

PHYSICS UNION  
MATHEMATICS

# Physics I

Kinematics



Supported by the National Science  
Foundation (DRL-0733140).

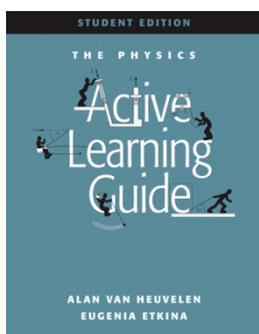
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**RUTGERS**  
THE STATE UNIVERSITY  
OF NEW JERSEY

# PUM Physics I

## Kinematics

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Most of the module activities were adapted from:

A. Van Heuvelen and E. Etkina, *Active Learning Guide*, Addison Wesley, San Francisco, 2006.

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Some activities are based on FMCE (Thornton and Sokoloff), on Ranking Task Exercises in Physics (O’Kuma, Maloney, and Hieggelke), and on the work of D. Schwartz.

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This material is based upon work supported by the National Science Foundation under Grant DRL-0733140. Any opinions, findings and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of the National Science Foundation (NSF).

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## Lesson 1: Motion is Relative



### 1.1 Observe and Find a Pattern

In this activity you will use a tube “telescope” to follow an object.

During the first two experiments you will be THE OBSERVER. The observer must always keep the object in sight through the telescope. Consider the following situations:



- a) In the first experiment, the teacher holds a ball and is standing still. **You are the observer.** Take note of the initial direction that the telescope points in order to see the ball through it.

**This is the original orientation of the telescope.**

- b) Next, the teacher travels from one corner of the classroom to another. Make sure that you see the ball through the telescope at all times. What happens to your telescope as you follow the object?
- c) For the next experiment, carefully observe your teacher. The teacher holds the ball the same way, but this time, **the teacher is the OBSERVER** and looks at the object through the telescope while traveling from right to left. What happens with the orientation of the teacher’s telescope during the experiment?
- d) Based on your experiences in part (a) through part (c):
- Can you say whether the ball was moving during the experiments in a) and b)? Explain your answer. (Hint: Compare what happened to the telescopes for the different observers.)
  - Based on this idea, is there anyone else that would observe the ball as not moving?
  - In general, how do you know whether something is moving or not?
- e) For the c) experiment, once again observe the orientation of the teacher’s telescope. The teacher points the telescope at a classmate and looks through it. The teacher travels in the same way.
- Does the orientation of the telescope change?
  - Does your classmate move? How do you know?



## 1.2 Describe

Consider the following situation.

- A blue car travels along a street with two passengers. One passenger sits in the front passenger seat of the car and the other passenger sits in the back seat.
- A red car travels in the same direction and is passing the blue car.
- There is a sidewalk along the road the cars are traveling and a pedestrian is standing on the sidewalk.

**Choose four students from your class to play the roles of the four people and recreate the situation.**

**Describe the movement of the front passenger in the blue car as seen by each of the following observers:**

Observer	Motion of the passenger in the blue car
a) The person sitting in the backseat of the blue car.	
b) The pedestrian standing on the sidewalk as the blue car passes.	
c) The driver of the red car moving in the same direction and passing the blue car.	

Review your analyses and answer the questions that follow.

- Do any of the observers say that the front passenger in the blue car was moving? Explain.
- Do any of the observers say that the front passenger in the blue car was **not moving**? Explain.
- Based on your answers in parts (a) through (e) and activity 1.1, explain what it means when someone says an object is “moving”.



### **Did You Know?**

**Motion:** An object is in *motion* with respect to another object (observer or reference object) if, as time progresses, its position is changing relative to the reference object.

**Reference frame:** A *reference frame* includes three essential components:

1. An object of reference with a point of reference selected as the origin of the coordinate system;
  2. A clearly defined coordinate system. The coordinate system includes the direction of the axis, such as north, south, east, west, or positive and negative direction. The reference frame also includes a unit for measuring distances.
  3. A zero clock reading that serves as a reference for future clock readings.
- 



### 1.3 Test Your Idea

At this point you may have concluded that,

*“Different observers see the same motion differently”.*

Another way of saying this is that,

*“Motion is relative”.*

Consider the following experiment: Your friend carries an object and walks in a westerly direction.

- a) Based on what you know about relative motion, **predict** what you would need to do to make the object appear stationary (not moving). Write your prediction and explain your thinking.
- b) Use the idea of relative motion to **predict** what you need to do to see the object as moving in the easterly direction. Write your prediction and explain it.
- c) Perform the experiment.
- d) Make a judgment about whether the idea that motion is relative was supported or disproved by your experiment. Explain.

**1.4 Relate** Use toy cars, marbles, and any other toys available in class. Come up with the observers who will see a car parked on your desk moving. Come up with observers who will see the car parked on your desk stationary. Repeat for another toy.

**1.5 Design an experiment** Design an experiment in which one observer sees a toy car or a marble moving 1.3 m along a straight line in the direction left to right, another observer sees the same car (marble) not moving at all and the third observer sees the same car (marble) moving 1.0 m left to right. Describe your experiment in words and draw a picture so a person who did not see you perform the experiment can repeat it.

## Homework



### 1.6 Describe

Consider the following situation.

- A person stands near a bus stop.
- Another person is in an approaching bus.
- A person riding in a car travels away from the bus stop.
- Another person stands at the bus stop.

a) Describe the motion of the person standing near the bus stop as seen *by the following observers*:

Observer	Motion of the person standing near the bus stop
The person sitting in the approaching bus.	
The person riding in the car moving away from the bus stop.	
The person standing at the bus stop.	

- b) Do any of the observers say that the person standing near the bus stop was moving? How do you know?
- c) Do any of the observers say that the person standing near the bus stop was not moving? How do you know?

## Lesson 2: Which Way is Which?



### 2.1 Observe and Describe

- Stand next to a partner so that you are side-by-side. Each of you should hold your left arm out to the side and then point with your finger. Describe whether you and your partner are pointing in the same direction.
- Now stand across from your partner so that you are face-to-face. Again, hold your left arm out to the side and then point with your finger. Describe whether you and your partner are pointing in the same direction.
- Was there a difference between the two situations? Explain why you believe this occurred.

### 2.2 Observe and Describe



For this portion of the experiment you and your partner will each need a compass.

- Stand next to a partner so that you are side-by-side and facing north. Each of you should hold out your arm so that you are pointing to the West. Describe whether you and your partner are pointing in the same direction.
- Now stand across from you partner so that you are face-to-face. One of you should be facing to the north and one of you facing to the South. Each of you should hold out your arm so that you are pointing to the West. Describe whether you and your partner are pointing in the same direction.
- Was there a difference between the two situations? Explain why you believe this occurred.

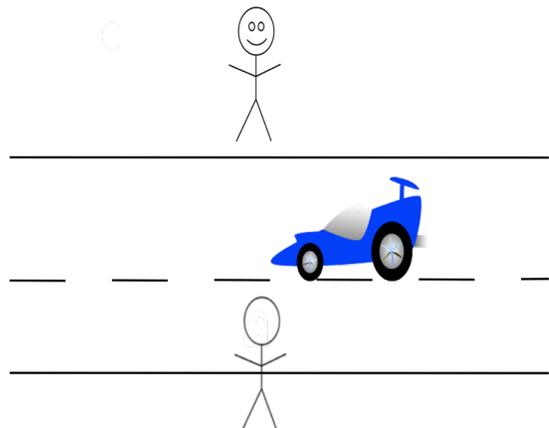
### 2.3 Explain

- Think about the activities you just performed. What was your reference frame in each of the situations?
- Which method of defining direction is more useful? Explain why you believe this.



### 2.4 Reason

- Tony and Alex are standing across the street from each other when they see a car travel in front of them. Tony says that the car is moving to the left while Alex believes it's moving to the right. Who is correct? Why do you believe this?



- b) How could you define directions on the pictures so that the two boys agree on the direction of motion?
- c) If you place a third observer behind the car, will she agree with the other two boys now?
- d) Why is it important to define a coordinate system when you are talking about motion?
- e) Think about coordinate systems in math class. How is direction defined on the coordinate grid?

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### ***Did You Know?***



The same  $-/+$  direction definition used on coordinate planes can be used to describe direction of motion in many situations. In physics, we always define directions on the pictures we draw as well as to include a zero mark to indicate our origin.

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### **2.5 Represent and Reason**



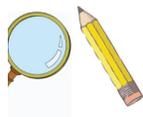
- a) In the picture below, define one direction as positive and one as negative. Then describe the motion of the sailboat as best you can. Explain how you know.



- b) Could you have defined the directions the opposite way? Why or why not?
  
  
- c) Draw another object in the picture that is moving in the negative direction. Use an arrow to show the way it's moving.



## Lesson 3: Constructing Dot Diagrams



### 3.1 Observe and represent

Place a ball on a flat metal track. With one of your teammates, practice rolling the ball along the track. (If you do not have a ball you can use other objects, for example a motorized car.)

Draw a picture of your experimental set up.



### 3.2 Hypothesize

Describe the motion of the ball as best as you can. Is the ball moving at a regular pace? Is it speeding up? Slowing? How do you know?

### 3.3 Test Your Idea



In the previous activity you came up with an idea about how the ball is moving. Let us test this idea by performing a new experiment. The goal is for your team to follow the ball as it travels in detail, the same way a real physicist would.

- With your teammates, think of a way to keep the clock reading and mark the location of the ball for each second as it rolls. You can use any marking method you think is appropriate. Describe your method and practice until you are comfortable.

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#### ***Here's an Idea!***

To practice your counting, you can look at a second hand on a stopwatch while tapping on a desk or shouting a word.

If you yell out, make sure that when you shout your counts for each second, they are brief; for example “yes”. Always practice enough so that you are comfortable keeping a consistent counting method going.

- Now that you have a method for marking the location of the ball for each second as it rolls, design an experiment to test the idea of how the ball rolls.
- Predict what the marks would look like if the ball **were moving according to the hypothesis** you came up with in the previous activity.
- Conduct the experiment.
- Explain how you can use the marks to describe the motion of the ball.

- f) Remember that after every testing experiment you must compare the actual outcome to your predicted outcome. How did yours compare? Did the outcome match your prediction? Explain how the evidence supports or disproves your hypothesis.
- g) What assumptions did you make when writing your prediction?



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### *Need Some Help?*

**Assumptions** are issues we take for granted in an experiment. They can help us explain why the results weren't exactly as we expected or could be something that wasn't taken into account when we designed the experiment. Despite the outcome of the experiment, it's important to consider the assumptions that may have been made.

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### **3.4 Represent and Reason**

- a) If the ball rolled faster than in (c), how would the marks be spaced? Use dots to represent the location of the ball and draw this dot diagram below.



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### *Did You Know?*

Much like the experiment you just conducted, motion can be represented by marks or dots. We refer to this as a **dot diagram**.

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- b) Imagine that you have a wind-up toy that first moved slowly and then faster and faster. How would the dot diagram look?
- c) If you had another toy that first moved fast and then slower and slower, how would the dot diagram look?

### **3.5 Relate**

- a) Imagine you are in a car that is coasting on a flat road at constant pace. Draw a dot diagram for it. Now imagine the same car moving at the same constant pace only uphill. Draw a dot diagram for it. How are the diagrams the same? How are they different?
- b) Now imagine that you are in a car speeding up from a street light. Draw a dot diagram for the motion of the car. Now imagine the same car rolling downhill and speeding up. Draw another dot diagram. How are these two diagrams the same? How are they different?
- c) Make a list of questions you can investigate about the motion of real objects using the dot diagrams.

## Homework

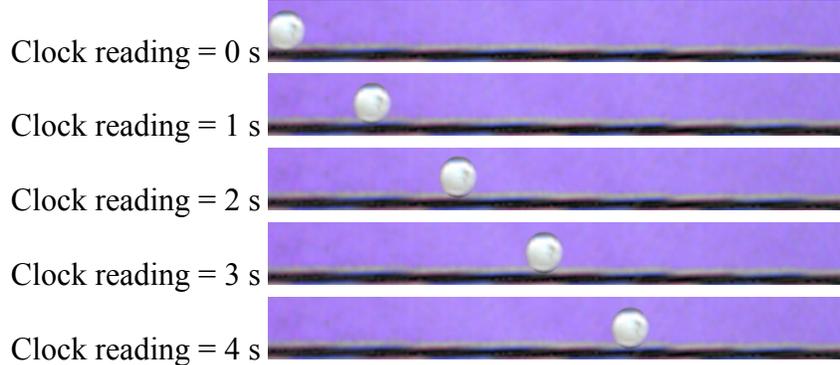
### 3.6 Design an experiment

Find 10 sugar packets or make 10 things that can serve as sugar packets for your experiment. Find an instrument you can use to measure distance – it can be anything as long as it is a rigid object – it will serve as a unit of length for you. Practice counting seconds using a clock with the second hand or a watch. Find any moving object in your house – can be a toy, a sibling or even you yourself. Design an experiment to record and describe the motion of this object using the sugar packets. Then perform the experiment and write a report about it. Include the data that you collected, how you collected them and explain what you learned during this experiment.



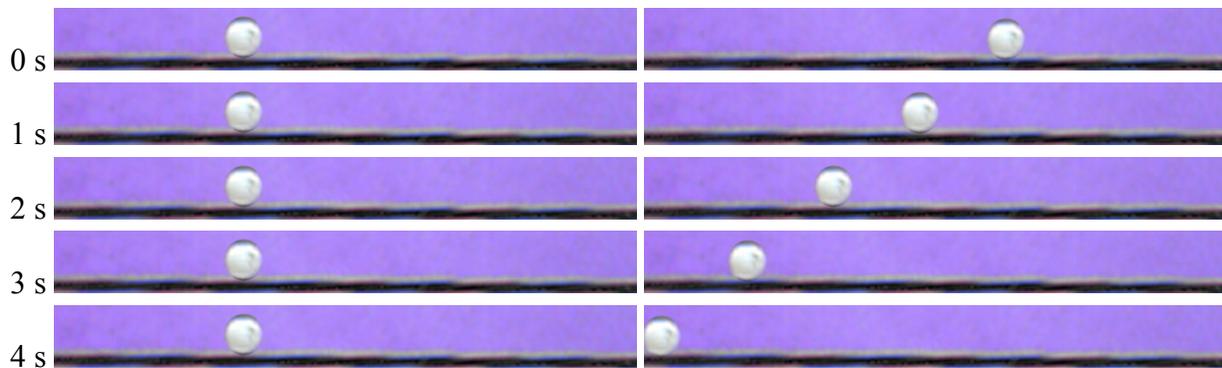
### 3.7 Represent and reason

Study the following images that Hector took with his camera of a ball rolling along a very long, dark, horizontal track.



- How would you describe the motion of this ball above?
- Draw a dot diagram for the rolling ball above

Tim and Crystal photographed the ball at the same time as Hector. Their pictures can be found below; Tim's on the left and Crystal's on the right.



- Explain how Tim, Crystal, and Hector took the pictures. They all photographed the same ball at the same time.

