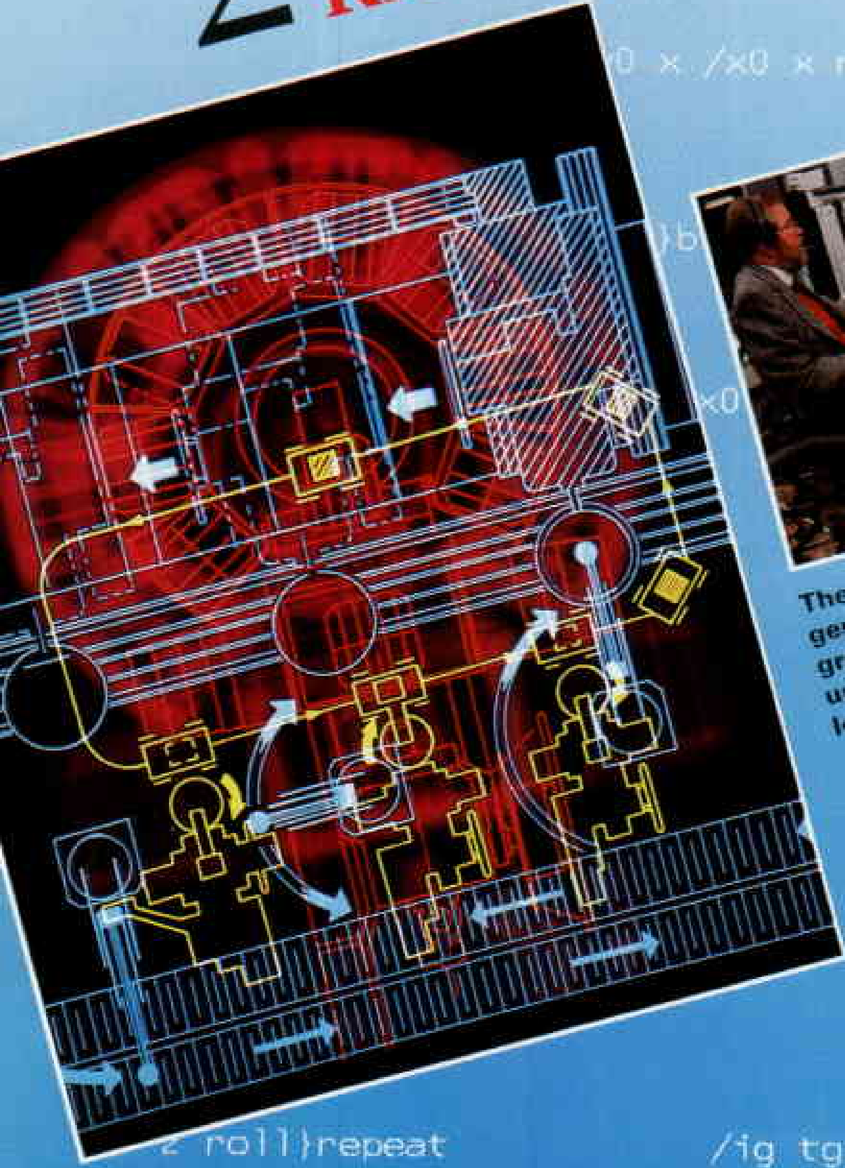
2 DEDUCTIVE REASONING



The computer program that generates a complex diagram such as this is made up of simple steps linked in logical sequence.

Using Deductive Reasoning

Objectives

1. Recognize the hypothesis and the conclusion of an if-then statement.
2. State the converse of an if-then statement.
3. Use a counterexample to disprove an if-then statement.
4. Understand the meaning of *if and only if*.
5. Use properties from algebra and properties of congruence in proofs.
6. Use the Midpoint Theorem and the Angle Bisector Theorem.
7. Know the kinds of reasons that can be used in proofs.

2-1 *If-Then Statements; Converses*

Your friend says, "If it rains after school, then I will give you a ride home."

A geometry student reads, "If B is between A and C, then AB + BC = AC."

These are examples of **if-then statements,** which are also called **conditional statements** or simply **conditionals.**

To represent an if-then statement symbolically, let p represent the **hypothesis,** shown in red, and let q represent the **conclusion,** shown in blue. Then we have the basic form of an if-then statement shown below:

If p, then q.
↑ ↑
p: hypothesis **q: conclusion**

The **converse** of a conditional is formed by interchanging the hypothesis and the conclusion.

Statement: If p, then q. Converse: If q, then p.

A statement and its converse say different things. In fact, some true statements have false converses.

Statement: If Ed lives in Texas, then he lives south of Canada.
False Converse: If Ed lives south of Canada, then he lives in Texas.

An if-then statement is false if an example can be found for which the hypothesis is true and the conclusion is false. Such an example is called a **counterexample.** It takes only one counterexample to disprove a statement. We know the converse above is false because we can find a counterexample: Ed could live in Kansas City, which *is* south of Canada and *is not* in Texas.

Some true statements have true converses.

Statement: If $4x = 20$, then $x = 5$.
True Converse: If $x = 5$, then $4x = 20$.

Conditional statements are not always written with the "if" clause first. Here are some examples. All these conditionals mean the same thing.

General Form	*Example*
If p, then q.	If $x^2 = 25$, then $x < 10$.
p implies q.	$x^2 = 25$ implies $x < 10$.
p only if q.	$x^2 = 25$ only if $x < 10$.
q if p.	$x < 10$ if $x^2 = 25$.

If a conditional and its converse are both true they can be combined into a single statement by using the words "if and only if." A statement that contains the words "if and only if" is called a **biconditional.** Its basic form is shown below.

p if and only if q.

Every definition can be written as a biconditional as the statements below illustrate.

Definition: Congruent segments are segments that have equal lengths.

Biconditional: Segments are congruent if and only if their lengths are equal.

Classroom Exercises

State the hypothesis and the conclusion of each conditional.

1. If $2x - 1 = 5$, then $x = 3$.
2. If she's smart, then I'm a genius.
3. $8y = 40$ implies $y = 5$.
4. $RS = \frac{1}{2}RT$ if S is the midpoint of $\overline{RT}$.
5. $\angle 1 \cong \angle 2$ if $m\angle 1 = m\angle 2$.
6. $\angle 1 \cong \angle 2$ only if $m\angle 1 = m\angle 2$.
7. Combine the conditionals in Exercises 5 and 6 into a single biconditional.

Provide a counterexample to show that each statement is false. You may use words or draw a diagram.

8. If $\overline{AB} \cong \overline{BC}$, then B is the midpoint of $\overline{AC}$.
9. If a line lies in a vertical plane, then the line is vertical.
10. If a number is divisible by 4, then it is divisible by 6.
11. If $x^2 = 49$, then $x = 7$.

State the converse of each conditional. Is the converse true or false?

12. If today is Friday, then tomorrow is Saturday.
13. If $x > 0$, then $x^2 > 0$.
14. If a number is divisible by 6, then it is divisible by 3.
15. If $6x = 18$, then $x = 3$.
16. Give an example of a false conditional whose converse is true.

Written Exercises

Write the hypothesis and the conclusion of each conditional.

A

1. If $3x - 7 = 32$, then $x = 13$.

2. I can't sleep if I'm not tired.

3. I'll try if you will.

4. If $m\angle 1 = 90$, then $\angle 1$ is a right angle.

5. $a + b = a$ implies $b = 0$.

6. $x = -5$ only if $x^2 = 25$.

Rewrite each pair of conditionals as a biconditional.

7. If B is between A and C, then $AB + BC = AC$.
If $AB + BC = AC$, then B is between A and C.

8. If $m\angle AOC = 180$, then $\angle AOC$ is a straight angle.
If $\angle AOC$ is a straight angle, then $m\angle AOC = 180$.

Write each biconditional as two conditionals that are converses of each other.

9. Points are collinear if and only if they all lie in one line.

10. Points lie in one plane if and only if they are coplanar.

Provide a counterexample to show that each statement is false. You may use words or a diagram.

11. If $ab < 0$, then $a < 0$.

12. If $n^2 = 5n$, then $n = 5$.

13. If point G is on $\overrightarrow{AB}$, then G is on $\overrightarrow{BA}$.

14. If $xy > 5y$, then $x > 5$.

15. If a four-sided figure has four right angles, then it has four congruent sides.

16. If a four-sided figure has four congruent sides, then it has four right angles.

Tell whether each statement is true or false. Then write the converse and tell whether it is true or false.

17. If $x = -6$, then $|x| = 6$.

18. If $x^2 = 4$, then $x = -2$.

19. If $b > 4$, then $5b > 20$.

20. If $m\angle T = 40$, then $\angle T$ is not obtuse.

21. If Pam lives in Chicago, then she lives in Illinois.

22. If $\angle A \cong \angle B$, then $m\angle A = m\angle B$.

B

23. $a^2 > 9$ if $a > 3$.

24. $x = 1$ only if $x^2 = x$.

25. $n > 5$ only if $n > 7$.

26. $ab = 0$ implies that $a = 0$ or $b = 0$.

27. If points D, E, and F are collinear, then $DE + EF = DF$.

28. P is the midpoint of $\overline{GH}$ implies that $GH = 2PG$.

29. Write a definition of congruent angles as a biconditional.

30. Write a definition of a right angle as a biconditional.

C

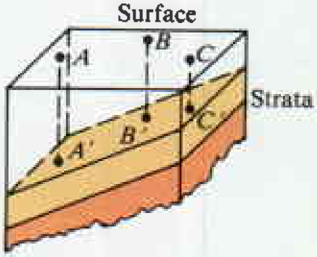
31. What can you conclude if the following sentences are all true?
(1) If p, then q. (2) p (3) If q, then not r. (4) s or r.

Career

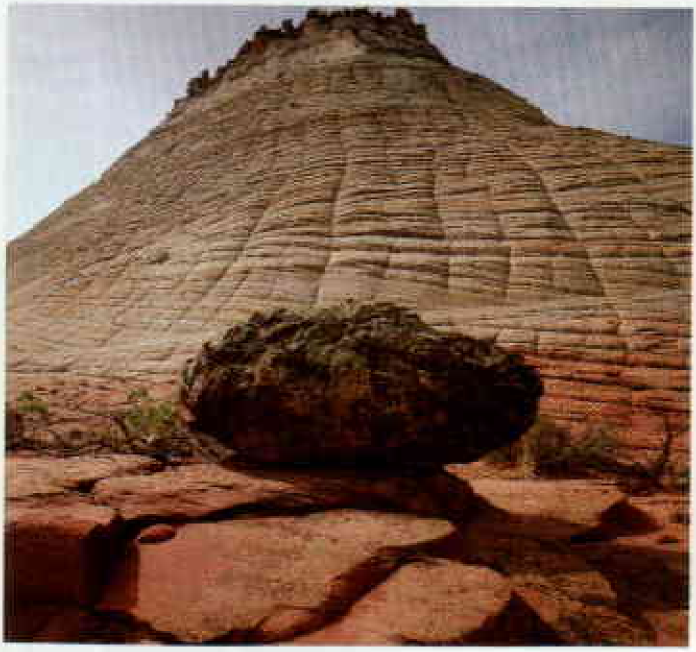
Geologist

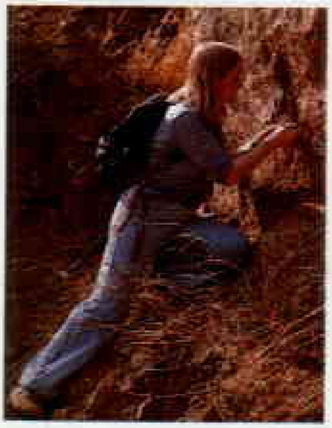
Geologists study rock formations like those at Checkerboard Mountain in Zion National Park. Rock formations often occur in *strata*, or layers, beneath the surface of the Earth. Earthquakes occur at *faults*, breaks in the strata. In search of a fault, how would you determine the position of a stratum of rock buried deep beneath the surface of the Earth?

A geologist might start by picking three noncollinear points, A, B, and C, on the surface and drilling holes to find the depths of points A', B', and C' on the stratum. These three points determine the plane of the surface of the stratum.



Geologists may work for industry, searching for oil or minerals. They may work in research centers, developing ways to predict earthquakes.





Today, geologists are trying to locate sources of geothermal energy, energy generated by the Earth's internal heat. A career in geology usually requires knowledge of mathematics, physics, and chemistry, as well as a degree in geology.

Mixed Review Exercises

Complete. You may find that drawing a diagram will help you.

