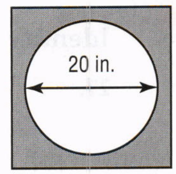
46. SHORT RESPONSE What are the zeros of the function graphed below?



47. Hinto invested \$300 into a savings account. The equation $A = 300(1.005)^{12t}$ models the amount in Hinto's account A after t years. How much will be in Hinto's account after 7 years?

A \$25,326 **C** \$385.01
B \$456.11 **D** \$301.52

48. GEOMETRY Ayana placed a circular piece of paper on a square picture as shown below. If the picture extends 4 inches beyond the circle on each side, what is the perimeter of the square picture?



F 64 in. **H** 94 in.
G 80 in. **J** 112 in.

49. Which of the following shows $4x^2 - 8x - 12$ factored completely?

A $4(x - 3)(x + 1)$
B $4(x + 3)(x - 1)$
C $(4x + 12)(x - 1)$
D $(x - 3)(4x + 4)$

Spiral Review

Solve each equation by using the Quadratic Formula. Round to the nearest tenth if necessary. (Lesson 9-5)

50. $6x^2 - 3x - 30 = 0$ **51.** $4x^2 + 18x = 10$ **52.** $2x^2 + 6x = 7$

Solve each equation by taking the square root of each side. Round to the nearest tenth if necessary. (Lesson 9-4)

53. $x^2 = 25$ **54.** $x^2 + 6x + 9 = 16$ **55.** $x^2 - 14x + 49 = 15$

Evaluate each product. Express the results in both scientific notation and standard form. (Lesson 7-3)

56. $(1.9 \times 10^2)(4.7 \times 10^6)$ **57.** $(4.5 \times 10^{-3})(5.6 \times 10^4)$ **58.** $(3.8 \times 10^{-4})(6.4 \times 10^{-8})$

59. DEMOLITION DERBY When a car hits an object, the damage is measured by the collision impact. For a certain car the collision impact I is given by $I = 2v^2$, where v represents the speed in kilometers per minute. What is the collision impact if the speed of the car is 4 kilometers per minute? (Lesson 7-1)

Use elimination to solve each system of equations. (Lesson 6-3)

60. $x + y = -3$
$x - y = 1$

61. $3a + b = 5$
$2a + b = 10$

62. $3x - 5y = 16$
$-3x + 2y = -10$

Skills Review

Find the next three terms of each arithmetic sequence. (Lesson 3-5)

63. $1, 3, 5, 7, \ldots$ **64.** $-6, -4, -2, 0, \ldots$ **65.** $6.5, 9, 11.5, 14, \ldots$

66. $10, 3, -4, -11, \ldots$ **67.** $\frac{1}{2}, \frac{5}{4}, 2, \frac{11}{4}, \ldots$ **68.** $1, \frac{3}{4}, \frac{1}{2}, \frac{1}{4}, \ldots$

9-7 Growth and Decay

Then
You analyzed exponential functions. (Lesson 9-6)

Now
- Solve problems involving exponential growth.
- Solve problems involving exponential decay.

New Vocabulary
exponential growth
compound interest
exponential decay

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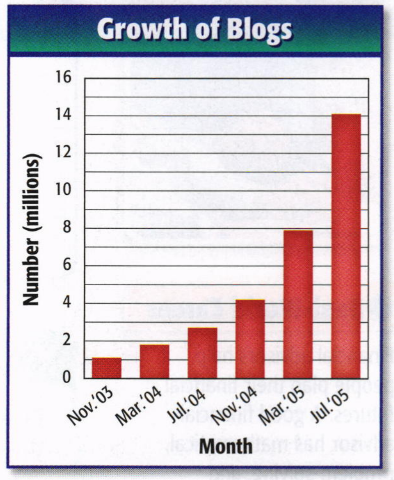
Why?

The number of Weblogs or blogs increased at a monthly rate of about 13.7% over 21 months. The average number of blogs per month can be modeled by $y = 1.1(1 + 0.137)^t$ or $y = 1.1(1.137)^t$, where y represents the total number of blogs in millions and t is the number of months since November 2003.



Exponential Growth The equation for the number of blogs is in the form $y = a(1 + r)^t$. This is the general equation for **exponential growth**.

Key Concept — Equation for Exponential Growth (For Your FOLDABLE)

$$y = a(1 + r)^t$$

a is the initial amount.
t is time.
y is the final amount.
r is the rate of change expressed as a decimal, $r > 0$.

Real-World EXAMPLE 1 Exponential Growth

CONTEST A radio station is sponsoring a contest. The prize begins as a $100 gift card. Once a day, the disc jockey announces a name, and the person has 15 minutes to call. If the person does not call within the allotted time, the prize increases by 2.5%.

a. Write an equation to represent the amount of the gift card in dollars after t days with no winners.

$y = a(1 + r)^t$	Equation for exponential growth
$y = 100(1 + 0.025)^t$	$a = 100$ and $r = 2.5\%$ or 0.025
$y = 100(1.025)^t$	Simplify.

In the equation $y = 100(1.025)^t$, y is the amount of the gift card and t is the number of days since the contest began.

b. How much will the gift card be worth if no one wins after 10 days?

$y = 100(1.025)^t$	Equation for amount of gift card
$= 100(1.025)^{10}$	$t = 10$
≈ 128.01	Use a calculator.

In 10 days, the gift card will be worth $128.01.

Check Your Progress

1. **TUITION** A college's tuition has risen 5% each year since 2000. If the tuition in 2000 was $10,850, write an equation for the amount of the tuition t years after 2000. Predict the cost of tuition for this college in 2015.

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Real-World Career

Financial advisors help people plan their financial futures. A good financial advisor has mathematical, problem-solving, and communication skills. A bachelor's degree is strongly preferred but not required.

Compound interest is interest earned or paid on both the initial investment and previously earned interest. It is an application of exponential growth.

Key Concept — Equation for Compound Interest

For Your FOLDABLE

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

A is the current amount.
P is the principal or initial amount.
r is the annual interest rate expressed as a decimal, $r > 0$.
n is the number of times the interest is compounded each year, and t is time in years.

Real-World EXAMPLE 2 — Compound Interest

FINANCE Maria's parents invested $14,000 at 6% per year compounded monthly. How much money will there be in the account after 10 years?

$A = P\left(1 + \frac{r}{n}\right)^{nt}$	Compound interest equation
$= 14{,}000\left(1 + \frac{0.06}{12}\right)^{12(10)}$	$P = 14{,}000$, $r = 6\%$ or 0.06, $n = 12$, and $t = 10$
$= 14{,}000(1.005)^{120}$	Simplify.
$\approx 25{,}471.55$	Use a calculator.

There will be about $25,471.55 in 10 years.

Check Your Progress

2. **FINANCE** Determine the amount of an investment if $300 is invested at an interest rate of 3.5% compounded monthly for 22 years.

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Exponential Decay In **exponential decay**, the original amount decreases by the same percent over a period of time. A variation of the growth equation can be used as the general equation for exponential decay.

StudyTip

Growth and Decay Since r is added to 1, the value inside the parentheses will be greater than 1 for exponential growth functions. For exponential decay functions, this value will be less than 1 since r is subtracted from 1.

Key Concept — Equation for Exponential Decay

For Your FOLDABLE

$$y = a(1 - r)^t$$

a is the initial amount.
t is time.
y is the final amount.
r is the rate of decay expressed as a decimal, $0 < r < 1$.

Real-World EXAMPLE 3 — Exponential Decay

SWIMMING A fully inflated child's raft for a pool is losing 6.6% of its air every day. The raft originally contained 4500 cubic inches of air.

a. Write an equation to represent the loss of air.

$y = a(1 - r)^t$	Equation for exponential decay
$= 4500(1 - 0.066)^t$	$a = 4500$ and $r = 6.6\%$ or 0.066
$= 4500(0.934)^t$	Simplify.

$y = 4500(0.934)^t$, where y is the air in the raft in cubic inches after t days.

b. Estimate the amount of air in the raft after 7 days.

$y = 4500(0.934)^t$ — **Equation for air loss**

$= 4500(0.934)^7$ — **$t = 7$**

≈ 2790 — **Use a calculator.**

The amount of air in the raft after 7 days will be about 2790 cubic inches.

Check Your Progress

3. POPULATION The population of Campbell County, Kentucky, has been decreasing at an average rate of about 0.3% per year. In 2000, its population was 88,647. Write an equation to represent the population since 2000. If the trend continues, predict the population in 2010.

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Check Your Understanding

Example 1 p. 573

1. SALARY Ms. Acosta received a job as a teacher with a starting salary of \$34,000. According to her contract, she will receive a 1.5% increase in her salary every year. How much will Ms. Acosta earn in 7 years?

Example 2 p. 574

2. MONEY Paul invested \$400 into an account with a 5.5% interest rate compounded monthly. How much will Paul's investment be worth in 8 years?

Example 3 p. 574

3. ENROLLMENT In 2000, 2200 students attended Polaris High School. The enrollment has been declining 2% annually.

a. Write an equation for the enrollment of Polaris High School t years after 2000.

b. If this trend continues, how many students will be enrolled in 2015?

Practice and Problem Solving

= **Step-by-Step Solutions** begin on page R12.
Extra Practice begins on page 815.

Example 1 p. 573

4. MEMBERSHIPS The Work-Out Gym sold 550 memberships in 2001. Since then the number of memberships sold has increased 3% annually.

a. Write an equation for the number of memberships sold at Work-Out Gym t years after 2001.

b. If this trend continues, predict how many memberships the gym will sell in 2020.

5. COMPUTERS The number of people who own computers has increased 23.2% annually since 1990. If half a million people owned a computer in 1990, predict how many people will own a computer in 2015.

6. COINS Camilo purchased a rare coin from a dealer for \$300. The value of the coin increases 5% each year. Determine the value of the coin in 5 years.

Example 2 p. 574

7. INVESTMENTS Theo invested \$6600 at an interest rate of 4.5% compounded monthly. Determine the value of his investment in 4 years.

8. COMPOUND INTEREST Paige invested \$1200 at an interest rate of 5.75% compounded quarterly. Determine the value of her investment in 7 years.

9. SAVINGS Brooke is saving money for a trip to the Bahamas that costs \$295.99. She puts \$150 into a savings account that pays 7.25% interest compounded quarterly. Will she have enough money in the account after 4 years? Explain.