### 3.8 Test an Idea



- While over at your friend Scott's house, you notice that he has 20 television sets. You formulate the hypothesis that your friend Scott must be a secret television thief. Think of a way to test this hypothesis. Then write a prediction using the *if/and/then* format.
- A classmate writes, "*If Scott is a secret thief then he will steal my TV too.*" Can we call this statement a prediction? Why or Why not? See the rubric below for help.
- What are the assumptions that your classmate may have been making?
- How is this activity related to the activity where you tested your hypothesis about the motion of the ball on the track? How is it different?

Rubric to self-assess your prediction				
Scientific Ability	Missing	An attempt	Needs some improvement	Acceptable
<b>Is able to distinguish between a hypothesis and a prediction.</b>	No prediction is made. The experiment is not treated as a testing experiment.	A prediction is made but it is identical to the hypothesis.	A prediction is made and is distinct from the hypothesis but does not describe the outcome of the designed experiment.	A prediction is made, is distinct from the hypothesis, and describes the outcome of the designed experiment.



#### *Need Some Help?*

A **Hypothesis** is a general belief, pattern, model, or rule developed from observations. Hypotheses can be used to develop predictions.

A **Prediction** is a statement that describes the outcome of a specific experiment (prior to conducting the experiment). Predictions must be based on the hypothesis being tested.

**REMEMBER!** – Your predicted outcome must be based on your hypothesis.

## Lesson 4: Graphing and Physical Quantities

### 4.1 Observe and Describe



Eva recorded the position of a motorized toy car using the origin as her reference point. She wrote this in the table below. Notice how she labeled the columns using physical quantities that she measured versus the units used.

Clock reading	Position	Time interval	Change in position
0 s	0 cm	-----	-----
1 s	2 cm	$1\text{ s} - 0\text{ s} = 1\text{ s}$	$2\text{ cm} - 0\text{ cm} = 2\text{ cm}$
2 s	4 cm	$2\text{ s} - 1\text{ s} = 1\text{ s}$	$4\text{ cm} - 2\text{ cm} = 2\text{ cm}$
3 s	6 cm	$3\text{ s} - 2\text{ s} = 1\text{ s}$	$6\text{ cm} - 4\text{ cm} = 2\text{ cm}$
4 s	8 cm	$4\text{ s} - 3\text{ s} = 1\text{ s}$	$8\text{ cm} - 6\text{ cm} = 2\text{ cm}$

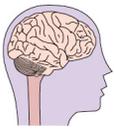
- What patterns do you see in the data?
- Explain the meaning of each column in the table. Make sure to specify the difference between the columns.
- What other units could Eva have used to measure the physical quantity of position? Clock reading?



### Did You Know?

**Physical quantity:** A tangible property that can be measured with a special instrument in specific units. For example, one unit for the physical quantity time  $t$  is a second.

**Change in a quantity:** Often, in science and mathematics, we are interested in the change in a quantity. The symbol  $\Delta$  is used to represent “change”. For example,  $Temperature_2 - Temperature_1 = \Delta T$ . Here  $Temperature_2$  stands for the temperature measured at some clock reading 2 which occurred after clock reading 1 when  $Temperature_1$  was measured.



## 4.2 Reason

a) Match the physical quantities with the units from the list below (a quantity can be measured in different units):

- mass • meter • temperature • second
- foot • year • centimeter • gram • kilogram • inches • Celsius • position • time interval • hour •

b) Give an example of another physical quantity and a possible unit of measurement for that quantity?

c) Label each of the physical quantities in Eva's table (activity 4.1) using the following symbols:  $x$ ,  $t$ ,  $\Delta x$ ,  $\Delta t$ .

Physical Quantity	Possible Units
<i>Ex: Temperature</i>	<i>Degrees Celsius</i>

## 4.3 Observe and describe

Use a motorized car to collect data similar to the data collected by Eva. Think of how will you conduct the experiment. What instruments do you need? How will you collect data? After you resolve all these questions with your group members, conduct the experiment and collect the data. Make a table similar to the table in activity 4.1 in which you record your data.

a) Represent the motion of the ball from your experiment (or Eva's experiment if you did not have the motorized car) with a graph. Write the clock reading (time) on the horizontal axis and position on the vertical axis.

### *Did You Know?*



In science, experimenters always put time on the horizontal axis when plotting.

b) Plot the points on the position-versus-time graph and draw a **trend line**.

### *Need Some Help?*



**Trend Line:** A trend line represents a trend in the data. To draw a trend line, try to draw a line that passes as close to each data point as possible. It is important to know that data points do not need to be on the line.

c) What information can you learn about the motion of the car (or the ball) from the graph? Explain.

## 4.4 Observe and Represent



- a) Now create two additional position-versus-time graphs using the tables below. In the first table Eva recorded the position of a toy car rolling down a ramp toward her. In the second table Eva recorded the position of a toy car as it slows to a stop in front of her.

Clock reading $t$	Position $x$	Time interval $\Delta t$	Change in position $\Delta x$
0 s	0 cm	-----	-----
1 s	1 cm	1 s – 0 s = 1 s	1 cm – 0 cm = 1 cm
2 s	4 cm	2 s – 1 s = 1 s	4 cm – 1 cm = 3 cm
3 s	9 cm	3 s – 2 s = 1 s	9 cm – 4 cm = 5 cm
4 s	16 cm	4 s – 3 s = 1 s	16 cm – 9 cm = 7 cm

Clock reading $t$	Position $x$	Time interval $\Delta t$	Change in position $\Delta x$
0 s	16 cm	-----	-----
1 s	23 cm	1 s – 0 s = 1 s	23 cm – 16 cm = 7 cm
2 s	28 cm	2 s – 1 s = 1 s	28 cm – 23 cm = 5 cm
3 s	31 cm	3 s – 2 s = 1 s	31 cm – 28 cm = 3 cm
4 s	32 cm	4 s – 3 s = 1 s	32 cm – 31 cm = 1 cm

- b) What are the differences in the motion of the cars for the three experiments Eva performed?
- c) What are the differences between the three graphs? (Specifically comment on what's happening to the line.)
- d) Develop a testable hypothesis that relates **type of motion** to **graphical appearance** (how the graph looks).



### Did You Know?

**Position, displacement, distance, and path length:** These refer to different things! Position  $x$  is the location of an object relative to a chosen zero on the coordinate axis. Displacement  $x_2 - x_1$  indicates a change in position and has a sign indicating the direction of the displacement. The magnitude of that position change is the distance and is always positive. Path length refers to the total length of the path that was travelled.

**Time:** The clock reading or *time* ( $t$ ) is the reading on a clock, on a stopwatch, or on some other time measuring instrument. Time can be measured in many different units, such as seconds, minutes, hours, days, years, and centuries, etc. In SI system it is measured in seconds.

**Time interval:** The difference between two clock readings is the *time interval*. If we represent one time reading as  $t_1$  and another reading as  $t_2$  then the time interval between those two clock readings is  $t_2 - t_1$ . Another way of writing this statement is:  $\Delta t = t_2 - t_1$

**Change:** The symbol  $\Delta$  is the Greek letter delta and in physics and mathematics it reads as delta  $t$  ( $\Delta t$ ) or the change in  $t$ .

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***In the module you will work on many activities that use these quantities, including graphs. Unless specifically mentioned or questioned, the unit for position, distance, displacement and path length is 1 meter and the unit for time and time interval is 1 second.***

#### 4.5 Relate

- Imagine that you are running up the steps keeping a constant pace (let's say you are doing 3 steps per second). Sketch a dot diagram and a position versus time graph for your motion.
- Now you finished climbing the stairs and enter a long hallway. You continue running at the same pace along the hallway. Sketch a dot diagram and a position versus time graph for your motion. How are they similar to the ones you made in a)? How is it different?

### *Homework*

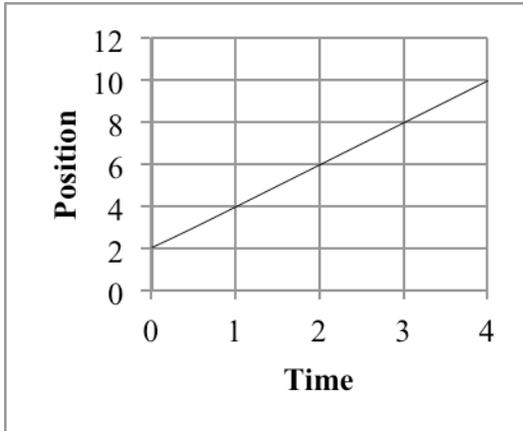
#### 4.6 Represent and Reason



Thus far, you have represented the motion of a ball with different representations: dot diagrams, words, pictures, tables, and now a graph. Choose a specific example of a moving object, specify the observer and explain how the different representations describe the same motion.

## 4.7 Represent and Reason

a) Use the graph to record data into the table provided.



Clock reading $t$	Position $x$	Time interval $\Delta t$	Change in position $\Delta x$
0 s		_____	_____
1 s			
2 s			
3 s			
4 s			

b) Create a dot diagram for the motion represented in the graph

c) What are the physical quantities in this problem? In what units are they measured?

d) You should notice that the two physical quantities in the graph did not have units of measure with them. Describe a real life situation for this motion if the units of measure were kilometers and seconds. Describe another situation if the units were centimeters and minutes. Think of what was happening

e) Draw a picture for each of the situations you described above.

## Lesson 5: The Truth Behind Graphic Representations



### 5.1 Hypothesize

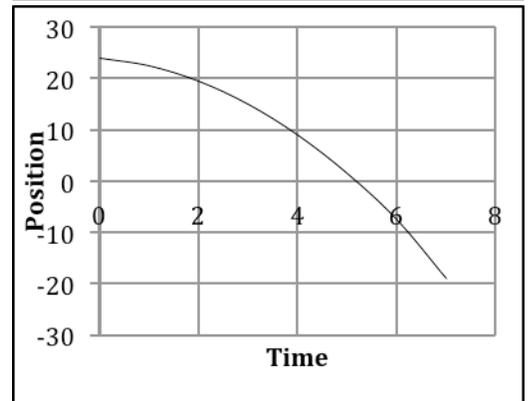
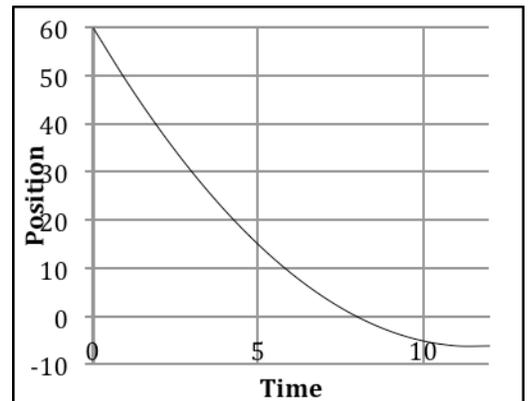
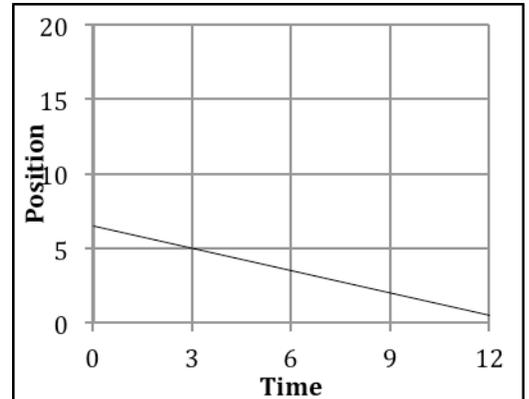
Let's review position versus time graphs. Use what you learned in the previous lesson to help you develop the following rules.

- What does constant pace motion look like on a position versus time graph?
- What does speeding up motion look like on a position versus time graph?
- What does slowing motion look like on a position versus time graph?

### 5.2 Test Your Idea

Use the graphs and descriptions below to test your rules.

- This graph represents an object moving at constant pace. Does this match your rule for constant pace? Explain why or why not. Modify your rule if necessary.
- This graph represents a slowing object. Does this match your rule for slowing? Explain why or why not. Modify your rule if necessary.
- This graph represents an object speeding up. Does this match your rule for speeding up? Explain why or why not. Modify your rule if necessary.



### 5.3 Test Your Idea (optional)

Use your newly modified hypotheses from the previous activity to predict how you'd have to move so that a motion detector creates position versus time graphs that match the previous graphs. Explain how your prediction compares to the outcome.

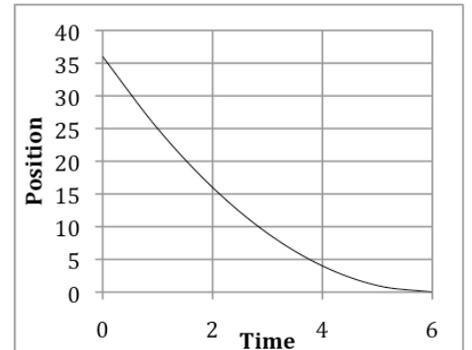
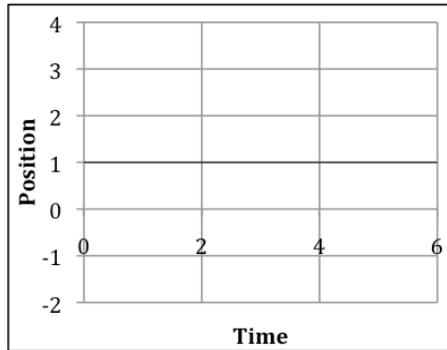
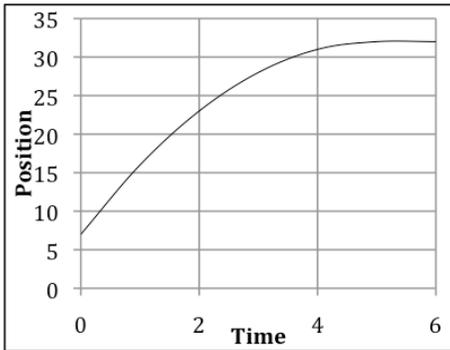
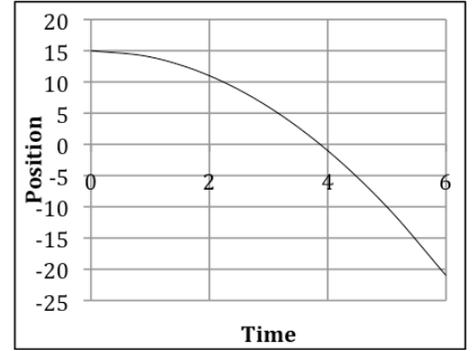
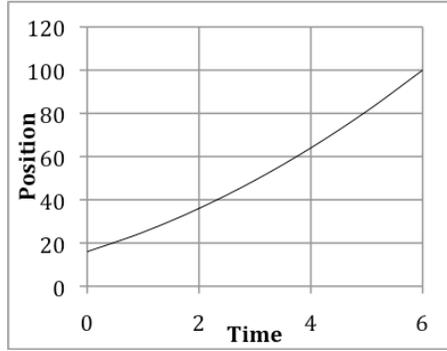
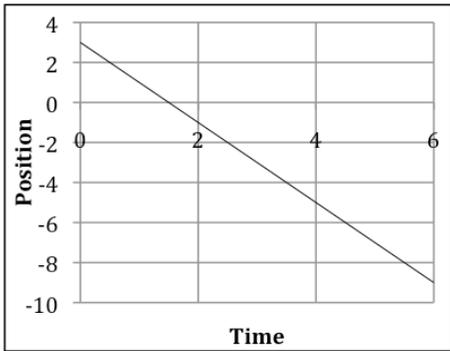
### 5.4 Reason

Examine the graphs below and then answer each of the questions below by recording the associated letters on the line provided.

