AP Calculus BC II
2021 Summer Assignment
Mr. Joseph Patti

The purpose of this assignments is to review all of the skills learned in Math Analysis, the prerequisite course for AP Calculus BC II. The estimated time of completion for this assignment should be 4-5 hours. Be sure to show all steps to your solutions. Also, use proper notation, give units of measure, and show all antiderivatives.

NAME:_____

Due: September 10, 2021

Part I. Find each derivative. Show all steps that lead to your answers. Derivatives should be simplified without any fractional exponents. (3 pts each)

1. $f(x) = 8\sqrt{x^4 - 4x^2}$

2. $y = -10x^2 \sin 3x$

3. $y = \left(x - \frac{1}{x}\right)\left(x^3 + \frac{1}{x^3}\right)$

4. $f(x) = \frac{x^6 - 3x^4 + 2x}{(x^2 - 2)^2}$

5. Find dy/dx given $x(t) = 4t^2$ and $y(t) = t^4 + 3t - 9$

6. $h(x) = sin^2(3x)$

7. $f(x) = \sqrt{\tan 2x}$

8. $y = sin^{-1}\left(\frac{1}{x}\right)$

9. $r = ln(1 - cos\theta)$

10. $y = tan(\sqrt{x})$

11. $h(x) = e^{\sin x^2}$

12. Find dy/dx for $xsin\ y + ysin\ x = \frac{\pi}{2\sqrt{2}}$ at $\left(\frac{\pi}{4}, \frac{\pi}{4}\right)$ (4 pts)

13. Find $\frac{d^2y}{dx^2}$ if $2x^2 - 3y^2 = 1$. (4 pts)

Part 2. Applications.
Show all steps to your solutions to receive full credit.

1. Find all extrema, intervals of increasing and decreasing, points of inflection, and intervals of concavity using the 1st and 2nd derivative tests. (8pts)

$$y = x^3 + 3x^2 - 9x$$

2. Apply the first derivative test to find the extrema and intervals of increasing and decreasing. (4pts)

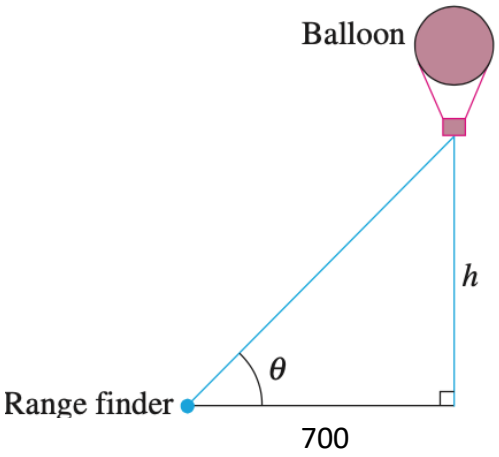
$$y = \frac{2x^2}{x^2-9}$$

3. A rectangular field is bounded by a building on one side, and is fenced in on the other 3 sides. If 3,000 feet of fencing is to be used, find the dimensions of the largest field that can be fenced in. (4 pts)

4. Two cars leave town at the same time. Car A travels due north at a rate of 70 km/hr and car B travels due west at a rate of 55 km/hr. How fast is the distance between the cars increasing after 4 hours? (5 pts)

5. A cylindrical tank with a radius of 10 feet is filling up with water at a rate of $96\pi\ ft^3/sec$. How fast is the water level rising? (5 pts)

6. A hot-air balloon is rising up from a level field and is tracked by a range finder positioned 700 feet from the lift-off point. At the moment the range finder's angle of elevation is $\theta = \frac{\pi}{5}$, the rate of increase in angle θ is 0.15 radian per minute. How fast is the balloon rising at that moment? (5 pts)



Part 3. Fundamental Theorem of Calculus.
Show all steps to your solutions to receive full credit.

1. Find the average value of $y = \sqrt{x}$ on $[0, 25]$. (4 pts)

In #2 – 3, find $\frac{d}{dx}$. (4 pts each)

2. $\int_1^{2x} sin^3\, t\, dt$

3. $\int_{5x}^{3} (t^2 - t)\, dt$

4. Approximate the area under the curve of $y = 4 - x^2$ on [-1,1] using MRAM with 4 subintervals. Be sure to draw a graph and show calculations. (4 pts)

5. Evaluate the integral using: a) the trapezoidal rule with n = 4 and b) find the true value using NINT. Be sure to draw a graph and show calculations. (5 pts)

$$\int_0^4 e^x \, dx$$

Evaluate each of the following using the Fundamental Theorem of Calculus. Be sure to identify each antiderivative. (3 pts each)

6. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} cosx \, dx$

7. $\int_1^0 2x\sqrt{x} \, dx$

8. $\int_{-2}^{2} (2x^3 - 4x + 1) \, dx$

9. $\int_{\frac{\pi}{3}}^{\frac{\pi}{4}} sec^2 x \, dx$

10. $\int_0^1 \frac{2}{1+x^2} \, dx$

11. $\int_0^2 \frac{1}{x+1} \, dx$