10. INVESTMENTS Jin's investment of \$4500 has been losing its value at a rate of 2.5% each year. What will his investment be worth in 5 years?

Example 3
p. 574

11. **POPULATION** Hawaii has been experiencing a 1.06% annual increase in population. In 2000, the population was 1,211,537. If this trend continues, what will be the population of Hawaii in 2020?

12. **CARS** Leonardo purchases a car for \$18,995. The car depreciates at a rate of 18% annually. After 6 years, Manuel offers to buy the car for \$4500. Should Leonardo sell the car? Explain.

Real-World Link

A car loses 15% to 20% of its value each year. Brand, model, and the condition of the car all contribute to a used car's value. Some brands depreciate much slower than other brands.

Source: Bankrate

13. **HOUSING** The median house price in the United States increased an average of 8.6% each year between 2002 and 2004. Assume that this pattern continues.

    a. Write an equation for the median house price for t years after 2004.

    b. Predict the median house price in 2015.

Median House Price	
2002	\$187,600
2003	\$195,000
2004	\$221,000

Source: *Real Estate Journal*

14. **ELEMENTS** A radioactive element's half-life is the time it takes for one half of the element's quantity to decay. The half-life of Plutonium-241 is 14.4 years. The number of grams A of Plutonium-241 left after t years can be modeled by $A = p(0.5)^{\frac{t}{14.4}}$, where p is the original amount of the element.

    a. How much of a 0.2-gram sample remains after 72 years?

    b. How much of a 5.4-gram sample remains after 1095 days?

15. **FINANCIAL LITERACY** Marta is planning to buy a new car. She will finance \$16,000 at an annual interest rate of 7% over a period of 60 months. In the formula, P is the amount of each payment, r is the annual interest rate in decimal form, and t is the time in years of the loan.

$$\text{Amount financed} = P\left[\frac{1-\left(1+\frac{r}{12}\right)^{-12t}}{\frac{r}{12}}\right]$$

    a. Use the formula to find her monthly payment.

    b. Assuming that she does not pay ahead, what will she have paid on the car?

H.O.T. Problems

Use **H**igher-**O**rder **T**hinking Skills

16. **REASONING** Determine the growth rate (as a percent) of a population that quadruples every year. Explain.

17. **CHALLENGE** Santos invested \$1200 into an account with an interest rate of 8% compounded monthly. Use a calculator to approximate how long it will take for Santos' investment to reach \$2500.

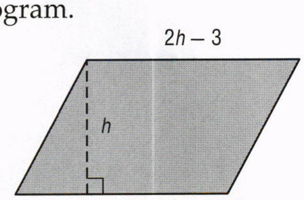
18. **REASONING** The amount of water in a container doubles every minute. After 8 minutes, the container is full. After how many minutes was the container half full? Explain.

19. **OPEN ENDED** Create a real-world situation that can be modeled by $y = 200(1.05)^t$.

20. **WRITING IN MATH** Compare and contrast the exponential growth formula and the exponential decay formula.

Standardized Test Practice

21. GEOMETRY The parallelogram has an area of 35 square inches. Find the height h of the parallelogram.



A 3.5 inches
B 4 inches
C 5 inches
D 7 inches

22. What are the roots of $x^2 + 2x = 48$?

F 6 and 8
G -6 and -8
H 6 and -8
J -6 and 8

23. Thi purchased a car for $22,900. The car depreciated at an annual rate of 16%. Which of the following equations models the value of Thi's car after 5 years?

A $A = 22{,}900(1.16)^5$
B $A = 22{,}900(0.16)^5$
C $A = 16(22{,}900)^5$
D $A = 22{,}900(0.84)^5$

24. GRIDDED RESPONSE A deck measures 12 feet by 18 feet. If a painter charges $2.65 per square foot, including tax, how much will it cost in dollars to have the deck painted?

Spiral Review

Graph each function. Find the y-intercept and state the domain and range. (Lesson 9-6)

25. $y = 3^x$
26. $y = \left(\frac{1}{2}\right)^x$
27. $y = 6^x$

Solve each equation by using the Quadratic Formula. Round to the nearest tenth if necessary. (Lesson 9-5)

28. $4x^2 + 15x = 25$
29. $3x^2 - 4x = 5$
30. $2x^2 = -2x + 11$
31. $4x^2 + 16x = -16$
32. $5x^2 + 5x = 60$
33. $2x^2 = 3x + 15$

34. EVENT PLANNING A hall does not charge a rental fee as long as at least $4000 is spent on food. For the prom, the hall charges $28.95 per person for a buffet. How many people must attend the prom to avoid a rental fee for the hall? (Lesson 5-2)

Determine whether the graphs of each pair of equations are *parallel*, *perpendicular*, or *neither*. (Lesson 4-4)

35. $y = -2x + 11$
$y + 2x = 23$

36. $3y = 2x + 14$
$-3x - 2y = 2$

37. $y = -5x$
$y = 5x - 18$

38. AGES The table shows equivalent ages for horses and humans. Write an equation that relates human age to horse age and find the equivalent horse age for a human who is 16 years old. (Lesson 3-4)

Horse age (x)	0	1	2	3	4	5
Human age (y)	0	3	6	9	12	15

Find the total price of each item. (Lesson 2-7)

39. umbrella: $14.00
tax: 5.5%

40. sandals: $29.99
tax: 5.75%

41. backpack: $35.00
tax: 7%

Skills Check

Graph each set of ordered pairs. (Lesson 1-6)

42. (3, 0), (0, 1), (−4, −6)
43. (0, −2), (−1, −6), (3, 4)
44. (2, 2), (−2, −3), (−3, −6)

9-8 Geometric Sequences as Exponential Functions

Then

You related arithmetic sequences to linear functions. (Lesson 3-5)

Now

- Identify and generate geometric sequences.
- Relate geometric sequences to exponential functions.

New Vocabulary

geometric sequence
common ratio

Math Online

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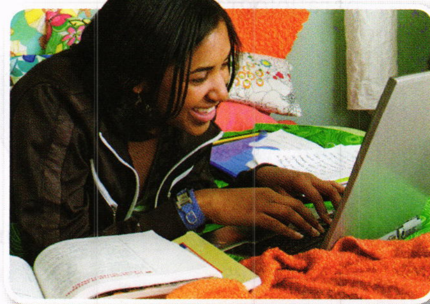
- Extra Examples
- Personal Tutor
- Self-Check Quiz
- Homework Help

Why?



You send a chain e-mail to a friend who forwards the e-mail to five more people. Each of these five people forwards the e-mail to five more people. The number of new e-mails generated forms a geometric sequence.

Recognize Geometric Sequences The first person generates 5 e-mails. If each of these people sends the e-mail to 5 more people, 25 e-mails are generated. If each of the 25 people sends 5 e-mails, 125 e-mails are generated. The sequence of e-mails generated, 1, 5, 25, 125, … is an example of a **geometric sequence**.

In a geometric sequence, the first term is nonzero and each term after the first is found by multiplying the previous term by a nonzero constant r called the **common ratio**. The common ratio can be found by dividing any term by its previous term.

EXAMPLE 1 Identify Geometric Sequences

Determine whether each sequence is *arithmetic*, *geometric*, or *neither*. Explain.

a. 256, 128, 64, 32, …

Find the ratios of consecutive terms.

256 128 64 32

$\frac{128}{256} = \frac{1}{2}$ $\frac{64}{128} = \frac{1}{2}$ $\frac{32}{64} = \frac{1}{2}$

Since the ratios are constant, the sequence is geometric. The common ratio is $\frac{1}{2}$.

b. 4, 9, 12, 18, …

Find the ratios of consecutive terms.

4 9 12 18

$\frac{9}{4} = 2\frac{1}{4}$ $\frac{12}{9} = 1\frac{1}{3}$ $\frac{18}{12} = 1\frac{1}{2}$

The ratios are not constant, so the sequence is not geometric.

Find the differences of consecutive terms.

4 9 12 18

$9 - 4 = 5$ $12 - 9 = 3$ $18 - 12 = 6$

There is no common difference, so the sequence is not arithmetic. Thus, the sequence is neither geometric nor arithmetic.

Check Your Progress

1A. 1, 3, 9, 27, … **1B.** −20, −15, −10, −5, … **1C.** 2, 8, 14, 22, …

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Once the common ratio is known, more terms of a sequence can be generated. The recursive formula can be rewritten as $a_n = a_1 r^{n-1}$, where n is a counting number and r is the common ratio.

StudyTip

Common Ratio If the terms of a geometric sequence alternate between positive and negative terms or vice versa, the common ratio is negative.

EXAMPLE 2 Find Terms of Geometric Sequences

Find the next three terms in each geometric sequence.

a. 1, −4, 16, −64, ...

Step 1 Find the common ratio.

$$1 \qquad -4 \qquad 16 \qquad -64$$

$$\frac{-4}{1} = -4 \qquad \frac{16}{-4} = -4 \qquad \frac{-64}{16} = -4$$

Step 2 Multiply each term by the common ratio to find the next three terms.

$$-64 \qquad 256 \qquad -1024 \qquad 4096$$

$$\times(-4) \qquad \times(-4) \qquad \times(-4)$$

The next three terms are 256, −1024, and 4096.

b. 9, 3, 1, $\frac{1}{3}$ ...

Step 1 Find the common ratio.

$$9 \qquad 3 \qquad 1 \qquad \frac{1}{3}$$

$$\frac{3}{9} = \frac{1}{3} \qquad \frac{1}{3} = \frac{1}{3} \qquad \frac{\frac{1}{3}}{3} = \frac{1}{3}$$

The value of r is $\frac{1}{3}$.

Step 2 Multiply each term by the common ratio to find the next three terms.

$$\frac{1}{3} \qquad \frac{1}{9} \qquad \frac{1}{27} \qquad \frac{1}{81}$$

$$\times\frac{1}{3} \qquad \times\frac{1}{3} \qquad \times\frac{1}{3}$$

The next three terms are $\frac{1}{9}$, $\frac{1}{27}$, and $\frac{1}{81}$.

Check Your Progress

2A. −3, 15, −75, 375, ...

2B. 24, 36, 54, 81, ...

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The Granger Collection, New York

Math History Link

Thomas Robert Malthus (1766–1834) Malthus studied populations and had pessimistic views about the future population of the world. In his work, he stated: "Population increases in a geometric ratio, while the means of subsistence increases in an arithmetic ratio."