- Which graphs represent objects moving at constant pace? \_\_\_\_\_



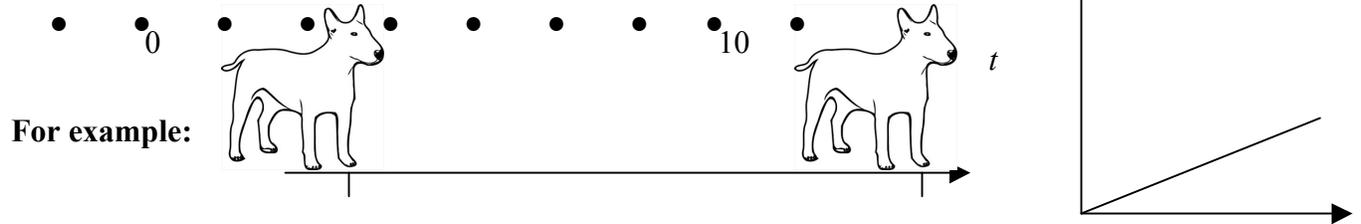
- b) Which graphs represents objects speeding up? \_\_\_\_\_
- c) Which graphs represent objects that are slowing? \_\_\_\_\_
- d) Which graphs represent an object moving in the negative direction? \_\_\_\_\_
- e) Do any of the graphs show an object that is not in motion? How do you know? Can we consider this a constant pace?



## *Homework*

### 5.5 Represent and Reason

Choose at least two objects from around your house and represent the motion by drawing a picture, making a dot diagram, and creating a graph. Then explain whether the motion is at constant pace, speeding up, or slowing.

$x$ 

My dog trotted down the road at a nearly constant pace. I know this because I could not see him visibly speeding up or slowing. In ten seconds he went about ten meters. This means equal distance per one second time interval.

cars travel from left to right



### 6.1 Observe and Reason

Study the dot diagram for two different toy cars. The time interval for the two toy cars is the same, one-second between each dot. They start at the same time.

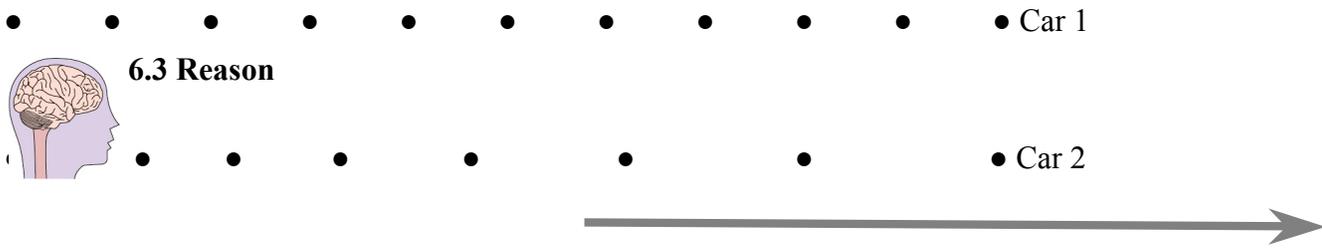


- Describe the motion of the two cars in words.
- Find where the cars were at the same location at the same time. Explain your answer!



### 6.2 Reason

If you graphed the motion of the two cars on a position-versus-time graph, what would the trend lines for the cars look?



**6.3 Reason**



Using the dot diagram above, answer the following questions.

- a) Describe the motion of the two cars in words.
- b) Find where the cars were at the same location at the same time. Explain your answer!

**6.4 Reason**



Your teacher, Mr. Hawking, has been observing space with his telescope for years. Recently he came to class with some exciting news. He's been tracking two asteroids beyond Mars for some time and now believes that they will eventually collide. Below is the data your teacher has been collecting. The

	Asteroid 1		Asteroid 2
Time	Position		Position
$t$	$x$		$x$
Day 0	20		200
Day 5	30		175
Day 10	40		150
Day 15	50		125
Day 20	60		100

asteroids' positions were recorded using a distant star as a reference point.

- a) Create a position-versus-time graph with the data and complete the table.
- b) Which asteroid is moving faster? How do you know? Explain two different ways you can determine this.
- c) What does it mean if the two trend lines intersect? If this occurs, when and where?

The same brand of popcorn pops at the same speed. So a brand of popcorn only gets a single popping index.

You have to use the exact same procedure for each brand to find its index.

## Lesson 7: Reasoning in Kinematics: Inventing an Index

A big index value should mean that the popcorn pops faster. A small index number should mean that the popcorn pops slower.

- *Miles per gallon* is an index of how well a car uses gas..
- *Batting average* is an index of how well a baseball player hits.
- *Grades* are an index of how well students perform on a test.

We want you to invent a procedure for computing an index that helps make comparisons.

### RK 7.1 The Popping Index

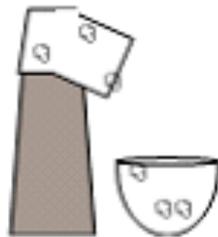
Three companies make popcorn.

They use different types of corn so the popping is fast or slow.

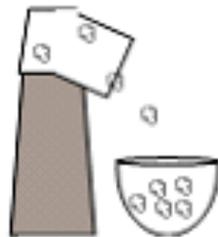
Invent a procedure for computing a “popping index” to let consumers know how fast each brand pops.



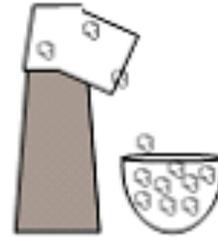
Poppomatic Popcorn = \_\_\_\_\_



12 seconds

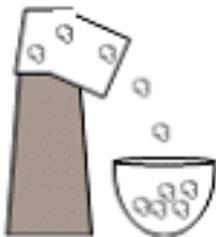


18 seconds

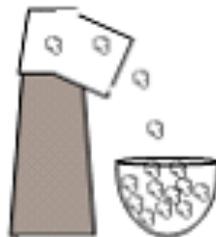


24 seconds

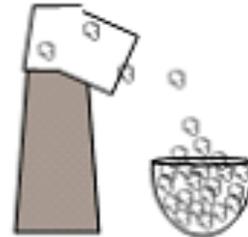
HotPops = \_\_\_\_\_



20 seconds

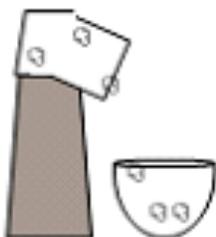


30 seconds

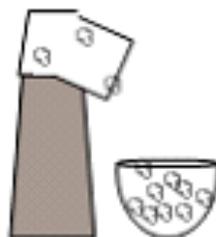


50 seconds

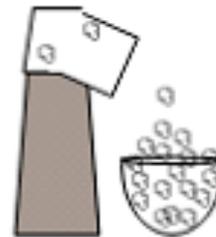
HipHop Popping Corn = \_\_\_\_\_



6 seconds



12 seconds



20 seconds

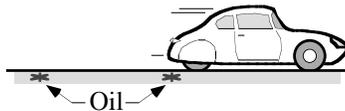
## RK 7.2 Fastness Index

Let's look at another kind of index.

Your task this time is to come up with a *fastness index* for cars with dripping oil.

You will see a bunch of cars, and you need to come up with one number to stand for each car's fastness.

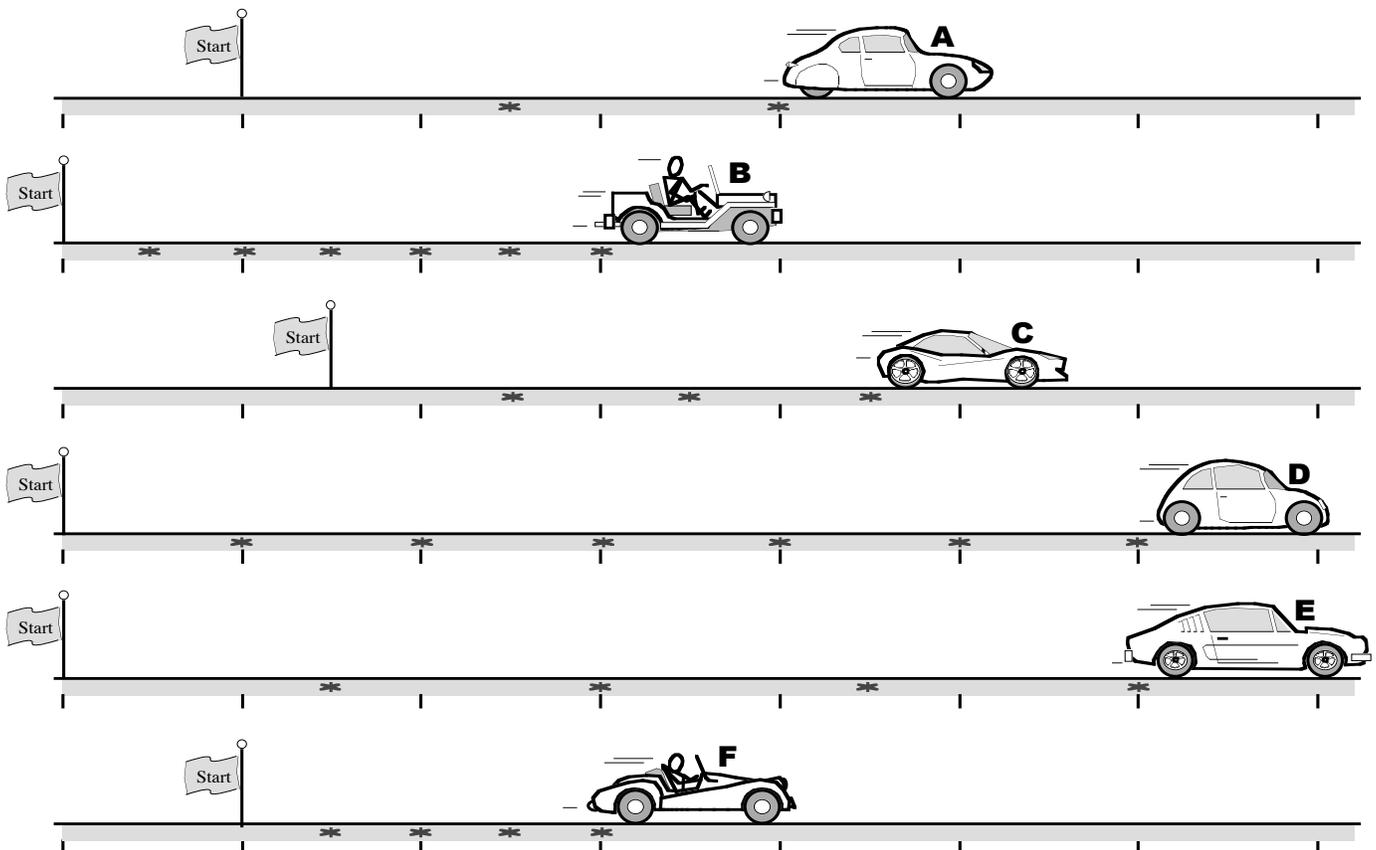
There is no watch or clock to tell you how long each car has been going. However, all the cars drip oil once a second. (They are not very good cars!)



You can look at the oil drops to help figure out how long a car has been traveling.

**This task is a little harder than before.**

- A company always makes its cars go the same fastness.
- We will not tell you how many companies there are.
- You have to decide which cars are from the same company. They may look different!



**To review:**

- (1) Make a fastness index for each car.
- (2) Decide how many companies there are.
- (3) To show the cars that are from the same company, draw a line that connects the cars.

**RK 7.3 Reason**

1. Which popcorn is fastest? Give an explanation why you picked that popcorn.
2. Which cars are fastest? Give an explanation why you picked that car.
3. For each question below, explain your reasoning:
  - A full bowl of popcorn has 60 popped corns in it. How long does it take the fastest popcorn to fill a bowl of popcorn?
  - How long does it take for the fastest car to travel 15 blocks?
  - How many popped corns will the fastest popcorn pop in 70 seconds?
  - How far will the fastest car travel in 20 seconds?
4. Another company, “Acme,” has an index of 2.5.
  - a) Let’s say that Acme makes popcorn. Using everyday language, describe the specific information that the number 2.5 tells about their popcorn.
  - b) Now let’s say that Acme makes cars. Using everyday language, describe the specific information that the number 2.5 tells about their car.

Each slide gets one steepness index because it has the same steepness all the way down.

You have to use the exact same procedure for each slide to find its index. Explain why you think there were less than 6 car companies, even though there were six different diagrams describing the car companies.

A big index value should mean that the slide is steep. A small index number should mean that the slide is less steep.

### RK 7.4 The Steepness Index

Let's try another kind of index.

Splash Attack

Super Soaker

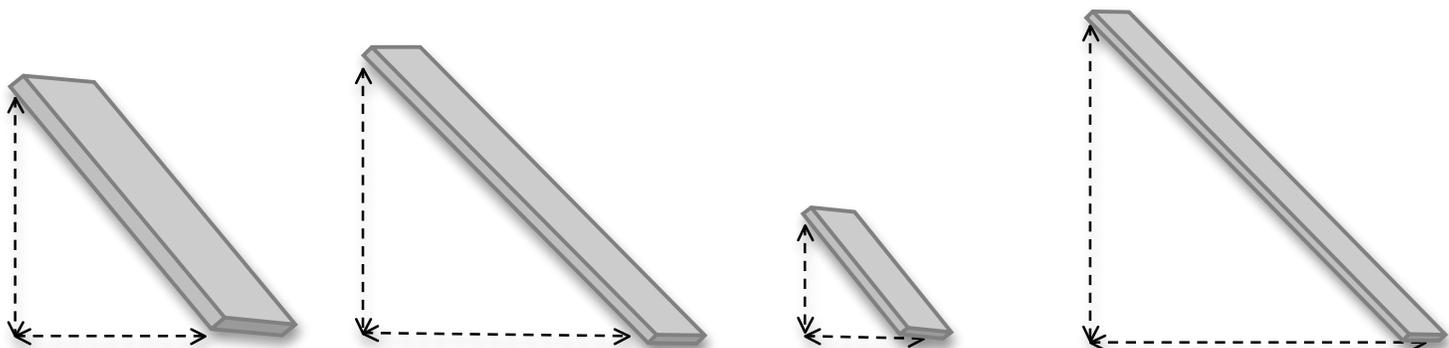
Tsunami

As the owner of 2-Die-4 Water Park, you are in charge of buying the slides. Most of your clients are teenagers, and they like the steepest slides they can find. You want to buy slides that will attract the most business. You are trying to choose between the slides shown below.