1. If M is the midpoint of $\overline{AB}$, then $\underline{\ ?\ } \cong \underline{\ ?\ }$.
2. If $\overrightarrow{BX}$ is the bisector of $\angle ABC$, then $\underline{\ ?\ } \cong \underline{\ ?\ }$.
3. If point B lies in the interior of $\angle AOC$, then $m\angle \underline{\ ?\ } + m\angle \underline{\ ?\ } = m\angle \underline{\ ?\ }$.
4. If $\angle POQ$ is a straight angle and R is any point not on $\overleftrightarrow{PQ}$, then $m\angle \underline{\ ?\ } + m\angle \underline{\ ?\ } = \underline{\ ?\ }$.

2-2 *Properties from Algebra*

Since the length of a segment is a real number and the measure of an angle is a real number, the facts about real numbers and equality that you learned in algebra can be used in your study of geometry. The properties of equality that will be used most often are listed below.

Properties of Equality

Addition Property	If $a = b$ and $c = d$, then $a + c = b + d$.
Subtraction Property	If $a = b$ and $c = d$, then $a - c = b - d$.
Multiplication Property	If $a = b$, then $ca = cb$.
Division Property	If $a = b$ and $c \neq 0$, then $\frac{a}{c} = \frac{b}{c}$.
Substitution Property	If $a = b$, then either a or b may be substituted for the other in any equation (or inequality).
Reflexive Property	$a = a$
Symmetric Property	If $a = b$, then $b = a$.
Transitive Property	If $a = b$ and $b = c$, then $a = c$.

Recall that $DE = FG$ and $\overline{DE} \cong \overline{FG}$ can be used interchangeably, as can $m\angle D = m\angle E$ and $\angle D \cong \angle E$. Thus the following properties of congruence follow directly from the related properties of equality.

Properties of Congruence

Reflexive Property	$\overline{DE} \cong \overline{DE}$ $\quad$ $\angle D \cong \angle D$
Symmetric Property	If $\overline{DE} \cong \overline{FG}$, then $\overline{FG} \cong \overline{DE}$. If $\angle D \cong \angle E$, then $\angle E \cong \angle D$.
Transitive Property	If $\overline{DE} \cong \overline{FG}$ and $\overline{FG} \cong \overline{JK}$, then $\overline{DE} \cong \overline{JK}$. If $\angle D \cong \angle E$ and $\angle E \cong \angle F$, then $\angle D \cong \angle F$.

The properties of equality and other properties from algebra, such as the **Distributive Property,**

$$a(b + c) = ab + ac,$$

can be used to justify your steps when you solve an equation.

Example 1 Solve $3x = 6 - \frac{1}{2}x$ and justify each step.

Solution

Steps	*Reasons*
1. $3x = 6 - \frac{1}{2}x$	1. Given equation
2. $6x = 12 - x$	2. Multiplication Property of Equality
3. $7x = 12$	3. Addition Property of Equality
4. $x = \frac{12}{7}$	4. Division Property of Equality

Example 1 shows a proof of the statement "If $3x = 6 - \frac{1}{2}x$, then x *must* equal $\frac{12}{7}$." In other words, when given the information that $3x = 6 - \frac{1}{2}x$ we can use the properties of algebra to conclude, or *deduce*, that $x = \frac{12}{7}$.

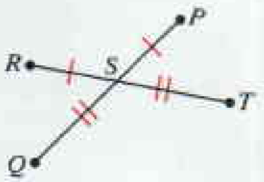
Many proofs in geometry follow this same pattern. We use certain given information along with the properties of algebra and accepted statements, such as the Segment Addition Postulate and Angle Addition Postulate, to show that other statements *must* be true. Often a geometric proof is written in two-column form, with statements on the left and a reason for each statement on the right.

In the following examples, congruent segments are marked alike and congruent angles are marked alike. For example, in the diagram below, the marks show that $\overline{RS} \cong \overline{PS}$ and $\overline{ST} \cong \overline{SQ}$. In the diagram for Example 3 the marks show that $\angle AOC \cong \angle BOD$.

Example 2

Given: $\overline{RT}$ and $\overline{PQ}$ intersecting at S so that $RS = PS$ and $ST = SQ$.

Prove: $RT = PQ$



Proof:

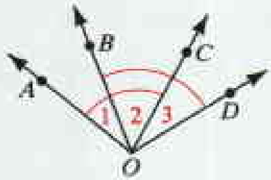
Statements	Reasons
1. $RS = PS$; $ST = SQ$	1. Given
2. $RS + ST = PS + SQ$	2. Addition Prop. of =
3. $RS + ST = RT$; $PS + SQ = PQ$	3. Segment Addition Postulate
4. $RT = PQ$	4. Substitution Prop.

In Steps 1 and 3 of Example 2, notice how statements can be written in pairs when justified by the same reason.

Example 3

Given: $m\angle AOC = m\angle BOD$

Prove: $m\angle 1 = m\angle 3$



Proof:

Statements	Reasons
1. $m\angle AOC = m\angle BOD$	1. Given
2. $m\angle AOC = m\angle 1 + m\angle 2$; $m\angle BOD = m\angle 2 + m\angle 3$	2. Angle Addition Postulate
3. $m\angle 1 + m\angle 2 = m\angle 2 + m\angle 3$	3. Substitution Prop.
4. $m\angle 2 = m\angle 2$	4. Reflexive Prop.
5. $m\angle 1 = m\angle 3$	5. Subtraction Prop. of =

Notice that the reason given for Step 4 is "Reflexive Property" rather than "Reflexive Property of Equality." Since the reflexive, symmetric, and transitive properties of equality are so closely related to the corresponding properties of congruence, we will simply use "Reflexive Property" to justify either

$$m\angle BOC = m\angle BOC \quad \text{or} \quad \angle BOC \cong \angle BOC.$$

Suppose, in a proof, you have made the statement that

$$m\angle R = m\angle S$$

and also the statement that

$$m\angle S = m\angle T.$$

You can then deduce that $m\angle R = m\angle T$ and use as your reason either "Transitive Property" or "Substitution Property." Similarly, if you know that

(1) $m\angle R = m\angle S$
(2) $m\angle S = m\angle T$
(3) $m\angle T = m\angle V$

you can go on to write (4) $m\angle R = m\angle V$

and use either "Transitive Property" or "Substitution Property" as your reason. Actually, you use the Transitive Property twice or else make a double substitution.

There are times when the Substitution Property is the simplest one to use. If you know that

(1) $m\angle 4 + m\angle 2 + m\angle 5 = 180$
(2) $m\angle 4 = m\angle 1$; $m\angle 5 = m\angle 3$

you can make a double substitution and get

(3) $m\angle 1 + m\angle 2 + m\angle 3 = 180.$

Note that you can't use the Transitive Property here.

Classroom Exercises

Justify each statement with a property from algebra or a property of congruence.

1. $\angle P \cong \angle P$

2. If $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{EF}$, then $\overline{AB} \cong \overline{EF}$.

3. If $RS = TW$, then $TW = RS$.

4. If $x + 5 = 16$, then $x = 11$.

5. If $5y = -20$, then $y = -4$.

6. If $\frac{z}{5} = 10$, then $z = 50$.

7. $2(a + b) = 2a + 2b$

8. If $2z - 5 = -3$, then $2z = 2$.

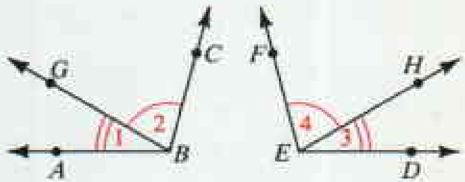
9. If $2x + y = 70$ and $y = 3x$, then $2x + 3x = 70$.

10. If $AB = CD$, $CD = EF$, and $EF = 23$, then $AB = 23$.

Complete each proof by supplying missing reasons and statements.

11. Given: $m\angle 1 = m\angle 3$;
$m\angle 2 = m\angle 4$

Prove: $m\angle ABC = m\angle DEF$



Proof:

Statements	Reasons
1. $m\angle 1 = m\angle 3$; $m\angle 2 = m\angle 4$	1. ?
2. $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 4$	2. ?
3. $m\angle 1 + m\angle 2 = m\angle ABC$; $m\angle 3 + m\angle 4 = m\angle DEF$	3. ?
4. $m\angle ABC = m\angle DEF$	4. ?

12. Given: $ST = RN$; $IT = RU$

Prove: $SI = UN$

Proof:

Statements	Reasons
1. $ST = RN$	1. ?
2. ? $= SI + IT$; ? $= RU + UN$	2. ?
3. $SI + IT = RU + UN$	3. ?
4. $IT = RU$	4. ?
5. ?	5. ?

Written Exercises

Justify each step.

A

1.
$$4x - 5 = -2$$
$$4x = 3$$
$$x = \frac{3}{4}$$

2.
$$\frac{3a}{2} = \frac{6}{5}$$
$$3a = \frac{12}{5}$$
$$a = \frac{4}{5}$$

3.
$$\frac{z + 7}{3} = -11$$
$$z + 7 = -33$$
$$z = -40$$

4.
$$15y + 7 = 12 - 20y$$
$$35y + 7 = 12$$
$$35y = 5$$
$$y = \frac{1}{7}$$

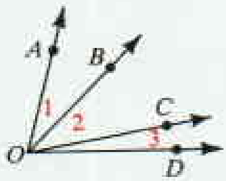
5.
$$\frac{2}{3}b = 8 - 2b$$
$$2b = 3(8 - 2b)$$
$$2b = 24 - 6b$$
$$8b = 24$$
$$b = 3$$

6.
$$x - 2 = \frac{2x + 8}{5}$$
$$5(x - 2) = 2x + 8$$
$$5x - 10 = 2x + 8$$
$$3x - 10 = 8$$
$$3x = 18$$
$$x = 6$$

Copy everything shown and supply missing statements and reasons.

7. Given: $\angle AOD$ as shown

Prove: $m\angle AOD = m\angle 1 + m\angle 2 + m\angle 3$



Proof:

Statements	Reasons
1. $m\angle AOD = m\angle AOC + m\angle 3$	1. ?
2. $m\angle AOC = m\angle 1 + m\angle 2$	2. ?
3. ?	3. ?

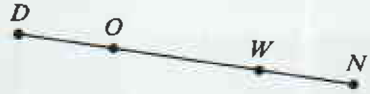
8. Given: $FL = AT$

Prove: $FA = LT$

Proof:

Statements	Reasons
1. ?	1. Given
2. $LA = LA$	2. ?
3. $FL + LA = AT + LA$	3. ?
4. $FL + LA = FA$; $LA + AT = LT$	4. ?
5. ?	5. Substitution Prop.

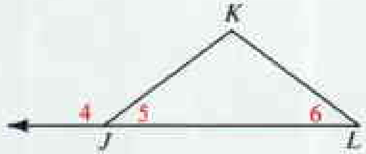
9. Given: $DW = ON$
Prove: $DO = WN$



Proof:

Statements	Reasons
1. $DW = ON$	1. ?
2. $DW = DO + OW$; $ON = \underline{?} + \underline{?}$	2. ?
3. ?	3. Substitution Prop.
4. $OW = OW$	4. ?
5. ?	5. ?

10. Given: $m\angle 4 + m\angle 6 = 180$
Prove: $m\angle 5 = m\angle 6$



Proof:

Statements	Reasons
1. $m\angle 4 + m\angle 6 = 180$	1. ?
2. $m\angle 4 + m\angle 5 = 180$	2. ?
3. $m\angle 4 + m\angle 5 = m\angle 4 + m\angle 6$	3. ?
4. $m\angle 4 = m\angle 4$	4. ?
5. ?	5. ?

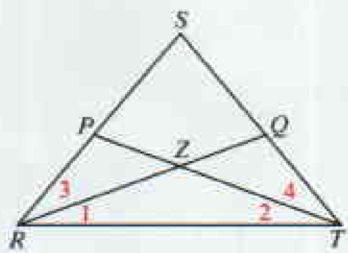
Copy everything shown and write a two-column proof.

B **11.** Given: $m\angle 1 = m\angle 2$; $m\angle 3 = m\angle 4$
Prove: $m\angle SRT = m\angle STR$

12. Given: $RP = TQ$; $PS = QS$
Prove: $RS = TS$

13. Given: $RQ = TP$; $ZQ = ZP$
Prove: $RZ = TZ$

14. Given: $m\angle SRT = m\angle STR$; $m\angle 3 = m\angle 4$
Prove: $m\angle 1 = m\angle 2$



Exs. 11–14

C **15.** Consider the following statements:

Reflexive Property: Robot A is as rusty as itself.
Symmetric Property: If Robot A is as rusty as Robot B, then Robot B is as rusty as Robot A.
Transitive Property: If Robot A is as rusty as Robot B and Robot B is as rusty as Robot C, then Robot A is as rusty as Robot C.