Geometric Sequences and Functions Finding the nth term of a geometric sequence would be tedious if we used the above method. The table below shows a rule for finding the nth term of a geometric sequence.

Position, n	1	2	3	4	...	n
Term, a_n	a_1	$a_1 r$	$a_1 r^2$	$a_1 r^3$	...	$a_1 r^{n-1}$

Notice that the common ratio between the terms is r. The table shows that to get the nth term, you multiply the first term by the common ratio r raised to the power $n - 1$. A geometric sequence can be defined by an exponential function in which n is the independent variable, a_n is the dependent variable, and r is the base. The domain is the counting numbers.

Key Concept — nth term of a Geometric Sequence

For Your FOLDABLE

The nth term a_n of a geometric sequence with first term a_1 and common ratio r is given by the following formula, where n is any positive integer and $a_1, r \neq 0$.

$$a_n = a_1 r^{n-1}$$

Watch Out!

Negative Common Ratio If the common ratio is negative, as in Example 3, make sure to enclose the common ratio in parentheses. $(-2)^8 \neq -2^8$

EXAMPLE 3 Find the nth Term of a Geometric Sequence

a. Write an equation for the nth term of the sequence $-6, 12, -24, 48, \ldots$.

The first term of the sequence is -6. So, $a_1 = -6$. Now find the common ratio.

$$-6 \qquad 12 \qquad -24 \qquad 48$$

$$\frac{12}{-6} = -2 \qquad \frac{-24}{12} = -2 \qquad \frac{48}{-24} = -2$$

The common ratio is -2.

$a_n = a_1 r^{n-1}$ — Formula for nth term

$a_n = -6(-2)^{n-1}$ — $a_1 = -6$ and $r = 2$

b. Find the ninth term of this sequence.

$a_n = a_1 r^{n-1}$ — Formula for nth term

$a_9 = -6(-2)^{9-1}$ — For the nth term, $n = 9$.

$= -6(-2)^8$ — Simplify.

$= -6(256)$ — $(-2)^8 = 256$

$= -1536$

Check Your Progress

3. Write an equation for the nth term of the geometric sequence $96, 48, 24, 12, \ldots$. Then find the tenth term of the sequence.

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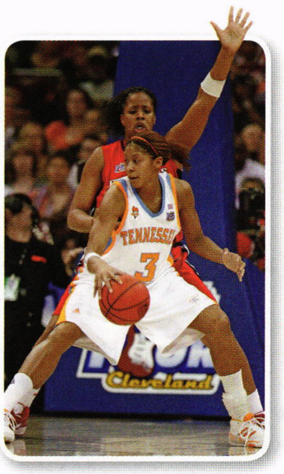


Real-World Link

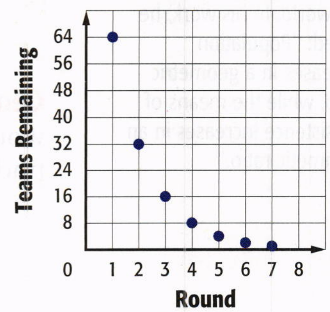
The first NCAA Division I women's basketball tournament was held in 1982. The University of Tennessee has won the most national titles with 8 titles as of 2008.

Source: NCAA Sports

Real-World EXAMPLE 4 Graph a Geometric Sequence

BASKETBALL The NCAA women's basketball tournament begins with 64 teams. In each round, one half of the teams are left to compete, until only one team remains. Draw a graph to represent how many teams are left in each round.

Compared to the previous rounds, one half of the teams remain. So, $r = \frac{1}{2}$. Therefore, the geometric sequence that models this situation is 64, 32, 16, 8, 4, 2, 1. So in round two, 32 teams compete, in round three 16 teams compete and so forth. Use this information to draw a graph.



Check Your Progress

4. TENNIS A tennis ball is dropped from a height of 12 feet. Each time the ball bounces back to 80% of the height from which it fell. Draw a graph to represent the height of the ball after each bounce.

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Check Your Understanding

Example 1 p. 580

Determine whether each sequence is *arithmetic*, *geometric*, or *neither*. Explain.

1. 200, 40, 8, … **2.** 2, 4, 16, … **3.** −6, −3, 0, 3, … **4.** 1, −1, 1, −1, …

Example 2 p. 581

Find the next three terms in each geometric sequence.

5. 10, 20, 40, 80, … **6.** 100, 50, 25, … **7.** $4, -1, \frac{1}{4}, \ldots$ **8.** −7, 21, −63, …

Example 3 p. 582

Write an equation for the *n*th term of each geometric sequence, and find the indicated term.

9. the fifth term of −6, −24, −96, …

10. the seventh term of −1, 5, −25, …

11. the tenth term of 72, 48, 32, …

12. the ninth term of 112, 84, 63, …

Example 4 p. 582

13. **EXPERIMENT** In a physics class experiment, Diana drops a ball from a height of 16 feet. Each bounce has 70% the height of the previous bounce. Draw a graph to represent the height of the ball after each bounce.

Practice and Problem Solving

● = **Step-by-Step Solutions** begin on page R12.
Extra Practice begins on page 815.

Example 1 p. 580

Determine whether each sequence is *arithmetic*, *geometric*, or *neither*. Explain.

14. 4, 1, 2, … **15.** 10, 20, 30, 40, … **16.** 4, 20, 100, …

17. 212, 106, 53, … **18.** −10, −8, −6, −4, … **19.** 5, −10, 20, 40, …

Example 2 p. 581

Find the next three terms in each geometric sequence.

20. 2, −10, 50, … **21** 36, 12, 4, … **22.** 4, 12, 36, …

23. 400, 100, 25, … **24.** −6, −42, −294, … **25.** 1024, −128, 16, …

Example 3 p. 582

26. The first term of a geometric series is 1 and the common ratio is 9. What is the 8th term of the sequence?

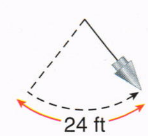
27. The first term of a geometric series is 2 and the common ratio is 4. What is the 14th term of the sequence?

28. What is the 15th term of the geometric sequence −9, 27, −81, …?

29. What is the 10th term of the geometric sequence 6, −24, 96, …?

Example 4 p. 582

30. **PENDULUM** The first swing of a pendulum is shown. On each swing after that, the arc length is 60% of the length of the previous swing. Draw a graph that represents the arc length after each swing.

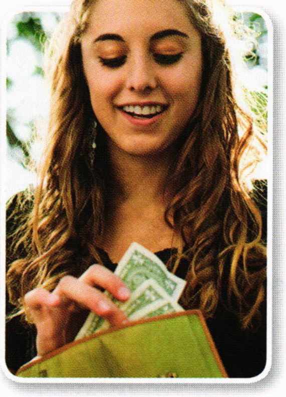


31. Find the eighth term of a geometric sequence for which $a_3 = 81$ and $r = 3$.

32. **MAPS** At an online mapping site, Mr. Mosley notices that when he clicks a spot on the map, the map zooms in on that spot. The magnification increases by 20% each time.

a. Write a formula for the *n*th term of the geometric sequence that represents the magnification of each zoom level. (*Hint:* The common ratio is not just 0.2.)

b. What is the fourth term of this sequence? What does it represent?



Real-World Link

The average American 9- to 14-year-old gets $9.15 each week for allowance.

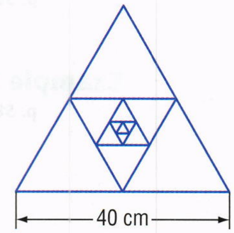
Source: *Money Magazine*

33. **ALLOWANCE** Danielle's parents have offered her two different options to earn her allowance for a 9-week period over the summer. She can either get paid $30 each week or $1 the first week, $2 for the second week, $4 for the third week, and so on.

a. Does the second option form a geometric sequence? Explain.

b. Which option should Danielle choose? Explain.

34. **SIERPINSKI'S TRIANGLE** Consider the inscribed equilateral triangles at the right. The perimeter of each triangle is one half of the perimeter of the next larger triangle. What is the perimeter of the smallest triangle?



35. If the second term of a geometric sequence is 3 and the third term is 1, find the first and fourth terms of the sequence.

36. If the third term of a geometric sequence is -12 and the fourth term is 24, find the first and fifth terms of the sequence.

37. **EARTHQUAKES** The Richter scale is used to measure the force of an earthquake. The table shows the increase in magnitude for the values on the Richter scale.

Richter Number (x)	Increase in Magnitude (y)	Rate of Change (slope)
1	1	—
2	10	9
3	100	
4	1000	
5	10,000	

a. Copy and complete the table. Remember that the rate of change is the change in y divided by the change in x.

b. Plot the ordered pairs (Richter number, increase in magnitude).

c. Describe the graph that you made of the Richter scale data. Is the rate of change between any two points the same?

d. Write an exponential equation that represents the Richter scale.

H.O.T. Problems

Use **H**igher-**O**rder **T**hinking Skills

38. **CHALLENGE** Write a sequence that is both geometric and arithmetic. Explain your answer.

39. **FIND THE ERROR** Haro and Matthew are finding the ninth term of the geometric sequence $-5, 10, -20, \ldots$. Is either of them correct? Explain your reasoning.

Haro

$$r = \frac{10}{-5} \text{ or } -2$$
$$a_9 = -5(-2)^{9-1}$$
$$= -5(512)$$
$$= -2560$$

Matthew

$$r = \frac{10}{-5} \text{ or } -2$$
$$a_9 = -5 \cdot (-2)^{9-1}$$
$$= -5 \cdot -256$$
$$= 1280$$

40. **REASONING** Write a sequence of numbers that form a pattern but are neither arithmetic nor geometric. Explain the pattern.

41. **OPEN ENDED** Write a geometric sequence that has a common ratio of $\frac{3}{4}$.

42. **WRITING IN MATH** Summarize how to find a specific term of a geometric sequence.

Standardized Test Practice

43. Find the eleventh term of the sequence 3, −6, 12, −24, … .

A 1024
B 3072
C 33
D −6144

44. What is the total amount of the investment shown in the table below if interest is compounded monthly?

Principal	\$500
Length of Investment	4 years
Annual Interest Rate	5.25%

F \$613.56
G \$616.00
H \$616.56
J \$718.75

45. SHORT RESPONSE Gloria has \$6.50 in quarters and dimes. If she has 35 coins in total, how many of each coin does she have?