Invent a procedure for computing a "steepness index" so that you can buy the best slides, and prove to your customers that you have the steepest slides in town.



Find the steepness index for the following portions of these slides:



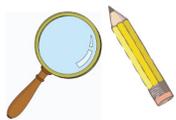
### RK 7.5 Reason

1. Which slide is steepest? Give an explanation why you picked that slide.
2. Another company, “Acme,” has a steepness index of 0.75. Using everyday language, describe the specific information that the number 0.75 tells about their slides.
3. A quality control officer for Hop-On Popcorn counted the number of kernels popped at several different times. The data are shown below in the table.

<b>Popped Kernels</b>	<b>time (sec)</b>
8	6
12	9
16	12
4	3

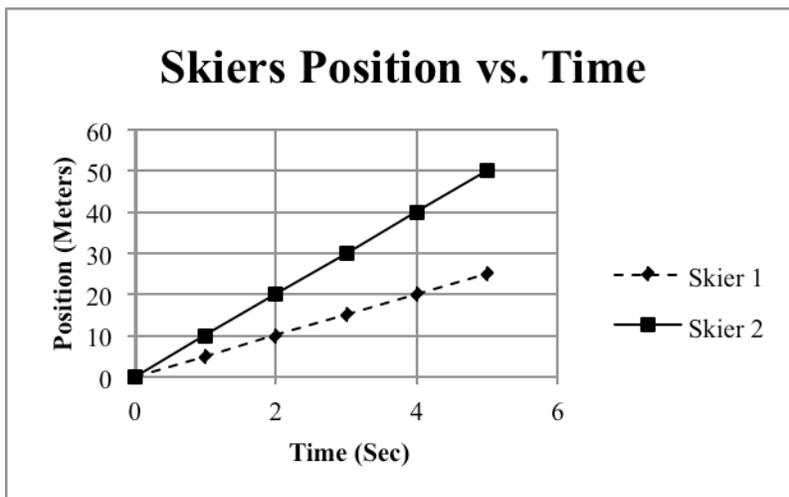
- a) Make a graph of the popped kernels vs. time.
- b) Find the steepness index of the best-fit line. What information does the steepness of the best-fit line tell you about Hop-On Popcorn?
- c) Which pops faster, the Hop-on Popcorn or HipHop Popping Corn? Explain.
- d) Sketch a bowl of Hop-On Popcorn after 15 seconds in an air popper.

## Lesson 7A: Slopes and Functions



### 7.1 Observe and Represent

Another way of comparing trend lines is by calculating the slope of each line and comparing the numerical values of the slopes.



- a) Use the graph above and calculate the slope of the line for each case. Explain how you calculated the slope. **How is the slope similar to the index?**

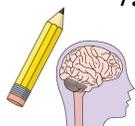


### *Need Some Help?*

**Slope:** Often used to describe the measurement of the steepness of a straight line. A higher slope value indicates a steeper incline. The slope is defined as the ratio of the change in the value of the dependent variable (vertical change) over the change in the value of the independent variable (horizontal change). In other words, vertical change divided by horizontal change!

- b) For the skiers, what do you think the slope of the line represents? Try to answer using your common sense.
- c) What are the units of slope? How do you know?
- d) Refer to the graphs to check if your answer makes sense. How do you know? Is there anything else you notice? Explain.

## 7.2 Represent and Reason



- a) Recall the asteroid activity 6.5. Write a **function** (expression) that will allow you to find position ( $x$ ) for any time ( $t$ ) for the asteroid.



### Need Some Help?

When mathematicians and physicists express patterns mathematically they use **functions**. A function is a rule that one uses to find a dependent variable when an independent variable is known. You may have met functions in a math class. There the independent variable was labeled  $x$  and the dependent variable is labeled  $y$ . The function then is  $y(x)$ . In science and math class you can actually use any labels as long as you agree on which was the independent and which is the dependent variable. **For the problem below**, the independent is  $t$ , and a dependent is  $x$ .

#### Example:

Examine: Describe the relationship between the two variables.

Define: Describe the variables used in the scenario

Represent: Write a mathematical equation using variables

*The object changes its position by 50 meters each second*

*$t$  = time elapsed  
 $x$  = position*

*$x = 50t$*

Time (second)	Position (meters)
1	50
2	100
3	150

This expression can be written in function notation as  $x(t) = 50t$ . **However, in physics** it is necessary to include **units of measure**  $x(t) = 50(\text{m/s}) t$  or  $x(t) = 50(\text{m/s}) * t$ .  $x(t)$  is read as “x of t.”



**REMEMBER!** When you graph a function the independent variable is *always* placed on the horizontal axis and the dependent variable on the vertical axis.

- b) Use the function provided to complete the tables:

Examine: Describe the relationship between the two variables.

Define: Describe the variables used in the scenario

Represent: Write a mathematical equation using variables

$$x(t) = \frac{1}{4} (\text{m/s}) * t$$

Table of Values	

### 7.3 Represent and Reason



- a) Imagine a man started his journey at 3 pm and ended at 3.30 pm. At 3 pm he was at a supermarket and at 3.30 pm he was at position 2.5 km away from the supermarket. How fast was he travelling? Explain how you got your answer. Express it in different units.
- 



#### *Did You Know?*

Variable and notation are very important in science and math. Specifically in physics, we must be clear about what each variable, notation, and expression means.

#### *Example:*

A woman started her journey at time  $t_1$  and ended at time  $t_2$ . At time  $t_1$ , she was at position,  $x(t_1) = x_1$  and at time  $t_2$ , she was at position  $x(t_2) = x_2$ .

As you can see, this notation is very overwhelming! This is why we omit  $x(t_1)$  and  $x(t_2)$  and simplify it as  $x_1$  and  $x_2$ . However, it is necessary to know that every time you see  $x$  with a subscript, the subscript indicates a position at a specific clock reading (time).

---

Use the variables (*no numbers*) from the information box above to answer each question:

- b) When did the woman start her trip?
- c) How far did the woman travel?
- d) How fast did the woman travel?
- 



**Here's an Idea!** – The format that you wrote in part (d) is typically how physicists define velocity.

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### 7.4 Hypothesize

Write a function  $(x)t$  for a man walking at a speed of 0.3 m/s who starts moving from position zero at a zero clock reading.



## 7.5 Test Your Idea

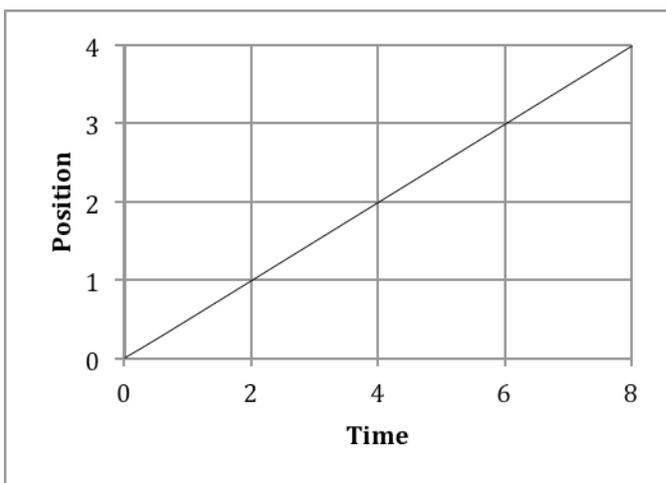
If these functions that you have been writing really do represent motion accurately, it should be possible to use them to predict position of a moving object if we know the type of motion that occurs. To do so we will use the PhET simulation, “The Moving Man.” You should navigate to the following web address and click, “Run Now.”

[http://phet.colorado.edu/simulations/sims.php?sim=The\\_Moving\\_Man](http://phet.colorado.edu/simulations/sims.php?sim=The_Moving_Man)

- Use the function for the man from 7.4 to predict where the man will be after 8 second
- In the “Moving Man” simulation, type “0.3” in the velocity box. Make sure that the man begin at position zero and zero clock reading.
- Then click “Go.” Just before the man gets to the wall click, “Stop.” This can be located at the bottom of the simulation.
- Move the gray bar on the graph to 8 seconds to check your mathematical prediction of the position.
- Did your prediction match the actual outcome? Make sure to check you work and discuss any problems you may have had. You can use the space below.



## 7.6 Represent and Reason



- Determine the slope of the trend line shown above using data from the graph. Would the slope change if you used different data points? Use two more examples to determine the slope.
  - Write an equation for the function that will allow you to find a value for  $x$  for any value of  $t$ .
- c) Sketch a graph that represents the motion of an object traveling at the same speed but in the opposite direction.

- d) Calculate the slope for the graph and write an equation for the function that will allow you to find  $x$  for any  $t$ .
- e) What is the same about the functions you wrote in part (b) and part (d). How are they similar? How are they different?

## 7.7 Represent and Reason



- a) A train is travelling at 22 m/s when it passes your town. Where will the train be 30 minutes later? Write a function  $x(t)$  for the train.
- b) Imagine that the train was 300 meters south of the your town when it was first observed. (We will assume that south is a negative direction) How would the function change? Write a new function for  $x(t)$ .



### Need Some Help?

The method we have been using to write functions is great, but it only works when an object's motion begins at the origin. If this is not the case, the starting position of the object must be included in our function! Consider the problem from earlier in the lesson.

Before the object started at 0 and our function was  $x(t) = 50(\text{m/s})t$ . Now that the object is starting at 1 meter from the origin, how could we change our function to fit the data?

Time (sec)	Position (m)
0	1
1	51
2	101
3	151

**Hint:** What could you add at the end of the function?

- c) Imagine that instead of motion, you are studying the amount of money in your pocket. Amount of money is the dependent variable and time is the independent variable. You start with 10 dollars and then spend 2 dollars every hour for 5 hours. Sketch a graph of number of dollars in your pocket versus time, then write a function representing the amount of money in your pocket as a function of time.



### Did You Know?

**Position, distance, displacement and path length:** These refer to different things! Position  $x$  is the location of an object relative to a chosen zero on the coordinate axis. Displacement  $x_2 - x_1$  indicates a change in position and has a sign indicating the direction of the displacement. The magnitude of that position change is the distance and is always positive. Path length refers to the total length of the path that was travelled.

**Velocity and speed for constant speed linear motion:** Velocity is the slope of the position versus clock reading graph or the ratio of the displacement of an object during a time interval divided by that time interval. The unit of velocity is m/s, miles/h, km/h, and so forth. Positive velocity means that the object is moving in the positive direction, negative means in the negative direction. Speed is the magnitude of velocity and is always positive. For constant speed, motion in a straight line, velocity can be written as:

$$v = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t}$$

$x_2 - x_1 = (\Delta x) = \text{change in position}$

$t_2 - t_1 = (\Delta t) = \text{time interval}$

## *Homework*



### 7.8 Reason

- (a) You walked the first mile in 13 minutes. Assuming you are walking at constant pace, how far will you walk in 26 minutes? 39 minutes? In an hour?
- (b) You keep walking at the same pace. You walked 5 miles. How long did it take you to do it? Express your answer in minutes, in second and in hours.
- (c) You walked at the same pace for 2 hours. How far did you walk?

### 7.9 Reason

A car is moving at the speed of 65 miles per hour. How many kilometers does it cover in one hour? How many meters does it cover in one hour? How many meters does it cover in one second?

### 7.10. Reason

A car is moving at 15 m/s. How many miles does it cover in 1 second? How many miles does it cover in 1 hour?

### 7.11. Hypothesize

Invent a rule that will allow you to convert a speed measured in mph into speed measure in m/s. Invent a rule that will allow you to convert the speed measure in m/s into the speed measured in mph. How can you test whether the rules are correct?

## Lesson 8: How Fast Do You Walk?



### 8.1 Represent and Reason

Since there is a relationship between position, time, and velocity, we can use any two of the values to determine the third. The table on the right shows the data for 5 different objects.

Object #	Velocity $v$	Time interval $\Delta t$	Change in position $\Delta x$
1	15 m/s	2 s	30 m
2	2 m/s	0.5 s	1 m
3	3 m/s	3 s	
4		5 s	500 m
5	0.6 m/s		36 m

- Complete the table on the right by looking for patterns in the data.
- How did you find  $\Delta x$  when  $\Delta t$  and  $v$  were given?
- How did you find  $v$  when  $\Delta t$  and  $\Delta x$  were given?
- How did you find  $\Delta t$  when  $v$  and  $\Delta x$  were given?



### 8.2 Represent and Reason

The following function describes the motion of a biker:  $x(t) = 3 \text{ m} + (-5 \text{ m/s}) t$

- Explain what each number in the equation means.
- What are the independent and dependent variables in the function?
- Where is the object at  $t = 0$ ? Where is the object at  $t = 3$  seconds?
- How far has the object travelled in 3 seconds?
- What symbol could you use to represent the physical quantity whose value you determined in the part (c)? What about part (d)?



### 8.3 Design an Experiment

- Design an experiment with your team members in which you determine the “normal” walking speed of each team member.
- Draw a picture of your experimental set up. Discuss how you will conduct the experiment.
- What quantities are you going to measure?

- d) Make a list of the equipment you will need.
- e) Write down the procedure you will follow.
- f) Perform the experiment and record your data.
- g) Discuss with your team different ways to represent the data.

### 8.4 Explain

It is very important for scientists to identify different experimental uncertainties and understand how these uncertainties impact their gathering of data and analysis. Experimentalists try to minimize uncertainties as much as possible.

- a) Think about the word “experimental uncertainty” and discuss its meaning with your teammates.
- b) What do you think the experimental uncertainty of measuring speed with a car speedometer?



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#### *Did You Know?*

**Uncertainties:** The margins within which you can possibly know the value of any physical quantity. Uncertainties depend on the instruments and the ways you conduct the experiment. ANY instrument has an uncertainty equal to half of the smallest division.

**Example:**  $10 \pm 0.5$  cm

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- c) What kind of measurement tool could have been used in the example above? What was the smallest interval for this measurement tool? How do you know?

Review your data in your walking experiment.

- d) Identify as many sources of uncertainty as you can. What would you do to reduce the uncertainties in this experiment?
- e) How did it affect your position measurement?
- f) How do you think you could represent uncertainty on a position versus time graph?

## *Homework*

### 8.5 Evaluate

Complete a lab report. Reflect on the experiment you performed and write a scientific report about your investigation. The report should describe what you did and what you found. It should include any data tables, graphs, etc. Also include the answers to the following questions in your report:

- How did you decide what was a normal walking speed for each person in your group?
- When a person is walking at his/her normal walking speed, are she/he always traveling at constant speed?
- Explain how the uncertainties in the position measurement affect the normal walking speed for each person. Are the uncertainties in measurement the same for each person? Explain.

Use the rubric below to help you write your report.