A *relation* such as "is as rusty as" that is reflexive, symmetric, and transitive is an *equivalence relation*. Which of the following are equivalence relations?

a. is rustier than **b.** has the same length as
c. is opposite (for rays) **d.** is coplanar with (for lines)

2-3 *Proving Theorems*

Chapter 1 included three *theorems*, statements that are proved. The theorems were deduced from *postulates*, statements that are accepted without proof. We will prove additional theorems throughout the book. When writing proofs, we will treat properties from algebra as postulates.

Suppose you are told that Y is the midpoint of $\overline{XZ}$ and that $XZ = 12$. You probably realize that $XY = 6$. Your conclusion about one particular situation suggests the general statement shown below as Theorem 2-1. The theorem uses the definition of a midpoint to prove additional properties of a midpoint that are not explicitly included in the definition. In this case, the theorem states something obvious. Later theorems may not be so obvious. In fact, some of them may surprise you.

Theorem 2-1 *Midpoint Theorem*

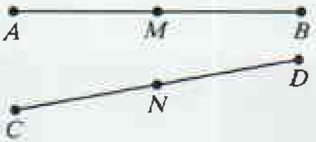
If M is the midpoint of $\overline{AB}$, then $AM = \frac{1}{2}AB$ and $MB = \frac{1}{2}AB$.

Given: M is the midpoint of $\overline{AB}$.
Prove: $AM = \frac{1}{2}AB$; $MB = \frac{1}{2}AB$

Proof:

Statements	Reasons
1. M is the midpoint of $\overline{AB}$.	1. Given
2. $\overline{AM} \cong \overline{MB}$, or $AM = MB$	2. Definition of midpoint
3. $AM + MB = AB$	3. Segment Addition Postulate
4. $AM + AM = AB$, or $2AM = AB$	4. Substitution Prop. (Steps 2 and 3)
5. $AM = \frac{1}{2}AB$	5. Division Prop. of =
6. $MB = \frac{1}{2}AB$	6. Substitution Prop. (Steps 2 and 5)

Example 1 Given: M is the midpoint of $\overline{AB}$;
N is the midpoint of $\overline{CD}$;
$AB = CD$
What can you deduce?



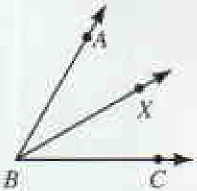
Solution Because M and N are midpoints, you know that $AM = MB$ and $CN = ND$.
From the Midpoint Theorem, you know that $AM = \frac{1}{2}AB$ and $CN = \frac{1}{2}CD$.
Since $AB = CD$, you know that $\frac{1}{2}AB = \frac{1}{2}CD$.
By substitution, you get $AM = CN$.
Thus you can deduce that AM, MB, CN, and ND are all equal.

The next theorem is similar to the Midpoint Theorem. It proves properties of the angle bisector that are not given in the definition. The proof is left as Classroom Exercise 10.

Theorem 2-2 *Angle Bisector Theorem*

If $\overrightarrow{BX}$ is the bisector of $\angle ABC$, then

$$m\angle ABX = \frac{1}{2}m\angle ABC \text{ and } m\angle XBC = \frac{1}{2}m\angle ABC.$$



Given: $\overrightarrow{BX}$ is the bisector of $\angle ABC$.

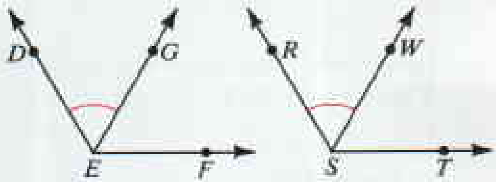
Prove: $m\angle ABX = \frac{1}{2}m\angle ABC$; $m\angle XBC = \frac{1}{2}m\angle ABC$

In addition to postulates and definitions, theorems may be used to justify steps in a proof. Notice the use of the Angle Bisector Theorem in Example 2.

Example 2

Given: $\overrightarrow{EG}$ is the bisector of $\angle DEF$;
$\overrightarrow{SW}$ is the bisector of $\angle RST$;
$m\angle DEG = m\angle RSW$

Prove: $m\angle DEF = m\angle RST$



Proof:

Statements	Reasons
1. $\overrightarrow{EG}$ is the bisector of $\angle DEF$; $\overrightarrow{SW}$ is the bisector of $\angle RST$.	1. Given
2. $m\angle DEG = \frac{1}{2}m\angle DEF$; $m\angle RSW = \frac{1}{2}m\angle RST$	2. Angle Bisector Theorem
3. $m\angle DEG = m\angle RSW$	3. Given
4. $\frac{1}{2}m\angle DEF = \frac{1}{2}m\angle RST$	4. Substitution Prop. (Steps 2 and 3)
5. $m\angle DEF = m\angle RST$	5. Multiplication Prop. of =

The two-column proofs you have seen in this section and the previous one are examples of **deductive reasoning.** We have proved statements by reasoning from postulates, definitions, theorems, and given information. The kinds of reasons you can use to justify statements in a proof are listed below.

Reasons Used in Proofs

Given information

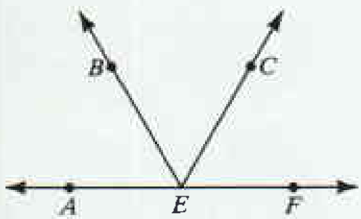
Definitions

Postulates (These include properties from algebra.)

Theorems that have already been proved

Classroom Exercises

What postulate, definition, or theorem justifies the statement about the diagram?



Exs. 1–9

1. $m\angle AEB + m\angle BEC = m\angle AEC$

2. $AE + EF = AF$

3. $m\angle AEB + m\angle BEF = 180$

4. If E is the midpoint of $\overline{AF}$, then $\overline{AE} \cong \overline{EF}$.

5. If E is the midpoint of $\overline{AF}$, then $AE = \frac{1}{2}AF$.

6. If E is the midpoint of $\overline{AF}$, then $\overrightarrow{EC}$ bisects $\overline{AF}$.

7. If $\overrightarrow{EB}$ bisects $\overline{AF}$, then E is the midpoint of $\overline{AF}$.

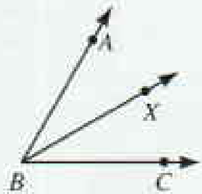
8. If $\overrightarrow{EB}$ is the bisector of $\angle AEC$, then $m\angle AEB = \frac{1}{2}m\angle AEC$.

9. If $\angle BEC \cong \angle CEF$, then $\overrightarrow{EC}$ is the bisector of $\angle BEF$.

10. Complete the proof of Theorem 2-2.

Given: $\overrightarrow{BX}$ is the bisector of $\angle ABC$.

Prove: $m\angle ABX = \frac{1}{2}m\angle ABC$; $m\angle XBC = \frac{1}{2}m\angle ABC$



Proof:

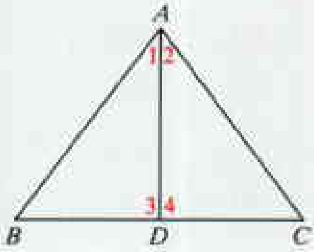
Statements	Reasons
1. $\overrightarrow{BX}$ is the bisector of $\angle ABC$.	1. ___?___
2. $\angle ABX \cong$ ___?___, or $m\angle ABX =$ ___?___	2. ___?___
3. $m\angle ABX + m\angle XBC = m\angle ABC$	3. ___?___
4. $m\angle ABX + m\angle ABX = m\angle ABC$, or $2m\angle ABX = m\angle ABC$	4. ___?___
5. $m\angle ABX = \frac{1}{2}m\angle ABC$	5. ___?___
6. $m\angle XBC = \frac{1}{2}m\angle ABC$	6. Substitution Prop. (Steps ___?___ and ___?___)

Written Exercises

Name the definition, postulate, or theorem that justifies the statement about the diagram.

A

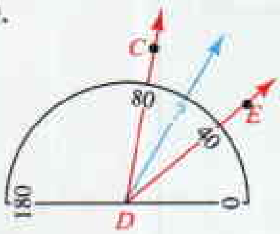
1. If D is the midpoint of $\overline{BC}$, then $\overline{BD} \cong \overline{DC}$.
2. If $\angle 1 \cong \angle 2$, then $\overrightarrow{AD}$ is the bisector of $\angle BAC$.
3. If $\overrightarrow{AD}$ bisects $\angle BAC$, then $\angle 1 \cong \angle 2$.
4. $m\angle 3 + m\angle 4 = 180$
5. If $\overline{BD} \cong \overline{DC}$, then D is the midpoint of $\overline{BC}$.
6. If D is the midpoint of $\overline{BC}$, then $BD = \frac{1}{2}BC$.
7. $m\angle 1 + m\angle 2 = m\angle BAC$
8. $BD + DC = BC$



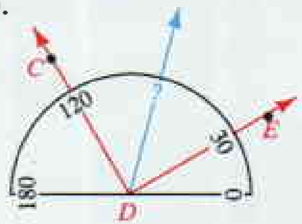
Exs. 1–8

Write the number that is paired with the bisector of $\angle CDE$.

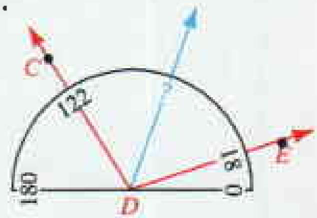
9.



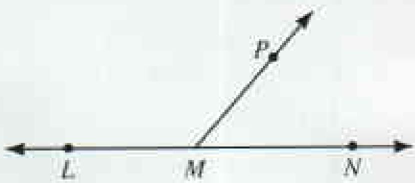
10.



11.

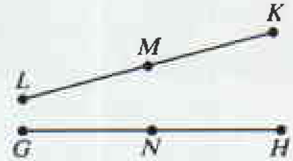


12. a. Draw a diagram similar to the one shown.
    b. Use a protractor to draw the bisectors of $\angle LMP$ and $\angle PMN$.
    c. What is the measure of the angle formed by these bisectors?
    d. Explain how you could have known the answer to part (c) without measuring.

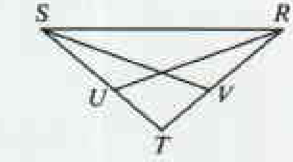


B

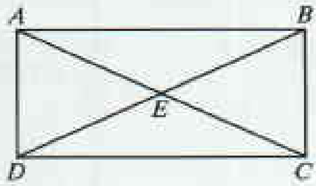
13. The coordinates of points L and X are 16 and 40, respectively. N is the midpoint of $\overline{LX}$, and Y is the midpoint of $\overline{LN}$. Sketch a diagram and find:
    a. LN b. the coordinate of N c. LY d. the coordinate of Y
14. $\overrightarrow{SW}$ bisects $\angle RST$ and $m\angle RST = 72$. $\overrightarrow{SZ}$ bisects $\angle RSW$, and $\overrightarrow{SR}$ bisects $\angle NSW$. Sketch a diagram and find $m\angle RSZ$ and $m\angle NSZ$.
15. a. Suppose M and N are the midpoints of $\overline{LK}$ and $\overline{GH}$, respectively. What segments are congruent?
    b. What additional information about the figure would enable you to deduce that $LM = NH$?



16. a. Suppose $\overrightarrow{SV}$ bisects $\angle RST$ and $\overrightarrow{RU}$ bisects $\angle SRT$. What angles are congruent?
    b. What additional information would enable you to deduce that $m\angle VSU = m\angle URV$?



What can you deduce from the given information?

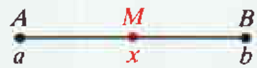


17. Given: $AE = DE$;
$CE = BE$

18. Given: $\overline{AC}$ bisects $\overline{DB}$;
$\overline{DB}$ bisects $\overline{AC}$;
$CE = BE$

19. Copy and complete the following proof of the statement: If points A and B have coordinates a and b, with $b > a$, and the midpoint M of $\overline{AB}$ has coordinate x, then $x = \frac{a+b}{2}$.

Given: Points A and B have coordinates a and b;
$b > a$; midpoint M of $\overline{AB}$ has coordinate x.