46. A sidewalk is being built along the inside edges of all four sides of a rectangular lawn. The lawn is 32 feet long and 24 feet wide. The remaining lawn will have an area of 425 square feet. How wide will the sidewalk be?

A 3.5 feet
B 17 feet
C 24.5 feet
D 25 feet

Spiral Review

Find the next three terms in each geometric sequence. (Lesson 9-7)

47. 2, 6, 18, 54, …

48. −5, −10, −20, −40, …

49. $1, -\frac{1}{2}, \frac{1}{4}, -\frac{1}{8}, \ldots$

50. −3, 1.5, −0.75, 0.375, …

51. 1, 0.6, 0.36, 0.216, …

52. 4, 6, 9, 13.5, …

Graph each function. Find the y-intercept and state the domain and range. (Lesson 9-6)

53. $y = \left(\frac{1}{4}\right)^x - 5$

54. $y = 2(4)^x$

55. $y = \frac{1}{2}(3^x)$

56. LANDSCAPING A blue spruce grows an average of 6 inches per year. A hemlock grows an average of 4 inches per year. If a blue spruce is 4 feet tall and a hemlock is 6 feet tall, write a system of equations to represent their growth. Find and interpret the solution in the context of the situation. (Lesson 6-2)

57. MONEY City Bank requires a minimum balance of \$1500 to maintain free checking services. If Mr. Hayashi is going to write checks for the amounts listed in the table, how much money should he start with in order to have free checking? (Lesson 5-1)

Check	Amount
750	\$1300
751	\$947

Write an equation in slope-intercept form of the line with the given slope and y-intercept. (Lesson 4-1)

58. slope: 4, y-intercept: 2

59. slope: −3, y-intercept: $-\frac{2}{3}$

60. slope: $-\frac{1}{4}$, y-intercept: −5

61. slope: $\frac{1}{2}$, y-intercept: −9

62. slope: $-\frac{2}{5}$, y-intercept: $\frac{3}{4}$

63. slope: −6, y-intercept: −7

Skills Check

Evaluate $a(1 + r)^t$ to the nearest hundredth for each of the given values. (Lesson 1-2)

64. $a = 20, r = 0.25, t = 5$

65. $a = 1000, r = 0.65, t = 4$

66. $a = 200, r = 0.35, t = 8$

67. $a = 60, r = 0.2, t = 10$

68. $a = 8, r = 0.5, t = 2$

69. $a = 500, r = 0.55, t = 12$

9-9 Analyzing Functions with Successive Differences

Then

You graphed linear, quadratic, and exponential functions. (Lessons 3-2, 9-1, 9-6)

Now

- Identify linear, quadratic, and exponential functions from given data.
- Write equations that model data.

Math Online

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- Extra Examples
- Personal Tutor
- Self-Check Quiz
- Homework Help

Why?

Every year the golf team sells candy to raise money for charity. By knowing what type of function models the sales of the candy, they can determine the best price of the candy.

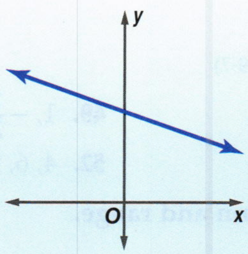
Identify Functions You can use linear functions, quadratic functions, and exponential functions to model data. The general forms of the equations and a graph of each function type are listed below.

Concept Summary — Linear and Nonlinear Functions

For Your FOLDABLE

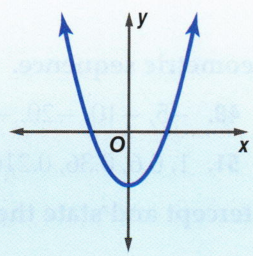
Linear Function

$y = mx + b$



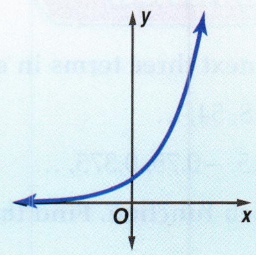
Quadratic Function

$y = ax^2 + bx + c$



Exponential Function

$y = ab^x$, when $b > 0$

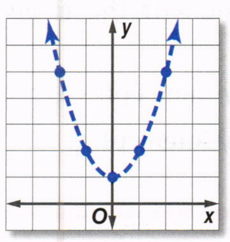


EXAMPLE 1 Choose a Model Using Graphs

Graph each set of ordered pairs. Determine whether the ordered pairs represent a *linear* function, a *quadratic* function, or an *exponential* function.

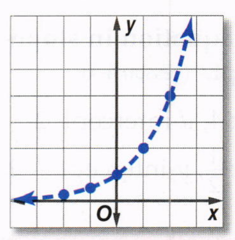
a. $\{(-2, 5), (-1, 2), (0, 1), (1, 2), (2, 5)\}$

The ordered pairs appear to represent a quadratic function.



b. $\left\{\left(-2, \frac{1}{4}\right), \left(-1, \frac{1}{2}\right), (0, 1), (1, 2), (2, 4)\right\}$

The ordered pairs appear to represent an exponential function.



Check Your Progress

1A. $(-2, -3), (-1, -1), (0, 1), (1, 3)$

1B. $(-1, 0.25), (0, 1), (1, 4), (2, 16)$

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Another way to determine which model best describes data is to use patterns. The differences of successive y-values are called *first differences*. The differences of successive first differences are called *second differences*.

- If the differences of successive y-values are all equal, the data represent a linear function.
- If the second differences are all equal, but the first differences are not equal, the data represent a quadratic function.
- If the ratios of successive y-values are all equal and $r \neq 1$, the data represent an exponential function.

> **Watch Out!**
>
> **x-Values** Before you check for successive differences or ratios, make sure the x-values are increasing by the same amount.

EXAMPLE 2 Choose a Model Using Differences or Ratios

Look for a pattern in each table of values to determine which kind of model best describes the data.

a.

x	−2	−1	0	1	2
y	−8	−3	2	7	12

First differences: $-8 \quad -3 \quad 2 \quad 7 \quad 12$

$5 \quad 5 \quad 5 \quad 5$

Since the first differences are all equal, the table of values represents a linear function.

b.

x	−1	0	1	2	3
y	8	4	2	1	0.5

First differences: $8 \quad 4 \quad 2 \quad 1 \quad 0.5$

$-4 \quad -2 \quad -1 \quad -0.5$

The first differences are not all equal. So, the table of values does not represent a linear function. Find the second differences and compare.

First differences: $-4 \quad -2 \quad -1 \quad -0.5$

Second differences: $2 \quad 1 \quad 0.5$

The second differences are not all equal. So, the table of values does not represent a quadratic function. Find the ratios of the y-values and compare.

$8 \quad 4 \quad 2 \quad 1 \quad 0.5$

Ratios: $\frac{4}{8} = \frac{1}{2} \quad \frac{2}{4} = \frac{1}{2} \quad \frac{1}{2} \quad \frac{0.5}{1} = \frac{1}{2}$

The ratios of successive y-values are equal. Therefore, the table of values can be modeled by an exponential function.

Check Your Progress

2A.

x	−3	−2	−1	0	1
y	−3	−7	−9	−9	−7

2B.

x	−2	−1	0	1	2
y	−18	−13	−8	−3	2

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Write Equations Once you find the model that best describes the data, you can write an equation for the function. For a quadratic function in this lesson, the equation will have the form $y = ax^2$.

EXAMPLE 3 Write an Equation

Determine which kind of model best describes the data. Then write an equation for the function that models the data.

x	-4	-3	-2	-1	0
y	32	18	8	2	0

Step 1 Determine which model fits the data.

32 18 8 2 0

First differences: -14 -10 -6 -2

Second differences: 4 4 4

Since the second differences are equal, a quadratic function models the data.

Step 2 Write an equation for the function that models the data.

The equation has the form $y = ax^2$. Find the value of a by choosing one of the ordered pairs from the table of values. Let's use $(-1, 2)$.

$y = ax^2$ **Equation for quadratic function**

$2 = a(-1)^2$ **$x = -1$ and $y = 2$**

$2 = a$ An equation that models the data is $y = 2x^2$.

Watch Out!

Finding a
In Example 3, the point (0, 0) cannot be used to find the value of a. You will have to divide each side by 0, giving you an undefined value for a.

Check Your Progress

3A.

x	-2	-1	0	1	2
y	11	7	3	-1	-5

3B.

x	-3	-2	-1	0	1
y	0.375	0.75	1.5	3	6

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Real-World EXAMPLE 4 Write an Equation for a Real-World Situation

BOOK CLUB The table shows the number of book club members for four consecutive years. Determine which model best represents the data. Then write a function that models the data.

Time (years)	0	1	2	3	4
Members	5	10	20	40	80

Understand We need to find a model for the data, and then write a function.

Plan Find a pattern using successive differences or ratios. Then use the general form of the equation to write a function.

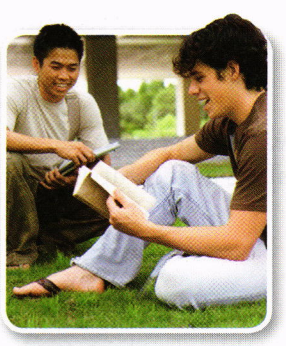
Solve The constant ratio is 2. This is the value of the base. An exponential function of the form $y = ab^x$ models the data.

$y = ab^x$ **Equation for exponential function**

$5 = a(2)^0$ **$x = 0$, $y = 5$, and $b = 2$**

$5 = a$ The equation that models the data is $y = 5 \cdot 2^x$.

Check You used (0, 5) to write the function. Verify that every other ordered pair satisfies the equation.



Real-World Link

A poll by the National Education Association found that 87% of all teens polled found reading relaxing, 85% viewed reading as rewarding, and 79% found reading exciting.

Source: *American Demographics*

Check Your Progress

4. ADVERTISING The table shows the cost of placing an ad in a newspaper. Determine a model that best represents the data and write a function that models the data.

No. of Lines	5	6	7	8
Total Cost ($)	14.50	16.60	18.70	20.80

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Check Your Understanding

Example 1 p. 586

Graph each set of ordered pairs. Determine whether the ordered pairs represent a *linear* function, a *quadratic* function, or an *exponential* function.