<b>Rubric to self-assess your report</b>				
<b>Ability</b>	<b>Missing</b>	<b>An attempt</b>	<b>Needs some improvement</b>	<b>Acceptable</b>
<b>Is able to formulate the question to be investigated</b>	No mention is made of the question to be investigated.	The question is posed but is not clear.	The question is posed, but it involves more than one variable.	The question is posed and it involves only one variable.
<b>Is able to design an experiment to answer the question</b>	The experiment does not answer the question.	The experiment is related to the question but will not help answer it.	The experiment investigates the question but might not produce the data to find a pattern.	The experiment investigates the question and might produce the data to find a pattern.
<b>Is able to describe what is observed in words, pictures, and diagrams.</b>	No description is mentioned.	A description is mentioned, but it is incomplete. No picture is present.	A description exists, but it is mixed up with explanations or other elements of the experiment. A labeled picture is present.	The report clearly describes what happens in the experiment, both verbally and by means of a labeled picture.
<b>Is able to construct a mathematical (if applicable) relationship that represents a trend in data</b>	No attempt is made to construct a relationship that represents a trend in the data.	An attempt is made, but the relationship does not represent the trend.	The relationship represents the trend but no analysis of how well it agrees with the data is included (if applicable), or some features of the relationship are missing.	The relationship represents the trend accurately and completely and an analysis of how well it agrees with the data is included (if applicable).

## Lesson 9: When Worlds Collide!



### 9.1 Design an experiment

You have two battery-operated toy cars. Your first goal is to determine whether they travel at constant speeds and what the value of that speed is for each.

- Decide what experiment you can perform to determine the speed of each car. Think about using dot diagrams, a data table, and a graph.
- Identify the different uncertainties for this experiment. How can you minimize the uncertainties?
- Make a table to record time ( $t$ ) and position ( $x$ ) for each car for different clock readings.
- Graph the position versus time data for each car. Calculate the speed for each car. Write the mathematical function that will allow you to find the position of each car for any clock reading.

### 9.2 Test Your Idea



- Select a random distance that each car will travel. Predict how long it will take each car to travel that distance. Show your work.
- Measure the distance on the floor and place each car at the starting position. Let the cars go and measure the time it takes each to arrive at the final position.
- How did the measured times compare to your predicted times from part (e)? Are they exactly the same? If not, why do you think that they do not agree?



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### *Did You Know?*

Scientists and mathematicians often compare theoretical predictions with experimental data by making statements such as:

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*“ to within experimental uncertainty, the experimental result agrees with the predicted value...”*

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- If you take into account your experimental uncertainties, do your experimental results agree with your predicted values? Defend your answer. Show your work.

### 9.3 Test Your Idea



In this activity, you will place the cars 3.5 meters apart (think of how certain you are in the distance that they are apart, it depends on the uncertainty in your measurement of distance). Make sure that your group uses the same cars as you did in activity 9.1. The cars will be pointing at each other and will be released at the same time.

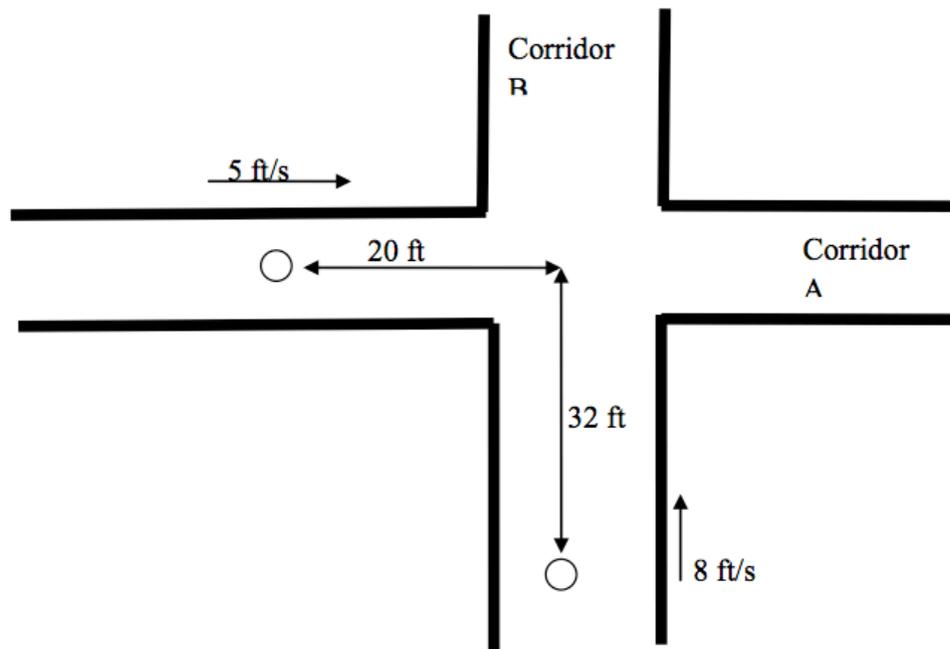
- Predict the time and location that the cars will collide. Clearly show how you calculated your result.
- After you have made the prediction, measure a distance of 3.5 meters. Place one car at position 0 and the other at position 3.5 meter. Face the cars at each other. Which is going to have a negative velocity?
- When your team is ready, release the cars at the same time and record the time and location where the cars collide.
- To within experimental uncertainty, how did the predicted values compare to your experimental results? Explain.

### *Homework*



#### 9.4 Represent and Reason

A student starts walking at 5 ft/s in a corridor A and is 20 ft away from the intersection of corridors A and B. A second student starts at the same time running at 8 ft/s in corridor B and is 32 feet away from the intersection.



- Create a dot diagram for the problem.
- Graph the motion of the two students on a position-versus-time graph.
- Represent the motion of each student with a function.

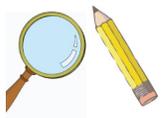
- d) How many different functions can you write for motion of each student? What will be different and what will be the same?
- e) Will the two students collide? Show your work and explain your reasoning.

### 9.5 Reason



- a) Abby and Kate decide they need to start exercising more. They begin walking every day for 30 minutes. If they are averaging a pace of 1.1 m/s, how far will they be walking each day? Show your work.
- b) After several weeks of conditioning, the girls are averaging a pace of 2.4 m/s. If they decided to walk from New York City to Los Angeles, how many days would it take them to get there? Show your work.

## Lesson 10 Motion Diagram: A New Tool



### 10.1 Observe and Represent

Use a small ball and a long tilted ramp at a very small angle. Let the ball roll down the ramp.

- Describe the motion of the ball.
- Compare the motion of this ball to the motion of a ball that was pushed once and then let go on a flat surface. How are they the same? How are they different?
- Draw a dot diagram for the motion of the ball.
- What happens to the distance between each pair of dots as the ball rolls down the ramp?
- If you were to make a position versus time graph for this motion how would the trend line look? Why does this make sense?

### 10.2 Explain



- If you showed your dot diagram to a random person would they be able to tell which direction the ball was moving? Why or why not?
- Examine the dot diagram below and describe the motion of the object it represents. Did you have any problem doing this?  

- Elizabeth and Matt are arguing about the motion that this dot diagram represents. Matt says it's slowing and Elizabeth says it's speeding up. Who is correct?
- How could we alter our dot diagram to better represent the motion of the object?



### 10.3 Represent and Reason

You decide to “borrow” your parent’s car and roll it down a hill so that it moves faster and faster. A friend sitting in the car drops beanbags out the window each second. The pattern of the beanbags looks similar to the dot diagram of the ball rolling down the tilted ramp from the previous activity (the dot pictures look the same).

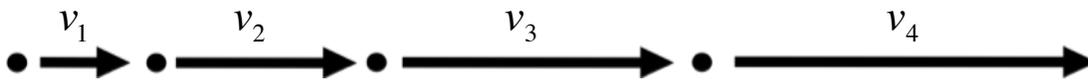
- Draw a dot diagram that shows the velocity during each time interval, direction of motion, and a directional axis.



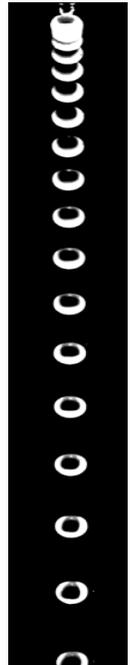
#### *Need Some Help?*

What you need to draw is something physicists call, a **motion diagram**. It is a sophisticated replacement for a dot diagram that conveys more information about a situation. If you are new to this representation, you may want to list or label the important features so that you are sure to include these when you draw one for yourself.

#### *Example:*



- What does the length of the arrow tell you about the motion of the object?
- What two ways is direction indicated on the representation? Why are both necessary? What do plus and minus signs mean here? Is the direction in which the object is going positive or negative?
- What does the length of each arrow tell you about the motion of the car at a particular dot or position?
- What does the length of each arrow tell you about the motion of the ball at a particular time?
- With a classmate, act out the motion represented in the motion diagram.



### 10.4 Represent

The multiple-exposure picture on the left shows a ball’s motion. The images of the exposure are  $\frac{1}{4}$  of a second apart.

- Describe the motion of the ball. Is it falling or was it thrown upward?

- b) Create a motion diagrams for the two different possibilities

### 10.5 Represent



Your friend is at the top of a hill on a sled. He is given a very small nudge to get him moving down the hill. Create four different representations for this motion (pictures, graphs, tables, and motion diagrams)

## Homework



### 10.6 Represent and Reason



- Describe the motion of the motorcycle in the horizontal direction and vertical direction.
- What major assumption do you have to make in order to complete this activity?
- Represent the horizontal motion of the motorcycle with a motion diagram on the edge of the picture.
- Represent the vertical motion of the motorcycle with a motion diagram on the edge of the picture.



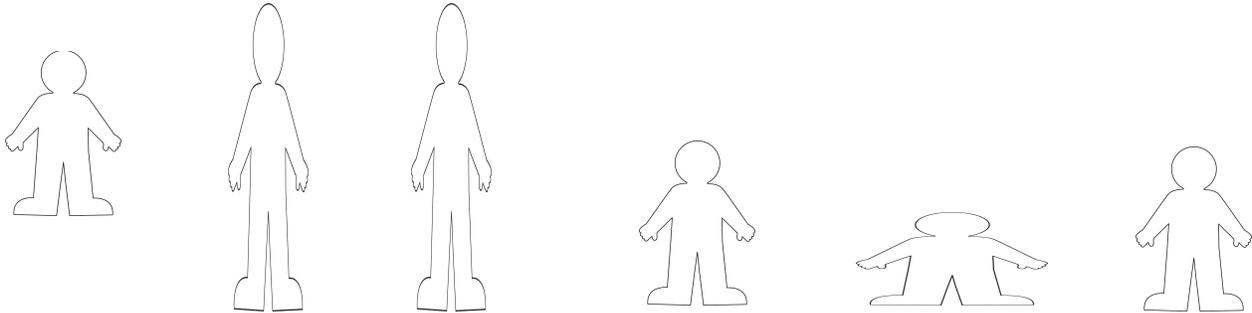
- e) Create a motion diagram to represent the moon's motion.
  - f) What additional assumption did you have to make for the moon that you did not have to make for the motorcyclist?
  - g) What conclusion might we make about the motion of the moon?
- h) What is the benefit of having / making many different representations for the same situation?

## Lesson 11: Time For Stretching



### 11.1 Describe

During art class you made a clay figure of a person and the art teacher was very impressed by your work. She even displayed your work for the whole class. However throughout the week you noticed that people were changing your artwork!



For each day, determine if your artwork was stretched up, squished down, or unchanged. Then **draw an arrow** to represent the direction of change. Make sure your arrow shows the amount of stretching or squishing.

a) Original to Day 1:

b) Day 1 to Day 2:

c) Day 2 to Day 3:

d) Day 3 to Day 4:

e) Day 4 to Day 5:



### *Here's An Idea!*

This activity may not have seemed like physics but it is design to help you understand **change arrows**. This activity serves as an **analogy** for you to refer back to when you're having trouble. Scientist often will make **analogies** for complex systems in order to better understand what may be occurring.

### 11.2 Observe and Represent



In this activity, we're going to examine the motion diagram provided in the previous lesson.

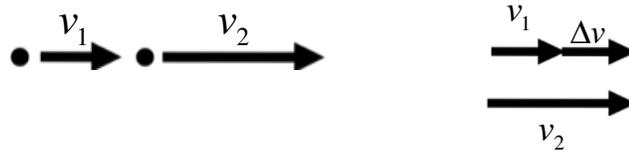
- Think about the previous activity. What must you do to velocity arrow one to get velocity arrow two? What direction? How much?
- Answer the questions in part (a) for arrows 2 and 3 as well as arrows 3 and 4. Is there any difference?



### Did you know?

The arrows that you drew to show the difference between velocity arrows are called  **$\Delta v$  arrows** or **change in velocity arrows**. We can line them up and compare the size of the arrows in order to determine the change. These arrows tell us the direction and the magnitude (size) of the change. It's just like the direction and amount of squishing/stretching from the previous activity! A complete motion diagram includes  $\Delta v$  arrows.

*Example:*



- What does the  $\Delta v$  arrow tell you about the motion of the object? Explain.
- Revisit the dot picture you drew for the ball rolling down an incline. Create a complete motion diagram for the motion of the ball.



### 1.3 Reason and Explain

Describe the motion represented in this motion diagram.



- Is the object moving in the positive or negative direction? Is it speeding up, slowing down, or moving at constant velocity? How do you know?
- What direction should the  $\Delta v$  arrow be pointing? Explain how you determined this.
- Is the change in motion ( $\Delta v$  arrows) in the same direction that the object is moving ( $v$  arrows)?
- Make a rule for speeding up and slowing down by comparing motion (velocity arrows) to change in motion ( $\Delta v$  arrows).



## 11.4 Test Your Idea

In order to test your hypothesis from the previous activity your group will be conducting the following experiment. To begin, you will place a basketball on the floor at rest (not moving). One member of your group will then hit the ball with a mallet, so that it starts moving at considerable speed. Another member will need to run beside it and mark the location of the ball every second.

Start by discussing the responsibilities of each team member and determining how you will mark to position of the ball at each second.

- Write a prediction for the direction of motion and the direction of the change in motion during the experiment.
- Create a **complete** motion diagram to predict the motion of the ball during the experiment.
- Draw a **picture** of the experimental setup. (This should include what each person's responsibility is.)
- Conduct the experiment.
- Create a motion diagram by examining the actual marks on the ground and the relative distances between marks. How does this compare to your predicted motion diagram?
- Measure the positions of the markers, and record the data in a table with the columns:  $t$ ,  $x$ ,  $\Delta t$ , and  $\Delta x$ . Make sure to include your uncertainty. What patterns do you see?
- Graph the data on a position-versus-time graph. **Check!** Should you be graphing the position or change in position? Did you represent uncertainty on your graph?
- Does the data from the experiment support or disprove your hypothesis? Did you make any additional assumptions when you made the prediction about the outcome of the experiment? Draw a conclusion about the relationship between motion and change in motion.



## 11.5 Represent and Reason

- Make up a story about the motion of some object and represent it with a motion diagram.
- Draw motion diagrams for: a car starting to speed up from rest next to a street light, for the car coasting along a street with a constant speed passing the street light along the way, and for the car slowing down to a stop next to a street light.
- Draw a picture of the three situations above and then sketch a position-versus-time graph for each. Assume that the streetlight is the origin.