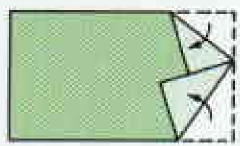


Prove: $x = \frac{a+b}{2}$

Proof:

Statements	Reasons
1. A, M, and B have coordinates a, x, and b respectively; $b > a$	1. ?
2. $AM = x - a$; $MB = b - x$	2. ?
3. M is the midpoint of $\overline{AB}$.	3. ?
4. $\overline{AM} \cong \overline{MB}$, or $AM = MB$	4. ?
5. $x - a = b - x$	5. ?
6. $2x =$?	6. ?
7. $x = \frac{a+b}{2}$	7. ?

C **20.** Fold down a corner of a rectangular sheet of paper. Then fold the next corner so that the edges touch as in the figure. Measure the angle formed by the fold lines. Repeat with another sheet of paper, folding the corner at a different angle. Explain why the angles formed are congruent.



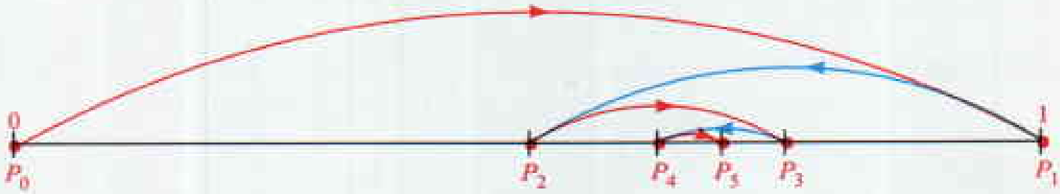
21. M is the midpoint of $\overline{AB}$, Q is the midpoint of $\overline{AM}$, and T is the midpoint of $\overline{QM}$. If the coordinates of A and B are a and b, find the coordinates of Q and T in terms of a and b.

22. Point T is the midpoint of $\overline{RS}$, W is the midpoint of $\overline{RT}$, and Z is the midpoint of $\overline{WS}$. If the length of $\overline{TZ}$ is x, find the following lengths in terms of x. (*Hint*: Sketch a diagram and let $y = WT$.)

a. RW **b.** ZS **c.** RS **d.** WZ

♦ Computer Key-In

A bee starts at point P_0, flies to point P_1, and lands. The bee then returns half of the way to P_0, landing at P_2. From P_2, the bee returns half of the way to P_1, landing at P_3, and so forth. Can you predict the bee's location after 10 trips?



Assuming that P_0 and P_1 have coordinates 0 and 1, respectively, the BASIC program below will compute and print the bee's location at the end of trips 2 through 10. P_n represents the position of the bee after n trips. Since P_n is the midpoint of the bee's previous two positions, P_{n-1} and P_{n-2}, line 50 calculates $P(N)$ by using the statement proved in Exercise 19, page 47.

```
10  DIM P(50)
20  LET P(0) = 0
30  LET P(1) = 1
40  FOR N = 2 TO 10
50  LET P(N) = (1/2) * (P(N - 2) + P(N - 1))
60  PRINT N, P(N)
70  NEXT N
80  END
```

Exercises

1. Enter the program on your computer and RUN it. Do you notice any patterns or trends in the coordinates? Change line 40 so that the computer will print the coordinates up to P_{40}. What simple fraction is approximated by P_{40}?

2. In line 50, $P(n)$ could instead be computed from the *series*

$$1 - \tfrac{1}{2} + \tfrac{1}{4} - \tfrac{1}{8} + \cdots + (-\tfrac{1}{2})^{n-1}$$

where each term of the series reflects the bee's return half of the way from P_{n-1} to P_{n-2}. Replace line 50 with the line below and RUN the new program.

```
50  LET P(N) = P(N - 1) + (-1/2) ↑ (N - 1)
```

Check that both programs produce the same results. (Some slight variations will be expected, due to rounding off.)

3. Suppose that on each trip the bee returned one third of the way to the previous point instead of half of the way. How would the series in Exercise 2 be modified? How would line 50 of Exercise 2 be modified? RUN a modified program for 30 trips and determine what point the bee seems to be approaching.

Self-Test 1

Use the conditional: If $\overline{AB}$ and $\overline{CD}$ intersect, then $\overrightarrow{AB}$ and $\overrightarrow{CD}$ intersect.

1. Write the hypothesis and the conclusion of the conditional.
2. Write the converse of the conditional. Is the converse true or false?
3. Rewrite the following pair of conditionals as a biconditional:
   $\overline{AB} \cong \overline{CD}$ if $AB = CD$; $\overline{AB} \cong \overline{CD}$ only if $AB = CD$.
4. Provide a counterexample to disprove the statement:
   If $m\angle A$ is less than 100, then $\angle A$ is an acute angle.
5. Given: $m\angle A + m\angle B = 180$; $m\angle C = m\angle B$
   What property of equality justifies the statement $m\angle A + m\angle C = 180$?
6. Point M is the midpoint of $\overline{RT}$. $RM = x$ and $RT = 4x - 6$. Find the value of x.
7. The measure of $\angle ABC$ is 108. $\overrightarrow{BD}$ is the bisector of $\angle ABC$, and $\overrightarrow{BE}$ is the bisector of $\angle ABD$. Find the measure of $\angle EBC$.
8. You can use given information and theorems as reasons in proofs. Name two other kinds of reasons you can use.

Biographical Note

Julia Morgan

Julia Morgan (1872–1959), the first successful woman architect in the United States, was born in San Francisco. Though best known for her design of San Simeon, the castle-like former home of William Randolph Hearst pictured at the left, she designed numerous public buildings and private homes. Even today, to own "a Julia Morgan house" carries considerable prestige.

To become an architect, Morgan needed great determination as well as a brilliant mind. Since the University of California did not have an architecture curriculum at that time, she prepared for graduate work in Paris by studying civil engineering. In Paris the École des Beaux-Arts, which had just begun to admit foreigners, was particularly reluctant to admit a foreign woman. She persisted, however, and became the school's first woman graduate.

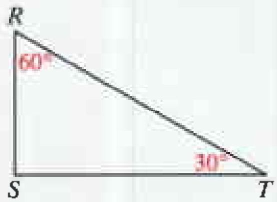
Theorems about Angles and Perpendicular Lines

Objectives

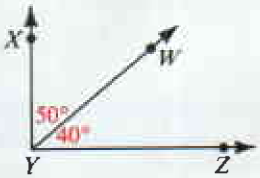
1. Apply the definitions of complementary and supplementary angles.
2. State and use the theorem about vertical angles.
3. Apply the definition and theorems about perpendicular lines.
4. State and apply the theorems about angles supplementary to, or complementary to, congruent angles.
5. Plan proofs and then write them in two-column form.

2-4 *Special Pairs of Angles*

Complementary angles (comp. $\angle$s) are two angles whose measures have the sum 90. Each angle is called a *complement* of the other.

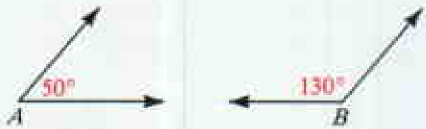


$\angle R$ and $\angle T$ are complementary.



$\angle XYW$ is a complement of $\angle WYZ$.

Supplementary angles (supp. $\angle$s) are two angles whose measures have the sum 180. Each angle is called a *supplement* of the other.



$\angle A$ and $\angle B$ are supplementary.

$\angle DEG$ is a supplement of $\angle GEF$.

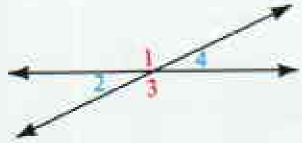
Example 1 A supplement of an angle is three times as large as a complement of the angle. Find the measure of the angle.

Solution Let x = the measure of the angle.
Then $180 - x$ = the measure of its supplement,
and $90 - x$ = the measure of its complement.

$$180 - x = 3(90 - x)$$
$$180 - x = 270 - 3x$$
$$2x = 90$$
$$x = 45$$

The measure of the angle is 45.

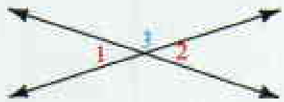
Vertical angles (vert. $\angle$s) are two angles such that the sides of one angle are opposite rays to the sides of the other angle. When two lines intersect, they form two pairs of vertical angles.
$\angle 1$ and $\angle 3$ are vert. $\angle$s. $\angle 2$ and $\angle 4$ are vert. $\angle$s.



Theorem 2-3

Vertical angles are congruent.

Given: $\angle 1$ and $\angle 2$ are vertical angles.
Prove: $\angle 1 \cong 2$

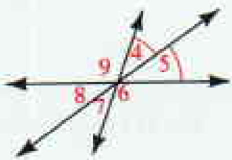


Proof:

Statements	Reasons
1. $m\angle 1 + m\angle 3 = 180$; $m\angle 2 + m\angle 3 = 180$	1. Angle Addition Postulate
2. $m\angle 1 + m\angle 3 = m\angle 2 + m\angle 3$	2. Substitution Prop.
3. $m\angle 3 = m\angle 3$	3. Reflexive Prop.
4. $m\angle 1 = m\angle 2$, or $\angle 1 \cong \angle 2$	4. Subtraction Prop. of =

Example 2 In the diagram, $\angle 4 \cong \angle 5$.
Name two other angles congruent to $\angle 5$.

Solution $\angle 8 \cong \angle 5$ since vertical angles are congruent.
Since $\angle 7 \cong \angle 4$ and $\angle 4 \cong \angle 5$, $\angle 7 \cong \angle 5$ by the Transitive Property.

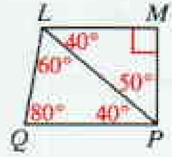


Classroom Exercises

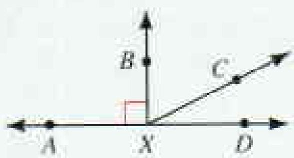
Find the measures of a complement and a supplement of $\angle A$.

1. $m\angle A = 10$ **2.** $m\angle A = 75$ **3.** $m\angle A = 89$ **4.** $m\angle A = y$

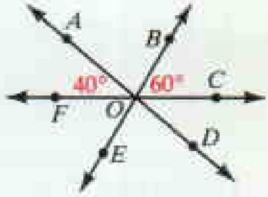
5. Name two right angles.
6. Name two adjacent complementary angles.
7. Name two complementary angles that are not adjacent.
8. a. Name a supplement of $\angle MLQ$.
b. Name another pair of supplementary angles.



9. In the diagram, $m\angle AXB = 90$. Name:
a. two congruent supplementary angles
b. two supplementary angles that are not congruent
c. two complementary angles
d. a straight angle



Complete.



10. $\angle AOB \cong$ ___?___ **11.** $\angle AOE \cong$ ___?___

12. $\angle FOB \cong$ ___?___ **13.** $\angle COA \cong$ ___?___

14. $m\angle FOE =$ ___?___ **15.** $m\angle COD =$ ___?___

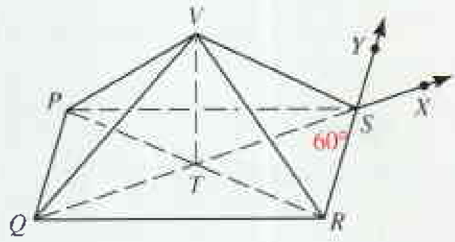
16. $m\angle DOB =$ ___?___ **17.** $m\angle AOB =$ ___?___

18. $m\angle COE =$ ___?___ **19.** $m\angle FOB =$ ___?___

20. The four angles of figure *PQRS* are right angles. $\angle VTR$ is a right angle. $m\angle QSR = 60$. Find the measures.

a. $m\angle VTP$ **b.** $m\angle XSY$

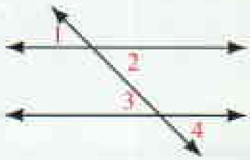
c. $m\angle RSX$ **c.** $m\angle PSY$



21. Given: $\angle 2 \cong \angle 3$

a. What can you deduce?

b. Explain how you would prove your conclusion.



Written Exercises

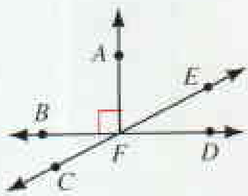
Find the measures of a complement and a supplement of $\angle K$.

A **1.** $m\angle K = 20$ **2.** $m\angle K = 72\frac{1}{2}$ **3.** $m\angle K = x$ **4.** $m\angle K = 2y$

5. Two complementary angles are congruent. Find their measures.

6. Two supplementary angles are congruent. Find their measures.

In the diagram, $\angle AFB$ is a right angle. Name the figures described.

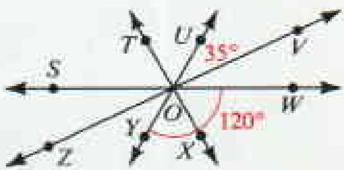


7. Another right angle **8.** Two complementary angles

9. Two congruent supplementary angles **10.** Two noncongruent supplementary angles

11. Two acute vertical angles **12.** Two obtuse vertical angles

In the diagram, $\overrightarrow{OT}$ bisects $\angle SOU$, $m\angle UOV = 35$, and $m\angle YOW = 120$. Find the measure of each angle.