1. $(-2, 8), (-1, 5), (0, 2), (1, -1)$
2. $(-3, 7), (-2, 3), (-1, 1), (0, 1), (1, 3)$
3. $(-3, 8), (-2, 4), (-1, 2), (0, 1), (1, 0.5)$
4. $(0, 2), (1, 2.5), (2, 3), (3, 3.5)$

Example 2 p. 587

Look for a pattern in each table of values to determine which kind of model best describes the data.

5.

x	0	1	2	3	4
y	5	8	17	32	53

6.

x	−3	−2	−1	0
y	−6.75	−7.5	−8.25	−9

7.

x	−1	0	1	2	3
y	3	6	12	24	48

8.

x	3	4	5	6	7
y	−1.5	0	2.5	6	10.5

Example 3 p. 588

Determine which kind of model best describes the data. Then write an equation for the function that models the data.

9.

x	−1	0	1	2	3
y	1	3	9	27	81

10.

x	−5	−4	−3	−2	−1
y	125	80	45	20	5

11.

x	−3	−2	−1	0	1
y	1	1.5	2	2.5	3

12.

x	−1	0	1	2
y	−1.25	−1	−0.75	−0.5

Examples 4 p. 588

13. **PLANTS** The table shows the height of a plant for four consecutive weeks. Determine which kind of function best models the height. Then write a function that models the data.

Week	0	1	2	3	4
Height (in.)	3	3.5	4	4.5	5

Practice and Problem Solving

= **Step-by-Step Solutions** begin on page R12.
Extra Practice begins on page 815.

Example 1 p. 586

Graph each set of ordered pairs. Determine whether the ordered pairs represent a *linear* function, a *quadratic* function, or an *exponential* function.

14. $(-1, 1), (0, -2), (1, -3), (2, -2), (3, 1)$
15. $(1, 2.75), (2, 2.5), (3, 2.25), (4, 2)$
16. $(-3, 0.25), (-2, 0.5), (-1, 1), (0, 2)$
17. $(-3, -11), (-2, -5), (-1, -3), (0, -5)$
18. $(-2, 6), (-1, 1), (0, -4), (1, -9)$
19. $(-1, 8), (0, 2), (1, 0.5), (2, 0.125)$

Examples 2 and 3 pp. 587–588

Look for a pattern in each table of values to determine which kind of model best describes the data. Then write an equation for the function that models the data.

20.

x	−3	−2	−1	0
y	−8.8	−8.6	−8.4	−8.2

21.

x	−2	−1	0	1	2
y	10	2.5	0	2.5	10

22.

x	−1	0	1	2	3
y	0.75	3	12	48	192

23.

x	−2	−1	0	1	2
y	0.008	0.04	0.2	1	5

24.

x	0	1	2	3	4
y	0	4.2	16.8	37.8	67.2

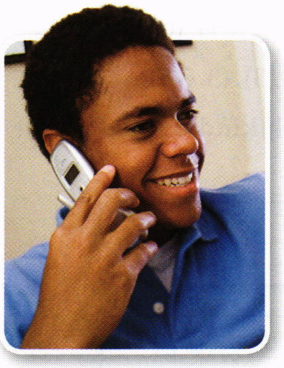
25.

x	−3	−2	−1	0	1
y	14.75	9.75	4.75	−0.25	−5.25

Example 4
p. 588

26. **WEBSITES** A company launched a new Web site. They tracked the number of visitors to its Web site over a period of 4 days. Determine which kind of model best represents the number of visitors to the Web site with respect to time. Then write a function that models the data.

Day	0	1	2	3	4
Visitors (in thousands)	0	0.9	3.6	8.1	14.4



27. **LONG DISTANCE** The cost of a long-distance telephone call depends on the length of the call. The table shows the cost for up to 6 minutes.

Length of call (min)	1	2	3	4	5	6
Cost ($)	0.12	0.24	0.36	0.48	0.60	0.72

a. Graph the data and determine which kind of function best models the data.

b. Write an equation for the function that models the data.

c. Use your equation to determine how much a 10-minute call would cost.

Real-World Link

The top three forms of communication used by teenagers are e-mail, cell phones, and landline telephones.

Source: Harris Interactive

28. **DEPRECIATION** The value of a car depreciates over time. The table shows the value of a car over a period of time.

Year	0	1	2	3	4
Value ($)	18,500	15,910	13,682.60	11,767.04	10,119.65

a. Determine which kind of function best models the data.

b. Write an equation for the function that models the data.

c. Use your equation to determine how much the car is worth after 7 years.

29. **BACTERIA** A scientist estimates that a bacteria culture with an initial population of 12 will triple every hour.

a. Make a table to show the bacteria population for the first 4 hours.

b. Which kind of model best represents the data?

c. Write a function that models the data.

d. How many bacteria will there be after 8 hours?

30. **PRINTING** A printing company charges the fees shown to print flyers. Write a function that models the total cost of the flyers, and determine how much 30 flyers would cost.

H.O.T. Problems

Use **H**igher-**O**rder **T**hinking Skills

31. **CHALLENGE** Write a function that has constant second differences, first differences that are not constant, a y-intercept of -5, and passes through the point at (2, 3).

32. **REASONING** What type of function will have a constant third differences but not constant second differences? Explain.

33. **OPEN ENDED** Write a linear function that has a constant first difference of 4.

34. **REASONING** If data can be modeled by a quadratic function, what is the relationship between the coefficient of x^2 and the constant second difference?

35. **WRITING IN MATH** Summarize how to determine whether a given set of data is modeled by a *linear* function, a *quadratic* function, or an *exponential* function.

Standardized Test Practice

36. SHORT RESPONSE Write an equation that models the data in the table.

x	0	1	2	3	4
y	3	6	12	24	48

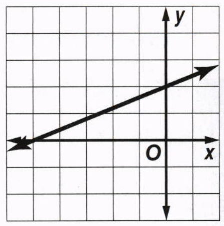
37. What is the equation of the line below?

A $y = \frac{2}{5}x + 2$

B $y = \frac{2}{5}x - 2$

C $y = \frac{5}{2}x + 2$

D $y = \frac{5}{2}x - 2$



38. The point $(r, -4)$ lies on a line with an equation of $2x + 3y = -8$. Find the value of r.

F -10

G 0

H 2

J 8

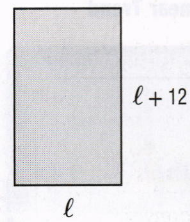
39. GEOMETRY The rectangle has an area of 220 square feet. Find the length ℓ.

A 8 feet

B 10 feet

C 22 feet

D 34 feet



Spiral Review

40. INVESTMENTS Joey's investment of \$2500 has been decreasing in value at a rate of 1.5% each year. What will his investment be worth in 5 years? (Lesson 9-8)

Write an equation for the *n*th term of each geometric sequence, and find the seventh term of each sequence. (Lesson 9-7)

41. 1, 2, 4, 8, …

42. $-20, -10, -5, \ldots$

43. $4, -12, 36, \ldots$

44. $99, -33, 11, \ldots$

45. 22, 44, 88, …

46. $\frac{2}{3}, \frac{1}{3}, \frac{1}{6}, \ldots$

Find each product. (Lesson 7-8)

47. $(x - 4)^2$

48. $(2y + 3)^2$

49. $(4x - 7)^2$

50. $(a - 5)(a + 5)$

51. $(5x - 6y)(5x + 6y)$

52. $(9c - 2d^2)(9c + 2d^2)$

53. CANOE RENTAL To rent a canoe, you must pay a daily rate plus \$10 per hour. Ilia and her friends rented a canoe for 3 hours and paid \$45. Write a linear equation for the cost C of renting the canoe for h hours, and determine how much it cost to rent the canoe for 8 hours. (Lesson 4-2)

Determine whether each equation is a linear equation. If so, write the equation in standard form. (Lesson 3-1)

54. $3x = 5y$

55. $6 - y = 2x$

56. $6xy + 3x = 4$

57. $y + 5 = 0$

58. $7y = 2x + 5x$

59. $y = 4x^2 - 1$

Skills Review

Graph each function. (Lesson 4-7)

60. $f(x) = |x - 2|$

61. $g(x) = |3x + 4|$

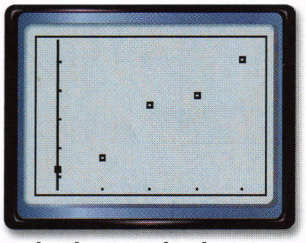
62. $f(x) = \left|\frac{1}{2}x + 5\right|$

Graphing Technology Lab
Curve Fitting

Math Online glencoe.com
- Other Calculator Keystrokes
- Graphing Technology Personal Tutor

If there is a constant increase or decrease in data values, there is a linear trend. If the values are increasing or decreasing more and more rapidly, there may be a quadratic or exponential trend.

Linear Trend



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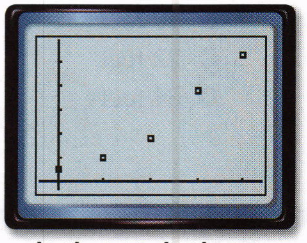
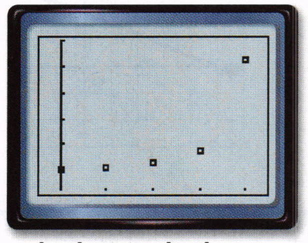
Quadratic Trend



[0, 5] scl: 1 by [0, 6] scl: 1

Exponential Trend



[0, 5] scl: 1 by [0, 6] scl: 1

With a graphing calculator, you can find the appropriate regression equation.

ACTIVITY

CHARTER AIRLINE **The table shows the average monthly number of flights made each year by a charter airline that was founded in 2000.**

Year	2000	2001	2002	2003	2004	2005	2006	2007
Flights	17	20	24	28	33	38	44	50

Step 1 Make a scatter plot.

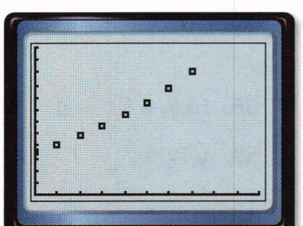
- Enter the number of years since 2000 in **L1** and the number of flights in **L2**.

KEYSTROKES: *Review entering a list on page 253.*

- Use **STAT PLOT** to graph the scatter plot.

KEYSTROKES: *Review statistical plots on page 254.*

Use Zoom 9 to graph.



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From the scatter plot we can see that the data may have either a quadratic trend or an exponential trend.