Draw dots to represent the position of object for equal time intervals.

Point **arrows** in the direction of motion and use the relative lengths

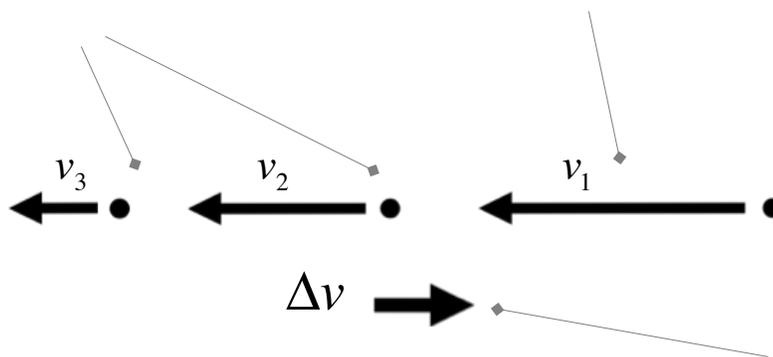
Draw **arrow** to indicate how the arrows are *changing*.

**Draw the arrows thicker than the arrows.**

**Did You Know?** to indicate how fast the object is moving between the points.

**Constructing a Motion Diagram:** Here is a tool to help you learn how to construct motion diagrams. Motion diagrams provide a concrete way to represent motion.

Motion diagrams help you to represent, visualize, and analyze motion. They are especially useful for checking the quantitative math descriptions of motion that you will learn later.



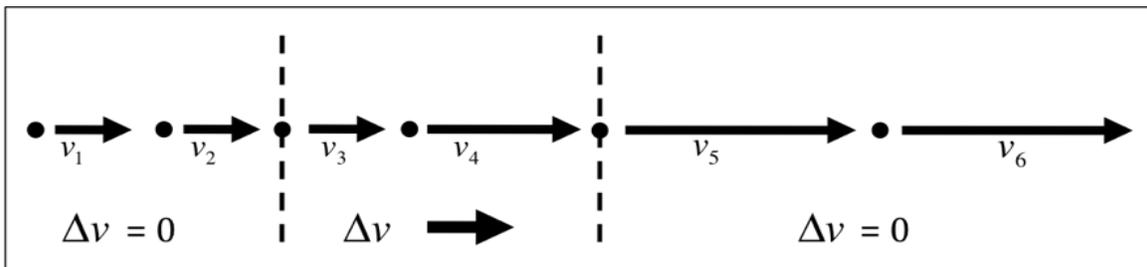
### Homework



#### 11.6 Reason

The illustration below depicts a motion diagram for a given object.

- Describe the motion in words by devising a story for some object's motion that is consistent with this diagram. Note that the process has three distinct parts: the vertical dashed lines separate the parts.



- Now imagine that the object moves exactly the same way, only in the opposite direction. Represent its motion with a motion diagram.

- c) Create a position versus time graph that represents this motion.
- d) Two racing cars start at the same time from the starting line but one moves ahead. Sketch two motion diagrams, one underneath the other, to represent this event. Then sketch position versus time graphs for both cars using the same set of axes.
- e) The same cars from part (c) start slowing down after they cross the finish line. One slows down faster than the other. Represent their motion with motion diagrams, one underneath the other. Then sketch position versus time graphs representing their motion; use the same set of axes.

## Lesson 12: Average Speed



### 12.1 Represent and Reason

A car stops for a red light.

The light turns green and the car moves forward for 3 seconds at a steadily increasing speed. During this time, it travels 20 meters. The car then travels at constant speed for another 3 seconds a distance of 30 meters. Finally, when approaching another red light, the car steadily slows to a stop during the next 3 s in 15 meters.

- Draw a motion diagram that describes this process.
- What is the average speed of the car?

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### *Need Some Help?*

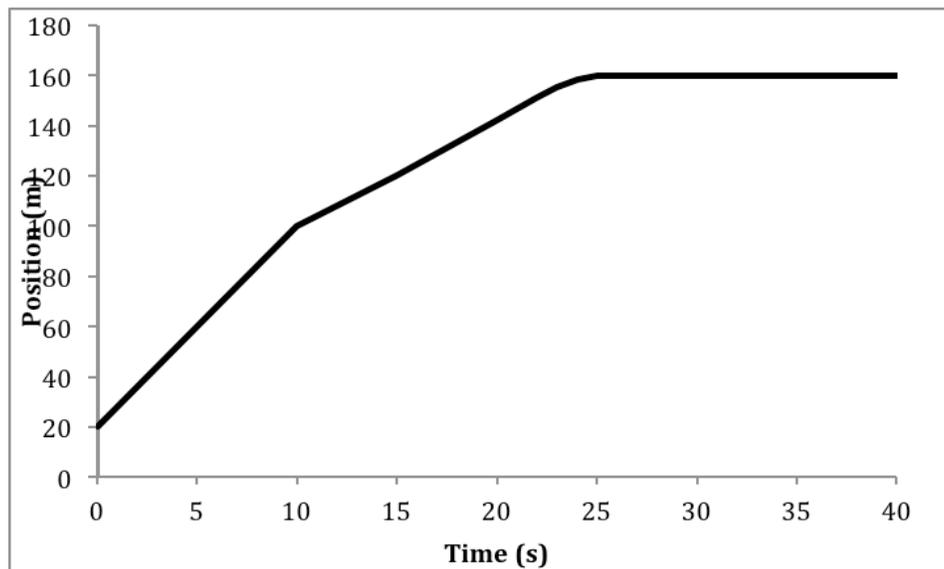
To find the **average speed**, you need to divide the total path length traveled by the total time travelled. **REMEMBER!** – A length can never be negative so path length must also be a positive value.

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- How does the total average speed for the entire 9 seconds compare to the average speed for each 3-second interval? Why are the average speeds different? Explain.



### 12.2 Represent and Reason

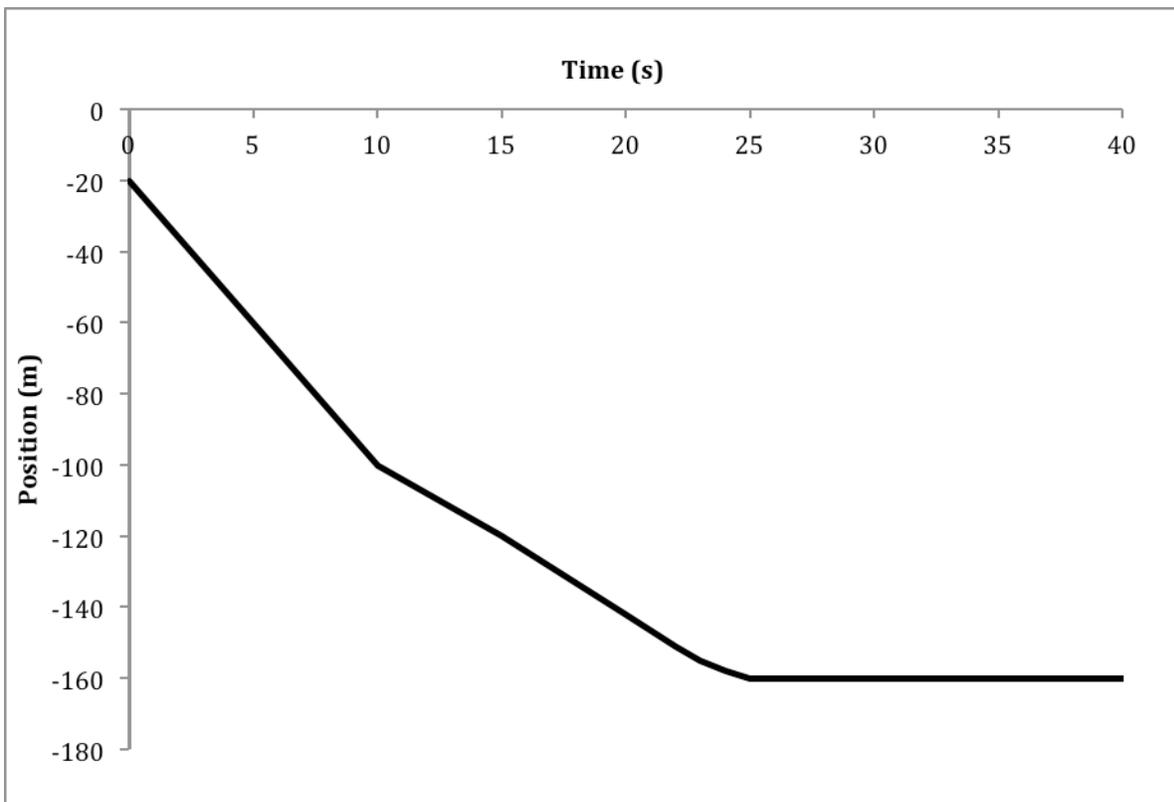


The position of an object over the course of 40 seconds is represented in the graph above.

- a) Describe the object's motion. Act it out – pretend that you are a moving object – what do you need to do to move according to the graph? If you have a motion detector, check your motion with it.
- b) What happened to the motion after 20 seconds?
- c) What is the average speed of the object for the different time intervals: 0-10 sec, 10-20 sec, and 20-30 sec?
- d) What is the average speed for the object during the entire 40 sec?



### 12.3 Represent and Reason



The position of an object over the course of 40 seconds is represented in the graph above.

- Create a story of the motion that this graph could represent. Act it out – pretend that you are a moving object – what do you need to do to move according to the graph? If you have a motion detector, check your motion with it.
- Draw a motion diagram to represent the object's motion.
- What is the average speed for the object during the entire 40 sec?
- What is the average velocity of the object during the entire 40 sec?



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### *Need Some Help?*

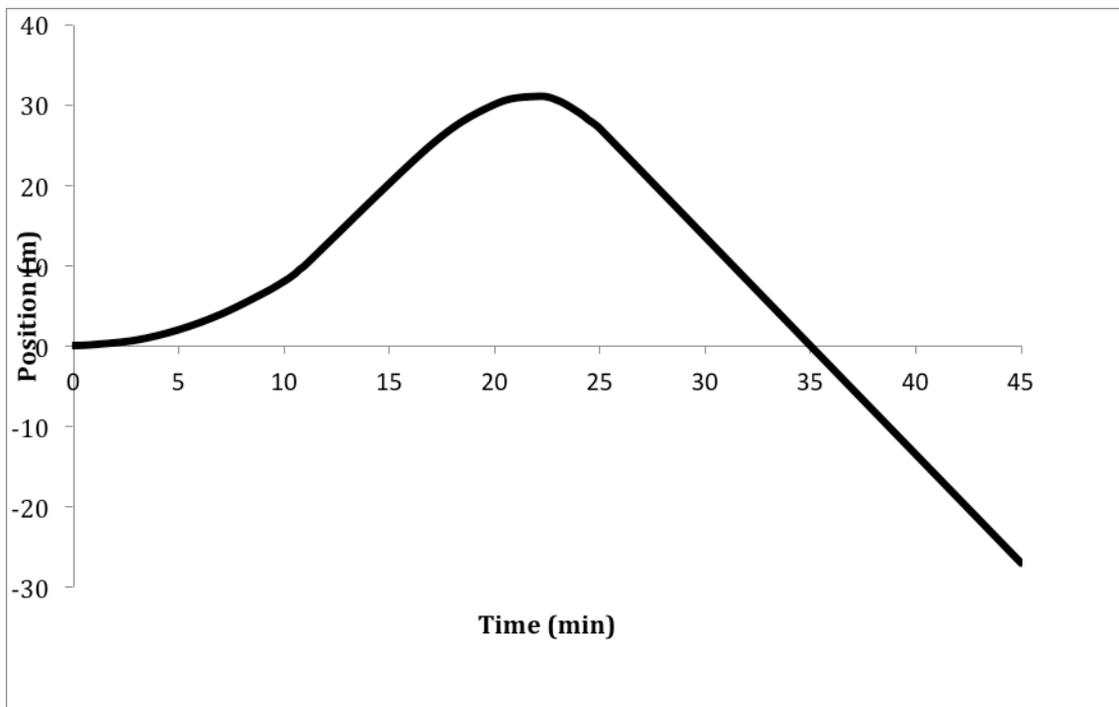
To find **average velocity**, you must divide total distance (change in position) by total time travelled. **REMEMBER!** Distance is the same thing as change in position. This means it can have a negative value.

---

- How is the average speed different from average velocity? How are they the same **in this situation**?



### 12.4 Represent and Reason



The position of an object over the course of 45 minutes is represented in the graph above.

- Describe the object's motion over the thirty minutes. Act it out – pretend that you are a moving object – what do you need to do to move according to the graph? If you have a motion detector, check your motion with it.
- What is the average speed of the object for the different time intervals: 0-15 min, 20-30 min, and 35-45 min?
- What is the average speed for the object during the entire 45 mins?
- What is the average velocity for the object during the entire 45 mins?
- How are the two values similar and how are they different? Explain.

### 12.5 Explain

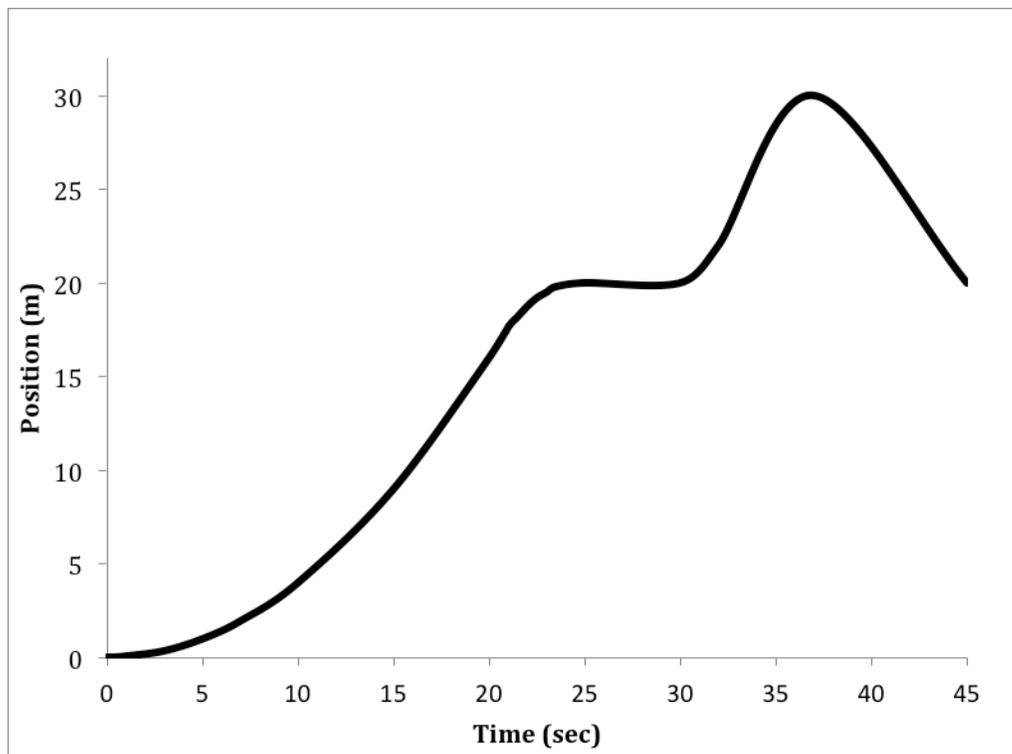


You and a friend have to run around the track for gym class. The requirement is 6 laps during the class period (the length of a track is 400 m). Your friend likes to take breaks so she sprints a lap and then walks the next. You like to lightly jog the whole thing. You jog 2 m/s and your friend sprints 6m/s and walks 1.2m/s. Who finished first? Who was running faster on average?