13. $m\angle ZOY$ **14.** $m\angle ZOW$

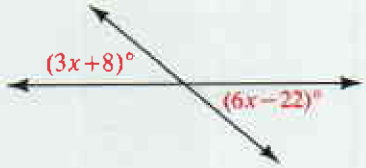
15. $m\angle VOW$ **16.** $m\angle SOU$

17. $m\angle TOU$ **18.** $m\angle ZOT$

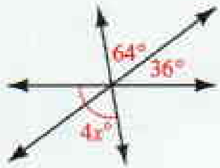
Find the value of *x*.

19.

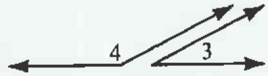
20.



21.



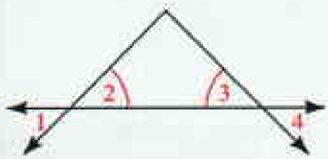
22. ∠1 and ∠2 are supplements.
∠3 and ∠4 are supplements.



a. If $m\angle 1 = m\angle 3 = 27$, find $m\angle 2$ and $m\angle 4$.
b. If $m\angle 1 = m\angle 3 = x$, find $m\angle 2$ and $m\angle 4$ in terms of x.
c. If two angles are congruent, must their supplements be congruent?

23. Copy everything shown. Complete the proof.

Given: $\angle 2 \cong \angle 3$
Prove: $\angle 1 \cong \angle 4$



Proof:

Statements	Reasons
1. $\angle 1 \cong \angle 2$	1. ?
2. $\angle 2 \cong \angle 3$	2. ?
3. $\angle 3 \cong \angle 4$	3. ?
4. ?	4. Transitive Property (used twice)

If ∠*A* and ∠*B* are supplementary, find the value of *x*, *m*∠*A*, and *m*∠*B*.

B **24.** $m\angle A = 2x$, $m\angle B = x - 15$

25. $m\angle A = x + 16$, $m\angle B = 2x - 16$

If ∠*C* and ∠*D* are complementary, find the value of *y*, *m*∠*C*, and *m*∠*D*.

26. $m\angle C = 3y + 5$, $m\angle D = 2y$

27. $m\angle C = y - 8$, $m\angle D = 3y + 2$

Use the given information to write an equation and solve the problem.

28. Find the measure of an angle that is twice as large as its supplement.

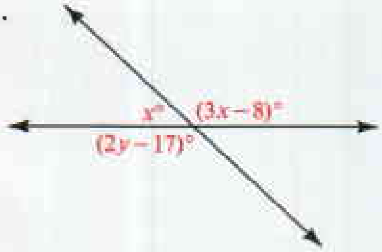
29. Find the measure of an angle that is half as large as its complement.

30. The measure of a supplement of an angle is 12 more than twice the measure of the angle. Find the measures of the angle and its supplement.

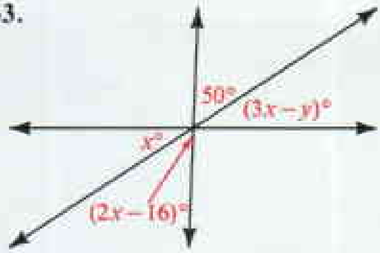
31. A supplement of an angle is six times as large as a complement of the angle. Find the measures of the angle, its supplement, and its complement.

Find the values of x and y for each diagram.

32.



33.



C **34.** Can the measure of a complement of an angle ever equal exactly half the measure of a supplement of the angle? Explain.

35. You are told that the measure of an acute angle is equal to the difference between the measure of a supplement of the angle and twice the measure of a complement of the angle. What can you deduce about the angle? Explain.

Application *Orienteering*

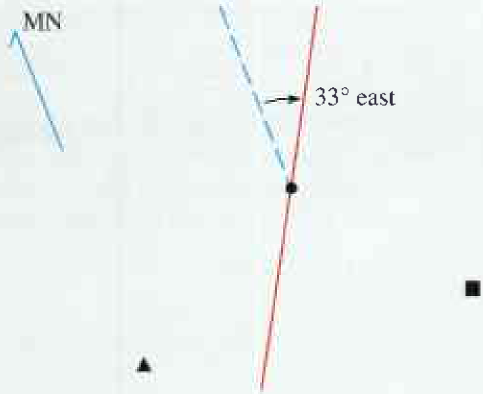
The sport of orienteering involves finding your way from control point to control point in a wilderness area, using a map and protractor-type compass. Similar methods can be used by hikers, hunters, boaters, and backpackers.

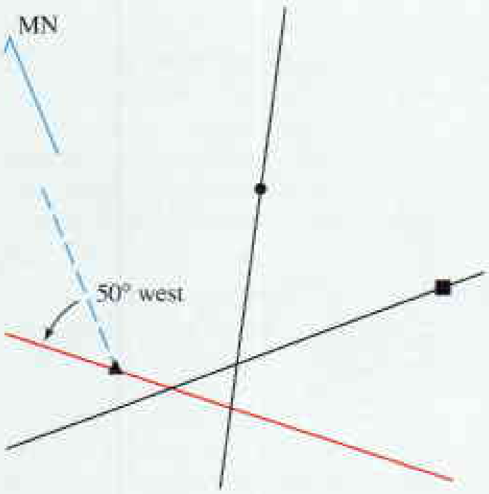
One thing you want to be able to do is locate your position on the map. This can be done by taking sightings of specific objects. For example, suppose you can see a lookout tower (on Number Four Mountain at • on the map shown below).

You sight across your compass and discover the tower is 33° east of magnetic north (MN). On your map you draw a line through the tower at a 33° angle to magnetic north. Be sure to use magnetic north rather than true north, for they may differ by as much as 20°. Hiking maps and nautical charts usually give both. All compass readings here are given in terms of magnetic north.

You are somewhere on the line you have drawn. If there is a feature near you (a trail, stream or pond), then your position is where the line crosses the feature on the map. Otherwise, you will need to take a second sighting, on the peak of Lily Bay Mountain (at ▲ on the map). It is 50° west of north. Draw a line on your map through the peak at a 50° angle with magnetic north. You are close to the point where the lines cross.

Since a third landmark is visible, the summit of Bluff Mountain (■ on the map), you can check your position with a third sighting. The three lines might cross at a single point. However, there is usually some error in sighting and drawing the angles, so instead of meeting exactly at a point, the three lines drawn often form a triangle. If the triangle is small, it gives you a good idea of your true position.





Exercises

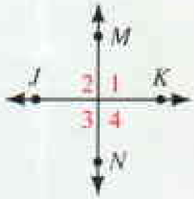
1. Another orienteering party sights on Lily Bay Mountain and the lookout tower and finds the following angles: mountain, 58° west of north; tower, 40° east of north. Are they north or south of you?
2. If you head due east from Lily Bay Mountain (90° east of magnetic north), will you pass Bluff Mountain on your right or on your left?
3. Lillian and Ray both sight Lily Bay Mountain at 70° west of north, but Lillian sees the lookout tower at 40° east of north, while Ray sees it at 20° east of north. Which person is closer to Bluff Mountain?
4. Sailors use this method of finding their position when they are navigating near shore, sighting on lighthouses, smokestacks, and other landmarks shown on their charts. They call the small triangle formed by the three sighting lines a "cocked hat," and usually mark their position at the corner closest to the nearest hazard. Why is this a sensible rule?

2-5 *Perpendicular Lines*

In the town shown, roads that run east-west are called streets, while those that run north-south are called avenues. Each of the streets is *perpendicular* to each of the avenues.

Perpendicular lines are two lines that intersect to form right angles (90° angles). Because lines that form one right angle always form four right angles (see Exercise 26, page 21), you can conclude that two lines are perpendicular, by definition, once you know that any one of the angles they form is a right angle. The definition of perpendicular lines can be used in the two ways shown below.

1. If $\overleftrightarrow{JK}$ is perpendicular to $\overleftrightarrow{MN}$ (written $\overleftrightarrow{JK} \perp \overleftrightarrow{MN}$), then each of the numbered angles is a right angle (a 90° angle).
2. If any one of the numbered angles is a right angle (a 90° angle), then $\overleftrightarrow{JK} \perp \overleftrightarrow{MN}$.



The word *perpendicular* is also used for intersecting rays and segments. For example, if $\overleftrightarrow{JK} \perp \overleftrightarrow{MN}$ in the diagram, then $\overline{JK} \perp \overline{MN}$ and the sides of $\angle 2$ are perpendicular.

The definition of perpendicular lines is closely related to the following theorems. Notice that Theorem 2-4 and Theorem 2-5 are *converses* of each other. For the proofs of the theorems, see the exercises.

Theorem 2-4

If two lines are perpendicular, then they form congruent adjacent angles.

Theorem 2-5

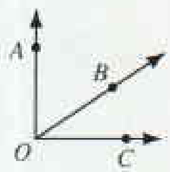
If two lines form congruent adjacent angles, then the lines are perpendicular.

Theorem 2-6

If the exterior sides of two adjacent acute angles are perpendicular, then the angles are complementary.

Given: $\overrightarrow{OA} \perp \overrightarrow{OC}$

Prove: $\angle AOB$ and $\angle BOC$ are comp. $\angle$s.

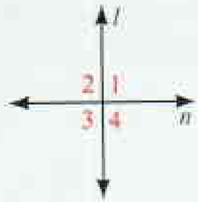


Classroom Exercises

1. Complete the proof of Theorem 2-4: If two lines are perpendicular, then they form congruent adjacent angles.

Given: $l \perp n$

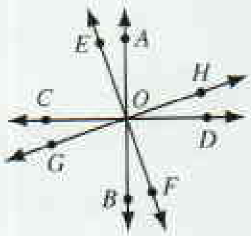
Prove: $\angle 1$, $\angle 2$, $\angle 3$, and $\angle 4$ are congruent angles.



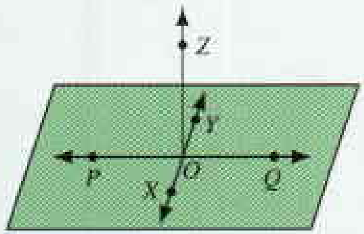
Proof:

Statements	Reasons
1. $l \perp n$	1. _?_
2. $\angle 1$, $\angle 2$, $\angle 3$, $\angle 4$ are 90° ∠s.	2. Definition of _?_
3. $\angle 1$, $\angle 2$, $\angle 3$, $\angle 4$ are ≅ ∠s.	3. Definition of _?_

2. In the diagram, $\overleftrightarrow{AB} \perp \overleftrightarrow{CD}$ and $\overleftrightarrow{EF} \perp \overleftrightarrow{GH}$. Name eight right angles.

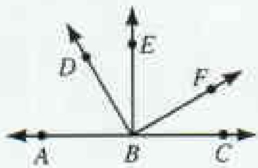


3. In the diagram, $\overrightarrow{OZ} \perp \overleftrightarrow{PQ}$, $\overrightarrow{OZ} \perp \overleftrightarrow{XY}$, and $\overleftrightarrow{PQ} \perp \overleftrightarrow{XY}$. Name eight right angles.



In the diagram, $\overrightarrow{BE} \perp \overleftrightarrow{AC}$ and $\overrightarrow{BD} \perp \overrightarrow{BF}$. Find the measures of the following angles.

	$m\angle CBF$	$m\angle EBF$	$m\angle DBE$	$m\angle DBA$	$m\angle DBC$
4.	40	?	?	?	?
5.	x	?	?	?	?



Name the definition or state the theorem that justifies the statement about the diagram.

6. If $\angle 6$ is a right angle, then $\overleftrightarrow{RS} \perp \overleftrightarrow{TV}$.

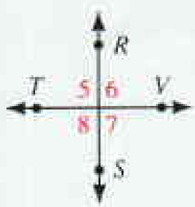
7. If $\overleftrightarrow{RS} \perp \overleftrightarrow{TV}$, then $\angle 5$, $\angle 6$, $\angle 7$, and $\angle 8$ are right angles.

8. If $\overleftrightarrow{RS} \perp \overleftrightarrow{TV}$, then $\angle 8 \cong \angle 7$.

9. If $\overleftrightarrow{RS} \perp \overleftrightarrow{TV}$, then $m\angle 6 = 90$.

10. If $\angle 5 \cong \angle 6$, then $\overleftrightarrow{RS} \perp \overleftrightarrow{TV}$.

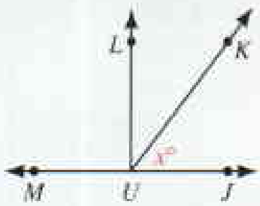
11. If $m\angle 5 = 90$, then $\overleftrightarrow{RS} \perp \overleftrightarrow{TV}$.