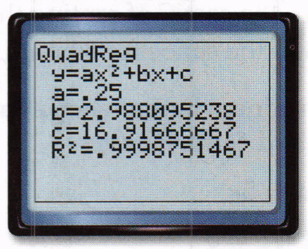
Step 2 Find the regression equation.

We will check both trends by examining their regression equations.

- Select DiagnosticOn from the **CATALOG**.
- Select QuadReg on the STAT menu.

KEYSTROKES: STAT ▶ 5 ENTER ENTER

The equation is in the form $y = ax^2 + bx + c$.



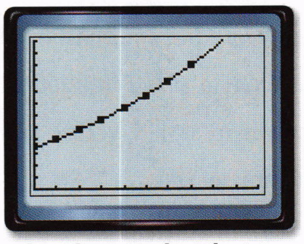
The equation is about $y = 0.25x^2 + 3x + 17$.

R^2 is the **coefficient of determination**. The closer R^2 is to 1, the better the model. To acquire the exponential equation select **ExpReg** on the STAT menu. To choose a quadratic or exponential model, fit both and use the one with the R^2 value closer to 1.

Step 3 Graph the quadratic regression equation.

- Copy the equation to the Y= list and graph.

KEYSTROKES: Y= VARS 5 ▶ ▶ 1 Zoom 9

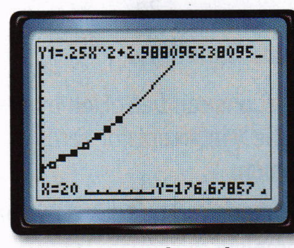


[0, 10] scl: 1 by [0, 60] scl: 5

Step 4 Predict using the equation.

If this trend continues, we can use the graph of our equation to predict the monthly number of flights the airline will make in a specific year. Let's check the year 2020. First adjust the window.

KEYSTROKES: 2nd CALC 1 At x = enter 20 ENTER.



[0, 25] scl: 1 by [0, 200] scl: 5

There will be approximately 177 flights per month if this trend continues.

Exercises

Plot each set of data points. Determine whether to use a *linear, quadratic* or *exponential* regression equation. State the coefficient of determination.

1.

x	y
1	30
2	40
3	50
4	55
5	50
6	40

2.

x	y
0.0	12.1
0.1	9.6
0.2	6.3
0.3	5.5
0.4	4.8
0.5	1.9

3.

x	y
0	1.1
2	3.3
4	2.9
6	5.6
8	11.9
10	19.8

4.

x	y
1	1.67
5	2.59
9	4.37
13	6.12
17	5.48
21	3.12

5. **BAKING Alyssa baked a cake and is waiting for it to cool so she can ice it. The table shows the temperature of the cake every 5 minutes after Alyssa took it out of the oven.**

Time (min)	Temperature (°F)
0	350
5	244
10	178
15	137
20	112
25	96
30	89

   a. Make a scatter plot of the data.

   b. Which regression equation has an R^2 value closest to 1? Is this the equation that best fits the context of the problem? Explain your reasoning.

   c. Find an appropriate regression equation, and state the coefficient of determination. What is the domain and range?

   d. Alyssa will ice the cake when it reaches room temperature (70°F). Use the regression equation to predict when she can ice her cake.

CHAPTER 9

Study Guide and Review

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- STUDY TO GO
- Vocabulary Review

Chapter Summary

Key Concepts

Graphing Quadratic Functions (Lesson 9-1)

- A quadratic function can be described by an equation of the form $y = ax^2 + bx + c$, where $a \neq 0$.
- The axis of symmetry for the graph of $y = ax^2 + bx + c$, where $a \neq 0$, is $x = -\frac{b}{2a}$.

Solving Quadratic Equations (Lessons 9-2, 9-4, and 9-5)

- Quadratic equations can be solved by graphing. The solutions are the x-intercepts or zeros of the related quadratic function.
- Quadratic equations can be solved by completing the square. To complete the square for $x^2 + bx$, find $\frac{1}{2}$ of b, square this result, and then add the result to $x^2 + bx$.
- Quadratic equations can be solved by using the Quadratic Formula, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Transformations of Quadratic Functions (Lesson 9-3)

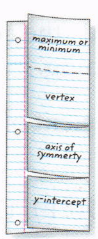
- $f(x) = x^2 + c$ translates the graph up or down.
- $f(x) = ax^2$ compresses or expands the graph vertically.

Exponential Functions (Lessons 9-6 and 9-7)

- An exponential function can be described by an equation of the form $y = ab^x$, where $a \neq 0$, $b > 0$ and $b \neq 1$.
- The general equation for exponential growth is $y = a(1 + r)^t$, where $r > 0$, and the general equation for exponential decay is $y = a(1 - r)^t$, where $0 < r < 1$. y represents the final amount, a is the initial amount, r represents the rate of change, and t is the time in years.

FOLDABLES® Study Organizer

Be sure the Key Concepts are noted in your Foldable.



Key Vocabulary

- axis of symmetry (p. 525)
- common ratio (p. 578)
- completing the square (p. 552)
- compound interest (p. 574)
- dilation (p. 545)
- discriminant (p. 561)
- double root (p. 538)
- exponential decay (p. 574)
- exponential function (p. 567)
- exponential growth (p. 573)
- geometric sequence (p. 578)
- maximum (p. 525)
- minimum (p. 525)
- parabola (p. 525)
- Quadratic Formula (p. 558)
- quadratic function (p. 525)
- reflection (p. 545)
- standard form (p. 525)
- symmetry (p. 526)
- transformation (p. 544)
- translation (p. 544)
- vertex (p. 525)

Vocabulary Check

State whether each sentence is *true* or *false*. If *false*, replace the underlined term to make a true sentence.

1. The <u>axis of symmetry</u> of a quadratic function can be found by using the equation $x = -\frac{b}{2a}$.
2. The <u>vertex</u> is the maximum or minimum point of a parabola.
3. The graph of a quadratic function is a <u>straight line</u>.
4. The graph of a quadratic function has a <u>maximum</u> if the coefficient of the x^2 is positive.
5. A quadratic equation with a graph that has two x-intercepts has <u>one</u> real root.
6. The expression $b^2 - 4ac$ is called the <u>discriminant</u>.
7. An example of an <u>exponential</u> function is $y = 3^x$.
8. The <u>exponential growth</u> equation is $y = C(1 - r)^t$.
9. The solutions of a quadratic equation are called <u>roots</u>.
10. The graph of the parent function is <u>translated down</u> to form the graph of $f(x) = x^2 + 5$.

Lesson-by-Lesson Review

9-1 Graphing Quadratic Functions (pp. 525–535)

Consider each equation.

a. Determine whether the function has a *maximum* or *minimum* value.

b. State the maximum or minimum value.

c. What are the domain and range of the function?

11. $y = x^2 - 4x + 4$

12. $y = -x^2 + 3x$

13. $y = x^2 - 2x - 3$

14. $y = -x^2 + 2.$

15. **BASEBALL** A baseball is thrown with an upward velocity of 32 feet per second. The equation $h = -16t^2 + 32t$ gives the height of the ball t seconds after it is thrown.

a. Determine whether the function has a *maximum* or *minimum* value.

b. State the maximum or minimum value.

c. State a reasonable domain and range for this situation.

EXAMPLE 1

Consider $f(x) = x^2 + 6x + 5$.

a. Determine whether the function has a *maximum* or *minimum* value.

For $f(x) = x^2 + 6x + 5$, $a = 1$, $b = 6$, and $c = 5$.

Because a is positive, the graph opens up, so the function has a minimum value.

b. State the *minimum* or *maximum* value of the function.

The minimum value is the y-coordinate of the vertex.

The x-coordinate of the vertex is $\frac{-b}{2a}$ or $\frac{-6}{2(1)}$ or -3.

$f(x) = x^2 + 6x + 5$	**Original function**
$f(-3) = (-3)^2 + 6(-3) + 5$	$x = -3$
$f(-3) = -4$	**Simplify.**

The minimum value is -4.

c. State the domain and range of the function.

The domain is all real numbers. The range is all real numbers greater than or equal to the minimum value, or $\{y \mid y \geq -4\}$.

9-2 Solving Quadratic Equations by Graphing (pp. 537–542)

Solve each equation by graphing. If integral roots cannot be found, estimate the roots to the nearest tenth.

16. $x^2 - 3x - 4 = 0$

17. $-x^2 + 6x - 9 = 0$

18. $x^2 - x - 12 = 0$

19. $x^2 + 4x - 3 = 0$

20. $x^2 - 10x = -21$

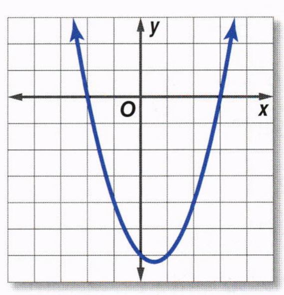
21. $6x^2 - 13x = 15$

22. **NUMBER THEORY** Find two numbers that have a sum of 2 and a product of -15.

EXAMPLE 2

Solve $x^2 - x - 6 = 0$ by graphing.

Graph the related function $f(x) = x^2 - x - 6$.



The x-intercepts of the graph appear to be at -2 and 3, so the solutions are -2 and 3.

CHAPTER 9 Study Guide and Review

9-3 Transformations of Quadratic Functions (pp. 544–549)

Describe how the graph of each function is related to the graph of $f(x) = x^2$.

23. $f(x) = x^2 + 8$

24. $f(x) = x^2 - 3$

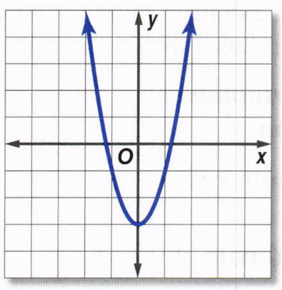
25. $f(x) = 2x^2$

26. $f(x) = 4x^2 - 18$

27. $f(x) = \frac{1}{3}x^2$

28. $f(x) = \frac{1}{4}x^2$

29. Write an equation for the function shown in the graph.



30. PHYSICS A ball is dropped off a cliff that is 100 feet high. The function $h = -16t^2 + 100$ models the height h of the ball after t seconds. Compare the graph of this function to the graph of $h = t^2$.

EXAMPLE 3

Describe how the graph of $f(x) = x^2 - 2$ is related to the graph of $f(x) = x^2$.