## Homework

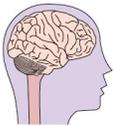


### 12.6 Represent and reason



The position of an object over the course of 40 seconds is represented in the graph above.

- a) Write a story about the object's motion. Act it out – pretend that you are a moving object – what do you need to do to move according to the graph? Move in front of your sibling, a friend or a relative and explain to them how your motion matches the graph.
- b) Create a motion diagram that represents the motion of the object.
- c) What is the average speed and average velocity of the object for the different time intervals: 0-10 min, 10-20 min, 20-30 min, and 30-40min?
- d) During which time interval is the average speed the greatest? Average velocity? Are the speeds and velocities the same? Explain.



### 12.7 Reason

Kim and Jessie run around the track (400m). Kim runs at a rate of 4 m/s, Jessie runs  $1\frac{1}{3}$  m/s for the first half and  $6\frac{2}{3}$  m/s for the second half. Who arrives first? Who has the faster average speed? Average velocity? Show your work!!

### 12.8 Relate

Use a newspaper or Internet to find a graph of some changing quantity. Copy and paste or redraw the graph in your notebook. Describe in words what the graph represents and determine the average value of the changing quantity. Why is it important to understand what an average value is?

### 12.9 Relate

If your family owns a car, record the readings of the odometer in the car every night or every morning. Graph the path length traveled by the car as a function of time. Determine the average path length that the car travels every day. What is the car's average displacement?

### 12.10 Relate

Estimate (estimate means roughly calculate or judge the value of something) the average number of steps you take every day. How did you do the estimate?

Health experts suggest that a person should walk about 10,000 steps every day to stay healthy. Is this number close to your estimate? If you were to walk 10,000 steps a day, how many miles would you walk?

# Lesson 13: When the speed is not constant

## Part I Speeding index

### I. Speeding Up Index

Let's look at another kind of index.

Your task this time is to come up with a *speeding up index* for cars with dripping oil.

You will see a bunch of cars, and you need to come up with one number to stand for each car's speeding up.

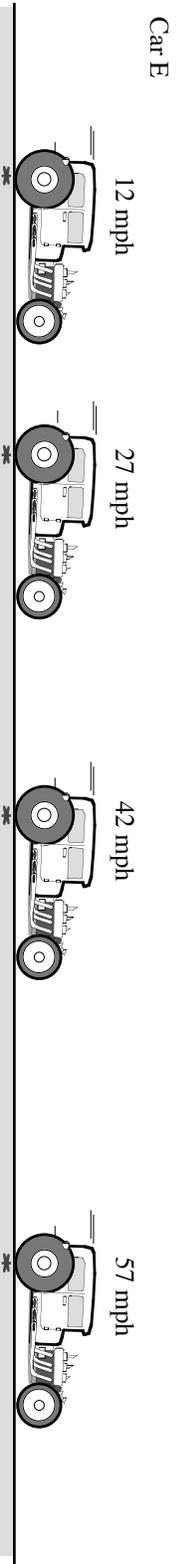
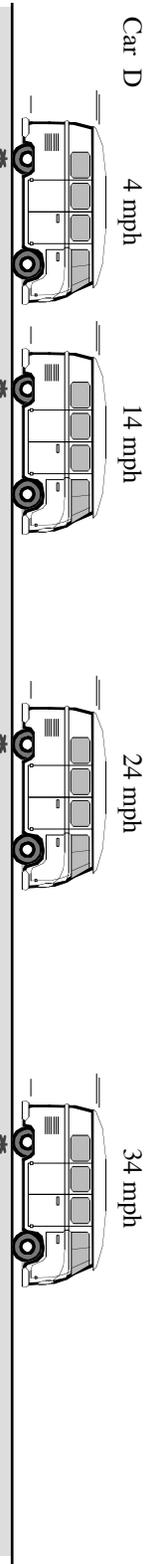
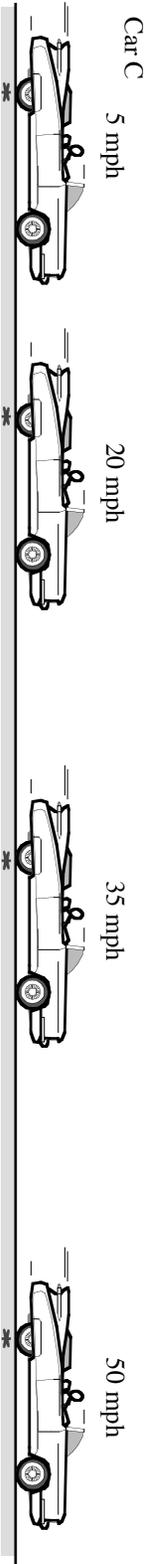
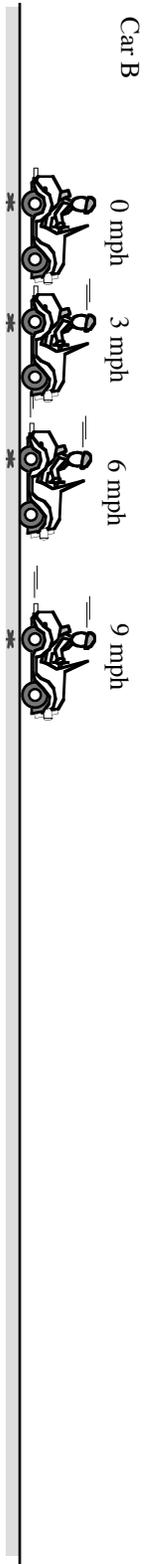
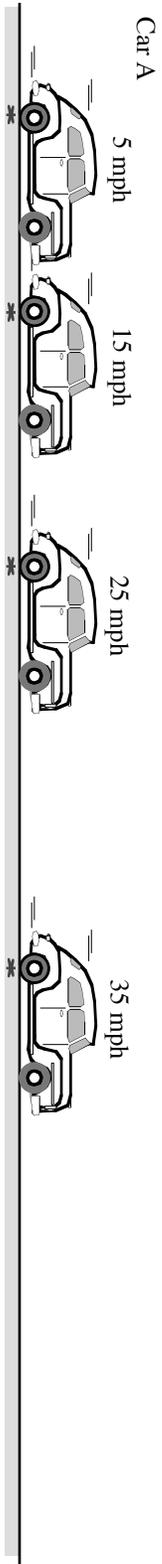
There is no watch or clock to tell you how long each car has been going. However, all the cars drip oil once a second. (They are not very good cars!) The speedometer reading tells you how fast the car is going when the oil drips.

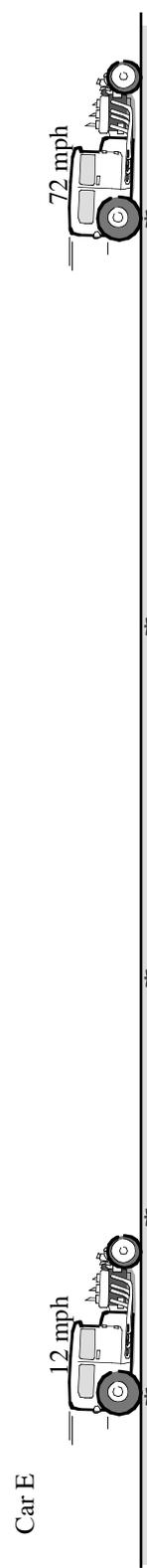
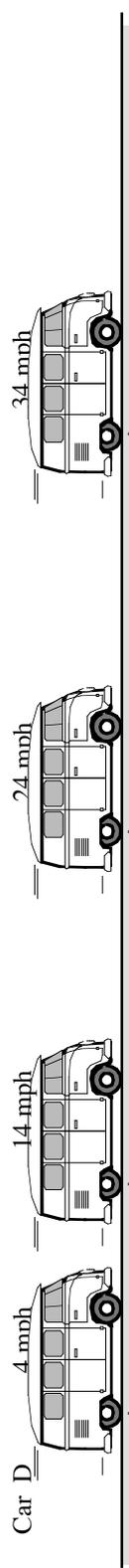
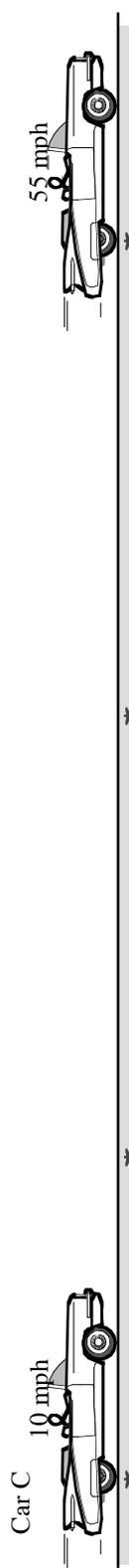
**This task is a little harder than before.**

- A company always makes its cars speed up in the same way.
- We will not tell you how many companies there are.
- You have to decide which cars are from the same company. They may look different!
- 

**To review:**

- (1) Make a speeding up index for each car. A bigger index means a car speeds up more.
- (2) Decide how many companies there are.
- (3) To show the cars that are from the same company, draw a line that connects the cars.





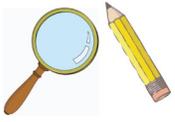
## II. Reason

1. What does the speeding up index tell you about the car (what does the number mean)?
2. How many car companies are there?
3. Why were there were less than 5 car companies, even though there were 5 different diagrams describing the car companies (circle all possible reasons)?
  - a) because some cars went as fast as others in each second.
  - b) because some cars sped up as much as others in each second.
  - c) because some cars sped up as much as others during the entire trip shown

Explain your choice(s).

4. What is the difference between *fast*, *slow*, *speeding up quickly*, and *speeding up slowly*? Use the following questions to help you:
  - a) Which car is the fastest? Explain what “fastest” means.
  - b) How long will it take the fastest car to speed up by 75 mph?
  - c) Which car is the slowest? Explain what “slowest” means.
  - d) How fast will the slowest car be going 10s after it starts?
  - e) Now write a short paragraph comparing “fast” to “speeding up quickly” and “slow” to “speeding up slowly”. Can a car be going fast and speed up slowly? Can a car be going slow and speed up quickly? Explain.

## Part II The physics of speeding and slowing



### 13.1 Observe and Represent

Your teacher decides to take the class rollerblading. During the trip she comes across a slight hill and asks your friend to collect data on her position each second. The data are recorded in the table on the right.

- Create a motion diagram for your teacher as she travels down the hill. How does the change in velocity arrow ( $\Delta v$ ) compare during each time interval?
- Calculate the change in position and record it in the table.
- Draw position versus time graph for your teacher. What can you say about her velocity?
- Use the data in the table above to determine the velocity of your teacher during the following time intervals: 1-4 s; 2-5 s; 2-6 s. What can you say about the velocity?

Clock reading $t$ (s)	Position $x$ (m)	Change in position $\Delta x$ (m)
0 s	0.0 m	-----
1 s	0.5 m	
2 s	3.0 m	
3 s	7.5 m	
4 s	14.0 m	
5 s	22.5 m	
6 s	33.0 m	



### 13.2 Observe and Find a Pattern

- Complete the table for the average velocity versus time data for this motion.
- Do you see any patterns with velocity data? How does the data relate to the motion diagram you originally drew?
- Create a velocity versus time graph for the data by labeling the vertical axis “velocity” and the horizontal axis “time” (clock reading).

Time Interval $t$ (s)	Average Clock Reading $t$ (s)	Average velocity $v$ (m/s)
0s – 1s	0.5 s	
1s – 2s	1.5 s	
2s – 3s	2.5 s	
3s – 4s	3.5 s	
4s – 5s	4.5 s	
5s – 6s	5.5 s	



### 13.3 Reason

- Find the slope of the line in two different places. Is the slope changing or is it constant?
- What does the slope of the line represent in terms of motion? What are the units for this quantity? What name can we give to the slope?
- Write a mathematical expression for acceleration. **Hint:** Think about how we defined velocity in terms of change in position and change in time

- d) Write a function (expression) that allows you to find the velocity of the teacher ( $v$ ) at any time ( $t$ ) while he's going down the hill. **Hint:** Again think back to how we wrote functions to find ( $x$ ) for any ( $t$ ).
- e) Check your function using data from your velocity versus time graph



### 13.4 Test Your Idea

If these functions really do represent motion accurately, it should be possible to use them to predict velocity of a moving object if we know the type of acceleration that occurs. To do so we will once again use the PhET simulation, “The Moving Man.” You should navigate to the following web address and click, “Run Now.”

[http://phet.colorado.edu/simulations/sims.php?sim=The\\_Moving\\_Man](http://phet.colorado.edu/simulations/sims.php?sim=The_Moving_Man)

- a) Write a function for the velocity of the man starting from rest (not moving) and accelerating at  $1.5 \text{ m/s}^2$ .
- b) Use the function to predict how fast the man will be moving after 7 seconds.
- c) In the “Moving Man” simulation, type “1.5” in the acceleration box. Then go into the top of the screen and click “special features.” Check the item that says “free range.” Make sure you are starting at a zero position and clock reading. When done click “Go.” After about ten seconds click “Stop.” This can be located at the bottom of the simulation.
- d) Move the gray bar on the graph to 7 seconds to check your mathematical prediction of the velocity. If you cannot see the graph you can adjust it using the arrows on the right hand side.
- e) Did your prediction match the actual outcome? Make sure to check your work and discuss any problems you may have had. You can use the space below.
- f) Use any two velocities from the graph and the associated time interval to confirm that the man was really accelerating a  $1.5 \text{ m/s}^2$ .



### Did You Know?

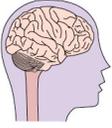
We define **acceleration** as the slope of the velocity versus time graph or the rate at which velocity of an object changes. This means how quickly the velocity of an object changes. This is similar to the definition of velocity, which is a rate at which position changes for an object. The rate of change of a particular quantity is defined as the change in the quantity divided by the time interval during which the change occurred:

$$\text{acceleration} = \frac{\text{velocity at the end} - \text{velocity at the beginning}}{\text{time interval during which velocity changed}} = \frac{\Delta v}{\Delta t} \quad \text{or} \quad a = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t}$$

$$v_2 - v_1 = (\Delta v) = \text{change in velocity}$$

$$t_2 - t_1 = (\Delta t) = \text{time interval}$$

---



### 13.5 Reason

- Imagine that the same change in velocity happened but occurred over a longer time interval. How would the velocity versus time graph be different? What would this mean for the acceleration?
- Imagine that the same change in velocity happened but occurred over a shorter time interval. How would the velocity versus time graph be different? What would this mean for the acceleration?
- The trend line on our graph starts at the origin. What would it mean if the trend line did not begin at the origin?
- Think about your rule for the direction of the motion and the direction of the change in motion. What does it mean to have a negative acceleration?
- James thinks that negative acceleration means slowing. Give an example of motion consistent with his idea and give an example that is inconsistent.
- On a motion diagram, we use the change in velocity arrow,  $\Delta v$ . How is the change in velocity different from acceleration? Discuss both the magnitude and the direction.