Written Exercises

A **1.** In the diagram, $\overrightarrow{UL} \perp \overleftrightarrow{MJ}$ and $m\angle JUK = x$. Express in terms of x the measures of the angles named.

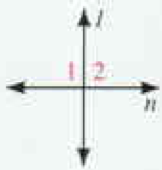
a. $\angle LUK$ **b.** $\angle MUK$



2. Copy and complete the proof of Theorem 2-5: If two lines form congruent adjacent angles, then the lines are perpendicular.

Given: $\angle 1 \cong \angle 2$

Prove: $l \perp n$



Proof:

Statements	Reasons
1. $\angle 1 \cong \angle 2$, or $m\angle 1 = m\angle 2$	1. ?
2. $m\angle 1 + m\angle 2 = 180$	2. ?
3. $m\angle 2 + m\angle 2 = 180$, or $2m\angle 2 = 180$	3. ?
4. $m\angle 2 = 90$	4. ?
5. ?	5. Def. of $\perp$ lines

Name the definition or state the theorem that justifies the statement about the diagram.

3. If $\angle EBC$ is a right angle, then $\overrightarrow{BE} \perp \overleftrightarrow{AC}$.

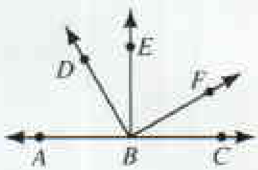
4. If $\overleftrightarrow{AC} \perp \overrightarrow{BE}$, then $\angle ABE$ is a right angle.

5. If $\overrightarrow{BE} \perp \overleftrightarrow{AC}$, then $\angle ABD$ and $\angle DBE$ are complementary.

6. If $\angle ABD$ and $\angle DBE$ are complementary angles, then $m\angle ABD + m\angle DBE = 90$.

7. If $\overrightarrow{BE} \perp \overleftrightarrow{AC}$, then $m\angle ABE = 90$.

8. If $\angle ABE \cong \angle EBC$, then $\overleftrightarrow{AC} \perp \overrightarrow{BE}$.



Exs. 3–12

In the diagram, $\overrightarrow{BE} \perp \overleftrightarrow{AC}$ and $\overrightarrow{BD} \perp \overrightarrow{BF}$. Find the value of x.

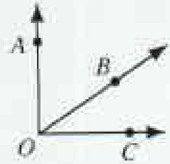
9. $m\angle ABD = 2x - 15$, $m\angle DBE = x$

10. $m\angle DBE = 3x$, $m\angle EBF = 4x - 1$

11. $m\angle ABD = 3x - 12$, $m\angle DBE = 2x + 2$, $m\angle EBF = 2x + 8$

12. $m\angle ABD = 6x$, $m\angle DBE = 3x + 9$, $m\angle EBF = 4x + 18$, $m\angle FBC = 4x$

13. Copy and complete the proof of Theorem 2-6: If the exterior sides of two adjacent acute angles are perpendicular, then the angles are complementary.

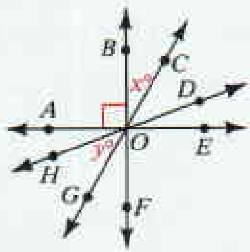


Given: $\overrightarrow{OA} \perp \overrightarrow{OC}$
Prove: $\angle AOB$ and $\angle BOC$ are comp. ∠s.

Proof:

Statements	Reasons
1. $\overrightarrow{OA} \perp \overrightarrow{OC}$	1. ?
2. $m\angle AOC = 90$	2. Def. of ⊥ lines
3. $m\angle AOB + m\angle BOC = m\angle AOC$	3. ?
4. ?	4. Substitution Prop.
5. ?	5. Def. of comp. ∠s

In the figure $\overleftrightarrow{BF} \perp \overleftrightarrow{AE}$, $m\angle BOC = x$, and $m\angle GOH = y$. Express the measure of the angle in terms of x, y, or both.



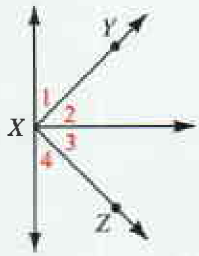
B **14.** $\angle COA$

15. $\angle COH$

16. $\angle HOF$

17. $\angle DOE$

Can you conclude from the information given for each exercise that $\overrightarrow{XY} \perp \overrightarrow{XZ}$?



18. $m\angle 1 = 46$ and $m\angle 4 = 44$

19. $\angle 1$ and $\angle 3$ are complementary.

20. $\angle 2 \cong \angle 3$

21. $m\angle 1 = m\angle 4$

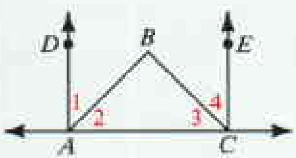
22. $\angle 1$ and $\angle 3$ are congruent and complementary.

23. $m\angle 1 = m\angle 2$ and $m\angle 3 = m\angle 4$

24. $\angle 1 \cong \angle 3$ and $\angle 2 \cong \angle 4$

25. $\angle 1 \cong \angle 4$ and $\angle 2 \cong \angle 3$

What can you conclude from the information given?



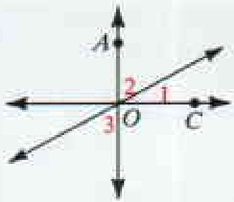
26. Given: $\overrightarrow{AB}$ bisects $\angle DAC$;
$\overrightarrow{CB}$ bisects $\angle ECA$;
$m\angle 2 = 45$;
$m\angle 3 = 45$

27. Given: $\overrightarrow{AD} \perp \overleftrightarrow{AC}$; $\overrightarrow{CE} \perp \overleftrightarrow{AC}$; $m\angle 1 = m\angle 4$

28. Copy everything shown and write a two-column proof.

Given: $\overleftrightarrow{AO} \perp \overleftrightarrow{CO}$

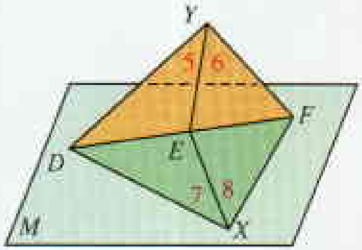
Prove: $\angle 1$ and $\angle 3$ are comp. $\angle$s.



C **29.** First find two lines (other than $\overleftrightarrow{YD}$ and $\overleftrightarrow{YF}$) that are perpendicular. Then write a two-column proof that the lines are perpendicular.

Given: $\overleftrightarrow{YD} \perp \overleftrightarrow{YF}$;
$m\angle 7 = m\angle 5$;
$m\angle 8 = m\angle 6$

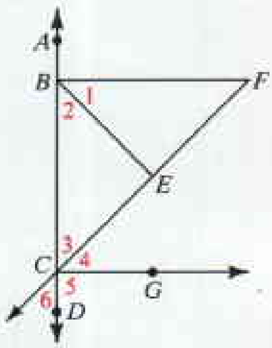
Prove: $\underline{\ ?\ } \perp \underline{\ ?\ }$



Mixed Review Exercises

Write something you can conclude from the given information.

1. Given: $m\angle 1 = m\angle 4$; $m\angle 2 = m\angle 3$
2. Given: $AB = CD$
3. Given: $m\angle 6 = m\angle 4$
4. Given: $\overline{FB} \perp \overline{AD}$; $\overrightarrow{BE}$ bisects $\angle FBC$.
5. Given: $BE = EF$; E is the midpoint of $\overline{FC}$.
6. Given: $\angle 1$ and $\angle 2$ are complements.
7. Given: $\angle 4$ and $\angle 6$ are complements.



2-6 *Planning a Proof*

As you have seen in the last few sections, a proof of a theorem consists of five parts:

1. *Statement* of the theorem
2. A *diagram* that illustrates the given information
3. A list, in terms of the figure, of what is *given*
4. A list, in terms of the figure, of what you are to *prove*
5. A series of *statements and reasons* that lead from the given information to the statement that is to be proved

In many of the proofs in this book, the diagram and the statements of what is given and what is to be proved will be supplied for you. Sometimes you will be asked to provide them.

When you draw a diagram, try to make it reasonably accurate, avoiding special cases that might mislead. For example, when a theorem refers to an angle, don't draw a *right* angle.

Before you write the steps in a two-column proof you will need to plan your proof. Sometimes you will read the statement of a theorem and see immediately how to prove it. Other times you may need to try several approaches before you find a plan that works.

If you don't see a method of proof immediately, try reasoning back from what you would like to prove. Think: "This conclusion will be true if __?__ is true. This, in turn, will be true if __?__ is true . . . ." Sometimes this procedure leads back to a given statement. If so, you have found a method of proof.

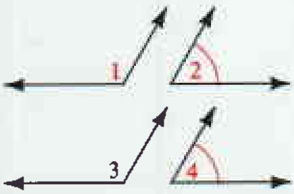
Studying the proofs of previous theorems may suggest methods to try. For example, the proof of the theorem that vertical angles are congruent suggests the proof of the following theorem.

Theorem 2-7

If two angles are supplements of congruent angles (or of the same angle), then the two angles are congruent.

Given: $\angle 1$ and $\angle 2$ are supplementary;
$\angle 3$ and $\angle 4$ are supplementary;
$\angle 2 \cong \angle 4$

Prove: $\angle 1 \cong \angle 3$



Proof:

Statements	Reasons
1. $\angle 1$ and $\angle 2$ are supplementary; $\angle 3$ and $\angle 4$ are supplementary.	1. Given
2. $m\angle 1 + m\angle 2 = 180$; $m\angle 3 + m\angle 4 = 180$	2. Def. of supp. $\angle$s
3. $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 4$	3. Substitution Prop.
4. $\angle 2 \cong \angle 4$, or $m\angle 2 = m\angle 4$	4. Given
5. $m\angle 1 = m\angle 3$, or $\angle 1 \cong \angle 3$	5. Subtraction Prop. of =

The proof of the following theorem is left as Exercise 18.

Theorem 2-8

If two angles are complements of congruent angles (or of the same angle), then the two angles are congruent.

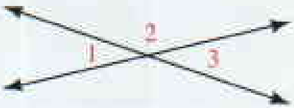
There is often more than one way to prove a particular statement, and the amount of detail one includes in a proof may differ from person to person. You should show enough steps so the reader can follow your argument and see why the theorem you are proving is true. As you gain more experience in writing proofs, you and your teacher may agree on what steps may be combined or omitted.

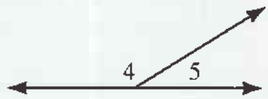
Classroom Exercises

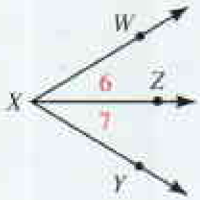
a. In each exercise use the information given to conclude that two angles are congruent.

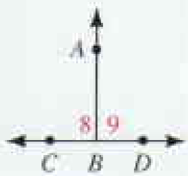
b. Name or state the definition or theorem that justifies your conclusion.

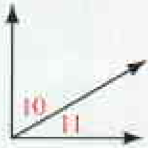
1. $\angle 6$ is comp. to $\angle 10$;
   $\angle 7$ is comp. to $\angle 10$.
2. $m\angle 5 = 31$; $m\angle 7 = 31$
3. $\overline{AB} \perp \overline{CD}$
4. $\overrightarrow{XZ}$ bisects $\angle WXY$.
5. $\angle 4$ is supp. to $\angle 6$;
   $\angle 2$ is supp. to $\angle 7$;
   $\angle 6 \cong \angle 7$
6. Given only the diagrams, and no additional information







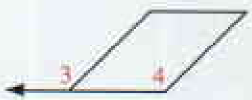


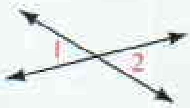


Describe your plan for proving the following. You don't need to give all the details.