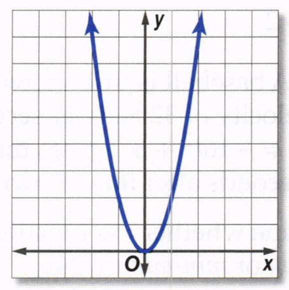
The graph of $f(x) = x^2 + c$ represents a translation up or down of the parent graph.

Since $c = -2$, the translation is down.

So, the graph is shifted down from the parent function.

EXAMPLE 4

Write an equation for the function shown in the graph.



Since the graph opens upward, the leading coefficient must be positive. The parabola has not been translated up or down, so $c = 0$. Since the graph is stretched vertically, it must be of the form of $f(x) = ax^2$ where $a > 1$. The equation for the function is $y = 2x^2$.

9-4 Solving Quadratic Equations by Completing the Square (pp. 552–557)

Solve each equation by completing the square. Round to the nearest tenth if necessary.

31. $x^2 + 6x + 9 = 16$

32. $-a^2 - 10a + 25 = 25$

33. $y^2 - 8y + 16 = 36$

34. $y^2 - 6y + 2 = 0$

35. $n^2 - 7n = 5$

36. $-3x^2 + 4 = 0$

37. NUMBER THEORY Find two numbers that have a sum of -2 and a product of -48.

EXAMPLE 5

Solve $x^2 - 16x + 32 = 0$ by competing the square. Round to the nearest tenth if necessary.

Isolate the x^2- and x-terms. Then complete the square and solve.

$$\begin{aligned}
x^2 - 16x + 32 &= 0 \\
x^2 - 16x &= -32 \\
x^2 - 16x + 64 &= -32 + 64 \\
(x - 8)^2 &= 32 \\
x - 8 &= \pm\sqrt{32} \\
x &= 8 \pm\sqrt{32} \\
x &= 8 \pm 4\sqrt{2}
\end{aligned}$$

The solutions are about 2.3 and 13.7.

MIXED PROBLEM SOLVING
For mixed problem-solving practice, see page 845.

9-5 Solving Quadratic Equations by Using the Quadratic Formula (pp. 558–564)

Solve each equation by using the Quadratic Formula. Round to the nearest tenth if necessary.

38. $x^2 - 8x = 20$

39. $21x^2 + 5x - 7 = 0$

40. $d^2 - 5d + 6 = 0$

41. $2f^2 + 7f - 15 = 0$

42. $2h^2 + 8h + 3 = 3$

43. $4x^2 + 4x = 15$

44. **GEOMETRY** The area of a square can be quadrupled by increasing the side length and width by 4 inches. What is the side length?

EXAMPLE 6

Solve $x^2 + 10x + 9 = 0$ by using the Quadratic Formula.

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ **Quadratic Formula**

$= \dfrac{-10 \pm \sqrt{10^2 - 4(1)(9)}}{2(1)}$ **$a = 1, b = 10, c = 9$**

$= \dfrac{-10 \pm \sqrt{64}}{2}$ **Simplify.**

$= \dfrac{-10 + 8}{2}$ or $\dfrac{-10 - 8}{2}$ **Separate the solutions.**

$= -1$ or -9 **Simplify.**

9-6 Exponential Functions (pp. 567–572)

Graph each function. Find the y-intercept, and state the domain and range.

45. $y = 2^x$

46. $y = 3^x + 1$

47. $y = 4^x + 2$

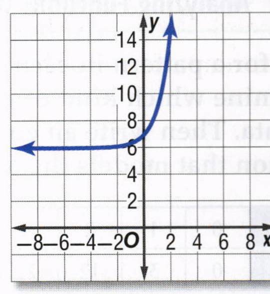
48. $y = 2^x - 3$

49. **BIOLOGY** The population of bacteria in a petri dish increases according to the model $p = 550(2.7)^{0.008t}$, where t is the number of hours and $t = 0$ corresponds to 1:00 P.M. Use this model to estimate the number of bacteria in the dish at 5:00 P.M.

EXAMPLE 7

Graph $y = 3^x + 6$. Find the y-intercept, and state the domain and range.

x	$3^x + 6$	y
−3	$3^{-3} + 6$	6.04
−2	$3^{-2} + 6$	6.11
−1	$3^{-1} + 6$	6.33
0	$3^{0} + 6$	7
1	$3^{1} + 6$	9



The y-intercept is $(0, 7)$. The domain is all real numbers, and the range is all real numbers greater than 6.

9-7 Growth and Decay (pp. 573–577)

50. Find the final value of \$2500 invested at an interest rate of 2% compounded monthly for 10 years.

51. **COMPUTERS** Zita's computer is depreciating at a rate of 3% per year. She bought the computer for \$1200.

a. Write an equation to represent this situation.

b. What will the computer's value be after 5 years?

EXAMPLE 8

Find the final value of \$2000 invested at an interest rate of 3% compounded quarterly for 8 years.

$A = P\left(1 + \frac{r}{n}\right)^{nt}$ **Compound interest equation**

$= 2000\left(1 + \frac{0.03}{4}\right)^{4(8)}$ **$P = 2000, r = 0.03, n = 4$, and $t = 8$**

$\approx \$2540.22$ **Use a calculator.**

There will be about \$2540.22 in 8 years.

CHAPTER 9

Study Guide and Review

9-8 Geometric Sequences as Exponential Functions (pp. 578–583)

Find the next three terms in each geometric sequence.

52. $-1, 1, -1, 1, \ldots$

53. $3, 9, 27, \ldots$

54. $256, 128, 64, \ldots$

Write the equation for the *n*th term of each geometric sequence.

55. $-1, 1, -1, 1, \ldots$

56. $3, 9, 27, \ldots$

57. $256, 128, 64, \ldots$

58. **SPORTS** A basketball is dropped from a height of 20 feet. It bounces to $\frac{1}{2}$ its height after each bounce. Draw a graph to represent the situation.

EXAMPLE 9

Find the next three terms in the geometric sequence 2, 6, 18, … .

Step 1 Find the common ratio. Each number is 3 times the previous number, so $r = 3$.

Step 2 Multiply each term by the common ratio to find the next three terms.

$18 \times 3 = 54,\ 54 \times 3 = 162,\ 162 \times 3 = 486$

The next three terms are 54, 162, and 486.

EXAMPLE 10

Write the equation for the *n*th term of the geometric sequence −3, 12, −48, … .

The common ratio is −4. So $r = -4$.

$a_n = a_1 r^{n-1}$ **Formula for the *n*th term**

$a_n = -3(-4)^{n-1}$ **$a_1 = -3$ and $r = -4$**

9-9 Analyzing Functions with Successive Differences (pp. 584–589)

Look for a pattern in each table of values to determine which kind of model best describes the data. Then write an equation for the function that models the data.

59.

x	0	1	2	3	4
y	0	3	12	27	48

60.

x	0	1	2	3	4
y	1	2	4	8	16

61.

x	0	1	2	3	4
y	0	−1	−4	−9	−16

62.

x	0	1	2	3	4
y	3	6	9	12	15

63. **SCHOOL SPIRIT** The table shows the cost to purchase school-spirit posters. Determine which kind of model best describes the data. Then write the equation.

No. of posters	2	4	6	8
Cost	4	7	10	13

EXAMPLE 11

Determine which kind of model best describes the data. Then write an equation for the function that models the data.

x	0	1	2	3	4
y	3	4	5	6	7

Step 1 Determine which model fits the data.

First differences: 3 4 5 6 7

1 1 1 1

Since the first differences are all equal, a linear function models the data.

Step 2 Write an equation for the function that models the data.

The equation has the form $y = mx + b$.

The slope is 1 and the y-intercept is 3, so the equation is $y = x + 3$.

Practice Test

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Chapter Test

Use a table of values to graph the following functions. State the domain and range.

1. $y = x^2 + 2x + 5$
2. $y = 2x^2 - 3x + 1$

Consider $y = x^2 - 7x + 6$.

3. Determine whether the function has a *maximum* or *minimum* value.
4. State the maximum or minimum value.
5. What are the domain and range?

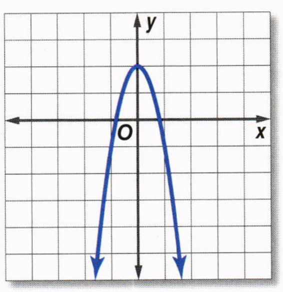
Solve each equation by graphing. If integral roots cannot be found, estimate the roots to the nearest tenth.

6. $x^2 + 7x + 10 = 0$
7. $x^2 - 5 = -3x$

Describe how the graph of each function is related to the graph of $f(x) = x^2$.

8. $g(x) = x^2 - 5$
9. $g(x) = -3x^2$
10. $h(x) = \frac{1}{2}x^2 + 4$

11. **MULTIPLE CHOICE** Which is an equation for the function shown in the graph?



A $y = -3x^2$

B $y = 3x^2 + 1$

C $y = x^2 + 2$

D $y = -3x^2 + 2$

Solve each equation by completing the square.

12. $x^2 + 2x + 5 = 0$
13. $x^2 - x - 6 = 0$
14. $2x^2 - 36 = -6x$

Solve each equation by using the Quadratic Formula. Round to the nearest tenth if necessary.

15. $x^2 - x - 30 = 0$
16. $x^2 - 10x = -15$
17. $2x^2 + x - 15 = 0$

18. **BASEBALL** Elias hits a baseball into the air. The equation $h = -16t^2 + 60t + 3$ models the height h in feet of the ball after t seconds. How long is the ball in the air?

Graph each function. Find the y-intercept, and state the domain and range.

19. $y = 2(5)^x$
20. $y = -3(11)^x$
21. $y = 3x + 2$

Find the next three terms in each geometric sequence.

22. 2, −6, 18, …
23. 1000, 500, 250, …
24. 32, 8, 2, …

25. **MONEY** Lynne invested \$500 into an account with a 6.5% interest rate compounded monthly. How much will Lynne's investment be worth in 10 years?

F \$600.00 H \$956.09

G \$938.57 J \$957.02

26. **INVESTMENTS** Shelly's investment of \$3000 has been losing value at a rate of 3% each year. What will her investment be worth in 6 years?