## *Homework*

### 13.6 Represent and Reason



- The acceleration of an object is 3 m/s/s. Devise a number statement that would give you this acceleration if you know the initial and final velocity of an object and the time interval during which the velocity changed.
- A hiker changed her velocity from -5.0 m/s to -3.0 m/s in 10 s. What was her acceleration? Was she speeding up or slowing down? Represent the motion with a motion diagram and sketch a velocity versus time graph. Can you say how long it would take her to stop?

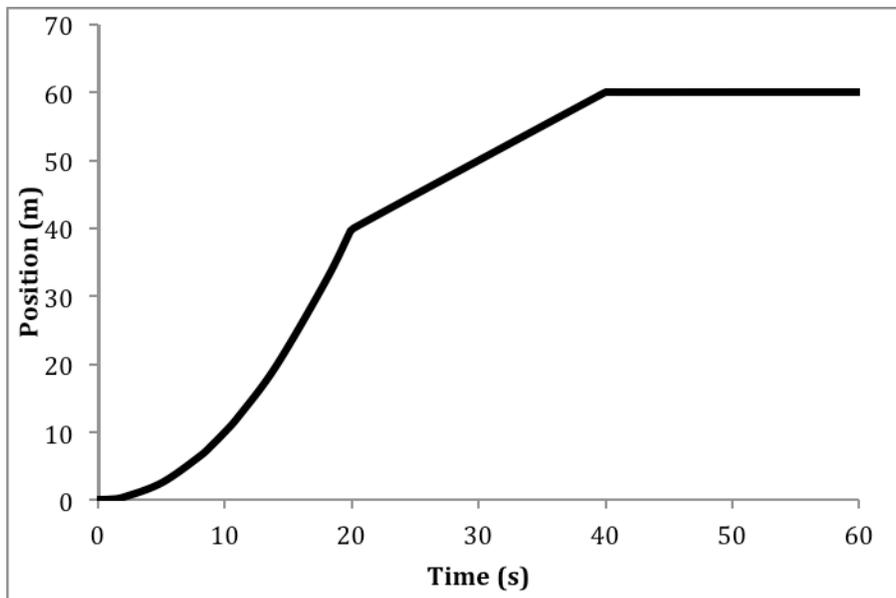
- c) A runner changed his velocity from  $+3 \text{ m/s}$  to  $+5 \text{ m/s}$  in  $4 \text{ s}$ . What was his acceleration? Was he speeding up or slowing down? Represent the motion with a motion diagram and sketch a velocity versus time graph.
- d) The same runner changed his velocity from  $+3 \text{ m/s}$  to  $+5 \text{ m/s}$  in  $3 \text{ s}$ . What was his acceleration? How is this case different from (a)?
- e) A biker changed her velocity from  $+6.3 \text{ m/s}$  to  $+2.7 \text{ m/s}$  in  $3 \text{ s}$ . What was her acceleration? Was she speeding up or slowing down? Represent the motion with a motion diagram and sketch a velocity versus time graph. How is the motion of the biker different from the motion of the runner in part (a)?
- f) Can you say that a positive acceleration means speeding up and negative means slowing down? How do you know?
- g) If you are moving with a velocity of  $-4 \text{ m/s}$  and you begin accelerating at  $-2 \text{ m/s}^2$ , what will happen to your motion? What if you were originally moving with a velocity of  $+4 \text{ m/s}$ . Describe both scenarios.
- h) A biker moving at a velocity of  $3 \text{ m/s}$  starts accelerating with an acceleration of  $-2 \text{ (m/s)/s}$ . What will be his velocity after  $2 \text{ s}$ ? Explain your reasoning. You can use a graph to help.

## Lesson 14: Putting It All Together



### 14.1 Reason and Explain

The graph below records the position versus time for an object.

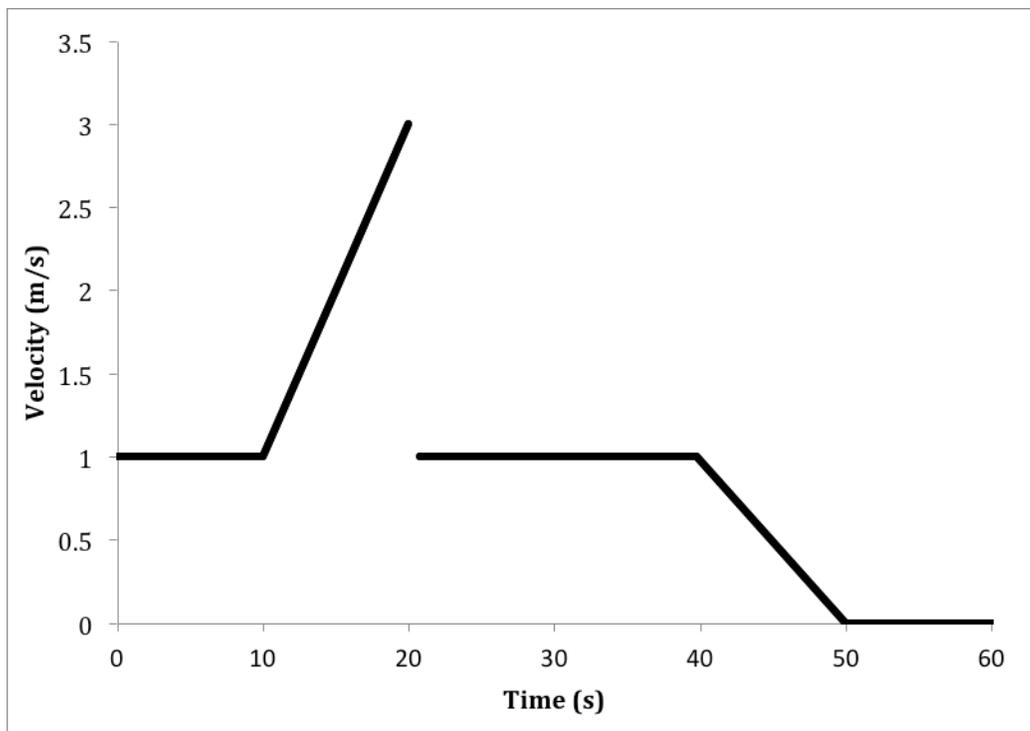


- Describe the motion in words.
- Create a motion diagram for this situation
- During what time interval(s) is the object at rest? Traveling at constant velocity? Accelerating?
- Rank the motion during the time intervals in order from fastest to slowest.  
0-10 s, 10-20 s, 30-40 s, 40-50 s, and 50-60 s?
- Find the average velocity for:
  - The first 40 s?
  - The first 50 s?
  - The first 60 s?
- Draw what you believe the velocity versus time graph should look like.

- g) If you have a motion detector, use it and your own motion to create a graph that matches the one above. Discuss why you were or were not able to reproduce this exact graph.



## 14.2 Represent and Reason



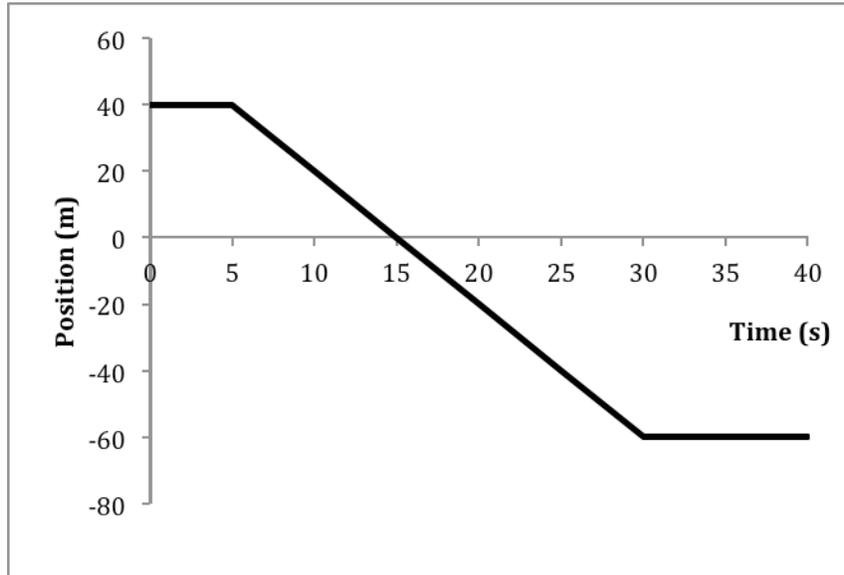
- a) Describe the motion in words.
- b) Create a motion diagram for this situation
- c) During what time interval(s) is the object accelerating? Acceleration zero? Object at rest? Traveling at constant velocity?
- d) What is the velocity of the object at the following clock readings? Explain how you know.  
10 s, 15 s, 20 s, 25 s, 45 s, and 55 s?
- e) Calculate the acceleration of the object for the following time intervals:  
0-10 s, 10-20s, 20-40 s, 40-50 s, and 50-60 s.

## Homework



### 14.3 Represent and Reason

The graph below records the position versus time for an object.



- Describe the motion of the object in words.
- Create a motion diagram
- During what time interval(s) is the object accelerating?
- During what time interval(s) is the object at rest?
- During what time interval(s) is the object traveling at constant velocity?
- What is the average velocity for the following time intervals:  
  
0-10 s, 10-20 s, 30-40 s, and 0-50 s?
- Draw the velocity versus time graph for this object.

## Lessons 15: Supplemental Review

### 15.1 Reason



You wear a pedometer (a device that counts steps). You know that you have to walk 2000 steps to cover a mile. Imagine that you have walked for 2 hours at an average speed of 4 miles/hour. How many steps will the pedometer read?

### 15.2 Reason



Doctors recommend walking 10,000 steps a day to stay healthy. How far do you need to walk to satisfy this requirement? How long does your friend, who only needs to walk 1700 steps to cover a mile, need to walk to satisfy the requirement? Explain how you know.

### 15.3 Reason



Eugenia goes running every morning. When she comes home, her pedometer reads 6400 steps. How far does she run if she needs to take 1700 steps to cover a mile? How long is her run if she runs, on average, a 10-minute mile? What assumptions did you make?

### 15.4 Reason



Light from the surface of the Sun reaches us in 8 min.

- How far away is the Sun if the speed of light is  $3 \times 10^8$  m/s?
- The next closest star, Proxima in the constellation of Centaurus, is about 4 light years away. How far away is it?
- How much farther is Proxima than the Sun from Earth?

### 15.5 Represent and reason



Your friend is going on a bicycle trip. She starts at her house, speeds up to 5 m/s in 2 seconds, continues to move at this speed for 10 seconds, then sees a red light and slows down to a stop in 4 seconds.

- Did she ever move at constant velocity? Constant acceleration? Explain how you know.
- Draw a picture of the situation. Decide where the zero is for the x-axis and what is the positive direction.
- Represent her motion with a motion diagram.
- Represent her motion with a position versus time graph and a velocity versus time graph.
- What was her acceleration during the first 2 seconds if you assume that it was constant? What was the acceleration during the last 4 seconds? What is the sign of this acceleration? What does this mean?
- How far did your friend move during the first 2 seconds? The first 12 seconds? The whole trip?

- g) Imagine that you choose the same origin, only you point the x-axis in the direction opposite to the direction of your friend's motion. Draw a picture of the motion, a position versus time graph, and a velocity versus time graph. What is the acceleration during the first 2 seconds? The last 4 seconds? What does it mean that the acceleration is negative during the first 2 seconds?

### 15.6 Represent and reason



A moonwalker is sending signals about its location to Earth, measuring its position with respect to the main beacon left by the astronauts. Here is the data that the scientists receive on Earth.

Clock reading (seconds)	Position reading (meters)
0	- 27
3	- 21
5	- 17
6	- 15
9	- 9
10	- 7
12	- 3
14	1

- Draw a picture of the moonwalker's motion. Where did you choose the zero for your position axis? Is the moonwalker moving away or towards the beacon?
- Draw a position versus time graph for the motion. What happens to the motion at the 13 s clock reading?
- Did the moonwalker move at constant or changing speed? Constant or changing velocity?
- What was the average velocity of the moonwalker? What was the average speed?

### 15.7 Represent and reason



Invent a table of data for a hypothetical motion at constant speed for an object starting at zero position and moving in the negative direction for some time. The clock reading data should be taken every second. After you invent the data, draw a position versus time graph.

### 15.8 Represent and reason



- The **position versus time function** describing the motion of a bicyclist is  $x = (4 \text{ m/s})t$ . Tell all you can about the motion and represent it in as many ways as possible.
- The position versus time function describing the motion of a bicyclist is  $x = (12 \text{ m}) + (4 \text{ m/s})t$ . Tell all you can about the motion and represent it in as many ways as possible.
- The position versus time function describing the motion of a bicyclist is  $x = (-12 \text{ m}) + (4 \text{ m/s})t$ . Tell all you can about the motion and represent it in as many ways as possible.

- d) The position versus time function describing the motion of a bicyclist is  $x = (-4 \text{ m/s})t$ . Tell all you can about the motion and represent it in as many ways as possible.
- e) The **velocity versus time function** describing the motion of a bicyclist is  $v = (4 \text{ m/s/s})t$ . Tell all you can about the motion and represent it in as many ways as possible.
- f) The velocity versus time function describing the motion of a bicyclist is  $v = (2 \text{ m/s}) + (4 \text{ m/s/s})t$ . Tell all you can about the motion and represent it in as many ways as possible.
- g) The velocity versus time function describing the motion of a bicyclist is  $v = -t$ . Tell all you can about the motion and represent it in as many ways as possible.

### 15.9 Reason

 You throw a ball upward. The ball leaves your hands at a speed of about 5 m/s. What is the acceleration of the ball while it is in contact with your hands if the time of contact is 0.7 seconds?

### 15.10 Represent

- 
- a) A hiker changed her velocity from -2 m/s to -3 m/s in 10 s. What was her acceleration? Was she speeding up or slowing down? Represent the motion with a motion diagram and sketch a velocity versus time graph.
  - b) A hiker changed her velocity from -2.75 m/s to -0.5 m/s in 15 s. What was her acceleration? Was she speeding up or slowing down? Represent the motion with a motion diagram and sketch a velocity versus time graph. Can you say how long it would take her to stop?

### 15.11 Represent and reason

 The graph below is a velocity versus time graph for an object thrown upward. State everything you can about the motion of the object. Explain why the graph has this particular shape.