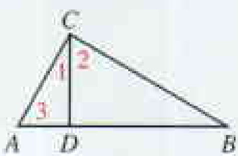
7. Given: $\angle 2 \cong \angle 3$
   Prove: $\angle 1 \cong \angle 4$

8. Given: $\angle 3$ is supp. to $\angle 1$;
   $\angle 4$ is supp. to $\angle 2$.
   Prove: $\angle 3 \cong \angle 4$





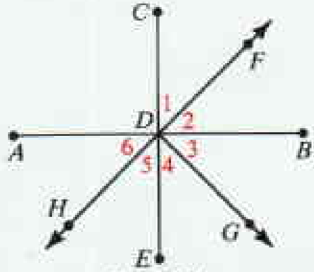
9. Given: $\overline{AC} \perp \overline{BC}$;
   $\angle 3$ is comp. to $\angle 1$.
   Prove: $\angle 3 \cong \angle 2$



10. Given: $m\angle 1 = m\angle 4$
    Prove: $m\angle 2 = m\angle 3$

Written Exercises

Write the name or statement of the definition, postulate, property, or theorem that justifies the statement about the diagram.



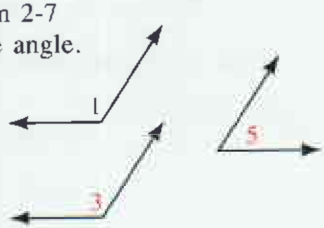
Exs. 1–14

A

1. $AD + DB = AB$
2. $m\angle 1 + m\angle 2 = m\angle CDB$
3. $\angle 2 \cong \angle 6$
4. If D is the midpoint of $\overline{AB}$, then $AD = \frac{1}{2}AB$.
5. If $\overrightarrow{DF}$ bisects $\angle CDB$, then $\angle 1 \cong \angle 2$.
6. $m\angle ADF + m\angle FDB = 180$
7. If $\overline{CD} \perp \overline{AB}$, then $m\angle CDB = 90$.
8. If $\angle 4 \cong \angle 3$, then $\overrightarrow{DG}$ bisects $\angle BDE$.
9. If $m\angle 3 + m\angle 4 = 90$, then $\angle 3$ and $\angle 4$ are complements.
10. If $\angle ADF$ and $\angle 4$ are supplements, then $m\angle ADF + m\angle 4 = 180$.
11. If $\overline{AB} \perp \overline{CE}$, then $\angle ADC \cong \angle ADE$.
12. If $\angle 4$ is complementary to $\angle 5$ and $\angle 6$ is complementary to $\angle 5$, then $\angle 4 \cong \angle 6$.
13. If $\angle FDG$ is a right angle, then $\overrightarrow{DF} \perp \overrightarrow{DG}$.
14. If $\angle FDG \cong \angle GDH$, then $\overrightarrow{DG} \perp \overleftrightarrow{HF}$.
15. Copy everything shown and complete the proof of Theorem 2-7 for the case where two angles are supplements of the same angle.

    Given: $\angle 1$ and $\angle 5$ are supplementary;
    $\angle 3$ and $\angle 5$ are supplementary.

    Prove: $\angle 1 \cong \angle 3$



Proof:

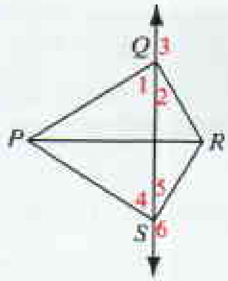
Statements	Reasons
1. $\angle 1$ and $\angle 5$ are supplementary; $\angle 3$ and _?_.	1. _?_
2. $m\angle 1 + m\angle 5 = 180$; $m\angle 3 + m\angle 5 = 180$	2. _?_
3. $m\angle 1 + m\angle 5 = m\angle 3 + m\angle 5$	3. _?_
4. $m\angle 5 = m\angle 5$	4. Reflexive Prop.
5. $m\angle 1 = m\angle 3$, or $\angle 1 \cong \angle 3$	5. _?_

16. a. Are there any angles in the diagram that must be congruent to $\angle 4$? Explain.
    b. If $\angle 4$ and $\angle 5$ are supplementary, name all angles shown that must be congruent to $\angle 4$.

17. a. Copy everything shown and complete the proof.

Given: $\overline{PQ} \perp \overline{QR}$;
$\overline{PS} \perp \overline{SR}$;
$\angle 1 \cong \angle 4$

Prove: $\angle 2 \cong \angle 5$



Proof:

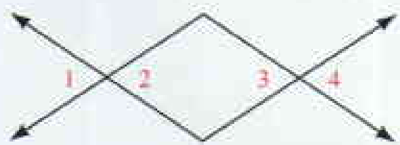
Statements	Reasons
1. $\overline{PQ} \perp \overline{QR}$; $\overline{PS} \perp \overline{SR}$	1. __?__
2. $\angle 2$ is comp. to $\angle 1$; $\angle 5$ is comp. to $\angle 4$.	2. __?__
3. $\angle 1 \cong \angle 4$	3. __?__
4. $\angle 2 \cong \angle 5$	4. __?__

b. After proving that $\angle 2 \cong \angle 5$ in part (a), tell how you could go on to prove that $\angle 3 \cong \angle 6$.

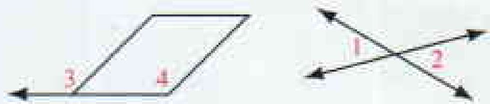
B **18.** Prove Theorem 2-8: If two angles are complements of congruent angles, then the two angles are congruent. *Note*: You will need to draw your own diagram and state what is given and what you are to prove in terms of your diagram. (*Hint*: See the proof of Theorem 2-7 on page 61.)

Copy everything shown and write a two-column proof.

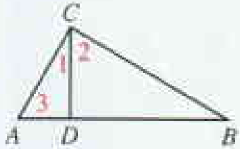
19. Given: $\angle 2 \cong \angle 3$
Prove: $\angle 1 \cong \angle 4$



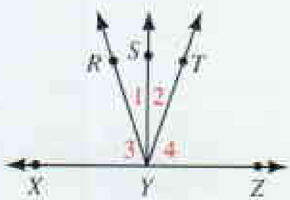
20. Given: $\angle 3$ is supp. to $\angle 1$;
$\angle 4$ is supp. to $\angle 2$.
Prove: $\angle 3 \cong \angle 4$



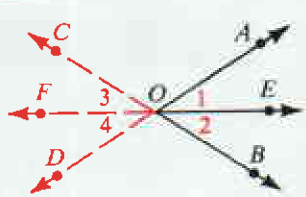
21. Given: $\overline{AC} \perp \overline{BC}$;
$\angle 3$ is comp. to $\angle 1$.
Prove: $\angle 3 \cong \angle 2$



22. Given: $m\angle 1 = m\angle 2$;
$m\angle 3 = m\angle 4$
Prove: $\overrightarrow{YS} \perp \overleftrightarrow{XZ}$

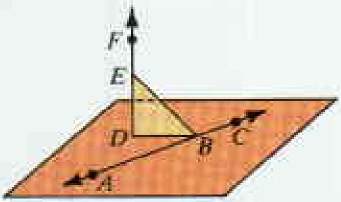


23. Draw any $\angle AOB$ and its bisector $\overrightarrow{OE}$. Now draw the rays opposite to $\overrightarrow{OA}$, $\overrightarrow{OB}$, and $\overrightarrow{OE}$. What can you conclude about the part of the diagram shown in red? Prove your conclusion.



C **24.** Make a diagram showing $\angle PQR$ bisected by $\overrightarrow{QX}$. Choose a point Y on the ray opposite to $\overrightarrow{QX}$.
Prove: $\angle PQY \cong \angle RQY$

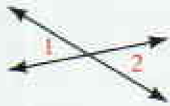
25. Given: $m\angle DBA = 45$;
$m\angle DEB = 45$
Prove: $\angle DBC \cong \angle FEB$



Self-Test 2

1. It is known that $\angle HOK$ has a supplement, but can't have a complement. Name one possible measure for $\angle HOK$.

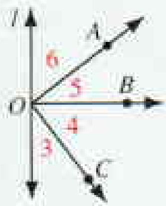
2. $m\angle 1 = 3x - 5$ and $m\angle 2 = x + 25$
a. $x = \underline{\ ?\ }$ **b.** $m\angle 1 = \underline{\ ?\ }$ (numerical value)



For Exercises 3 and 4 you are given that $\overrightarrow{OB} \perp l$ and $\overrightarrow{OA} \perp \overrightarrow{OC}$.

3. If $m\angle 3 = 37$, complete:
$m\angle 4 = \underline{\ ?\ }$ $m\angle 5 = \underline{\ ?\ }$ $m\angle 6 = \underline{\ ?\ }$

4. If $m\angle 3 = t$, express the measures of the other numbered angles in terms of t.

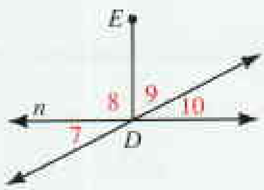


In the diagram, $\overline{DE} \perp n$. State the theorem or name the definition that justifies the statement about the diagram.

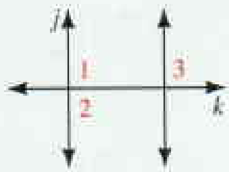
5. $\angle 8$ is a 90° angle.

6. $\angle 7 \cong \angle 10$

7. $\angle 9$ and $\angle 10$ are complementary.



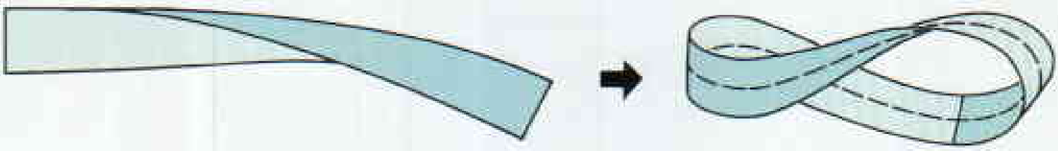
8. Give a plan for the following proof.
Given: $\angle 1$ is supp. to $\angle 3$;
$\angle 2$ is supp. to $\angle 3$.
Prove: $j \perp k$



9. Write a proof for Exercise 8 in two-column form.

Extra — *Möbius Bands*

Take a long narrow strip of paper. Give the strip a half-twist. Tape the ends together. The result is a *Möbius band*.



Exercises

1. Make a Möbius band. Color one side of the Möbius band. How much of the band is left uncolored? The original strip of paper had two sides. How many sides does a Möbius band have?
2. Cut the Möbius band lengthwise down the middle. (Start at a point midway between the edges and cut around the band.) What is the result? Cut the band a second time down the middle. Write a sentence or two describing what happens.
3. Give a full twist to a long, narrow strip of paper. Tape the ends together. How many sides does this band have? Cut this band lengthwise down the middle. Write a brief description of what is formed.
4. Make a Möbius band. Let the band be 3 cm wide. Make a lengthwise cut, staying 1 cm from the right-hand edge. Describe the result.
5. Take two long narrow strips of paper. Fasten them together so they are perpendicular and form a plus sign. Twist one strip so it is a Möbius band and fasten its ends together. Don't twist the other strip at all, just fasten its ends. Cut the Möbius band down the middle lengthwise. Then cut the other band down the middle. Describe the final result.

Chapter Summary

1. *If p, then q* is a conditional statement. p is the hypothesis and q is the conclusion. *If q, then p* is the converse. The statement *p if and only if q* is a biconditional that means both the conditional and its converse are true.
2. Properties of algebra (see page 37) can be used to reach conclusions in geometry. Properties of congruence are related to some of the properties of equality.
3. Deductive reasoning is a process of proving conclusions. Given information, definitions, postulates, and previously proved theorems are the four kinds of reasons that can be used to justify statements in a proof.

4. When $m\angle A + m\angle B = 90$, $\angle A$ and $\angle B$ are complementary. When $m\angle C + m\angle D = 180$, $\angle C$ and $\angle D$ are supplementary. Complements (or supplements) of the same angle or of congruent angles are congruent.
5. Vertical angles are congruent.
6. Perpendicular lines are two lines that form right angles (90° angles). If two lines are perpendicular, then they form congruent adjacent angles. If two lines form congruent adjacent angles, then the lines are perpendicular.
7. If the exterior sides of two adjacent acute angles are perpendicular, then the angles are complementary.
8. The proof of a theorem consists of five parts, which are listed on page 60.

Chapter Review

Use the conditional: If $m\angle 1 = 120$, then $\angle 1$ is obtuse.

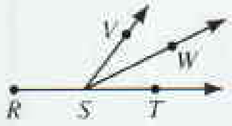
1. Write the hypothesis and the conclusion of the conditional. **2–1**
2. Write the converse of the conditional.
3. Provide a counterexample to disprove the converse.
4. Write a definition of a straight angle as a biconditional.

Justify each statement with a property from algebra or a property of congruence.