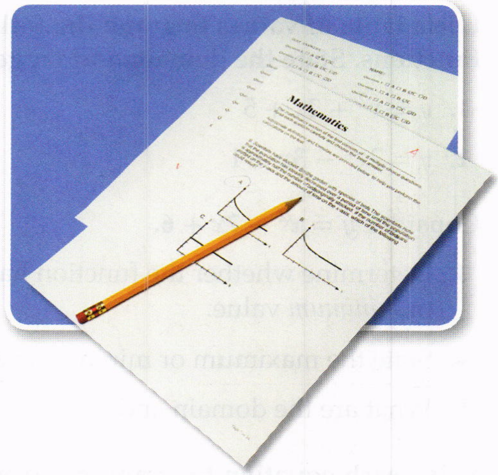
27. Graph {(−2, 4), (−1, 1), (0, 0), (1, 1), (2, 4)}. Determine whether the ordered pairs represent a *linear function*, a *quadratic function*, or an *exponential function*.

28. Look for a pattern in the table to determine which kind of model best describes the data.

x	0	1	2	3	4
y	1	3	5	7	9

Use a Formula

A *formula* is an equation that shows a relationship among certain quantities. Many standardized test problems will require using a formula to solve them.



Strategies for Using a Formula

Step 1

Become familiar with common formulas and their uses. You may or may not be given access to a formula sheet to use during the test.

- **If given a formula sheet,** be sure to practice with the formulas on it before taking the test so you know how to apply them.
- **If not given a formula sheet,** study and practice with common formulas such as perimeter, area, and volume formulas, the Distance Formula, the Pythagorean Theorem, the Midpoint Formula, the Quadratic Formula, and others.

Step 2

Choose a formula and solve.

- **Ask Yourself:** What quantities are given in the problem statement?
- **Ask Yourself:** What quantities am I looking for?
- **Ask Yourself:** Is there a formula I know that relates these quantities?
- **Write:** Write the formula out that you have chosen each time.
- **Solve:** Substitute known quantities into the formula and solve for the unknown quantity.
- **Check:** Check your answer if time permits.

EXAMPLE

Read the problem. Identify what you need to know. Then use the information in the problem to solve.

Find the exact roots of the quadratic equation $-2x^2 + 6x + 5 = 0$.

A $\frac{3 \pm \sqrt{17}}{4}$ **C** $\frac{3 \pm \sqrt{19}}{2}$

B $\frac{4 \pm \sqrt{17}}{3}$ **D** $\frac{3 \pm \sqrt{19}}{4}$

Read the problem carefully. You are given a quadratic equation and asked to find the exact roots of the equation. Use the **Quadratic Formula** to find the roots.

$-2x^2 + 6x + 5 = 0$	**Original equation**
$a = -2, b = 6, c = 5$	**Identify the coefficients of the equation.**
$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	**Quadratic Formula**
$= \dfrac{-(6) \pm \sqrt{(6)^2 - 4(-2)(5)}}{2(-2)}$	**$a = -2$, $b = 6$, and $c = 5$**
$= \dfrac{-6 \pm \sqrt{36 - (-40)}}{-4}$	**Simplify.**
$= \dfrac{-6 \pm \sqrt{76}}{-4}$	**Subtract.**
$= \dfrac{-6 \pm 2\sqrt{19}}{-4}$	**$\sqrt{76} = \sqrt{4 \cdot 19}$ or $2\sqrt{19}$.**
$= \dfrac{-2(3 \pm \sqrt{19})}{-2(2)}$	**Factor out -2 from the numerator and denominator.**
$= \dfrac{3 \pm \sqrt{19}}{2}$	**Simplify.**

The roots of the equation are $\dfrac{3 + \sqrt{19}}{2}$ and $\dfrac{3 - \sqrt{19}}{2}$. The correct answer is C.

Exercises

Read each problem. Identify what you need to know. Then use the information in the problem to solve.

1. Find the exact roots of the quadratic equation $x^2 + 5x - 12 = 0$.

   A $\dfrac{-5 \pm \sqrt{73}}{2}$ C $\dfrac{-3 \pm \sqrt{73}}{4}$

   B $\dfrac{4 \pm \sqrt{61}}{3}$ D $\dfrac{-1 \pm \sqrt{61}}{2}$

2. The area of a triangle in which the length of the base is 4 centimeters greater than twice the height is 80 square centimeters. What is the length of the base of the triangle?

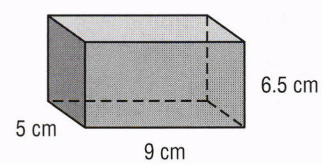
   F -10

   G 8

   H 16

   J 20

3. Find the volume of the figure below.



   A 18.5 cm^3 C 272 cm^3

   B 91 cm^3 D 292.5 cm^3

4. Myron is traveling 263.5 miles at an average rate of 62 miles per hour. How long will it take Myron to complete his trip?

   F 5 h 25 min

   G 4 h 15 min

   H 5 h 10 min

   J 4 h 25 min

CHAPTER 9

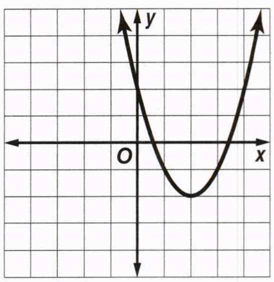
Standardized Test Practice

Cumulative, Chapters 1 through 9

Multiple Choice

Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.

1. What is the vertex of the parabola graphed below?



A (2, 0)

B (0, 2)

C (−2, 2)

D (2, −2)

2. Write an equation in slope-intercept form with a slope of $\frac{9}{10}$ and y-intercept of 3.

F $y = 3x + \frac{9}{10}$

G $y = \frac{9}{10}x + 3$

H $y = \frac{9}{10}x - 3$

J $y = 3x - \frac{9}{10}$

3. Use the Quadratic Formula to find the exact solutions of the equation $2x^2 - 6x + 3 = 0$.

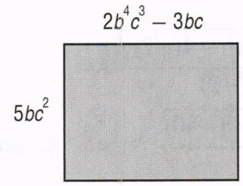
A $\frac{3 \pm \sqrt{3}}{2}$

B $\frac{3 \pm \sqrt{2}}{4}$

C $\frac{2 \pm \sqrt{5}}{3}$

D $\frac{5 \pm \sqrt{2}}{2}$

4. Write an expression for the area of the rectangle below.



F $10b^5c^5 - 3bc$

G $10b^5c^5 - 15b^2c^3$

H $2b^5c^5 - 3b^2c^3$

J $10b^4c^6 - 15bc^2$

5. Solve the quadratic equation below by graphing.

$$x^2 - 2x - 15 = 0$$

A −1, 4

B −3, 5

C 3, −5

D ∅

6. Jason is playing games at a family fun center. So far he has won 38 prize tickets. How many more tickets would he need to win to place him in the gold prize category?

Number of Tickets	Prize Category
1–20	bronze
21–40	silver
41–60	gold
61–80	platinum

F $2 \leq t \leq 22$

G $3 \leq t \leq 22$

H $1 \leq t \leq 20$

J $3 \leq t \leq 20$

Test-Taking Tip

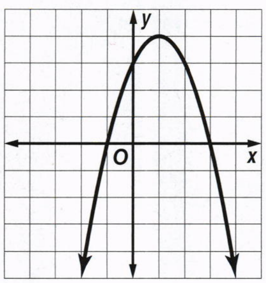
Question 5 If permitted, you can use a graphing calculator to quickly graph an equation and find its roots.

Short Response/Gridded Response

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

7. **GRIDDED RESPONSE** Misty purchased a car several years ago for $21,459. The value of the car depreciated at a rate of 15% annually. What was the value of the car after 5 years? Round your answer to the nearest whole dollar.

8. Use the graph of the quadratic equation shown below to answer each question.



a. What is the vertex?

b. What is the y-intercept?

c. What is the axis of symmetry?

d. What are the roots of the corresponding quadratic equation?

9. The cost of 5 notebooks and 3 pens is $9.75. The cost of 4 notebooks and 6 pens is $10.50. Which of the following systems can be used to find the cost of a notebook n and a pen p?

a. Write a system of equations to model the situation.

b. Solve the system of equations. How much does each item cost?

10. The table shows the total cost of renting a canoe for n hours.

Number of Hours (n)	Rental Cost (C)
1	$15
2	$20
3	$25
4	$30

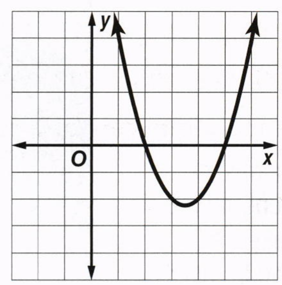
a. Write a function to represent the situation.

b. How much would it cost to rent the canoe for 7 hours?

Extended Response

Record your answers on a sheet of paper. Show your work.

11. Use the equation and its graph to answer each question.



a. Factor $x^2 - 7x + 10$.

b. What are the solutions of $x^2 - 7x + 10 = 0$?

c. What do you notice about the graph of the quadratic equation and where it crosses the x-axis? How do these values compare to the solutions of $x^2 - 7x + 10 = 0$? Explain.

Need Extra Help?											
If you missed Question...	1	2	3	4	5	6	7	8	9	10	11
Go to Lesson or Page...	9-1	4-2	9-5	7-6	9-2	5-1	9-7	9-1	6-4	3-5	8-3

CHAPTER 10 Radical Functions and Geometry

Then

In Chapters 8 and 9, you solved quadratic equations.

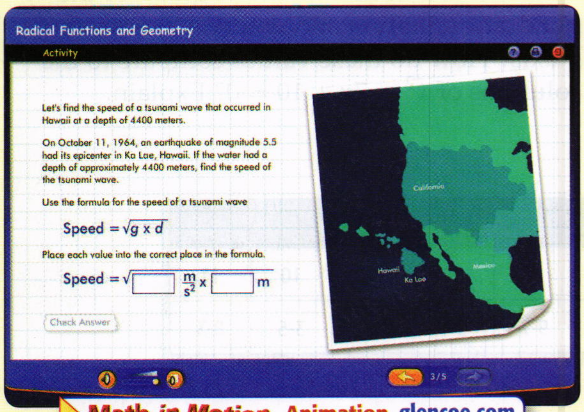
Now

In Chapter 10, you will:

- Graph and transform radical functions.
- Simplify, add, subtract, and multiply radical expressions.
- Solve radical equations.
- Use the Pythagorean Theorem.
- Find trigonometric ratios.

Why?

OCEANS Tsunamis, or large waves, are generated by undersea earthquakes. A radical equation can be used to find the speed of a tsunami in meters per second or the depth of the ocean in meters.



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