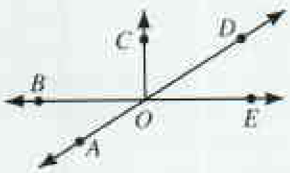
5. If $m\angle A + m\angle B + m\angle C = 180$ and $m\angle C = 50$, then $m\angle A + m\angle B + 50 = 180$. **2–2**
6. If $m\angle A + m\angle B + 50 = 180$, then $m\angle A + m\angle B = 130$.
7. If $6x = 18$, then $x = 3$.
8. If $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{EF}$, then $\overline{AB} \cong \overline{EF}$.

Name the definition, postulate, or theorem that justifies the statement.

9. If $\overline{RS} \cong \overline{ST}$, then S is the midpoint of $\overline{RT}$. **2–3**
10. If $\overrightarrow{SW}$ bisects $\angle VST$, then $\angle VSW \cong \angle WST$.
11. If $\overrightarrow{SW}$ bisects $\angle VST$, then $m\angle WST = \frac{1}{2}m\angle VST$.



12. If $\angle BOC$ is a right angle and $m\angle COD = 58$, then $m\angle DOE = \underline{\ ?\ }$, $m\angle BOA = \underline{\ ?\ }$, and $m\angle AOC = \underline{\ ?\ }$. **2–4**
13. Name a supplement of $\angle AOE$.



14. A supplement of a given angle is four times as large as a complement of the angle. Find the measure of the given angle.

Name the definition or state the theorem that justifies the statement about the diagram.

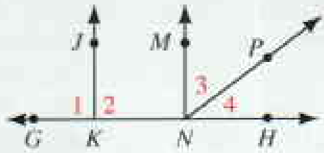
2–5

15. If $\overrightarrow{KJ} \perp \overleftrightarrow{GH}$, then $\angle 1$ is a right angle.

16. If $\angle 2$ is a $90°$ angle, then $\overrightarrow{KJ} \perp \overleftrightarrow{GH}$.

17. If $\overrightarrow{NM} \perp \overleftrightarrow{GH}$, then $\angle MNK \cong \angle MNH$.

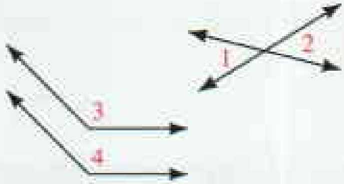
18. If $\overrightarrow{NM} \perp \overleftrightarrow{GH}$, then $\angle 3$ and $\angle 4$ are complementary.



2–6

19. Write a plan for a proof.
Given: $\angle 3$ is a supplement of $\angle 1$;
$\angle 4$ is a supplement of $\angle 2$.
Prove: $\angle 3 \cong \angle 4$

20. Write a proof in two-column form for Exercise 19.



Chapter Test

1. Use the conditional: Two angles are congruent if they are vertical angles.
a. Write the hypothesis. **b.** Write the converse.

2. Provide a counterexample to disprove the statement:
If $x^2 > 4$, then $x > 2$.

3. Write the biconditional as two conditionals that are converses of each other:
Angles are congruent if and only if their measures are equal.

4. Supply reasons to justify the steps:

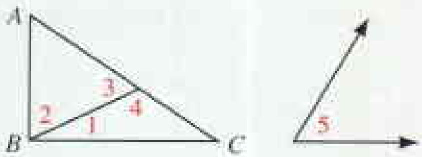
Steps	*Reasons*
1. $y = 12$	1. Given
2. $5x = 2x + y$	2. Given
3. $5x = 2x + 12$	3. __?__
4. $3x = 12$	4. __?__
5. $x = 4$	5. __?__

5. $\overrightarrow{OB}$ is the bisector of $\angle AOC$ and $\overrightarrow{OC}$ is the bisector of $\angle BOD$. $m\angle AOC = 60$. Find $m\angle COD$.

6. S is the midpoint of $\overline{RT}$ and W is the midpoint of $\overline{ST}$. If $RT = 32$, find ST, WT, and RW.

7. In the diagram, $\overline{AB} \perp \overline{BC}$. Name:
a. two supplementary angles
b. two complementary angles

8. Given: $\angle 5$ is supplementary to $\angle 4$.
a. What can you conclude about $\angle 5$ and $\angle 3$?
b. State the theorem that justifies your conclusion.

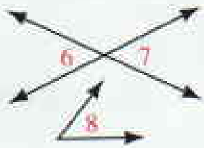


Exs. 7–9

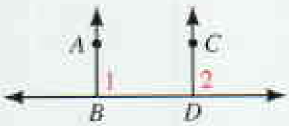
9. Suppose $m\angle 3 = 3x + 5$ and $m\angle 4 = 6x + 13$. Find the value of x.

10. State the theorem that justifies the statement $\angle 6 \cong \angle 7$.

11. Suppose you have already stated that $\angle 6 \cong \angle 7$ and $\angle 7 \cong \angle 8$. What property of congruence justifies the conclusion that $\angle 6 \cong \angle 8$?



12. Write a proof in two-column form.
Given: $\overrightarrow{DC} \perp \overleftrightarrow{BD}$; $\angle 1 \cong \angle 2$
Prove: $\overrightarrow{BA} \perp \overleftrightarrow{BD}$



Algebra Review: *Systems of Equations*

Solve each system of equations by the substitution method.

Example 1 (1) $y = 5 - 2x$
(2) $5x - 6y = 21$

Solution Substitute $5 - 2x$ for y in (2): $5x - 6(5 - 2x) = 21$
$17x - 30 = 21;\ x = 3$

Substitute 3 for x in (1): $y = 5 - 2(3) = -1$

The solution is $x = 3$, $y = -1$.

1. $y = 3x$
$5x + y = 24$

2. $y = 2x + 5$
$3x - y = 4$

3. $x = 8 + 3y$
$2x - 5y = 8$

4. $3x + 2y = 71$
$y = 4 + 2x$

5. $4x - 5y = 92$
$x = 7y$

6. $y = 3x + 8$
$x = y$

7. $8x + 3y = 26$
$2x = y - 4$

8. $x - 7y = 13$
$3x - 5y = 23$

9. $3x + y = 19$
$2x - 5y = -10$

Solve each system by the method of addition or subtraction.

Example 2 (1) $3x - y = 13$
(2) $4x + y = 22$

Solution Add (1) and (2):
$7x = 35;\ x = 5$

Substitute 5 for x in (2):
$4(5) + y = 22;\ y = 2$

The solution is $x = 5$, $y = 2$.

Example 3 (1) $6x + 15y = 90$
(2) $6x - 14y = 32$

Solution Subtract (2) from (1):
$29y = 58;\ y = 2$

Substitute 2 for y in (1):
$6x + 15(2) = 90;\ x = 10$

The solution is $x = 10$, $y = 2$.

10. $5x - y = 20$
$3x + y = 12$

11. $x + 3y = 7$
$x + 2y = 4$

12. $3x - 2y = 11$
$3x - y = 7$

13. $7x + y = 29$
$5x + y = 21$

14. $8x - y = 17$
$6x + y = 11$

15. $9x - 2y = 50$
$6x - 2y = 32$

16. $7y = 2x + 35$
$3y = 2x + 15$

17. $2y = 3x - 1$
$2y = x + 21$

18. $19 = 5x + 2y$
$1 = 3x - 4y$

Preparing for College Entrance Exams

Strategy for Success

When you are taking a college entrance exam, be sure to read the directions, the questions, and the answer choices very carefully. In the test booklet, you may want to underline important words such as *not, exactly, false, never,* and *except,* and to cross out answer choices that are clearly incorrect.

Indicate the best answer by writing the appropriate letter.

1. On a number line, point M has coordinate -3 and point R has coordinate 6. Point Z is on $\overrightarrow{RM}$ and $RZ = 4$. Find the coordinate of Z.
(A) -7 **(B)** 1 **(C)** 2 **(D)** 10 **(E)** cannot be determined

2. $\angle 1$ and $\angle 2$ are complementary. $m\angle 1 = 5x + 15$ and $m\angle 2 = 10x$. The measure of $\angle 1$ is:
(A) 5 **(B)** 11 **(C)** 40 **(D)** 70 **(E)** 30

3. Vertical angles are never:
(A) complementary **(B)** supplementary **(C)** right angles
(D) adjacent **(E)** congruent

4. A reason that cannot be used to justify a statement in a proof is:
(A) a postulate **(B)** a definition **(C)** given information
(D) yesterday's theorem **(E)** tomorrow's theorem

5. Which of the following must be true?
(I) If two lines form congruent adjacent angles, then the lines are perpendicular.
(II) If two lines are perpendicular, then they form congruent adjacent angles.
(III) If the exterior sides of two adjacent obtuse angles are perpendicular, then the angles are complementary.
(A) I only **(B)** II only **(C)** III only
(D) I and II only **(E)** I, II, and III

6. $\angle 1$ and $\angle 2$ are congruent angles. $m\angle 1 = 10x - 20$ and $m\angle 2 = 8x + 2$. $\angle 1$ is a(n) __?__ angle.
(A) acute **(B)** right **(C)** obtuse **(D)** straight
(E) answer cannot be determined

7. If you know that $m\angle A = m\angle B$ and $m\angle B = m\angle C$, then what reason can you give for the statement that $m\angle A = m\angle C$?
(I) Reflexive Property (II) Transitive Property (III) Substitution Property
(A) I only **(B)** II only **(C)** III only
(D) either I or II **(E)** either II or III

8. Which of the following is *not* the converse of the statement: If b, then c.
(A) If c, then b. **(B)** b if c. **(C)** c if and only if b.
(D) c only if b. **(E)** c implies b.

Cumulative Review: Chapters 1 and 2

Name or state the postulate, property, definition, or theorem that justifies the statement.

A **1.** If $8x = 16$, then $x = 2$.

2. If $\angle K \cong \angle L$ and $\angle L \cong \angle M$, then $\angle K \cong \angle M$.

3. If $\angle AOB$ is a right angle, then $\overleftrightarrow{OA} \perp \overleftrightarrow{OB}$.

4. If $a + 7 = b$ and $b = 4$, then $a + 7 = 4$.

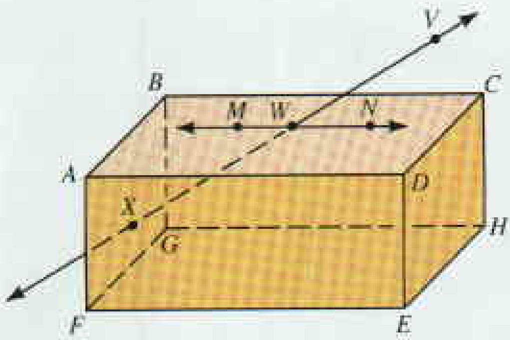
5. If $a + 7 = 4$, then $a = -3$.

6. There is a line through F and H.

7. The intersection of plane $CDEH$ and plane $FGHE$ is $\overleftrightarrow{EH}$.

8. If W is the midpoint of $\overline{XV}$, then $XW = \frac{1}{2}XV$.

9. $MW + WN = MN$



Exs. 6–14

Classify each statement as true or false.

10. $\overrightarrow{WV}$ contains point X.

11. $\overline{MN}$ lies in plane $ABCD$.

12. $\overleftrightarrow{WV}$ intersects plane $ABGF$.

13. F, E, H, and C are coplanar.

14. A, B, and V are coplanar.

Classify each statement as true or false. If it is false, provide a counterexample.

15. Through any three points, there is exactly one plane.

16. Perpendicular lines form congruent adjacent angles.

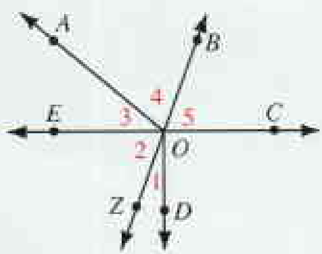
17. If points A and B are in plane M, then $\overline{AB}$ is in plane M.

18. Complementary angles must be adjacent.

B **19.** If $m\angle A = 45$, then the complement of $\angle A$ is one third of its supplement.

20. If $m\angle RUN = m\angle SUN$, then $\overrightarrow{UN}$ is the bisector of $\angle RUS$.

In the diagram, $\overrightarrow{OB}$ bisects $\angle AOC$ and $\overleftrightarrow{EC} \perp \overrightarrow{OD}$. Find the value of x.



21. $m\angle 5 = 2x$, $m\angle 3 = x$

22. $m\angle 1 = 2x$, $m\angle 2 = 6x + 2$

23. $m\angle 2 = 6x + 9$, $m\angle 5 = 2x + 49$

24. $m\angle 2 = 3x$, $m\angle 3 = 2x - 4$

25. $m\angle 1 = x - 8$, $m\angle 2 = 2x + 5$, $m\angle 4 = 3x - 26$